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One damage law for different mechanisms

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Abstract We consider here a general three-dimensional kinetic damage law. It uses the thermodynamic of irreversible processes formalism and the phenomenological aspects of isotropic damage. It gives the damage rate as a function of its associated variable, the strain energy density release rate and the accumulated plastic strain rate.

Associated with different plastic constitutive equations, this damage law takes into account brittle damage, ductile damage, low and high cycle fatigue and creep damage. In this paper we mainly focus on creep-fatigue interaction and high cycle fatigue. Associated to a viscoplastic constitutive equation having kinematic hardening, the damage law gives the non linear creep-fatigue interaction. The agreement with experiments is good. Associated to plastic constitutive equations also having kinematic hardening but introduced in a micromechanical two scale model based on the self-consistent scheme, it models the non linear accumulation of damage induced by a succession of sequences of different amplitudes as well as the effect of the mean stress and the influence of non proportional loading.

1 Introduction

A kinetic law of Damage evolution is considered here as a possibility to predict the conditions of crack initiation in any structure submitted to any static or dynamic loading by means of post processing after a classical structure calculation, such as Finite Element Analysis.

In the past decade a big amount of work has been done about damage or crack behavior but mostly from the point of view of micro-mechanics to analyze the damage state (Krajcinovic 1980) and not too much to describe its evolution.

Damage concerns the initiation of brittle, ductile, creep and fatigue cracks in metals (Lemaitre 1992). It concerns also brittle decohesion in concrete (Bazant-Pijeaudier Cabot 1987) and delamination in composites (Ladeveze-Allix 1992). These different phenomena correspond to different physical mechanisms often studied separately by different scientists as it was at the beginning of elasticity then plasticity then viscoplasticity. For example, it has been recognized only recently that kinematic and isotropic hardening theory may model plasticity resulting from such different mechanisms as slipping or climbing of dislocations in crystals, slips and twinning in metals, permutation of bonds in molecules of polymers, breaking of atomic bonds in concrete.

It is the purpose of this paper to establish that the same formalism may be applied to model the main features of different kinds of damage in different materials submitted to different kinds of loading. First of all, it is assumed that a structure calculation gives the history of the state of stress and strain of the structure, for a given history of loading especially in the zones of stress concentration. Secondly, it is assumed that the state of damage is localized enough in order to neglect its influence on the calculated state of stress and strain. This is the case of "small scale damage" for which no coupling between damage, elasticity and plasticity is introduced in the structure calculation. Thirdly, restriction is made here to small deformations and isotropic damage.

Within these hypothesis, damage post-processing may be incorporated at low cost to human and computer time in any Finite Element Code.

2 Damage constitutive equations

The following damage law has been formulated through the thermodynamics of irreversible processes by means of a state potential to define the energy involved in each process represented by a state variable, and a dissipation potential in order to derive the kinetic laws of evolution of the state variables by the normality rule (Lemaitre 1992)

$$\dot{D} = \left(\frac{Y}{S}\right)^{s} \dot{p} \quad \text{if } p \geq p_D$$

$$D = D_c \rightarrow \text{ crack initiation}$$

- D is the scalar isotropic damage variable, it is the surface density of micro defects in any plane of a Representative Volume Element.

  $0 \leq D \leq D_c \leq 1$

  Micro defects means microcracks or microcavities of any kind, whatever is the physical mechanism brittle or ductile, fatigue or debonding, etc.
- $\dot{p}$ is the accumulated plastic strain rate $\dot{p} = \left(\frac{2}{3}\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij}\right)^{1/2}$

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- Y is the strain energy density release rate, and the product $Y.\dot{D}$ is the power involved in damage process

  $$Y = -\frac{\partial W_e}{\partial D}$$

  where W_e is the elastic strain energy density and for linear isotropic elasticity

  $$Y = \frac{\sigma_{\text{eq}}^2 R_v}{2E(1-D)^2}$$

  where σ_{eq} is the von Mises equivalent stress and R_v the triaxiality function

  $$R_v = \frac{2}{3}(1+v) + 3(1-2v)\left(\frac{\sigma_H}{\sigma_{\text{eq}}}\right)^2,$$

  $$\sigma_H = \frac{1}{3}\sigma_{kk}, \quad \sigma_{\text{eq}} = \left(\frac{3}{2}\sigma_{ij}^D\sigma_{ij}^D\right)^{\frac{1}{2}}$$

In order to take into account the different effects of the damage in tension and in compression on the damage behavior, the effective stress in the sense of (Kachanov 1958) is taken as $\tilde{\sigma} = \frac{\sigma}{1-D}$ in tension, $\tilde{\sigma} = \frac{\sigma}{1-Dh}$ in compression, where h is the crack closure parameter $0 \le h \le 1$, often $h = 0.2$.

In three dimensions the strain energy density is written with the positive and negative parts of the principal stresses

$$\langle\sigma_{ij}\rangle = \sigma_{ij} \text{ if } \quad \sigma_{ij} \ge 0, \quad \langle\sigma_{ij}\rangle = 0 \quad \text{if } \sigma_{ij} < 0$$

which leads to

$$Y = \frac{1+v}{2E}\left[\frac{\langle\sigma_{ij}\rangle\langle\sigma_{ij}\rangle}{(1-D)^2} + \frac{h\langle-\sigma_{ij}\rangle\langle-\sigma_{ij}\rangle}{(1-Dh)^2}\right] - \frac{v}{2E}\left[\frac{\langle\sigma_{kk}\rangle^2}{(1-D)^2} + \frac{h\langle-\sigma_{kk}\rangle^2}{(1-Dh)^2}\right]$$

- the damage strength S, the damage exponent s (sometimes $s = 1$), the damage threshold p_D related to the density of energy stored in the material and the critical damage D_c (sometimes $D_c = 1$) are parameters to be identified for each material at each considered temperature.

  $$p_D = \varepsilon_{p_D}\frac{\sigma_u - \sigma_f}{\sigma_{\text{eq}_{\text{MAX}}} - \sigma_f}$$

  where ε_{p_D} is the threshold in tension, σ_u the ultimate stress and σ_f the fatigue limit. Then, all together, 5 damage parameters at most need to be identified.

The post processing consists in solving this non linear differential equation step by step in time for given histories of $Y_{\left(\sigma_{ij}(t)\right)}$ and $\dot{p}_{\left(\dot{\varepsilon}_{ij}^p(t)\right)}$.

3 Application to ductile type of damage

Damage is of a ductile type when plastic strain exists at the mesoscale, that is when the structure calculation exhibits plastic strain in the stress concentration zones.

3.1 Ductile fracture

Ductile crack initiation generally occurs when the strain hardening is saturated, then the yield criterion is of a perfectly plastic type with a plastic threshold σ_s

$$\frac{\sigma_{\text{eq}}}{1-D} = \sigma_s = \text{const}$$

Then $\dot{D} = \left(\frac{\sigma_s^2 R_v}{2ES}\right)^s \dot{p} \quad \text{if } p > p_D$

- For proportional loading, giving rise to $R_v = \text{const}$, by integration

  $$D_c = \left(\frac{\sigma_s^2 R_v}{2ES}\right)^s (p_R - p_D)$$

Comparing with the uniaxial case $R_v = 1$, $p_R = \varepsilon_{p_R}$, p_R being the accumulated plastic strain to crack initiation, $\frac{p_R - p_D}{\varepsilon_{p_R} - \varepsilon_{p_D}} = \frac{1}{R_v^s}$.

This shows that the STRAIN TO FRACTURE STRONGLY DECREASES WHEN THE TRIAXIALITY RATIO INCREASES.

- In plane fracture stress it corresponds to the limit curves of deep drawing: the set of in-plane strains which induce crack initiation. Figure 1 gives an example of the curve corresponding to proportional loadings with two points in non proportional loadings. It is obvious that the effect of non proportionality is not obtained by the damage law.

  $$E = 200\,000 \text{ MPa}, \quad v = 0.3, \quad \sigma_f = 200 \text{ MPa},$$
  $$C = 2000 \text{ MPa}, \quad \sigma_u = 600 \text{ MPa}, \; S = 0.5 \text{ MPa},$$
  $$s = 2, \quad D_c = 1, \quad \varepsilon_{pd} = 0.1, \quad h = 0.2.$$

Plastic strain calculated with linear kinematic hardening.

3.2 Low cycle fatigue

The same damage law applies when the loading varies periodically or not to model fatigue crack initiation at less than 10^4 cycles in which the amplitude of plastic strain is of the same order of magnitude as the amplitude of elastic strain (Dufailly et al. 1995).

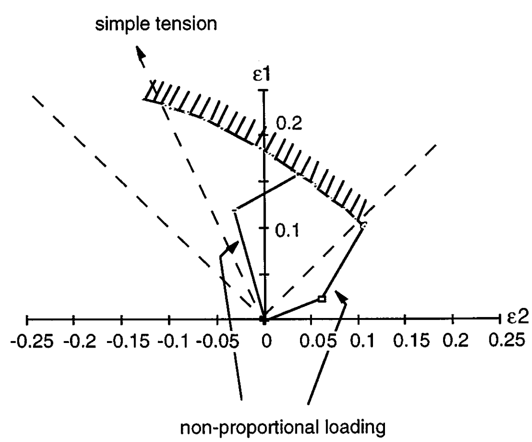

Fig. 1. Limit curves of deep drawing 2 1/4 CrMo Steel at room temperature

3.3
Creep fatigue interaction

At temperature above about one third of the absolute melting temperature most metals and alloys show a strong non-linear interaction between creep damage and fatigue damage that is difficult to model. Figure 2 compares on the classical interaction diagram the Taira linear rule $\frac{N_R}{N_{RC}} + \frac{N_R}{N_{RF}} = 1$ (Taira 1952) with the nonlinear results, close to experiments, of the integration of the damage law for a tension-compression fatigue loading with hold time. N_{RF} is the number of cycles to failure in pure fatigue $(\Delta t = 0), N_{RC}$ is the time to failure in pure creep tension divided by the holding time.

$$E = 170\,000 \text{ MPa}, \quad v = 0.3, \quad S = 0.2, \quad s = 1,$$
$$h = 0.2, \; p_D = 0.005, \quad D_c = 0.3,$$
$$N = 7.5, \quad K_v = 450 \text{ MPa},$$
$$\sigma_y = 30 \text{ MPa}, \quad X_\infty = 116 \text{ MPa}, \quad \gamma = 1200.$$

Plastic strain is calculated with elasto-visco-plastic constitutive equations having isotropic and kinematic hardenings.

4
Application to quasi-brittle type of damage

Damage is called quasi-brittle when plastic strain is negligible at the mesoscale. The structure calculation is fully elastic even in the stress concentration zones. As from physical and thermodynamical points of view, damage is governed by plasticity, which means that plasticity exists only at the micro-scale with practically no influence on the behaviour at the meso scale.

To model this situation a micro mechanical two scale model is considered as a weak micro-defect or inclusion embedded in an elastic meso-matrix as shown in Fig. 3. This inclusion is elastic, plastic and subjected to damage. Its weakness is its yield stress taken equal to the fatigue limit of the material.

The constitutive equations are the following.

i) Linear elasticity for the matrix

$$\varepsilon_{ij} = \frac{1+v}{E}\sigma_{ij} - \frac{v}{E}\sigma_{kk}\delta_{ij}$$

where E and v are the Young's modulus and the Poisson's ratio.

ii) Linear elasticity and plasticity with linear kinematic hardening coupled to damage for the inclusion. The different effects of the damage in tension and compression apply for elasticity by a proper determination of the effective stress $\tilde{\sigma}^\mu_{ij}$ but do not apply for plasticity which is physically driven by slips.

$$\varepsilon^{\mu E}_{ij} = \frac{1+v}{E}\tilde{\sigma}^\mu_{ij} - \frac{v}{E}\tilde{\sigma}_{kk}\delta_{ij}, \qquad \tilde{\sigma}^\mu_{ij} = \frac{\sigma^\mu_{ij}}{1-D}$$

$$\mathrm{d}\varepsilon^{\mu p}_{ij} = \frac{\partial f^\mu}{\partial \sigma^\mu_{ij}}\mathrm{d}\lambda$$

$$\text{with } f^\mu = \left(\frac{\sigma^D}{1-D} - \boldsymbol{X}^D\right)_{\text{eq}} - \sigma_f = 0$$

and $\mathrm{d}f^\mu = 0$ to determine $\mathrm{d}\lambda$

$$\mathrm{d}\boldsymbol{X}^{\mu D}_{ij} = \frac{2}{3}C\mathrm{d}\varepsilon^{\mu p}_{ij}(1-D)$$

where C and σ_f are the strain hardening modulus and the fatigue limit of the material identified at mesoscale, $\boldsymbol{X}^\mu$ is the kinematic back stress, $\boldsymbol{X}^{\mu D}$ its deviator.

iii) The damage law exists only at microscale

$$\mathrm{d}D = \left(\frac{Y^\mu}{S^\mu}\right)^{S^\mu} \mathrm{d}p^\mu \quad \text{ if } p^\mu \geq p^\mu_D,$$
$$\text{with } p^\mu_D = \varepsilon^\mu_{pD}\frac{\sigma_u - \sigma_f}{\sigma^\mu_{\mathrm{eq_{Max}}} - \sigma_f}$$

the damage strength S^μ, the damage exponent s^μ and the threshold ε^μ_{pD} should be identified indirectly from experiments at the mesoscale: high cycle fatigue for example.

The crack initiation criterion at the microscale is $D_c = 1$ but it corresponds to a mesocrack initiation in the scope of fracture mechanics if the representative volume element has a size $1 = G_c/Y_c$ which is the size of the crack initiated.

G_c is the toughness, Y_c is the critical value of the strain energy release rate identified at the mesoscale as

$$Y_c = \frac{\sigma_u^2}{2E(1-D_c)^2}$$

σ_u being the ultimate stress and D_c the critical value of the damage at the mesoscale $(0.2 < D_c < 0.5)$.

– The relation between macro and micro stresses comes from the self-consistent scheme (Kröner 1961).

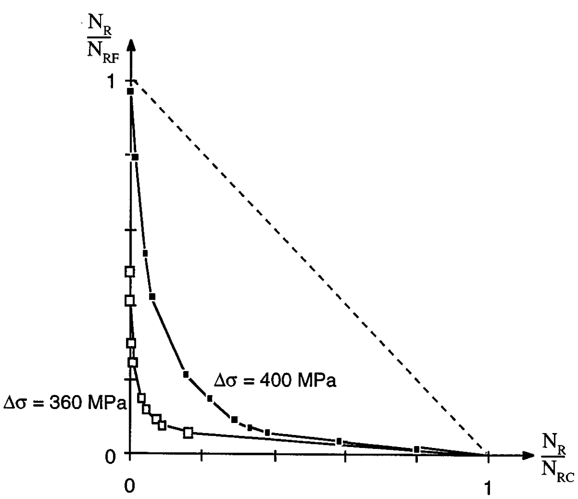

Fig. 2. Creep-fatigue interaction diagram In 100 Refractory alloy $T = 1000°\text{C}, \Delta t = 100\;\text{s}$

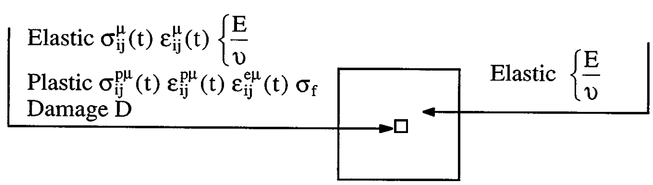

Fig. 3. Two scale model

$$\sigma_{ij}^{\mu} = \sigma_{ij} - aE(1-D)\varepsilon_{ij}^{\mu p}$$

where $a - \frac{1-\beta}{1+v}, \beta = \frac{2(4-5v)}{15(1-v)}$ is a coefficient coming from the Eshelby (Eshelby 1961) analysis ($a \approx 0.4$ for metals) but it is better to consider it as a coefficient to be identified from experiments (fatigue).

– Then a damage processor consists in solving simultaneously all the above equations where the data are:

* The material parameters
$E, v, \sigma_f, C, \sigma_u$ for elastoplasticity; $S^{\mu}, s^{\mu}, \varepsilon^{\mu}_{p_D}$ for the damage
h for the quasi-unilateral conditions ($h \approx 0.2$); a for the self-consistent scheme ($a \approx 0.4$)

* The history of the mesostress $\sigma_{ij}(t)$ at each Gauss point of a Finite Element Analysis for example.

The resolution is classical: a strain driven incremental in time algorithm using an elastic predictor and a plastic corrector (Benallal et al. 1988).

For a periodic high cycle fatigue, it is possible to avoid a too large number of calculations by a "jump in cycle" procedure where the damage is considered as piece wise linear (Lemaitre et al. 1994).

4.1 Brittle fracture

The resolution of the set of equations is able to predict brittle fracture for any kind of loading, proportional or not.

– Figure 4 shows the graph of the fracture criterion $D = D_c$ for a case of plane stresses showing the influence of the "compression" behavior and of a non proportional loading for the point denoted by *.σ_1 and σ_2 are the principal stresses, σ_R the stress in tension failure

$$E = 200\,000 \text{ MPa}, \quad v = 0.3, \quad \sigma_f = 200 \text{ MPa},$$
$$C = 2000 \text{ MPa}, \quad \sigma_u = 600 \text{ MPa}, \quad S^{\mu} = 0.5 \text{ MPa},$$
$$s^{\mu} = 2, \quad D_c = 1, \quad \varepsilon_{pd} = 0.1, \quad h = 0.2, \quad a = 0.4.$$

4.2 High cycle fatigue

The same set of equations applies to fatigue modeling for its main important properties.

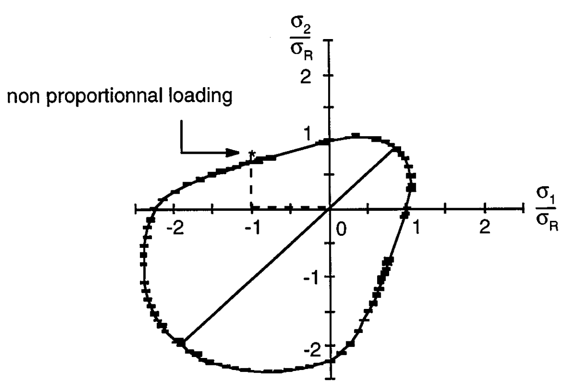

Fig. 4. Fracture criterion of a brittle quenched steel 2 1/4 CrMo Steel at room temperature

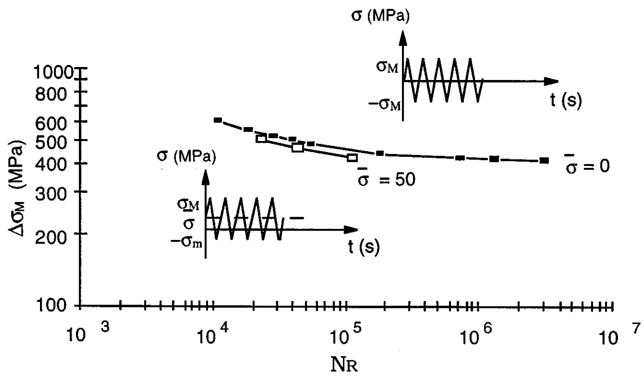

Fig. 5. Wöhler curves of a 2 1/4 CrMo Steel at room temperature

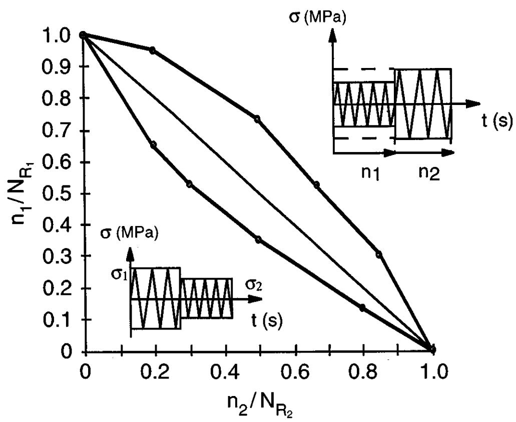

Fig. 6. Accumulation of damage on a 2 1/4 CrMo Steel at room temperature

– The effect of the mean stress $\bar{\sigma}$ in a one dimensional loading defined by $\sigma = \bar{\sigma} + \frac{\Delta\sigma}{2} \sin \omega t$ (Fig. 5)

$$E = 200\,000 \text{ MPa}, \quad v = 0.3, \quad \sigma_f = 200 \text{ MPa},$$
$$C = 2000 \text{ MPa}, \quad \sigma_u = 600 \text{ MPa}, \quad S^{\mu} = 0.5 \text{ MPa},$$
$$s^{\mu} = 2, \quad D_c = 1, \quad \varepsilon_{pd} = 0.1, \quad h = 0.2, \quad a = 0.4.$$

– The non linear accumulation of damage induced by a succession of sequences of two different amplitudes. Figure 6 compares these sequences to the linear Palmgren-Miner rule (Miner 1945).

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