Comput. Mech. (2006) 37: 470–478
DOI 10.1007/s00466-005-0731-0

ORIGINAL PAPER

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Buckling in Thin Walled Micro and Meso Structures of Lightweight Materials and Material Compounds

Received: 25 February 2005 / Accepted: 25 May 2005 / Published online: 31 August 2005

Abstract Modern materials and material compounds for application in lightweight structures exhibit, in addition to the use of constituents of high specific stiffness and strength, very special micro- and meso-structures. Typical representatives of such material compounds are sandwiches with thin homogeneous or composite face layers and structured core materials (for instance honeycombs and closed or open cell foams). The load carrying capacity of lightweight structures made of such materials and material compounds, respectively, is limited by a considerable number of rather different but interconnected instability modes occurring at length scales which are several orders of magnitude smaller than the size of the structural part. These non-global instabilities are the subject of the presented key-note paper.

Keywords Foams · Instability · Sandwich structures · Wrinkling · Dimpling

1 Introduction

Lightweight structures are frequently made of materials or material compounds exhibiting very special micro- and meso-structures, such as sandwiches composed of thin layers of homogeneous or composite materials, and core materials which also show some microstructure as, e.g., honeycombs and closed or open cell foams. In order to calculate the load carrying capacity of lightweight structures made of such materials and material compounds not only strength and stability considerations on the structural level, but also on the meso and micro-level must be taken into account. Regarding stability loss a considerable number of rather different, but interconnected instability modes occurring at length scales which are several orders of magnitude smaller than the size of the structural part can be the starting points of failure of the complete structure. For instance, on the micro-scale, i.e. on the level of material's microstructure, bifurcation of equilibrium paths can be observed, such as buckling of struts and cell walls of polymer or metallic foams [1–3]. Similar phenomena appear in structured core materials (e.g., honeycombs [4]) of sandwich plates or shells. Buckling or kinking of the reinforcing phase in fibre reinforced composites [5] or in layer-wise micro-structured materials are also examples for instabilities on this length scale. Such phenomena are called "micro-instabilities".

On a larger length scale, the meso-scale, instability phenomena can be related to ply-buckling caused by delamination in layered composites [6–8]. Other typical kinds of meso-instabilities, studied in the present paper, are localizations in the form of bands of buckled foam cells leading to phenomena similar to material instabilities such as the formation of necks or shear bands by plastic localization in polycrystalline materials.

Wrinkling of face layers of sandwich plates under macroscopic in-plane loading or bending [9–11] is also studied as a particular form of "meso-instability" of lightweight material compounds. A semi-analytical-numerical approach has been developed which is able to take anisotropy effects that are typical for modern lightweight sandwich structures into account realistically. Although wrinkling starts as a bifurcation process with a distinct periodic eigenfunction, the post-critical behaviour shows the formation of single localized folds being the result of crushing of the microstructure of the core.

Whether face layer wrinkling or face layer dimpling appears as the relevant meso-instability of sandwiches with honeycomb cores depends on the meso-geometrical parameters and on material properties. Accounting for this, dimpling is also studied by computational means. It is shown that this kind of mesoscopic structural instability represents an interactive buckling of cell walls and faces.

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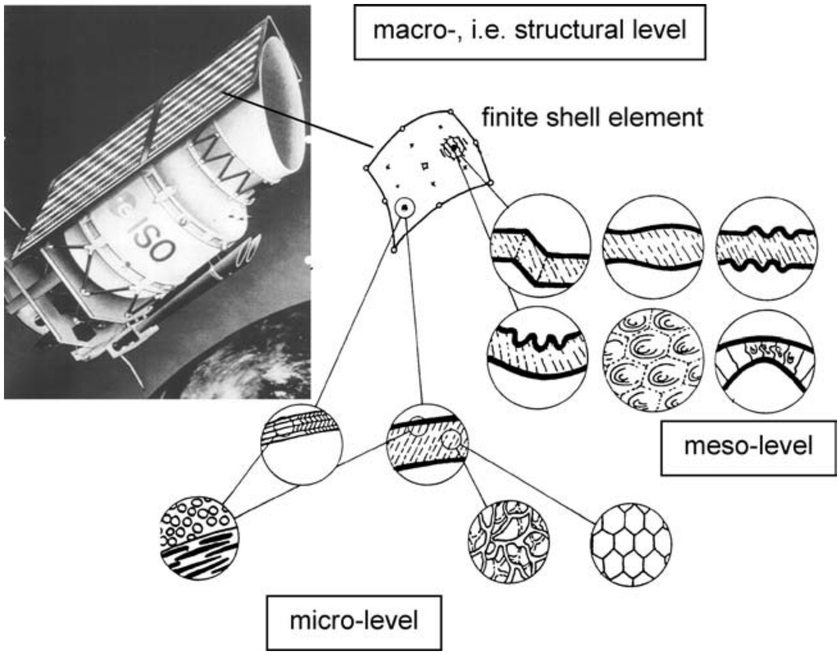

Fig. 1 Different length scales

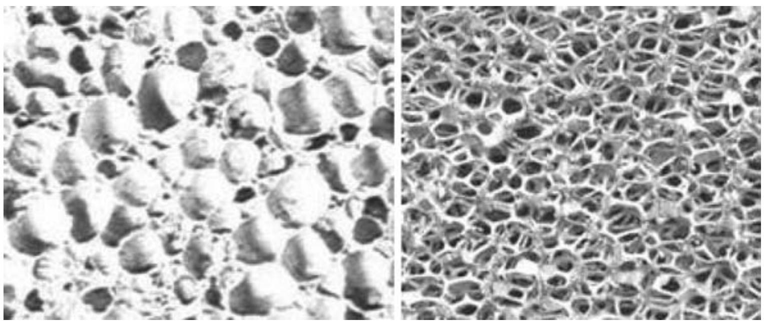

Fig. 2 Closed cell and open cell aluminium foams

The computational models on the micro and on the meso level are based on unit-cell concepts, well known from the field of micromechanics of materials. Hence, the principal features of unit-cell models are discussed, too.

2 Hierarchical computational analysis concept

In Fig. 1, a schematic representation of a general hierarchical computational analysis concept is given together with some typical micro- and mesoscopic phenomena. The general features of the analysis concept follow the procedure developed one decade ago, see [12], but they are extended to capture phenomena such as the ones described here. The overall, i.e., macroscopic or structural deformation and stability behaviour is treated by special finite elements (in most cases finite shell elements) which at the integration point level take into account mesoscopic effects such as meso-instabilities. These meso-instabilities depend on the behaviour at the micro-level, which again can be influenced by micro-instabilities.

From the macro-level loading configurations for the meso-level are obtained and from this the loading situation at the micro-level can be gained. As long as no instability happens at any length scale the coupling between the different length scales can easily be handled. Thus, up to the first appearance of instability the analysis is straightforward. In the post-critical regime, non-linear fully coupled analyses are required, regardless of the level where instability appears first.

3 Micro-Instabilities in metallic foams

Metallic foams can show an open or a closed cell topology, compare Fig. 2. The computational treatment of the micro-mechanical behaviour of metallic foams is summarized in a review article published recently [3].

Regarding micro-instabilities in closed cell foams, cell wall buckling is the typical mode; in open cell foams buckling of the struts is the analogous mechanism. Figure 3 shows the corresponding post-critical local deformations in real foams.

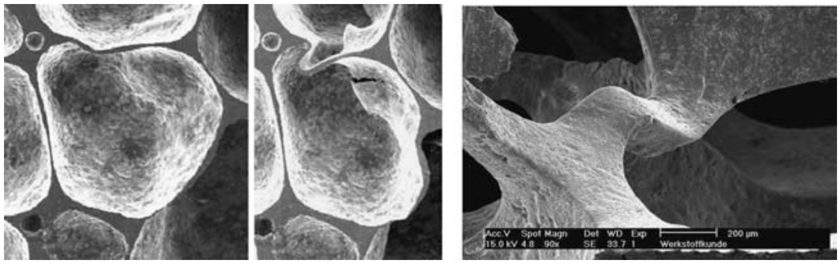

Fig. 3 Cell wall buckling in closed cell foams and strut buckling in open cell foams

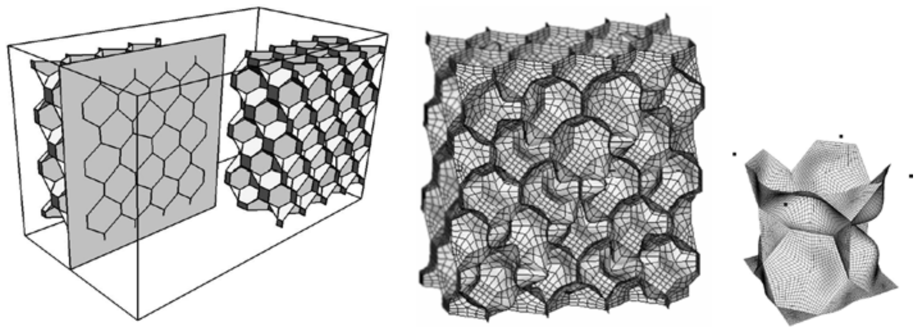

Fig. 4 Unit-cell models of a closed cell foam, micro-buckling under hydrostatic pressure

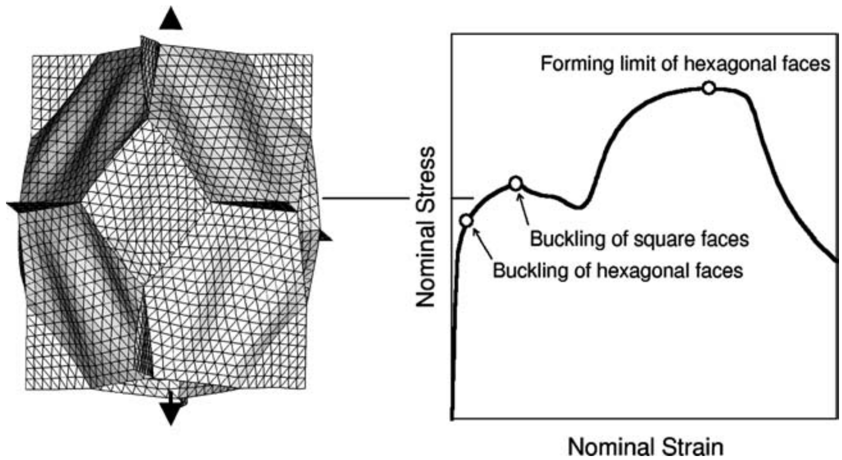

Fig. 5 Nominal stress-strain diagram of a model foam under tensile loading

Unit-cell analyses are currently the most powerful tools for studying micromechanical phenomena in microstructured materials [13].

Figure 4 shows a typical large perturbed unit-cell and a small, periodic unit-cell [14, 15] representing a closed cell aluminium foam. The microstructure is modelled by a perturbed tetrakaidecahedra topology, the walls are represented by thin shell elements. In this figure the unit-cell model as well as the buckling pattern under hydrostatic pressure is shown. In this case, micro-buckling means buckling of the cell walls.

Quite interestingly, micro-buckling is also a deformation mechanism of thin walled closed cell foams under macroscopic tensile loading. Figure 5 shows the macroscopic load-displacement path (expressed in terms of nominal stress-nominal strain diagram) of the foam material under uniaxial tension, the bulk material of which is modelled as elastic-plastic (using Ludwick's law).

The unit-cell for this investigation was formed on the basis of surfaces of minimum surface energy according to [16], leading to slightly curved cell walls. This is the reason why no clear bifurcation instability is depicted in the nonlinear analysis. The wrinkles in the deformed configuration of the hexagonal faces shown in Fig. 5 are the result of the post-buckling deformation of these faces. A further local buckling appears in the square faces when increasing the tensile load, leading to a snap-through behaviour of the cell followed by a stiffening up to the rupture finally caused by material instability in terms of strain localization (necking) in the wrinkled hexagonal faces, when the *Considère* condition as extended by [17] is met.

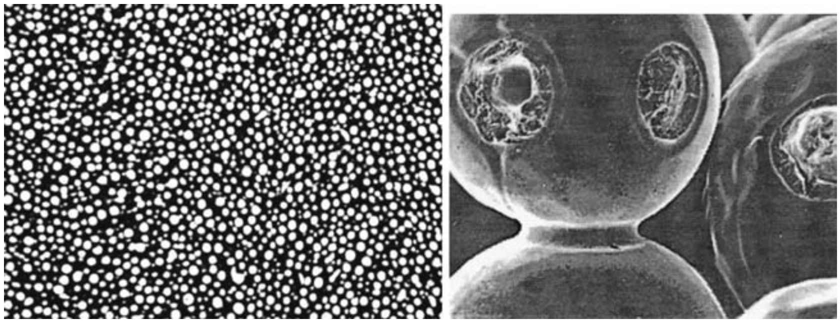

Fig. 6 Hollow spheres foam produced by sintering

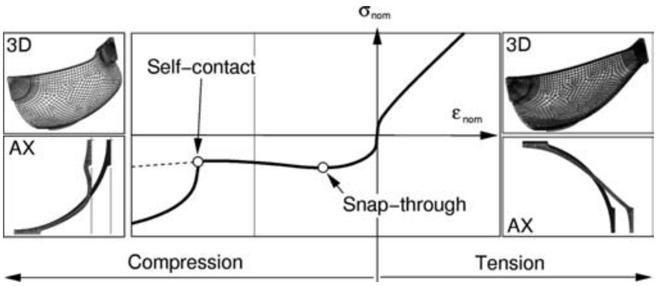

Fig. 7 Nominal stress vs. nominal strain behaviour of hollow spheres foams

4 Micro-Instabilities in hollow spheres foams

One rather sophisticated way for producing metallic foams is the pre-production of extremely small, very thin walled spheres which are subsequently sintered together, compare Fig. 6.

In order to optimize the micro-geometry of these foams with respect to weight-effective strength it is important to consider different failure modes as, e.g., failure by cracking along the sinter bridges (see Fig. 6 right picture), crack formation in the spheres' walls, or buckling of the spherical shells [18]. Figure 7 shows results of a numerical unit-cell simulation (simple cubic topology was chosen for the sake of simplicity) of a typical foam structure. One can see that snap-through buckling under compression can take place leading to the typical plateau-like behaviour of closed cell foams in the compression stress vs. strain diagram followed by densification due to self-contact. It can be seen, that full 3D-analyses for the considered cases can be replaced by axisymmetric analyses without too much loss of information and accuracy when a modelling technique as introduced in [19] is applied.

5 Micro and Meso-instabilities in materials with a honeycomb topology

Materials exhibiting a micro-topology reminding of honeycombs have the potential of developing manifold kinds of micro- and meso-instabilities. Three typical micro-buckling modes calculated by using two-cell and four-cell models, respectively, with sophisticated periodic boundary conditions [20] are shown in Fig. 8 and by using multi-cell models of regular honeycombs, see Fig. 9. The calculated buckling modes are compared to post-buckling deformations observed in real honeycomb structures.

Since in such microstructured materials often plastic micro yielding takes place before micro-buckling starts, multi-cell models were used to study the onset of micro-yielding [20]. Figure 10 shows the limits of linear macroscopic behaviour being expressed in terms of macroscopic compressive stresses in two orthogonal directions. Following radial macroscopic loading paths in this diagram leads to a linear elastic response as long as the corresponding stress states remain in the grey area of the diagram. This area is bounded by curves which represent elastic micro-buckling of cell walls and the onset of micro-yielding, respectively. In this sense, "failure initiation curves" for a perfect and an imperfect configuration of a typical metallic honeycomb micro-structure under bi-axial macroscopic stress fields are shown. In the perfect case the failure initiation envelope consists of two nearly parallel lines indicating onset of micro yielding. These lines converge at very high, biaxial "hydrostatic" macroscopic stress states, which cause only membrane stresses in a perfect honeycomb leading to a delayed occurrence of micro-yielding. This micro-yielding envelope is truncated by a curve which corresponds to compressive macroscopic stress states that lead to elastic micro-buckling.

Since both micro-yielding and micro-buckling initiate a non-linear mechanical response only the enclosed (grey) stress regime can be considered "safe" with respect to maintaining purely linear macroscopic behaviour.

The failure initiation envelope shrinks drastically as soon as imperfections, e.g., in the form of wiggles, are imposed. This is mainly due to the onset of micro-yielding in the bent cell walls. In this case, the linear regime shrinks most dramatically along the axis of biaxial hydrostatic compression, because the curved cell walls in the imperfect honeycombs experience bending deformations.

The appearance of plastic strain localization under tensile loading in elastic plastic homogeneous metals is well known and widely studied. It is, however, not as widely known, that similar material instabilities happen in materials with a honeycomb-like micro-topology, i.e. localized mesoscopic deformations appear under compressive loads, see e.g. [21]. Large deformation analyses using multi-cell models with imperfect honeycomb topology were studied in [20].

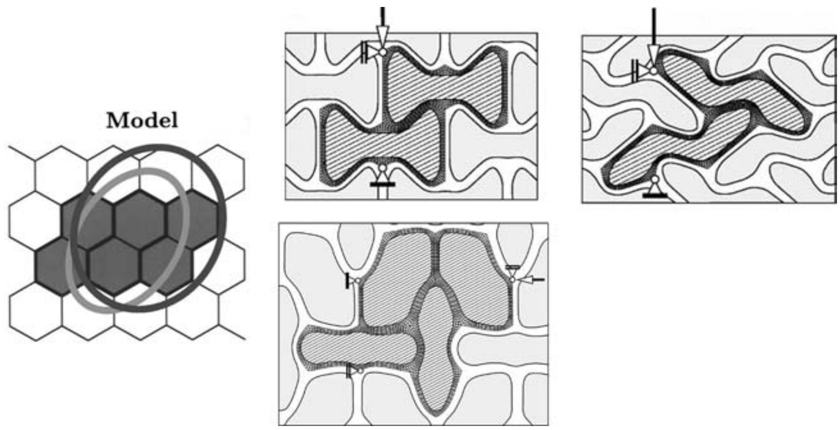

Fig. 8 Micro-buckling modes, calculated by two-cell and four-cell models

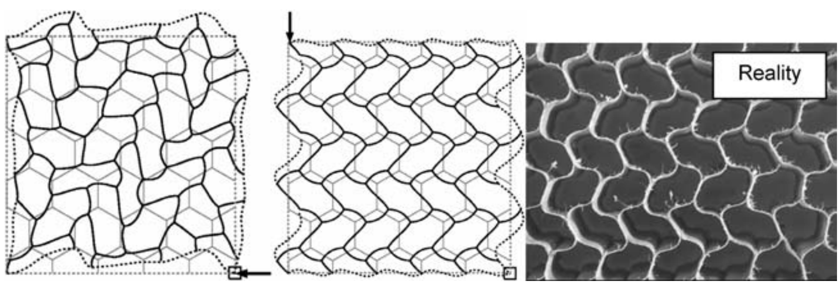

Fig. 9 Micro-buckling modes, calculated by multi-cell models

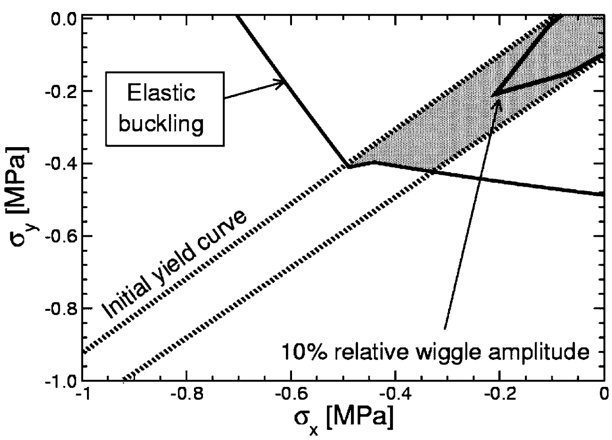

Fig. 10 Limitation of linear macroscopic behaviour by micro-yielding and buckling

The subsequent formation of deformation bands, as shown in Fig. 11, may be a possible explanation of the waviness of the stress-strain curves found not just in computational simulations but also in experimental compression tests. These waves indicate the collapse of single cell rows.

6 Wrinkling of face layers in sandwich compounds

Another mesoscopic instability is the formation of wrinkles in face layers of sandwich material compounds due to compressive in-plane forces or bending moments acting in the sandwich plate or shell. This phenomenon has been treated in [9, 22] as coupled bifurcation buckling of two parallel thin plates with elastic or elastic-plastic homogenized material between them, see Fig. 12.

One of the faces is defined as the "upper face layer", and the core together with the other face layer (called "lower face layer") acts as its "foundation". As far as elastic buckling is considered the upper face layer is treated as an infinite, thin plate on an elastic foundation under in-plane loading, the out-of-plane displacement w^u being described by

$$B_x^u \frac{\partial^4 w^u}{\partial x^4} + 2B_{xy}^u \frac{\partial^4 w^u}{\partial x^2 \partial y^2} + B_y^u \frac{\partial^4 w^u}{\partial y^4} + P_x^u \frac{\partial^2 w^u}{\partial x^2} + 2P_{xy}^u \frac{\partial^2 w^u}{\partial x \partial y} + P_y^u \frac{\partial^2 w^u}{\partial y^2} + w^u k^{\text{thin}} = 0, \quad (1)$$

with P_{ij}^u being the membrane forces in the upper face layer having effective bending and twisting stiffnesses B_{ij}^u, which in the case of layered composite faces are calculated by using classical lamination theory.

Since short wave buckling must be expected, the foundation, with the stiffness k^{thin} , must not be idealized by simple models such as, e.g., Winkler or Pasternak foundations. k^{thin} can be determined on the basis of continuum mechanics by using an analytical model as shown in [22]. It depends, in addition to the properties of the core (using effective core

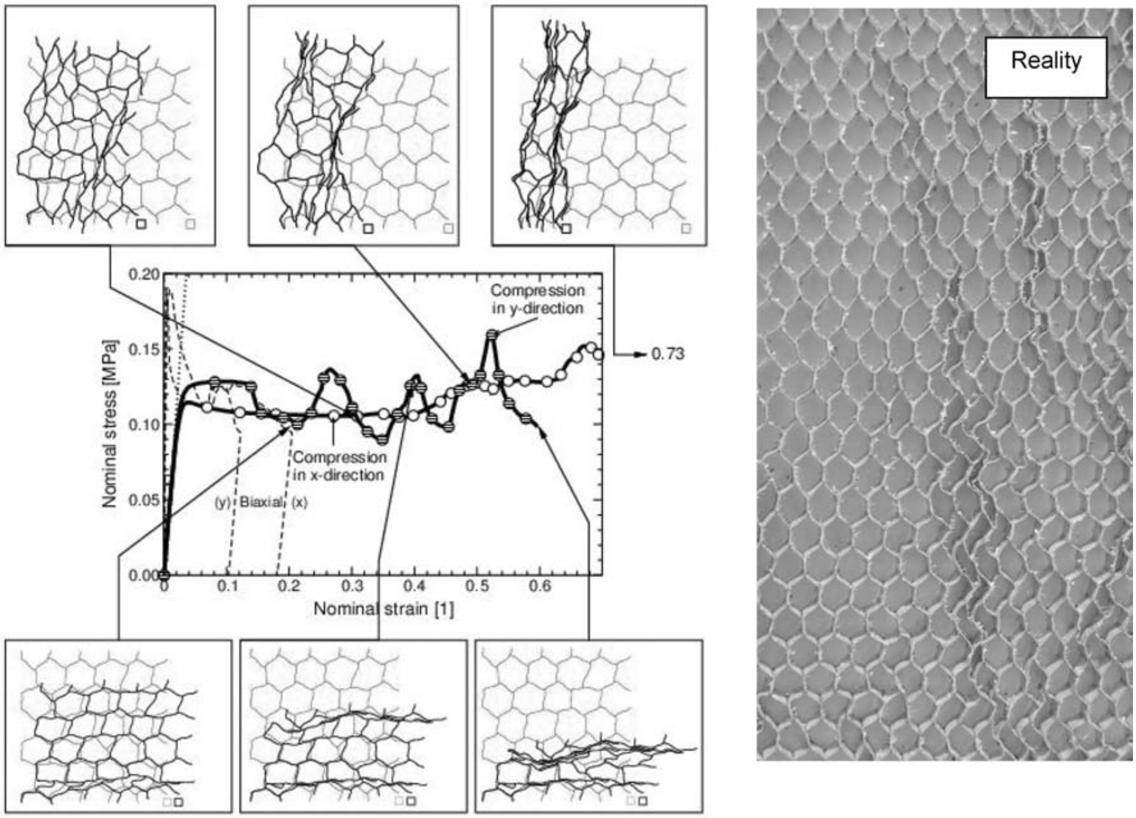

Fig. 11 Mesoscopic strain localizations under compressive loading

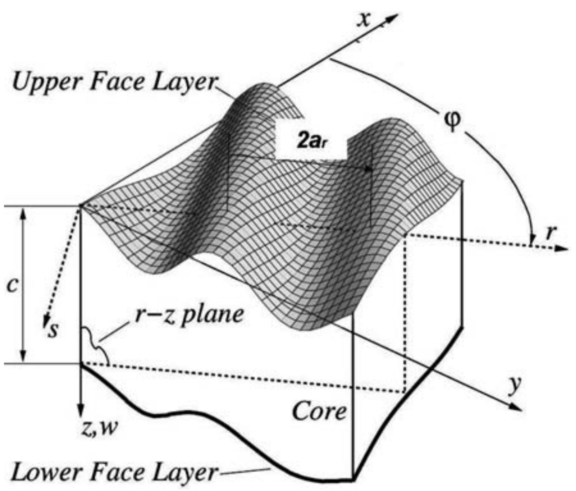

Fig. 12 Schematic representation of the analytical model

material properties) and of the lower face layer, on the combined buckling mode, i.e., $k^{\text{thin}} = k^{\text{thin}}(a_r)$.

The search for the first appearance of a non-trivial solution of (1),

$$w^u = w^{u*} \sin\left(\frac{\pi r}{a_r}\right) = w^{u*} \sin\left(\frac{\pi(x\cos\varphi + y\sin\varphi)}{a_r}\right) \quad (2)$$

leads to the critical membrane force intensity.

Based on this model eigenvalue analyses can be performed for calculating the critical load configurations for given wavelengths a_r and fold angles φ. Certainly, the physically relevant eigenvalue is that one which is minimal with respect to a_r and φ. Therefore, as described in detail in [22], an optimization procedure is required for finding critical winkling loads. This optimization can hardly be done analytically but is performed numerically. This is the reason why the approach is called a semi-analytical one. It allows the determination of critical mesoscopic stress states in the face layers as well as the prediction of the buckling mode for very general material and load combinations, see [9] and Fig. 13.

In Fig. 13 results of finite element unit-cell analyses are presented, too, and compared with the results obtained by the semi-analytical approach.

In the post-bifurcation regime (which is typically an unstable combination of local and global instability modes) starting from the periodic buckling mode a strong localization in the deformation pattern leading to single folds can be observed experimentally as well as numerically, see Fig. 14.

7 Dimpling of face layers, crushing of honeycomb cores

In the wrinkling models described above the core material is assumed to be homogeneous (although anisotropic), i.e. the micro structure of the core material (foam or honeycombs)

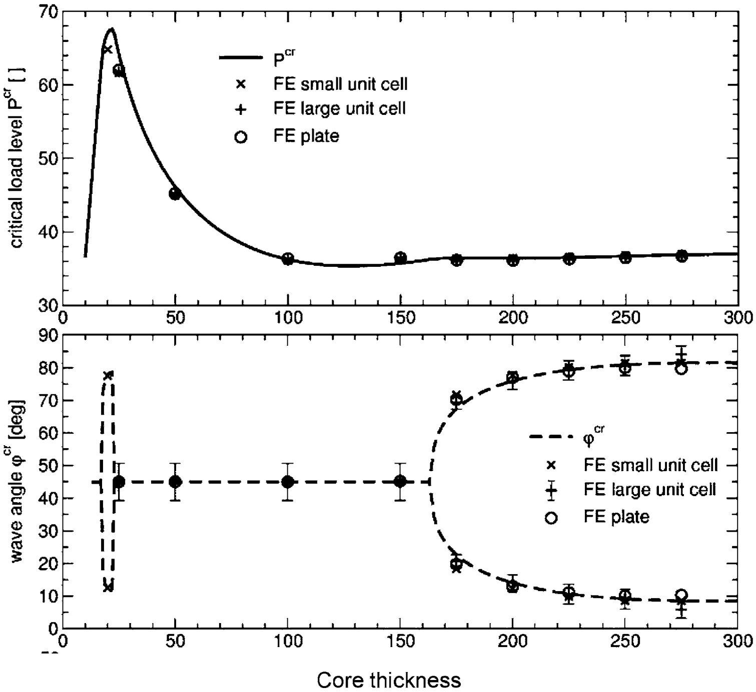

Fig. 13 Comparison between analytically derived results with results of finite element unit-cell analyses

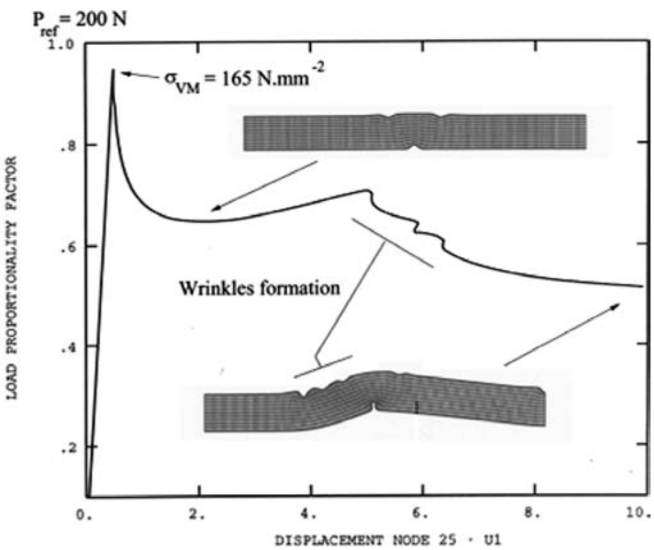

Fig. 14 Wrinkling in sandwich compounds - Formation of single folds (post-bifurcation regime)

is not represented explicitly but in a homogenised form. This is a suitable assumption as long as the buckling wavelength of the faces is sufficiently large, say, more then ten times the characteristic length of the microstructure of the core material (cell size). For very thin faces and for relatively large cells of the honeycomb core another type of meso-instability can appear under global in-plane and bending loads acting on the sandwich plate or shell: dimpling of the faces. There exists a number of estimating formulae based on formulae for buckling of rectangular or circular plates. The report [23], e.g., estimates a critical membrane force in the face layer by:

$$P_{crit} = \frac{1}{3} E_f t_f \sqrt{\left(\frac{2t_f}{SW}\right)^3}, \tag{3}$$

and report [24] leads to

$$P_{crit} = \frac{2E_f t_f}{1-\nu_f^2} \left(\frac{2t_f}{SW}\right)^2, \tag{4}$$

which are still in use. In these formulae E_f means Young's modulus of face layer material, t_f is the face layer thickness and SW is the diameter of the inscribed circle of the hexagons. More precise values for critical loads can be obtained by unit-cell analyses. Figure 15 shows dimpling modes obtained by using the smallest allowable unit-cell models for this problem, i.e. containing two honeycomb cells and periodic boundary conditions for all displacement and rotational degrees of freedom.

The quality and efficiency of the above described unit-cells is proven by performing stability analyses with unit-cells containing a much larger number of honeycomb cells, leading to identical results – see Fig. 16. The buckling mode as well as the critical membrane force coincides fully with that obtained by the small unit-cell. In Fig. 16 real dimpling patterns are also shown.

One of the essential outcomes of these analyses is the fact that this kind of mesoscopic structural instability represents an interaction buckling of the honeycomb cell walls and the face layers. This means that disregarding the

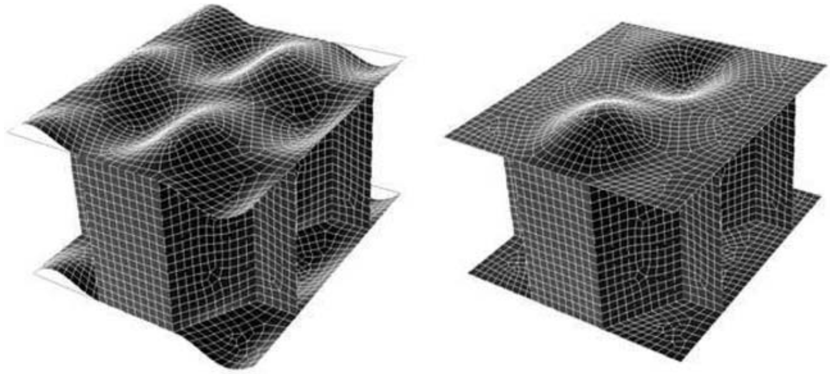

Fig. 15 Unit-cell model showing two typical dimpling modes

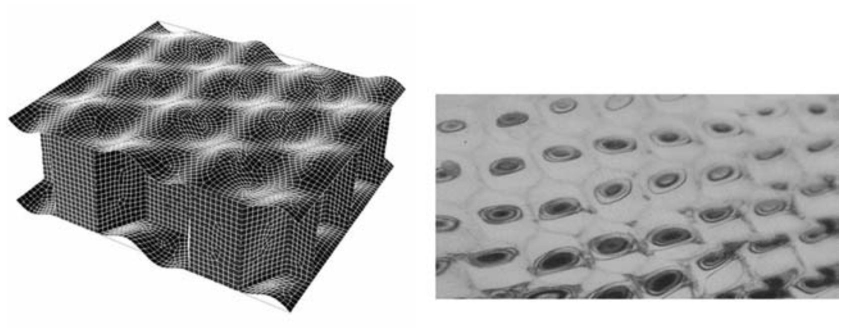

Fig. 16 Computed and experimentally observed dimpling patterns

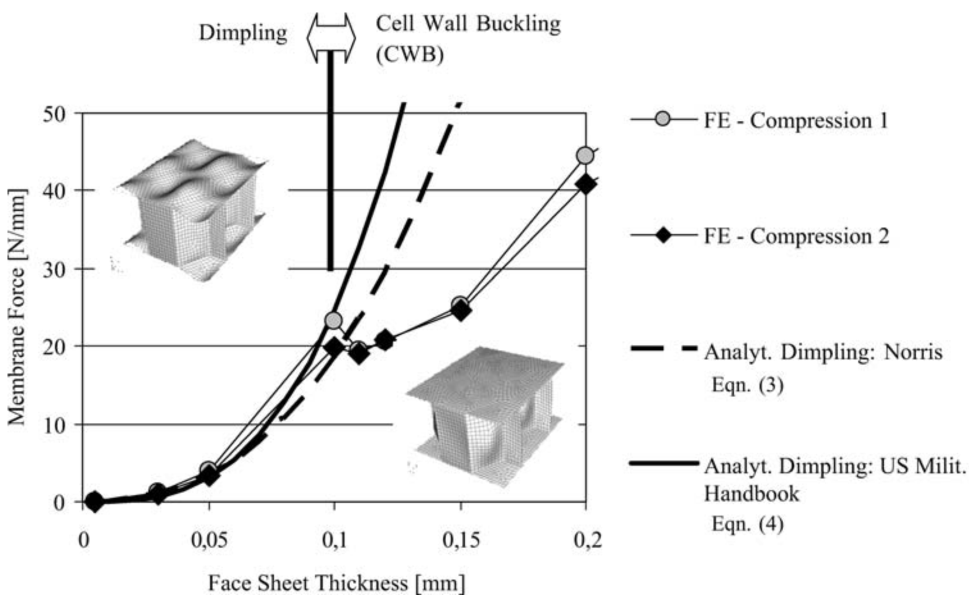

Fig. 17 Critical membrane forces in the face layers leading either to dimpling or to core buckling

coupling effect, as simplified formulae do, can lead to considerable errors in estimating critical loading situations. Figure 17 shows how the buckling mode switches from dimpling to core buckling, i.e., buckling of the honeycomb walls with increasing ratio between face layer thickness and honeycomb wall thickness.

8 Conclusions

Modern lightweight materials and materials compounds are very often discrete structures at different length scales. Therefore, structural mechanics and mechanics of materials are interwoven in considering the micro-, meso-, and macromechanical behaviors of these materials and components made of such materials. This is important in particular when instability phenomena at different length scales play a dominant role with respect to failure considerations. Unit-cell concepts, in combination with analytical and discretization methods, provide suitable procedures for capturing such instability phenomena.

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