Comput. Mech. (2006) 38: 163–170
DOI 10.1007/s00466-005-0735-9

ORIGINAL PAPER

K. Zhou · X. Li

Topology optimization of structures under multiple load cases using a fiber-reinforced composite material model

Received: 12 August 2004 / Accepted: 28 June 2005 / Published online: 31 August 2005

Abstract This paper presents a method to optimize the topology of structures under multiple load cases with stress constraints. Fiber-reinforced orthotropic composite is employed as the material model to simulate the constitutive relation of truss-like continua. The fiber densities and orientations at the nodes are taken as design variables. First, for each load case, the fiber orientations are aligned with the orientations of principal stress and the fiber densities are adjusted according to the strains along the fiber orientations. Then, to optimize the structure, the fiber densities and orientations under multiple load cases are determined by constraining its elastic matrix to approach the elastic matrix of the optimum structures defined for each single load case. Finally the member distribution in the optimal structure is suggested by the continuous lines formed according to the fiber densities and orientations. Several examples are presented to demonstrate the effectiveness of the proposed approach.

Keywords Topology optimization · Composite · Stress constraints · Multiple load cases

1 Introduction

The optimal topology design of structures has received the increasing attention of researchers. Xie and Steven [17] introduced an evolutionary structural optimization method using an isotropic material model. Its structural topology is changed iteratively by deleting or adding structural elements based on their stress level [16, 13]. Bendsϕe and Kikuchi introduced a homogenization method [2] where the material property of each design cell is computed and the optimal topology is achieved by solving a material distribution problem. The homogenization method can deal with flexure and stress constraints [1, 5]. In these papers, penalization of intermediate densities, perimeter control or post-processing techniques are commonly used to get distinct (0–1) topology designs. The optimal topology is represented by perforated plate. These optimal results depend on the finite elements and penalization function. For a further overview, see the reviews [6, 14].

In the free material optimization [3, 4, 7, 11, 15], anisotropic material models are used and the components of the elastic tensor are considered as design variables. Such a material model has much more design freedom. These approaches mainly focus on compliance optimization. Generally, their optimal results are non-manufacturable for most structures, without the assistance of other techniques. Currently, no thoroughly reliable technique is available to convert these types of result to a useable engineering structure. Some of the research papers [10] tend to interpret the density information only in a zero-one sense. Therefore, most of the information obtained in the results is actually ignored. Expressing the optimal results by principal stress vectors may give an improper image for multiple load cases. According to Michell's theory, the members in optimal structures under single load case are coincident with the principal stress directions. So, continuous lines along the principal stress directions may give a reasonable image under single load case [9, 18]. However, these lines may not represent the appropriate locations of members in optimal structures under multiple load cases. Such plots also depend on the manner of choosing starting points of lines, because they require the starting points to be carefully determined.

This paper suggests a new method to optimize the structural topology. First, the optimum truss-like continuum is determined. Then the truss-like continuum is interpreted as discrete truss structures. The process as described in this paper consists of three parts.

First, a fiber-reinforced orthotropic composite is introduced as a suitable material model. The constitutive relation of this composite is a reasonable simulation of a truss-like continuum. The fibers in the composite simulate the members in the structure. The final optimal structures are constructed by infinite number of members of infinitesimal spacing. The model is designated as a "truss-like continuum" because its

K. Zhou (✉) · X. Li
College of Civil Engineering,
Huaqiao University, 362021 Fujian, P. R. China
Tel.: 86-595-22691468
Fax: 86-595-22692097
E-mail: zhoukm@hqu.edu.cn

nature is similar to a truss although its constituent material is actually distributed continuously. This continuum structure is equivalent to truss in mechanics. These characteristics differ from the conventional anisotropic material model.

Next, a process to establish the optimum truss-like continuum under multiple load cases with stress constraints is suggested. Fiber distributions in the design region are iteratively changed according to the magnitudes and orientations of principal stresses obtained by finite element analysis.

Last, the topologically optimum structure is suggested by the continuous lines formed according to the fiber densities and orientations at the nodes. These continuous lines represent the actual distribution of members in the truss structures that it is intended to build. Although the final optimum structures still are a continuum without holes, their images suggest the optimal configuration of members. By choosing a few of the members, as few as we need in fact, it is feasible to build nearly optimal discrete truss structures. Althought some errors are caused in this process, some previous researches have indicated that the volume of a structure so represented will be very close to an exact solution [12,19].

This paper may be considered a generalization of recent work [18] on multiple load cases.

2 Elastic matrixes

2.1 Elastic matrix along the principal axes of the material

The optimal structure with stress constraints under a single load case is called a Michell truss, which frequently is an orthotropic truss-like continuum structure. To describe such a structure, an orthotropic fiber-reinforced composite material model is employed. Two groups of continuously distributed orthotropic fibers, which form the continuous or discrete structures, are embedded in matrix. The stresses and strains of two groups of orthogonal fibers are denoted by $\sigma_i', \varepsilon_i (i = 1, 2)$, respectively. The two planes perpendicular to the fiber orientations are defined as principal material planes. The symbol t_i $(i = 1, 2)$ is defined as fiber density. This definition means that the area of fiber in the infinitesimal area of $\mathrm{d}A_i$ in principal material planes i is $t_i\,\mathrm{d}A_i$. So t_i is non-dimensional. It is assumed that the elastic modulus of the matrix in the fiber orientation vanishes. This assumption means that the matrix does not bear normal stress on principal material planes. Therefore, the force acting on $\mathrm{d}A_i$ is $\sigma_i' t_i\,\mathrm{d}A$, and the average stress σ_i acting on $\mathrm{d}A_i$ can be expressed as

$$\sigma_i = t_i \sigma_i' \ (i = 1, 2), \quad \tau_{12} = 0. \tag{1}$$

Since we expect to get the isotropic structure in the end, the elastic modulus E of the two groups of fibers should be identical. So, the stress-strain relations of fibers can be expressed by

$$\sigma_i' = E\varepsilon_i, \quad (i = 1, 2). \tag{2}$$

The combination of Eqs. (1) and (2) leads to the relations between stress and strain along the fiber orientations,

$$\sigma_i = E t_i \varepsilon_i, \quad (i = 1, 2). \tag{3}$$

In truss-like continuum structure, there is no interaction between adjacent parallel members. Therefore the Poisson's ratios are assumed as zero. For truss-like continuum structure, the shear modulus in the principal material plane should vanish. In the process of optimizing, however, the shear modulus should not vanish. Otherwise, the stiffness matrix would become singular and equilibrium would be unstable or even impossible. Additionally, convergence will slow with too little shear modulus. From the above discussion, the relation between shear stress τ_{12} and shear strain γ_{12} in principal material plane is assumed as

$$\tau_{12} = 0.25\lambda E(t_1 + t_2)\gamma_{12}, \quad (0 \le \lambda \le 1), \tag{4}$$

where λ is defined as shear modulus ratio, which will affect the shear modulus and be changed from 1 to 0 in iteration. The Eqs. (3) and (4) can be rewritten in matrix form

$$[\,\sigma_1\ \sigma_2\ \tau_{12}\,] = \bar{\boldsymbol{D}}(t_1, t_2)[\,\varepsilon_1\ \varepsilon_2\ \gamma_{12}\,], \tag{5}$$

where $\bar{\boldsymbol{D}}(t_1, t_2)$ is elastic matrix,

$$\bar{\boldsymbol{D}}(t_1, t_2) = E \cdot \mathrm{diag}[\,t_1\ t_2\ 0.25\lambda(t_1 + t_2)\,]. \tag{6}$$

Initially, for $\lambda = 1$, $t_1 = t_2 = 1$, the material model is isotropic.

2.2 Elastic matrix off the principal axes of the material

If the fiber orientation is rotated the angle α positive anti-clockwise relative to the global coordinate systems, the relation between stress σ and strain ε in global coordinate systems can be expressed by [8]

$$\sigma = \boldsymbol{D}(t_1, t_2, \alpha)\varepsilon, \tag{7}$$

where $\boldsymbol{D}(t_1, t_2, \alpha)$ is the elastic matrix in the global coordinate systems,

$$\begin{aligned}\boldsymbol{D}(t_1, t_2, \alpha) &= \boldsymbol{T}^{\mathrm{T}}(\alpha)\bar{\boldsymbol{D}}(t_1, t_2)\boldsymbol{T}(\alpha)\\ &= E t_m \begin{bmatrix} 1 + R_t \cos 2\alpha & 0 & 0.5R_t \sin 2\alpha \\ & 1 - R_t \cos 2\alpha & 0.5R_t \sin 2\alpha \\ \text{sym.} & & 0.5 \end{bmatrix}\\ &+ \frac{1}{2}(\lambda - 1)E t_m \begin{bmatrix} \sin^2 2\alpha & -\sin^2 2\alpha & -0.5\sin 4\alpha \\ & \sin^2 2\alpha & 0.5\sin 4\alpha \\ \text{sym.} & & \cos^2 2\alpha \end{bmatrix},\end{aligned} \tag{8}$$

$$t_m = \frac{t_1 + t_2}{2}, \quad R_t = \frac{t_1 - t_2}{t_1 + t_2}$$

where $\boldsymbol{T}(\alpha)$ is the frame rotation matrix

$$\boldsymbol{T}(\alpha) = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 0.5\sin 2\alpha \\ \sin^2\alpha & \cos^2\alpha & -0.5\sin 2\alpha \\ -\sin 2\alpha & \sin 2\alpha & \cos 2\alpha \end{bmatrix}. \tag{9}$$

3 Material model of optimal structure under multiple load cases

In optimal structures under multiple load cases, the members may exist in more than two directions and be non-orthogonal to each other. In order to simplify the problem here, a two phase orthotropic composite material model is employed. In the coordinate systems rotated the angle θ positive anti-clockwise relative to the global coordinate systems, the elastic matrix at every point in optimal structures under multiple load cases can then be expressed as

$$\boldsymbol{D}(\theta; x_1, x_2, \varphi) = \boldsymbol{T}^{\mathrm{T}}(\alpha - \varphi)\bar{\boldsymbol{D}}(x_1, x_2)\mathbf{T}(\alpha - \varphi), \tag{10}$$

where x_1, x_2 and φ are the fiber densities and the angle respectively.

The optimization problem we discussed can be described as

$$\begin{aligned} &\text{Find } x_1 \geq 0, x_2 \geq 0, \varphi \\ &\text{Min} \int_\Omega (x_1 + x_2)\mathrm{d}\Omega \\ &\text{s.t.} \quad |\varepsilon_l| \leq \varepsilon_p, \quad (l = 1, 2, \ldots, L) \\ &\quad \text{Equilibrium and compatibility conditions} \end{aligned} \tag{11}$$

where ε_l is principal strain under load case l, $\varepsilon_p = \sigma_p/E$ the permissible strain and L the number of load cases.

In fact, we are able to optimize the fiber densities t_{1l}, t_{2l} and angle α_l as ref [18] under single load case l. Further more, we can determine its elastic matrix $\boldsymbol{D}(t_{1l}, t_{2l}, \alpha_l)$. Under multiple load cases, it is much more difficult to solve the problem (11). We try to estimate the elastic matrix $\boldsymbol{D}(x_1, x_2, \varphi)$ of optimum structures under multiple load case basing on the elastic matrix $\boldsymbol{D}(t_{1l}, t_{2l}, \alpha_l)$ of the optimum structure under single load case. As we know, the strains along the members (with non-zero area of across section) in the optimum structures under single load case are just the permissible strain. We managed to adjust the fiber densities x_1, x_2 and angle φ so as to make the maximum strains along the fiber orientation under all load cases not exceed the permissible strain. Therefore, the stiffness along the fiber orientatioin should not be less than the stiffness of every optimum structure under single load case

$$\begin{aligned} &\boldsymbol{D}(\theta; x_1, x_2, \varphi) \geq \boldsymbol{D}(\theta; t_{1l}, t_{2l}, \alpha_l), \\ &\quad (\theta = \alpha_l, \alpha_l + \pi/2; l = 1, 2, \ldots, L). \end{aligned} \tag{12}$$

Larger stiffness requires much more material. To minimize material volume, it is better to make the stiffness be close to the maximun stiffness of all optimum structures under every load case

$$\begin{aligned} &\boldsymbol{D}(\theta; x_1, x_2, \varphi) \approx \max_{l=1,2,\ldots,L} \boldsymbol{D}(\theta; t_{1l}, t_{2l}, \alpha_l), \\ &\quad (\theta = \alpha_l,\ \alpha_l + \pi/2; l = 1, 2, \ldots, L). \end{aligned} \tag{13}$$

Since the components of $\boldsymbol{D}$ being not independent, it is impossible to make all the components of elastic matrix strictly satisfy the Eq. (13) simultaneously. So, we replace the Eq. (13) with the equation of its main components

$$\begin{cases} D_{11}(\theta; x_1, x_2, \varphi) \\ = \max\limits_{l=1,2,\ldots,L} D_{11}(\theta; t_{1l}, t_{2l}, \alpha_l) \\ D_{22}(\theta; x_1, x_2, \varphi) \\ = \max\limits_{l=1,2,\ldots,L} D_{22}(\theta; t_{1l}, t_{2l}, \alpha_l), \end{cases} \quad (\theta = \alpha_l; l = 1, 2, \ldots, L). \tag{14}$$

Noting that

$D_{22}(\theta; x_1, x_2, \varphi) = D_{11}(\theta + \pi/2; x_1, x_2, \varphi)$,

Eq. (14) is unified as

$$\begin{aligned} &D_{11}(\theta; x_1, x_2, \varphi) = S_m(\theta), \\ &\quad (\theta = \alpha_l, \alpha_l + \pi/2;\ l = 1, 2, \ldots, L), \end{aligned} \tag{15}$$

where

$$S_m(\theta) = \max_{l=1,2,\ldots,L} D_{11}(\theta; t_{1l}, t_{2l}, \alpha_l). \tag{16}$$

To simplify the solving of Eq. (15), we let the difference between the two sides as little as possible,

$$\min_{x_1, x_2, \varphi} \delta^2, \tag{17}$$

where

$$\begin{aligned} \delta^2 &= \|D_{11}(\theta; x_1, x_2, \varphi) - S_m(\theta)\|_2^2 \\ &= \int_0^\pi [D_{11}(\theta; x_1, x_2, \varphi) - S_m(\theta)]^2 \mathrm{d}\theta. \end{aligned} \tag{18}$$

Furthermore, δ^2 can be calculated by

$$\delta^2 = \frac{\pi}{128}\delta_1^2 + \|S_m(\theta)\|_2^2, \tag{19}$$

where

$$\begin{aligned} \delta_1^2 = &(x_1^2 + x_2^2)(35 + 10\lambda + 3\lambda^2) \\ &+64(x_1 - x_2)(I_1 \sin 2\varphi + I_2 \cos 2\varphi) \\ &+16(x_1 + x_2)[(3 + \lambda)I_1 \\ &+(1 - \lambda)(I_3 \sin 4\varphi + I_4 \cos 4\varphi) \\ &+2x_1 x_2(3 + 10\lambda + 3\lambda^2)]. \end{aligned} \tag{20}$$

For $\|S_m(\theta)\|_2^2$ being independent of design variables, to minimize the Eq. (17), we differentiate Eq. (20) with respect to design variables x_1, x_2 and φ,

$$\frac{\partial \delta_1^2}{\partial x_i} = 0, \quad (i = 1, 2); \qquad \frac{\partial \delta_1^2}{\partial \varphi} = 0. \tag{21}$$

From Eq. (21) we gain

$$\begin{aligned} &x_1 + x_2 = 16\frac{(3 + \lambda)I_0 + (1 - \lambda)(I_3 \sin 4\varphi + I_4 \cos 4\varphi)}{19 + 10\lambda + 3\lambda^2} \\ &x_1 - x_2 = 4(I_1 \sin 2\varphi + I_2 \cos 2\varphi), \end{aligned} \tag{22}$$

and

$$\begin{aligned} &19 + 10\lambda + 3\lambda^2)[(I_1^2 - I_2^2) \sin 4\varphi + 2I_1 I_2 \cos 4\varphi] \\ &+4(1 - \lambda)(I_3 \cos 4\varphi - I_4 \sin 4\varphi)[(3 + \lambda)I_0 \\ &+(1 - \lambda)(I_3 \sin 4\varphi + I_4 \cos 4\varphi)] = 0 \end{aligned} \tag{23}$$

where

$$\begin{aligned} &I_{[0,1,2,3,4]} \\ &\quad = \frac{1}{\pi E} \int_0^\pi S_m(\theta)[1, \sin 2\theta, \cos 2\theta, \sin 4\theta, \cos 4\theta]\mathrm{d}\theta. \end{aligned} \tag{24}$$

These parameters I_0–I_4 are independent of x_1, x_2 and φ. The fiber angle φ can be obtained by solving the non-linear Eq. (23) by numerical methods. The fiber densities x_1, x_2 can be obtained by introducing φ into Eq. (22). For cases where the solutions of Eq. (23) are not unique, the solution which generates δ_1 minimum in Eq. (20) is selected.

4 Finite element analysis

4.1 Element stiffness

The fiber densities x_{1n}, x_{2n} and angles φ_n at nodes n $(n = 1, 2, \ldots, N)$ are taken as design variables, where N is the number of nodes. The elastic matrix of element e can be calculated by the weighted mean of elastic matrix at all nodes of element e,

$$\boldsymbol{D}_e(\xi, \eta) = \sum_{n \in S_e} N_n(\xi, \eta) \boldsymbol{D}_n, \tag{25}$$

where S_e is the set of all nodes of element e, $\boldsymbol{D}_n$ elastic matrix at node n, and N_n shape function that is as a weight factor. In this paper, 4-node isoparametric finite elements are employed. So, the shape function is

$$N_n(\xi, \eta) = (1 + \xi_n \xi)(1 + \eta_n \eta)/4, \quad (n = 1, 2, 3, 4), \tag{26}$$

where ξ_n, η_n are coordinates of node n in local coordinate systems $o\xi\eta$. The element stiffness matrix can be calculated by

$$\boldsymbol{k}_{ij} = \sum_{n \in S_e} \int_{A_e} N_n \boldsymbol{B}_i^T \boldsymbol{D}_n \boldsymbol{B}_j \, \mathrm{d}A, \tag{27}$$

where $\boldsymbol{B}_i$ and $\boldsymbol{B}_j$ are the geometry matrices.

4.2 Iteration algorithm

(i) The design domain is partitioned by finite elements;

(ii) Initial values at every node are set as

$$x_{1n}^0 = x_{2n}^0 = 1, \ \varphi_n^0 = 0, \quad (n = 1, 2, \ldots, N) \tag{28}$$

where the superscript is the iteration index. The changing rate $\Delta\lambda$ of shear modulus ratio is set as $\Delta\lambda = 0.1$ in this paper.

(iii) The stresses and strains at nodes under every single load case are calculated by finite element method. The principal stress directions under load case l are calculated by

$$\beta_{nl}^i = \frac{1}{2} \tan^{-1} \frac{2\tau_{xy,nl}^i}{\sigma_{x,nl}^i - \sigma_{y,nl}^i}, \quad (n = 1, 2, \ldots, N, \ l = 1, 2, \ldots, L) \tag{29}$$

The absolute value of directional strains in the principal stress directions is

$$\varepsilon_{nl}^i(\beta_{nl}^i) = \left| \varepsilon_{x,nl}^i + \varepsilon_{y,nl}^i + (\varepsilon_{x,nl}^i - \varepsilon_{y,nl}^i) \cos 2\beta_{nl}^i + \gamma_{xy,nl}^i \sin 2\beta_{nl}^i \right| /2, \quad (n = 1, 2, \ldots, N, \ l = 1, 2, \ldots, L). \tag{30}$$

(iv) The fiber distribution in every optimal structure under each single load case is determined as ref [18].

The fiber orientations are aligned along the principal stress directions

$$\alpha_{nl}^i = \begin{cases} \beta_{nl}^i & \left|\cos(\alpha_{nl}^{i-1} - \beta_{nl}^i)\right| \ge \sqrt{2}/2 \\ \beta_{nl}^i + \frac{\pi}{2} & \left|\cos(\alpha_{nl}^{i-1} - \beta_{nl}^i)\right| < \sqrt{2}/2 \end{cases}, \quad (n = 1, 2, \ldots, N, \ l = 1, 2, \ldots, L) \tag{31}$$

The fiber densities are adjusted by

$$\begin{aligned} t_{1nl}^i &= t_{1nl}^{i-1} \cdot \varepsilon_{nl}^i(\beta_{nl}^i)/\varepsilon_p \\ t_{2nl}^i &= t_{2nl}^{i-1} \cdot \varepsilon_{nl}^i(\beta_{nl}^i + \pi/2)/\varepsilon_p \end{aligned}, \quad (n = 1, 2, \ldots, N, \ l = 1, 2, \ldots, L) \tag{32}$$

which is similar to the stress ratio method.

(v) The fiber densities x_{1n}^i, x_{2n}^i and angle φ_n^i are obtained by solving Eqs. (22) and (23). To avoid the stiffness matrix becoming singular, too small densities are avoided

$$x_{jn}^i = \max\left(x_c^i, \ x_{jn}^i\right), \quad (j = 1, 2; \ n = 1, 2, \ldots, N), \qquad x_c^i = R \times \max_{j,n}\{x_{jn}^i\}, \tag{33}$$

where R is a small value to define the minimum permitted density, taken as $R = 10^{-7}$ in this paper.

(vi) Return to Step (iii) unless the relative change of volume in two successive iterations is less than a given tolerance

$$\left|1 - V^i/V^{i-1}\right| \le r, \tag{34}$$

where r is tolerance, which is taken as 1% in this paper, V the total volume of all fibers in the structures

$$V^i = \frac{1}{4} \sum_{e=1}^{N_e} A_e \sum_{n \in S_e} (x_{1n}^i + x_{2n}^i), \tag{35}$$

where A_e is the area of element e, S_e set of all nodes in element e.

(vii) The calculation ends if λ vanishes. Otherwise the shear modulus ratio λ is decreased by

$$\lambda^i = \lambda^{i-1} - \Delta\lambda. \tag{36}$$

before returning Step (iii).

In the final result, the shear modulus in the principal plane vanishes. This character is similar to truss-like continuum. The fibers in the continuum are then equivalent to members.

5 Forming optimum trusses

5.1 Starting points and fiber orientation at any point

According to equilibrium conditions, there must be members passing through these points acted at by point forces. Similarly, there must be distributed members crossing curves of nonvanishing curvature. So, the points acted upon by point forces may be selected as the starting points of continuum lines. Other starting points may be chosen along the curve with discrete spacing. The lines starting from the points along curve are orthogonal to the curve.

The angle of fiber at any point (x, y) in an element is calculated by the weighted average of the angles at four nodes around the point. The weights are the values of the shape functions at the point,

$$\varphi(x, y) = \sum_{n \in S_e} N_n(x, y)\varphi_n, \tag{37}$$

where φ_n is one of two fiber angles at node n, which is close to the fiber angle at the former point of the continuous line.

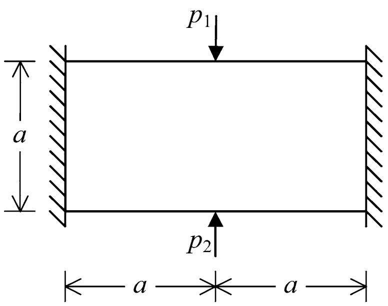

Fig. 1 Geometry and boundary condition in Example 1

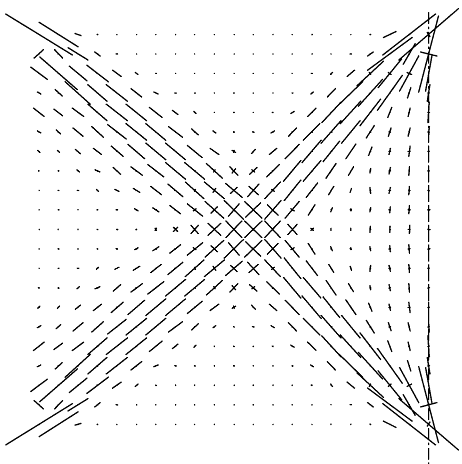

Fig. 2 Fiber densities and orientations

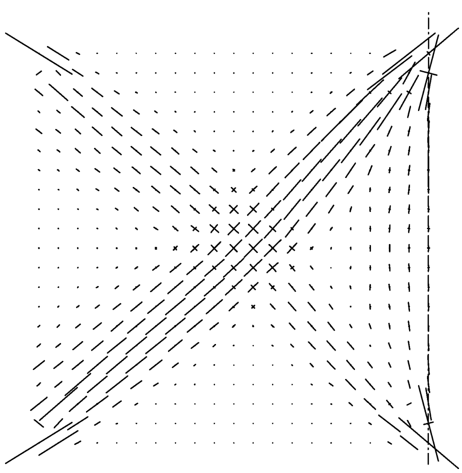

Fig. 3 Principal stresses in load case 1

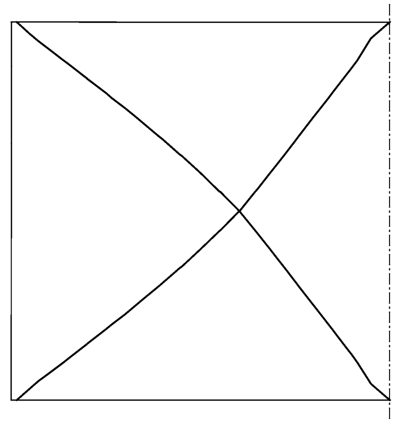

Fig. 4 Continuous fiber lines

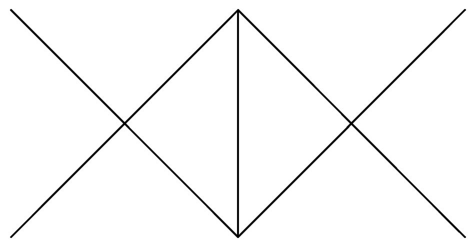

Fig. 5 The final optimum truss

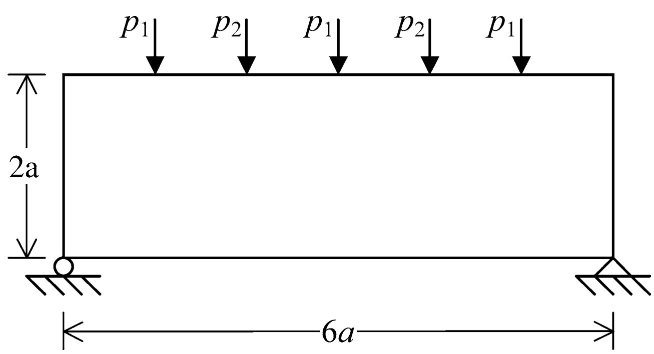

Fig. 6 Geometry and boundary condition in Example 2

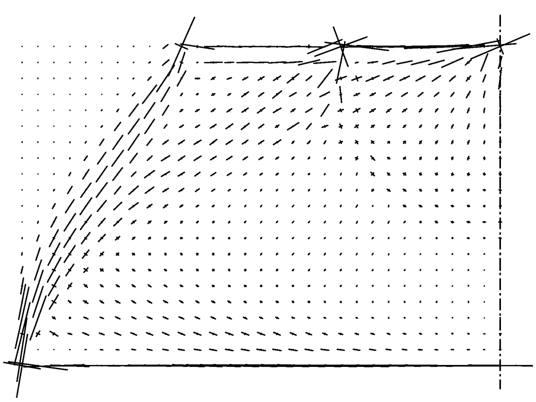

Fig. 7 Fiber densities and orientations

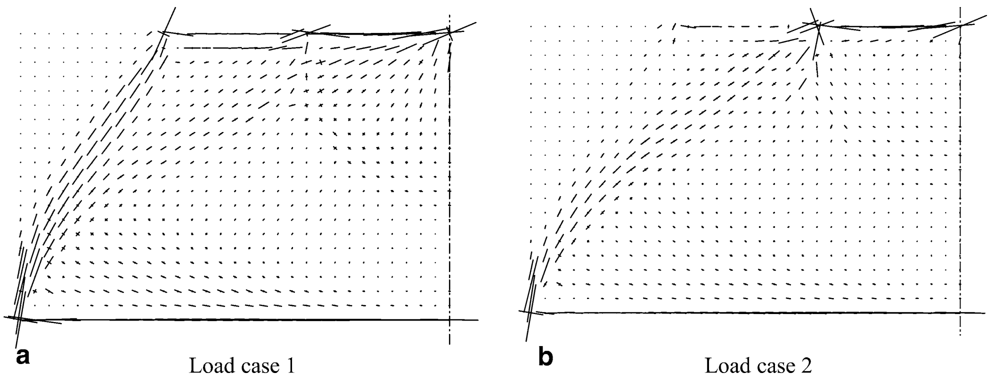

Fig. 8 Principal stresses

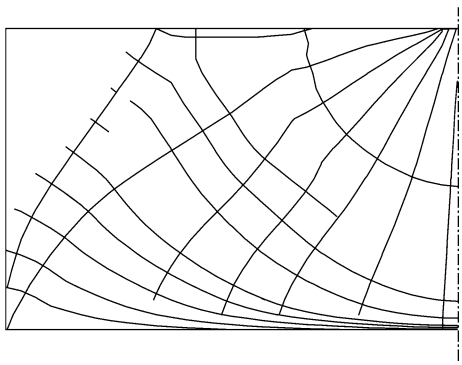

Fig. 9 Continuous fiber lines

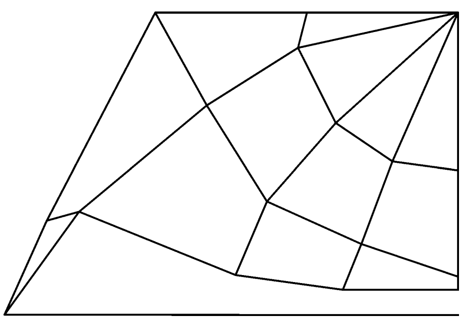

Fig. 10 The final optimum truss

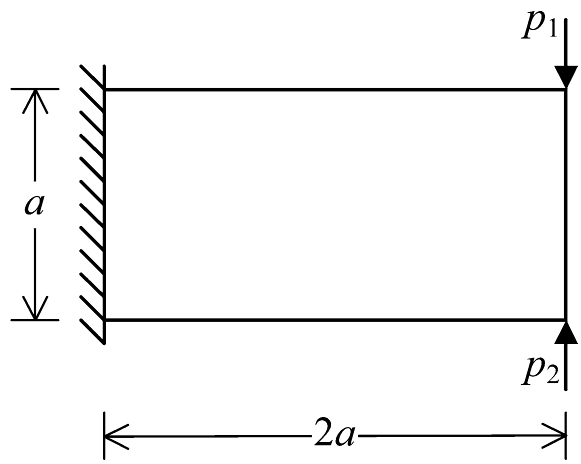

Fig. 11 Geometry and boundary condition in Example 3

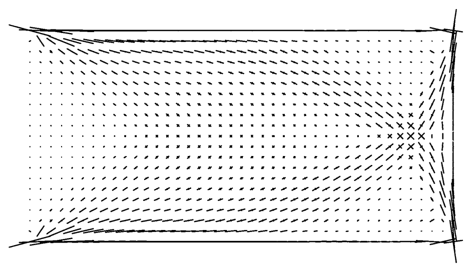

Fig. 12 Fiber densities and orientations

5.2 The procedure for forming the continuous lines of the fibers

(i) A starting point (x_s, y_s) is chosen properly as discussed above.

(ii) The line with the fiber angle $\bar{\varphi}_s$ at the point (x_s, y_s)

$$y = y_s + (x - x_s) \tan \bar{\varphi}_s \tag{38}$$

will intersect the boundary of an element at point $(\bar{x}_e, \bar{y}_e)$. The fiber angle $\bar{\varphi}_e$ at the intersection is calculated by Eq. (37).

(iii) To improve the precision of the line, the angle $\bar{\varphi}_s$ of the line is replaced by the average of the angles at the starting and ending points

$$\varphi_s = (\bar{\varphi}_s + \bar{\varphi}_e)/2. \tag{39}$$

The line from starting point (x_s, y_s) with angle φ_s will intersect the boundary of the element at another point (x_e, y_e), which is taken as the ending point of this line.

(iv) The line ends if the boundary of design domain is reached, or if the fiber density at the considered point is smaller than the critical value t_c defined in Eq. (33). Otherwise the ending point (x_e, y_e) is taken as the starting point (x_s, y_s) of the next line and return to Step (ii).

All continuous fiber lines are drawn by repeating the above process. Then these lines with too little spacing are deleted to make the plot distinct. The remaining continuous lines will represent the members in the structure.

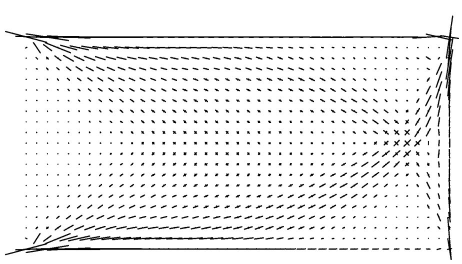

Fig. 13 Principal stresses in load case 1

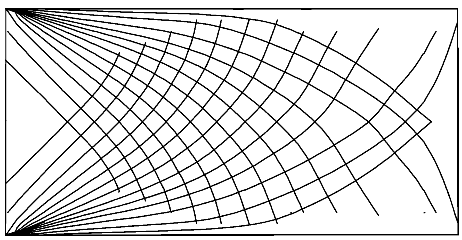

Fig. 14 Continuous fiber lines

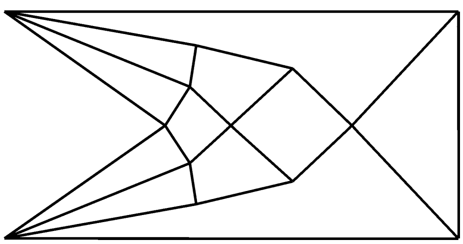

Fig. 15 The final optimum truss

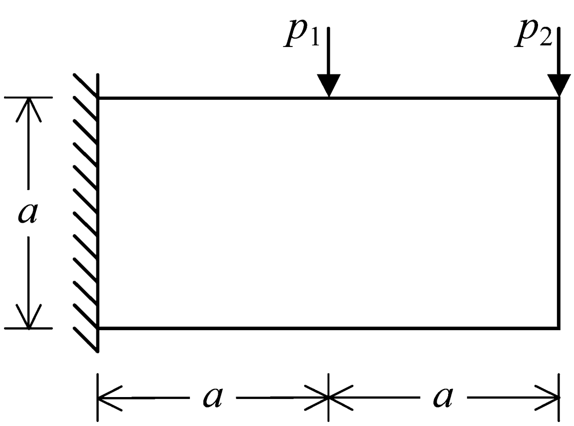

Fig. 16 Geometry and boundary condition in Example 4

5.3 Forming optimum truss

Parts of continuous lines mentioned above are selected as the members in the truss we intend to build. The positions of nodes and the cross section areas of members are taken as design variables. The size and shape of the truss are optimized. After that, the final optimum structure is established.

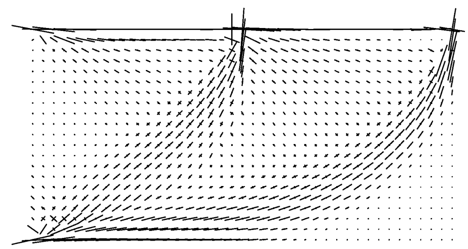

Fig. 17 Fiber densities and orientations

6 Examples

Four examples are presented to demonstrate the application of the method presented in this paper. Square 4-noded isoparametric elements are used. The two (groups of) loads p_1 and p_2 act on the structures independently as separate load cases. Other parameters are $E = 210\,\text{GPa}$, $a = 1\,\text{m}$, $\sigma_p = 160\,\text{MPa}$ and $p_1 = p_2 = 10\,\text{kN}$.

The figures given display four aspects of the solutions: fiber orientations and densities, principal stress, continuous fiber lines and final optimum truss. In the fiber orientations and densities plots, the lines with length proportional to the fiber densities are drawn along the fiber orientations. In the principal stress plots, the lines with length proportional to the magnitudes of principal stress are drawn along the principal stress directions for every single load case. In the above two plots, excessively long lines are cut to make the plots distinct. The plots of continuous fiber lines are drawn basing on the approach discussed in Section 5.1 and 5.2. The final optimum trusses are obtained as discussed in Section 5.3.

6.1 Example 1[3]

A rectangular design domain is supported and loaded as shown in Fig. 1. Half of a structure with 20 × 20 elements is calculated, taking account of its symmetry. The optimal results after 23 iterations are shown in Figs. 2–5.

6.2 Example 2[3]

A rectangular plate is supported and loaded as shown in Fig. 6. Half of the structure comprising 20 × 30 elements is calculated on account of its symmetry. The optimal results after 21 iterations are shown in Figs. 7–10.

6.3 Example 3[3]

A rectangular plate is supported and loaded as shown in Fig. 11. 20 × 40 elements are used. The optimal results after 21 iterations are shown in Figs. 12–15.

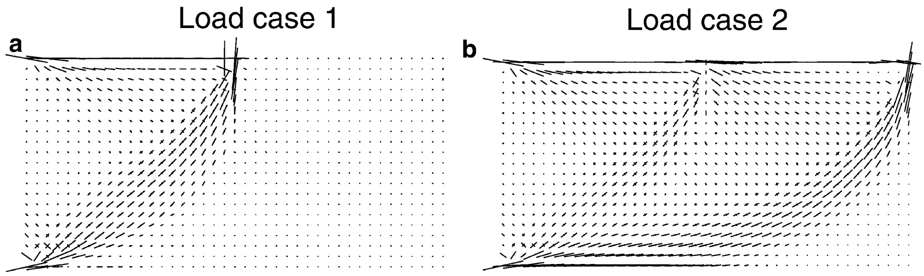

Fig. 18 Principal stresses

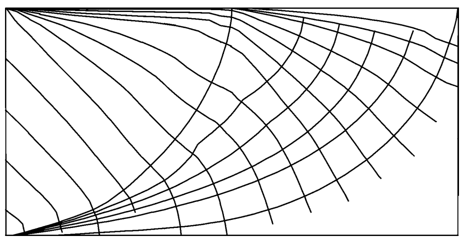

Fig. 19 Continuous fiber lines

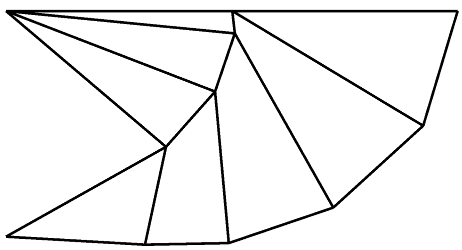

Fig. 20 The final optimum truss

6.4 Example 4

A rectangular plate is supported and loaded as shown in Fig. 16. 20 × 40 elements are used. The optimal results after 22 iterations are shown in Figs. 17–20.

7 Conclusions

A procedure to optimize the topology of structures under multiple load cases with stress constraints is presented. The existence of shear stresses and strains in the principal plane of the material, which vanish in optimal structures acted upon by a single load case, introduces significant complication to the problems of choosing an appropriate material model and displaying the analytical results. An evolutionary procedure of elastic matrix is introduced to overcome this difficulty. The different visualization methods are compared. The plots of continuous fiber lines are rather valuable to illustrate optimal results and to translate the truss-like continuum to discrete structure.

Acknowledgements This work was supported by scientific research fund of Huaqiao University.

References

1. Allaire G, Jouve F, Maillot H (2004) Topology optimization for minimum stress design with the homogenization method. Struct Opt 28:87–98
2. Bendsφe MP, Kikuchi N (1988) Generating optimal topologies in structural design using a homogenization method. Comput Meth Appl Mech Engng 71:197–224
3. Bendsφe MP, Diaz AR, Lipton R, Taylor JE (1995) Optimal design of material properties and material distribution for multiple loading conditions. Int J Numer Meth Engng 38:1149–1170
4. Bendsφe MP, Guedes JM, Haber RB etc. (1994) An analytical model to predict optimal material properties in the context of optimal structural design. J Appl Mech 61:930–937
5. Duysinx P, Bendsφe MP (1998) Topology optimization of continuum structures with local stress constraints. Int J Numer Meth Engng 43:1453–1478
6. Eschenauer HA, Olhoff N (2001) Topology optimization of continuum structures: A review. Appl Mech Rev 54:331–389
7. Guedes JM, Taylor JE (1997) On the prediction of material properties and topology for optimum continuum structures. Struct Opt 14:193–199
8. Hull D (1981) An introduction to composite materials, Cambridge University Press, New York
9. Hörnlein HREM, Kočvara M, Werner R (2001) Material optimization: bridging the gap between conceptual and preliminary design. Aerosp Sci Technol 5:541–554
10. Hsu Y, Sho M, Chen C (2001) Interpreting results from topology optimization using density contours. Comput Struct 79:1049–1058
11. Pedersen P (2004) Examples of density, orientation, and shape-optimal 2D-design for stiffness and/or strength with orthotropic materials. Struct Opt 24:37–49
12. Prager W (1974) A note on discretized Michell structures. Comput Meth Appl Mech Engrg 3:349–355
13. Querin OM, Young V, Steven GP, Xie YM (2000) Computational efficiency and validation of bi-directional evolutionary structural optimization. Comput Meth Appl Mech Engrg 189:559–573
14. Rozvany GIN (2001) Aims, scope, method, history and unified terminology of computer-aided topology optimization in structural mechanics. Struct Opt 21:90–108
15. Taylor JE (1998) An energy model for optimum design of linear continuum structures. Struct Opt 16:116–127
16. Xie YM, Steven GP (1994) Optimal design of multiple load case structures using an evolutionary procedure. Eng Comput 11:295–302
17. Xie YM, Steven, GP (1993) A simple evolutionary procedure for structures optimization. Comput Struct 49(5):885–896
18. Zhou K, Hu Y (2002) A method of constructing Michell truss using finite element method. Acta Mechanica Sinica 34:935–940 (in Chinese)
19. Zhou K, Li J (2004) The exact weight of discretized Michell trusses for a central point load. Struct Opt 28: 69–72