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Simultaneous identification of structural parameters and input time history from output-only measurements

J. Chen, J. Li

Abstract A new time-domain method is suggested in this paper for simultaneous identification of the structural parameters and the time history of the input excitation using output-only measurements. The proposed method is based on an iterative identification procedure consisting of the least-squares identification technique and a modification process between each iterative step. The modification process is introduced to convert the spatial information of the external excitation into mathematical conditions. First, the unknown force vector is conjectured through the equation of motion using the initial guess of the structural parameters and the measured structural responses. The estimated input force vector is then modified to force it comply with the spatial distribution of the external excitations. The modified input force vector is further used to provide new estimation of structural parameters. Repeat the aforementioned procedure until the structural parameters satisfy the preset convergence criterion. Numerical examples as shear building and truss bridge model are employed to evaluate the feasibility of the proposed method. In the numerical examples, typical scenario of complete and noise-free as well as incomplete and noise-contaminated output measurements are considered. The results demonstrate that the proposed method can accurately identify both the structural parameters and the input time history for the cases that the structural responses are not polluted or slightly contaminated by measurement noise.

Keywords Structural identification, Unknown input, Time domain approach, Incomplete output measurements, Shear building

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1 Introduction

Structural health monitoring (SHM) technology has recently received increasing attention in different disciplines such as civil, mechanical and aerospace engineering. A typical SHM system consists of several components of different functions, including network of data acquisition instrument, signal processing and analysis scheme. Among them, the system identification algorithm is regarded as the key part of a SHM system [1], which elicits the structural physical or modal parameters, such as stiffness, mass or natural frequency, mode shape and modal damping ratio, from the raw measurement data. Significant changes in the identified parameters will be then used to unveil damage of structures.

Many system identification methods, either in frequency-domain or in time-domain, have been proposed in the past decades. Since input excitation to structure is difficult to be precisely measured in practice, the frequency domain approach is commonly used to identify natural frequencies, modal damping ratios and mode shapes of a structure from measured structural responses without requiring the information on the external excitation but assuming it to be a white noise random process. However, this assumption is not tenable for practical input situations such as earthquake-induce ground motion, strong wind and impact force etc. Improper treatment of the input excitation can thus significantly affect the identification accuracy of the structural parameters, and the excitation itself may be of interest to the researcher in many cases. Furthermore, the natural frequencies and mode shapes are not sensitive indicators to the damage of individual structural members. Thus, the simultaneous identification of both structural parameters at the element level and the input excitation time history in the time domain attracts more and more attentions from researchers and engineers.

In contrast to plenty of methods applicable for identifying structural parameters when the input information is known, there is a paucity of methods available for situation that the time history or the statistical characteristic of the input excitation is unknown. As far as methods in time domain are concerned, they can be categorized according to the identification techniques involved, which are mainly the random decrement technique (RDT), the extended Kalman filter (EKF) and the recursive least-squares method (RLS). The Ibrahim time domain (ITD) method [2] can be adopted to identify the structural parameters when free-decay response of the structure is measured.

The free-decay response, however, is hard to be measured for structures under operational condition. The random decrement technique (RDT) developed by Cole [3] is probably the most commonly used method for solving the unknown input identification problem. Through this method, segments of the measured responses of a linear structure under random excitation (white noise time series) are ensemble averaged to form a signature, which is the representative of the free vibration decay curve of the structures, by removing the responses due to the excitation and the initial velocity. Vandiver et al. [4] proposed the mathematical basis for applying the random decrement technique and concluded that if the input excitation is a stationary Guassian white noise process, then the randomdec signatures of the displacement responses of the system are equivalent to the free decay response of the system. Spanos and Zeldin [5] investigated some theoretical and computational issues involving in the application of RDT. They pointed out that the randomdec signature of a linear system is influenced by the parameters of the input excitation and it will converge to the system's free vibration curve with the excitation tends to be a white noise process. Another widely used technique is the extended Kalman filter (EKF). Hoshiya and Saito [6] developed an approach to identify optimal structural parameters using the extended Kalman filter by taking the system parameters as unknown state variables. They [7] then used the weighted global iteration (WGI) technique to improve the identification accuracy of EKF, and applied the EKF-WGI to physical parameters identification of a static finite-element model. A further extension of this work [8] was to identify the input seismic excitation and system parameters of the shear-type building. More recently, Shi et al. [9] successfully applied the EKF to identification problem of single-degree-of-freedom system with unknown input. Numerical and experimental investigation showed that the input characteristics as well as the natural frequency and damping parameters of the system can be accurately estimated. The time-history of the input excitation, however, cannot be directly obtained by their approach. As for the RLS method, Wang and Haldar [10] used it to identify structural parameters at element level when the input excitation is unknown. They further extended the method by combining with the EKF-WGI technique to deal with the situation of limited measurement data [11]. Li and Chen [12–15] developed a different method termed statistical average algorithm based on RLS for simultaneous identification of structural parameters and time-history of seismic ground motion.

Most of the above-mentioned methods, however, are generally developed for the case of ambient vibration surveys of structure, in which environmental excitation are considered. These methods are therefore not readily applied to the case of forced vibration surveys of structure system, where the structure is excited to vibration by actuators installed on several key locations of the structure. In this connection, this paper thus proposes a method in time domain for simultaneous identification of structural parameters and the time history of input excitation using measurements from forced vibration surveys of structure. With the inspiration of the previous work [12–15], an iterative identification approach is developed based on the least-squares technique to solve this problem. The basic assumption of this method is that the external forces are applied on limited number of degree-of-freedom (DOF) of the structure and are less than the number of DOF whose response are measured. The locations of the applied force are known though its time history is unknown. The spatial distribution of the external excitation is then taken as additional information for the parameter identification. The implementation procedure of the proposed method is discussed in detail in the following section. Followed are numerical investigations to evaluate the feasibility of the method.

2 Methodology

2.1 Basic equations

Consider a discrete N DOF linear system (see Fig. 1) whose equation of motion at certain time instant t_i is given by

$$\mathbf{M}\ddot{\mathbf{X}}(t_i) + \mathbf{C}\dot{\mathbf{X}}(t_i) + \mathbf{K}\mathbf{X}(t_i) = \mathbf{F}(t_i) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ is, respectively, the mass, damping and stiffness matrices respectively, each of order N by N; $\mathbf{X}$, $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$ is the system displacement, velocity and acceleration vector of order $N \times 1$; $\mathbf{F}$ is the external dynamic force applied on the system of order $N \times 1$. The Rayleigh damping model is adopted in this study to describe the energy dissipation mechanism of the system. The damping matrix is thus expressed as

$$\mathbf{C} = a\mathbf{M} + b\mathbf{K} \tag{2}$$

In the traditional structural identification problem, the mass matrix, the external excitation and the dynamic responses are assumed to be known, while the damping and stiffness parameters are to be identified. In this study, the known quantities are the mass matrix and the dynamic responses, the damping and stiffness parameters as well as the time history of the input force are targeted.

From Eqs. (1) and (2), one has

$$\mathbf{M}\ddot{\mathbf{X}}(t_i) + a\mathbf{M}\dot{\mathbf{X}}(t_i) + \mathbf{K}\mathbf{X}(t_i) + b\mathbf{K}\dot{\mathbf{X}}(t_i) = \mathbf{F}(t_i) \tag{3}$$

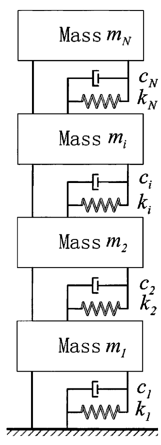


Fig. 1. N degrees of freedom spring–mass system

It is seen from Eq. (3) that the unknown damping parameters a, b and the stiffness parameters in matrix $\mathbf{K}$ are coupled with each other, leading Eq. (3) to be a non-linear equation regarding to the unknown parameters. In order to avoid solving non-linear identification problem, Eq. (3) are rearranged into the following formulas

$$\mathbf{K}\left(\mathbf{X}(t_i) + b\dot{\mathbf{X}}(t_i)\right) = \mathbf{F}(t_i) - \mathbf{M}\ddot{\mathbf{X}}(t_i) - a\mathbf{M}\dot{\mathbf{X}}(t_i) \tag{4}$$

$$a\mathbf{M}\dot{\mathbf{X}}(t_i) + b\mathbf{K}\dot{\mathbf{X}}(t_i) = \mathbf{F}(t_i) - \mathbf{M}\ddot{\mathbf{X}}(t_i) - \mathbf{K}\mathbf{X}(t_i) \tag{5}$$

Rearrange Eq. (4), the following identification equation can be obtained

$$\mathbf{H}_K(t_i)\boldsymbol{\theta}_K = \mathbf{P}_K(t_i) \tag{6}$$

Assembling Eq. (6) at all sampling time instants t_i, $i = 1, \ldots, L$ together, one has

$$\mathbf{H}_K\boldsymbol{\theta}_K = \mathbf{P}_K \tag{7}$$

in which

$$\mathbf{H}_K = [\mathbf{H}_K(t_1), \mathbf{H}_K(t_2), \ldots, \mathbf{H}_K(t_L)]^{\mathrm{T}} \tag{8}$$

$$\boldsymbol{\theta}_K = [k_1, k_2, \ldots, k_J]^{\mathrm{T}} \tag{9}$$

$$\mathbf{P}_K = [\mathbf{P}_K(t_1), \mathbf{P}_K(t_2), \ldots, \mathbf{P}_K(t_L)]^{\mathrm{T}} \tag{10}$$

where matrix $\mathbf{H}_K$ contains the velocity, displacement responses and the damping coefficient b. $\mathbf{H}_K$ is actually a rectangular matrix of order $(L \times N) \times J$, where L is the number of sampling points in the measured structural response time history and J is the number of unknown stiffness parameters. Vector $\boldsymbol{\theta}$ of order $(J \times 1)$ contains all the unknown stiffness parameters, and vector $\mathbf{P}_K$ of order $(L \times N) \times 1$ is related to the input force $\mathbf{F}$, the inertia forces and the parameter of a. In particular, for the N-story shear building considered here, J equals N and at any sample time instant $t_i (1 \le i \le L)$ one has

$$\mathbf{H}_K(t_i) = \begin{bmatrix} x_1 + b\dot{x}_1 & (x_1 + b\dot{x}_1) - (x_2 + b\dot{x}_2) & 0 & \vdots & 0 \\ 0 & (x_2 + b\dot{x}_2) - (x_1 + b\dot{x}_1) & (x_2 + b\dot{x}_2) - (x_3 + b\dot{x}_3) & \vdots & 0 \\ 0 & 0 & (x_3 + b\dot{x}_3) - (x_2 + b\dot{x}_2) & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & \vdots & (x_N + b\dot{x}_N) - (x_{N-1} + b\dot{x}_{N-1}) \end{bmatrix}_{N \times J} \tag{11}$$

$$\mathbf{P}_K(t_i) = \begin{bmatrix} f_1 - m_1\ddot{x}_1 - am_1\dot{x}_1 \\ f_2 - m_2\ddot{x}_2 - am_2\dot{x}_2 \\ \cdots \\ f_N - m_N\ddot{x}_N - am_N\dot{x}_N \end{bmatrix}_{N \times 1} \tag{12}$$

where $x_j = x_j(t_i)$ and $f_j = f_j(t_i)$ are the displacement response and external excitation force of the jth DOF $(j = 1, \ldots, N)$ at the time instant t_i.

From, Eq. (7) the stiffness parameters can be identified by the least-squares technique [16] as

$$\boldsymbol{\theta}_K = \left[\mathbf{H}_K^{\mathrm{T}}\mathbf{H}_K\right]^{-1}\mathbf{H}_K^{\mathrm{T}}\mathbf{P}_K \tag{13}$$

Similarly, the identification equation for damping coefficients can be derived from Eq. (5) as

$$\mathbf{H}_C\boldsymbol{\theta}_C = \mathbf{P}_C \tag{14}$$

in which,

$$\mathbf{H}_C = [\mathbf{H}_C(t_1), \mathbf{H}_C(t_2), \ldots, \mathbf{H}_C(t_L)]^{\mathrm{T}} \tag{15}$$

$$\boldsymbol{\theta}_C = [a, b]^{\mathrm{T}} \tag{16}$$

$$\mathbf{P}_C = [\mathbf{P}_C(t_1), \mathbf{P}_C(t_2), \ldots, \mathbf{P}_C(t_L)]^{\mathrm{T}} \tag{17}$$

where $\mathbf{H}_C$ is a matrix of order $(L \times N) \times 2$ and vector $\mathbf{P}_C$ is related to the input force $\mathbf{F}$, the inertia force and the spring force. Similarly, at any sample time instant $t_i (1 \le i \le L)$ one has

$$\mathbf{H}_C(t_i) = \begin{bmatrix} m_1\dot{x}_1 & k_1\dot{x}_1 + k_2(\dot{x}_1 - \dot{x}_2) \\ m_2\dot{x}_2 & k_2(\dot{x}_2 - \dot{x}_1) + k_3(\dot{x}_2 - \dot{x}_3) \\ \cdots & \cdots \\ m_N\dot{x}_N & k_N(\dot{x}_N - \dot{x}_{N-1}) \end{bmatrix}_{N \times 2} \tag{18}$$

$$\mathbf{P}_C(t_i) = \begin{bmatrix} f_1 - m_1\ddot{x}_1 - k_1x_1 - k_2(x_1 - x_2) \\ f_2 - m_2\ddot{x}_2 - k_2(x_2 - x_1) - k_3(x_2 - x_3) \\ \cdots \\ f_N - m_N\ddot{x}_N - k_N(x_N - x_{N-1}) \end{bmatrix}_{N \times 1} \tag{19}$$

Finally, the damping parameters can be identified from Eq. (13) by

$$\boldsymbol{\theta}_C = \left[\mathbf{H}_C^{\mathrm{T}}\mathbf{H}_C\right]^{-1}\mathbf{H}_C^{\mathrm{T}}\mathbf{P}_C \tag{20}$$

Problem faced here is that Eqs. (13) and (20) cannot be readily solved to determine the stiffness and damping parameters since there are unknown quantities involved in calculating $\mathbf{H}_K$, $\mathbf{H}_C$, $\mathbf{P}_K$ and $\mathbf{P}_C$. The following method is thus suggested to deal with this problem.

2.2 Identification procedure

The identification method developed here is based on the assumption that the locations of the external forces are known even though their time histories are unknown. And the number of DOF with applied force is less than the number of DOF whose responses are measured. This assumption reflects the situation of forced vibration survey of structure, where generally a small and often limited number of actuators are installed on key locations of the structure to excite it. In this regard, the input excitation vector $\mathbf{F}$ in Eq. (1) can be expressed as

$$\mathbf{F} = [\mathbf{F}_u, \mathbf{0}]^{\mathrm{T}} \tag{21}$$

where $\mathbf{F}_u$ denotes those DOFs with unknown external excitation, and '$\mathbf{0}$' stands for those DOFs without applied force. Consequently, the vector $\mathbf{P}_K$ and $\mathbf{P}_C$ in Eqs. (7) and (14) can be expressed as

$$\mathbf{P}_K = [\mathbf{P}_K(\mathbf{F}_u), \mathbf{P}_K(\mathbf{0})]^{\mathrm{T}} \tag{22}$$

$$\mathbf{P}_C = [\mathbf{P}_C(\mathbf{F}_u), \mathbf{P}_C(\mathbf{0})]^{\mathrm{T}} \tag{23}$$

Noting that in Eqs. (22) and (23) only the entries $\mathbf{P}_K(\mathbf{F}_u)$ and $\mathbf{P}_C(\mathbf{F}_u)$ are related to the unknown external force.

Using the above-mentioned equations, the implementation procedure of the suggested identification method has been divided into the following eight steps.

Step 1: Assign initial values for the unknown structural parameters $\boldsymbol{\theta}_K^0$ and $\boldsymbol{\theta}_C^0$. For instance, let $\boldsymbol{\theta}_K^0, \boldsymbol{\theta}_C^0 = (1, 1, \ldots, 1)^{\mathrm{T}}$, where the superscript '0' denotes the number of iteration. The matrix $\mathbf{H}_K$ and $\mathbf{H}_C$ are then computed by Eqs. (11) and (18) using $\boldsymbol{\theta}_K^0$, $\boldsymbol{\theta}_C^0$ and the measured responses $\mathbf{X}$, $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$.

Step 2: Assuming first that all the damping coefficients $\boldsymbol{\theta}_C$ are known as $\boldsymbol{\theta}_C^0$, the vector $\mathbf{P}_K$ can be computed by Eq. (7) resulting in $\tilde{\mathbf{P}}_K = \mathbf{H}_K\boldsymbol{\theta}_K^0$. Hereafter, symbol '$\sim$' means estimated value of vector $\mathbf{P}_K$ or $\mathbf{P}_C$, whilst symbol '$\wedge$' denotes modified value of the vector $\mathbf{P}_K$ or $\mathbf{P}_C$.

Step 3: From Eq. (22) it is clear that the estimated vector $\tilde{\mathbf{P}}_K = \left[\tilde{\mathbf{P}}_K(\mathbf{F}_u), \tilde{\mathbf{P}}_K(\mathbf{0})\right]^{\mathrm{T}}$. Note that $\mathbf{P}_K(\mathbf{0})$ can be directly computed using Eq. (12) because the external force on these DOFs are exactly zero at each sample point. We can thus replace the estimated value of $\tilde{\mathbf{P}}_K(\mathbf{0})$ by the computed value of $\mathbf{P}_K(\mathbf{0})$ leading to the modified vector $\hat{\mathbf{P}}_K$, which is

$$\hat{\mathbf{P}}_K = \left[\tilde{\mathbf{P}}_K(\mathbf{F}_u), \mathbf{P}_K(\mathbf{0})\right]^{\mathrm{T}} \tag{24}$$

A schematic diagram of the modification process of $\tilde{\mathbf{P}}_K$ is shown in Fig. 2.

Step 4: Then, the structural stiffness parameters can be estimated by the modified vector $\hat{\mathbf{P}}_K$ from Eq. (13)

$$\boldsymbol{\theta}_K^1 = \left(\mathbf{H}_K^{\mathrm{T}}\mathbf{H}_K\right)^{-1}\mathbf{H}_K^{\mathrm{T}}\hat{\mathbf{P}}_K \tag{25}$$

Step 5: Now, assuming that all the stiffness parameters $\boldsymbol{\theta}_K^1$ are known, the vector $\mathbf{P}_C$ can be computed by Eq. (14) resulting in $\tilde{\mathbf{P}}_C = \mathbf{H}_C\boldsymbol{\theta}_C^0$.

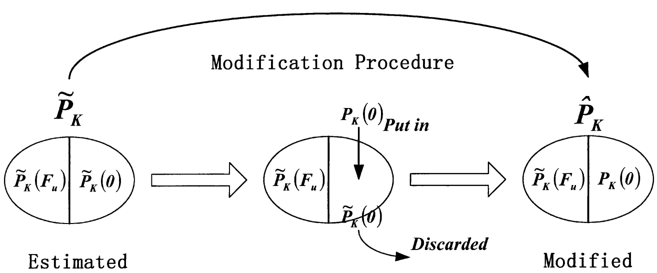


Fig. 2. The modification procedure of the identified vector $\mathbf{P}$

Step 6: Similarly, vector $\tilde{\mathbf{P}}_C$ consists of two parts, which is $\tilde{\mathbf{P}}_C = \left[\tilde{\mathbf{P}}_C(\mathbf{F}_u), \tilde{\mathbf{P}}_C(\mathbf{0})\right]^{\mathrm{T}}$. The component $\tilde{\mathbf{P}}_C(\mathbf{0})$ is then replaced by $\mathbf{P}_C(\mathbf{0})$, which is computed from Eq. (19) using the measured responses and the updated stiffness parameter $\boldsymbol{\theta}_K^1$, giving the modified vector $\hat{\mathbf{P}}_C$

$$\hat{\mathbf{P}}_C = \left[\tilde{\mathbf{P}}_C(\mathbf{F}_u), \mathbf{P}_C(\mathbf{0})\right]^{\mathrm{T}} \tag{26}$$

Step 7: A new estimation of the damping parameter $\boldsymbol{\theta}_C^1$ can be obtained by Eq. (20) as

$$\boldsymbol{\theta}_C^1 = \left(\mathbf{H}_C^{\mathrm{T}}\mathbf{H}_C\right)^{-1}\mathbf{H}_C^{\mathrm{T}}\hat{\mathbf{P}}_C \tag{27}$$

Step 8: Replace $\boldsymbol{\theta}_K^0$ and $\boldsymbol{\theta}_C^0$ by $\boldsymbol{\theta}_K^1$ and $\boldsymbol{\theta}_C^1$ in Step 1 and repeat Steps 2–8 until the following convergence criterion is satisfied.

$$\max\left|\frac{\boldsymbol{\theta}_{\mathrm{iter}}(l)-\boldsymbol{\theta}_{\mathrm{iter}-1}(l)}{\boldsymbol{\theta}_{\mathrm{iter}}(l)}\right|<\varepsilon,\ \boldsymbol{\theta}=\boldsymbol{\theta}_K\,\text{or}\,\boldsymbol{\theta}_C \tag{28}$$

where the subscript iter stands for the current iterative step, l means the lth element of vector $\boldsymbol{\theta}$; ε is the predetermined convergence index for the structural parameter, which is generally a small number of the magnitude between 10^{-4} and 10^{-6}.

Once the iterative procedure converges, the updated parameter vectors identified in Step 4 and 7 give the final identification result of all the structural parameters, whilst the time history of the input $\mathbf{F}$ can be easily determined by Eq. (1). For cases that the damping coefficients a and b are already known, only the stiffness parameters and the input excitation are to be identified, the identification procedure can be adjusted by skipping Step 5 to Step 7, i.e. skipping the steps for identifying the damping parameters. The convergence and uniqueness of the proposed method can be mathematically proved by the similar proof procedure as adopted in Ref. [13] for simultaneous identification of structural parameters and seismic ground motion. And it will be illustrated later by the numerical examples that convergence and uniqueness of the identification problem can also be achieved when enough measurement data are used.

As can be seen from the above procedure, the proposed method is actually an iterative identification procedure that consists of the least-squares technique for parameter identification and a modification procedure to force the identified force to comply with the spatial distribution of the external force along the structure. The key point of this method is the modification procedure as used in Steps 3 and 6, which provides a measure to convert the spatial location of the external force into additional information for parameter identification. The proposed method has no limitation on the type of the external excitation. Besides, the modification procedure is independent from the parameter identification algorithm chose, which implies that different parameter identification methods can be used instead of the least-squares technique. For instance, when Eqs. (7) and (14) became ill-conditioned due to the presence of measurement noise in matrix $\mathbf{H}_K$ and $\mathbf{H}_C$, direct solution to these equations by least-squares technique may produce instable and wrong results. In that case, regularization methods [17] can be used for the treatment of ill-conditioned problem and to improve the identification accuracy.

3
Numerical examples

3.1
General
The applicability of the proposed method is demonstrated in this section through numerical examples of a 4-DOF undamped dynamic system, a 15-storey tall building and a truss bridge model. For all the examples, the mass matrix of system is assumed to be known. The theoretical responses of all DOFs of the numerical model are first calculated in terms of displacement, velocity and acceleration using the Wilson-$\boldsymbol{\theta}$ method [18]. For the situation of complete and noise-free output measurements, all these calculated responses quantities (acceleration, velocity and displacement) will be used by the proposed method to identify the parameters and input excitation. On the other hand, to simulate practical situation of incomplete and noise-polluted output measurements, only the calculated acceleration are taken as measurements. It is further polluted by adding numerically generated zero-mean Gaussian white noise time series. The noise level is controlled by the root-mean-square ratio between the noise process and the corresponding acceleration response time-history to be polluted. The velocity and displacement responses at each DOF are then obtained by integrating the contaminated acceleration response using the SIMULINK toolbox in MATLAB. During the numerical integration, a high-pass filter is applied in order to remove the linear trend in the resulting time histories. All the following calculations are carried out on the same computational hardware platform. Therefore, the CPU time of each case is presented to evaluate the numerical cost of the method.

3.2
Example1: 4-DOF dynamic system under harmonic excitation
Figure 3 shows a four-degree-of-freedom undamped spring-mass dynamic system which is excited by actuators

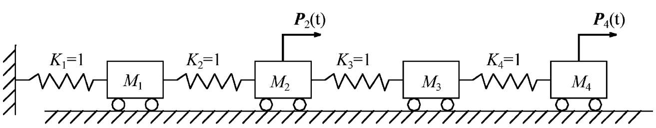


Fig. 3. Four-degree-of-freedom dynamic system

on Mass2 and Mass4. The equation of motion of this system is given as follow.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_1 \\ \ddot{\mathbf{x}}_2 \\ \ddot{\mathbf{x}}_3 \\ \ddot{\mathbf{x}}_4 \end{Bmatrix} + \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathbf{p}_2(t) \\ 0 \\ \mathbf{p}_4(t) \end{Bmatrix}$$

The applied forces are sinusoidal excitation, which are $\mathbf{p}_2(t) = 2\sin(0.8\pi t)\mathbf{N}$ and $\mathbf{p}_4(t) = \sin(1.2\pi t)\mathbf{N}$ respectively. The component $\mathbf{F}_u$ of the input excitation vector $\mathbf{F}$ in Eq. (21) is therefore $\mathbf{F}_u = \{\mathbf{p}_2(t), \mathbf{p}_4(t)\}$ for this example.

For this example, parameter identification results of four cases with complete and noise-free output measurements are given in Table 1. It is seen from Table 1 that for all the cases the unknown parameters can be accurately identified by the proposed method with short duration of measurements and very poor initial values. For Case 2, where only 100 sampling points are used, the maximum identification error is 0.14% for k_4 and the CPU time is 8.62 sec. When more sampling points are used, as that for Cases 1, 3, and 4, the identification accuracy can be significantly improved to about 0.01% for all parameters and the numerical cost can also be greatly reduced. Furthermore, different initial values are selected in Cases 2, 3, and 4, comparison between the results suggests that the proposed method is robust to the initial values, which is an attractive potential for practical application.

To learn the evolution of the identified time history of input excitation with the iteration steps, the identified time-histories at the 2nd and 4th DOF for Case 1 in iterative steps 1, 22 and 66 are depicted in Fig. 4a and b respectively. The solid line in Fig. 4 is the time history of the actual input excitation, while the dash, dot and dash-dot line is the identified input excitation at Steps 1, 22 and 66, respectively. Figure 4 is a double y-axis plot in which the right y-axis corresponds to the identification results at the first iterative step, while the left y-axis corresponds to the results at iterative Steps 22 and 66. It is seen from Fig. 4 that due to the poor initial guess of the parameters, the estimated input at the Step 1 deviates from the actual one significantly. The maximum amplitude of the inversed

Table 1. Identification results with complete noise-free measurements (Example 1)

Parameter and its real values		Case 1		Case 2		Case 3		Case 4	
		θ_K^0	$\hat{\theta}_k$	θ_K^0	$\hat{\theta}_k$	θ_K^0	$\hat{\theta}_k$	θ_K^0	$\hat{\theta}_k$
k_1	1	100	1.0001	100	1.0010	−1000	1.0001	0.001	1.0000
k_2	1	100	1.0000	100	1.0001	1000	1.0000	0.001	1.0000
k_3	1	100	1.0000	100	1.0007	0.001	1.0000	0.001	1.0000
k_4	1	100	1.0000	100	0.9986	−2000	1.0000	0.001	1.0000
CPU time		0.53 s		8.62 s		0.58 s		0.78 s	
L		200		100		200		200	

θ_K^0: Initial values of the parameters, $\hat{\theta}_k$: identified values, L: number of sampling points used, $\varepsilon = 10^{-5}$ for all cases

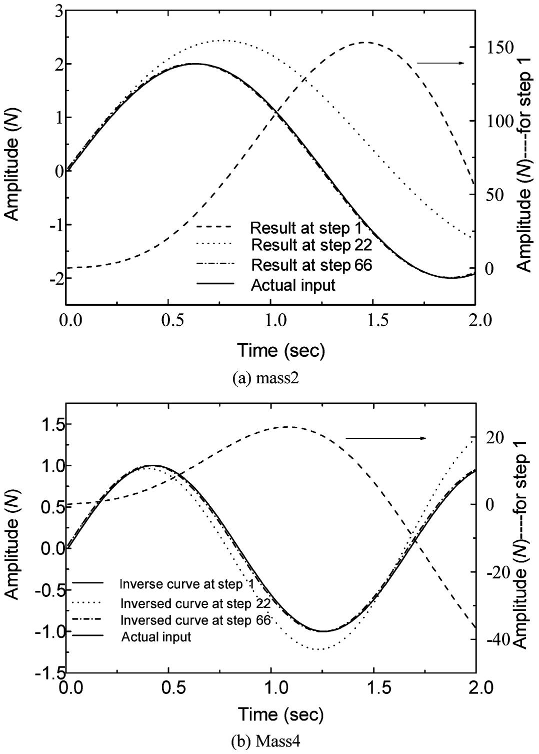


Fig. 4. Identified input time history at different iteration step. **a** Mass2. **b** Mass4

input is about 60 times larger than that of the actual input force. With the evolution of the iteration procedure, the inversed input time history will gradually and steadily approach to the real input curve, and at last overlap with the real input when the algorithm converges.

Let us then consider the situation of incomplete and noise-included output measurements. Table 2 shows parameter identification results for four cases with various measurement noise levels. In Case 1 of Table 2, the noise level is 1% and the sampling point used is 200. It is seen that the maximum identification error is 11.8% for this case which is unsatisfied. In Case 2 the same noise level of 1% is considered but 900 sampling points are used. It is seen that the identification accuracy has been significantly improved when more sampling points are involved. The maximum identification error is lowered to about 1.2%. From results of other cases listed in Table 2, one may see that the proposed method can accurately identify the stiffness parameter using incomplete output measurements with slight noise contamination.

3.3
Example 2: 15-storey high rise building

To show the feasibility of the proposed method for complex structure, it is applied to an existing 15-storey high rise building, which is idealized as 15-DOF shear building model consisting of lumped mass and massless springs. The concentrated mass of each story is calculated as $m_1 = 30 \times 10^3$ kg, m_2–$m_{14} = 28.896 \times 10^3$ kg, $m_{15} = 27.741 \times 10^3$ kg. The elastic shear stiffness of each story is computed as $k_1 = 43,051$, $k_2 = 42,776$, $k_3 = 42,761$, $k_4 = 42,536$, $k_5 = 42,496$, $k_6 = 42,422$, $k_7 = 42,398$, $k_8 = 42,372$, $k_9 = 42,291$, $k_{10} = 42,172$, $k_{11} = 42,114$, $k_{12} = 42,093$, $k_{13} = 41,898$, $k_{14} = 41,649$ and $k_{15} = 41,464$ kN/m, respectively. The two damping coefficients of the Rayleigh damping model are chosen as $a = 0.2936$ and $b = 6.406 \times 10^{-3}$ providing approximately 5% damping ratio for the first two modes of vibration. The structure is assumed to be excited at the top floor by a sinusoidal force $p = 1000 * \sin(5 * t)$ kN.

The measured dynamic responses without noise contamination are first used to identify the unknown structural stiffness and damping parameters and to estimate the time history of the input excitation. The identification results are summarized in Table 3, in which the conditions in Cases 1, 3 and 4 are all the same except for the initial values for the structural parameters selected, whereas Cases 1 and 2 are the cases using the different number of sampling points L but with the same initial value. For all the cases, the convergence indices ε are set as 10^{-5}. It is seen from Table 3 that the structural parameters identified by the proposed method are almost the same as the true values for all the cases including Case 2 where the sample points of only 200 are used. It can also been seen that even for very poor initial values selected as in Case 4, the structural parameters can be still identified. Thus, one may conclude that the accuracy of the identification method is not sensitive to the initial values of the structural parameters. For all the cases, the identified time history of the input excitation is found the same as the actual one.

To assess the capacity of the proposed method against the measurement noise in the response, the proposed method is then applied to incomplete measurements contaminated by noise. Three typical noise levels of 1%, 3% and 5% are considered. The identification results are

Table 2. Identification results with incomplete noise-polluted measurements (Example 1)

Parameter and its real values		Case 1 (Noise = 1%)		Case 2 (Noise = 1%)		Case 3 (Noise = 3%)		Case 4 (Noise = 5%)	
		$\hat{\theta}_k$	Error	$\hat{\theta}_k$	Error	$\hat{\theta}_k$	Error	$\hat{\theta}_k$	Error
k_1	1	1.118	11.8%	0.996	0.38%	0.985	1.53%	0.975	2.53%
k_2	1	1.023	2.31%	1.000	0.01%	1.003	0.25%	1.004	0.42%
k_3	1	0.972	2.80%	0.994	0.63%	0.993	0.75%	0.989	1.03%
k_4	1	1.122	12.2%	1.012	1.21%	1.013	1.25%	1.016	1.55%
CPU time		0.52 s		2.31 s		2.26 s		2.15 s	
L		200		900		900		900	

$\hat{\theta}_k$: Identified values, $\theta_K^0 = [100, 100, 100, 100]^{\mathrm{T}}$ and $\varepsilon = 10^{-5}$ for all cases

Table 3. Identification results with complete and noise-free measurements (Example 2)

Real value of unknown parameter ($k \times 10^4$ kN/m)		Case 1		Case 2		Case 3		Case 4	
		θ^0	$\hat{\theta}$	θ^0	$\hat{\theta}$	θ^0	$\hat{\theta}$	θ^0	$\hat{\theta}$
k_1	4.3051	1E5	4.3051	1E4	4.3056	1E4	4.3051	10	4.3051
k_2	4.2766	1E5	4.2776	1E4	4.2780	1E4	4.2776	10	4.2776
k_3	4.2761	1E5	4.2761	1E4	4.2763	1E4	4.2761	10	4.2761
k_4	4.2536	1E5	4.2536	1E4	4.2537	1E4	4.2536	10	4.2536
k_5	4.2496	1E5	4.2496	1E4	4.2496	1E4	4.2496	10	4.2496
k_6	4.2422	1E5	4.2422	1E4	4.2422	1E4	4.2422	10	4.2422
k_7	4.2398	1E5	4.2398	1E4	4.2398	1E4	4.2398	10	4.2398
k_8	4.2372	1E5	4.2372	1E4	4.2371	1E4	4.2372	10	4.2372
k_9	4.2291	1E5	4.2291	1E4	4.2290	1E4	4.2291	10	4.2291
k_{10}	4.2172	1E5	4.2127	1E4	4.2126	1E4	4.2127	10	4.2127
k_{11}	4.2114	1E5	4.2114	1E4	4.2113	1E4	4.2114	10	4.2114
k_{12}	4.2093	1E5	4.2093	1E4	4.2092	1E4	4.2093	10	4.2093
k_{13}	4.1898	1E5	4.1897	1E4	4.1896	1E4	4.1898	10	4.1898
k_{14}	4.1649	1E5	4.1648	1E4	4.1647	1E4	4.1649	10	4.1649
k_{15}	4.1464	1E5	4.1463	1E4	4.1462	1E4	4.1464	10	4.1464
a	0.2936	1	0.2935	100	0.2934	100	0.2936	1	0.2936
b	0.0064	0.1	0.0064	100	0.0064	100	0.0064	0.1	0.0064
CPU time		725 s		3710 s		2937 s		1440 s	
L		900		200		900		900	

θ^0: Initial values of the parameters, $\hat{\theta}$: identified values, L: number of sampling points used, $\varepsilon = 10^{-5}$ for all cases

Table 4. Identification results with incomplete and noise-polluted measurements (Example 2)

Real value of unknown parameter ($k \times 10^4$ kN/m)		Case 1 (Noise = 1%)		Case 2 (Noise = 3%)		Case 3 (Noise = 5%)	
		Identified	Error (%)	Identified	Error (%)	Identified	Error (%)
k_1	4.3051	4.2920	0.30	4.2434	1.43	4.1560	3.46
k_2	4.2766	4.2647	0.30	4.2166	1.43	4.1301	3.45
k_3	4.2761	4.2632	0.30	4.2155	1.42	4.1295	3.43
k_4	4.2536	4.2409	0.29	4.1938	1.41	4.1088	3.40
k_5	4.2496	4.2371	0.29	4.1904	1.39	4.1061	3.38
k_6	4.2422	4.2299	0.29	4.1838	1.38	4.1003	3.34
k_7	4.2398	4.2279	0.28	4.1823	1.36	4.0996	3.31
k_8	4.2372	4.2257	0.27	4.1810	1.33	4.0992	3.26
k_9	4.2291	4.2184	0.25	4.1748	1.28	4.0943	3.19
k_{10}	4.2172	4.2031	0.23	4.1613	1.22	4.0826	3.09
k_{11}	4.2114	4.2038	0.18	4.1646	1.11	4.0881	2.93
k_{12}	4.2093	4.2055	0.09	4.1708	0.91	4.0981	2.64
k_{13}	4.1898	4.1935	0.09	4.1674	0.54	4.1016	2.11
k_{14}	4.1649	4.1786	0.33	4.1641	0.02	4.1074	1.38
k_{15}	4.1464	4.1638	0.42	4.1567	0.25	4.1055	0.99
a	0.2936	0.2829	3.63	0.2314	18	0.1536	–
b	0.0064	0.0070	9.37	0.0089	39	0.0114	–
CPU time		2295 s		2178 s		1766 s	
L		2500		2500		2500	

For all cases, identical initial value of 1E5 is used for all stiffness parameter, $\theta_c^0 = [0.1, 0.01]^{\mathrm{T}}$ and $\varepsilon = 10^{-5}$

summarized in Table 4. From Table 4, one may see that the proposed method can accurately identify the stiffness parameters even though the noise contamination in the responses reaches a 5% level. The maximum identification error of the stiffness parameter is lower than 1%, 4% and 9% for noise level of 1%, 3% and 5%. For the damping coefficients, the capacity of the method against noise contamination seems to be relative weak.

It is interesting to compare the inverse input time history of the top floor with that of the first floor on which no external force was applied. The identified input forces of these two floors for 5% noise level are shown in Fig. 5, where the solid line is the actual force curve, the dash line is the inversed curve of the first floor and the dotted line is the identified input force time history of the top floor. It can be seen that the inverse curve of the top floor matches well with the actual force. Amplitude of the inverse curve of the first floor, on the other hand, is approximately zero compared with that of the top floor.

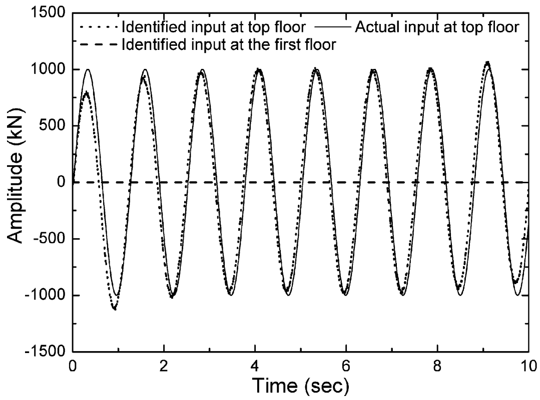


Fig. 5. Inversed time history at noise level of 5%

3.4
Example 3: truss bridge

In this example, a bridge truss supported at two ends as shown in Fig. 6 is considered. The mass and stiffness matrix of this structure can be computed by

$$\mathbf{K} = \sum_{e=1}^{n} \mathbf{K}_e = \sum_{e=1}^{n} a_e \mathbf{L}_e \mathbf{k}_e \mathbf{L}_e^{\mathrm{T}}$$

$$\mathbf{M} = \sum_{e=1}^{n} \mathbf{M}_e = \sum_{e=1}^{n} \rho_e \mathbf{L}_e \mathbf{m}_e \mathbf{L}_e^{\mathrm{T}}$$

where, n is the number of element ($n = 11$ for this example), ρ_e = mass density of truss element, $\mathbf{a}_e = \mathbf{E}_e\mathbf{A}_e$ axial stiffness, $\mathbf{L}_e$ = locating vector of element. $\mathbf{k}_e$ and $\mathbf{m}_e$ are respectively

$$\mathbf{k}_e = \frac{1}{l}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

$$\mathbf{m}_e = \frac{1}{6}\begin{bmatrix} 2C^2 & 2CS & C^2 & CS \\ 2CS & 2S^2 & CS & S^2 \\ C^2 & CS & 2C^2 & 2CS \\ CS & S^2 & 2CS & 2S^2 \end{bmatrix}$$

where $C \equiv \cos(\phi)$ and $S \equiv \sin(\phi)$, l = the length of the element. The used parameters in calculating the dynamic characteristics of this structure are listed in Table 5.

The structure is excited by $f_1(t)$ and $f_2(t)$ acting on node 4 and 5 in the y-direction. A proportional damping assumption is used here again. The two damping coefficients a and b are chosen as 0.9619 and 0.0024 respectively resulting in 5% damping ration for the first two mode of vibration. The objective is to determine the axial stiffness (i.e. E_iA_i/l_i, see Table 5) of all members, two damping coefficents and the time histories of $f_1(t)$ and $f_2(t)$ from the response measurements.

Table 6 shows the identification results for Cases 1 to 3 with complete and noise-free output measurements, and Case 4 with incomplete and noise-polluted output measurements. It is seen from Table 6 that for situation of noise-free and complete measurements the proposed

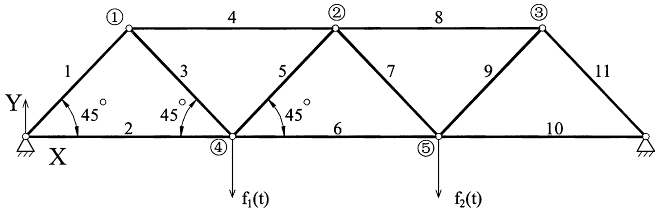


Fig. 6. Truss bridge model

Table 5. Structural properties of Example 3

No. of Element	E_iA_i/l_i	$\rho_i l_i/6$	ϕ
1	14,142	540	$\frac{\pi}{4}$
2	11,000	520	0
3	16,617	530	$3\frac{\pi}{4}$
4	11,500	510	0
5	15,627	550	$\frac{\pi}{4}$
6	10,000	560	0
7	15,627	550	$3\frac{\pi}{4}$
8	11,500	510	0
9	16,617	530	$\frac{\pi}{4}$
10	11,000	520	0
11	14,142	540	$3\frac{\pi}{4}$

method can identify accurately the unknown structural parameters with very limited number of sampling points no matter what initial values are selected. For Case 1, the convergence curve, which is defined as the variation of the identified value normalized by the actual value against the iterative number, is depicted in Figs. 7 and 8 for the stiffness and the damping parameters respectively. It is clearly demonstrated that with the increase of iteration times the estimated parameters converge rapidly to the true values. Figure 9a–c further show the identified time histories of the external excitations applied on Nodes 4, 5 and 1 in the y-direction. The external forces identified at Nodes 4 and 5 are found to be the same as the real input forces, whereas the amplitude of the identified force at Node 1, as well as that at Nodes 2 and 3, is nearly zero compared with that of Nodes 4 and 5. Thus, the external force time history can be accurately identified. The results of Cases 2 and 3 support this observation. For situation of incomplete and slight noise contamination output measurements, as that of Case 4, the parameter identification results are broadly acceptable.

4
Concluding remarks

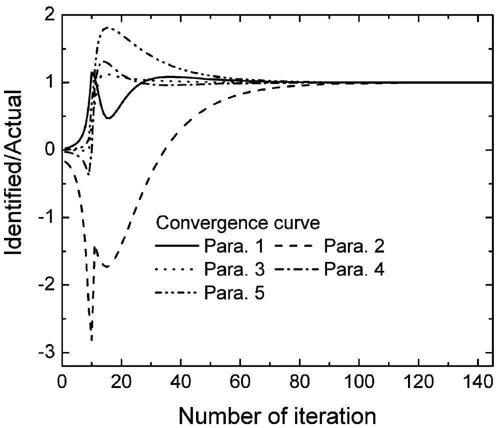
This paper describes a time domain identification method for simultaneously identification of structural parameters and the time-history of the external excitation. The proposed method is an iterative identification procedure consisting of the least-squares technique and a modification process between each recursive step. Numerical simulations are carried out to evaluate the feasibility of the proposed method. Main observations from this study are summarized as follows:

(1) The core idea of this method is the introduction of a modification procedure that converts the spatial

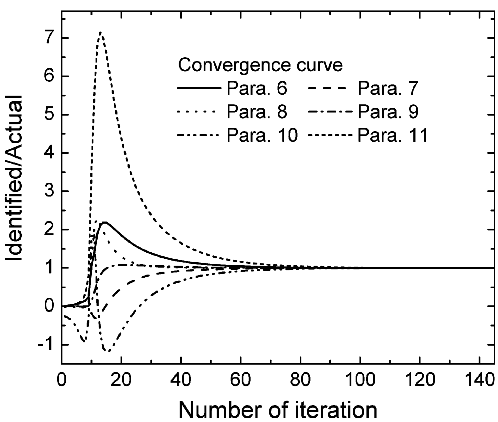
Table 6. Identification results for Example 3

Real value of unknown parameter ($k \times 10^4$ kN/m)		Case 1 (Noise-free)		Case 2 (Noise-free)		Case 3 (Noise-free)		Case 4 (Noise = 1%)	
		θ^0	$\hat{\theta}$	θ^0	$\hat{\theta}$	θ^0	$\hat{\theta}$	$\hat{\theta}$	Error (%)
k_2	4.3051	1	1.4142	1	1.4142	−100	1.4142	1.3021	7.93
k_2	4.2766	1	1.1000	1	1.1000	−100	1.1000	0.9980	9.27
k_3	4.2761	1	1.6616	1	1.6616	−100	1.6616	1.5656	5.78
k_4	4.2536	1	1.1500	1	1.1500	−100	1.1500	1.0752	6.50
k_5	4.2496	1	1.5626	1	1.5626	−100	1.5626	1.4884	4.75
k_6	4.2422	1	1.0000	1	1.0000	−100	1.0000	0.9282	7.18
k_7	4.2398	1	1.5626	1	1.5626	−100	1.5626	1.5522	0.67
k_8	4.2372	1	1.1500	1	1.1500	−100	1.1500	1.0751	6.51
k_9	4.2291	1	1.6615	1	1.6615	−100	1.6615	1.7695	6.50
k_{10}	4.2172	1	1.1000	1	1.1000	−100	1.1000	1.0301	6.35
k_{11}	4.2114	1	1.4142	1	1.4142	−100	1.4142	1.3131	7.14
a	0.9619	10	0.9619	100	0.9618	−100	0.9618	0.9181	4.55
b	0.0024	0.1	0.0024	100	0.0024	−100	0.0024	0.0027	12.5
CPU time		81.1 s		121.5 s		107.8 s		698 s	
L		800		800		800		2000	

θ^0: Initial values of the parameters, $\hat{\theta}$: identified values, L: number of sampling points used, $\varepsilon = 10^{-5}$ for all cases



(a) Convergence curve of stiffness parameter 1 to 5



(b) Convergence curve of stiffness parameter 6 to 11

Fig. 7. Convergence curve for stiffness parameter. **a** Convergence curve of stiffness parameter 1 to 5. **b** Convergence curves of stiffness parameter 6 to 11

information of the external forces into a mathematical condition to grantee the convergence of the method. This modification procedure has no limitation on the type of the excitation.

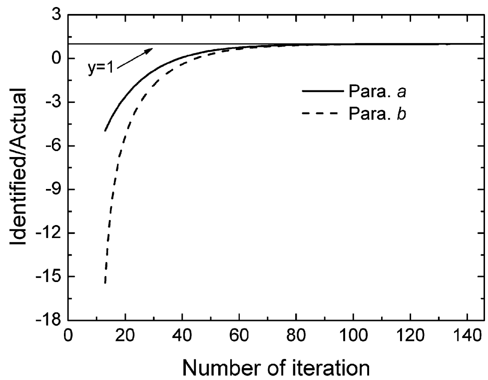
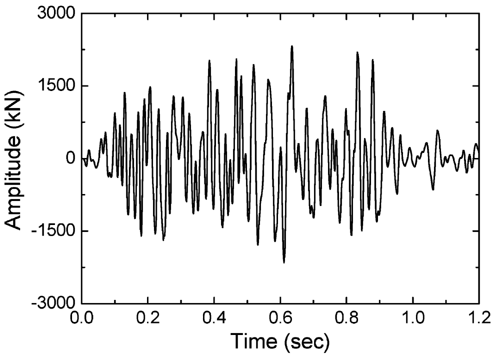
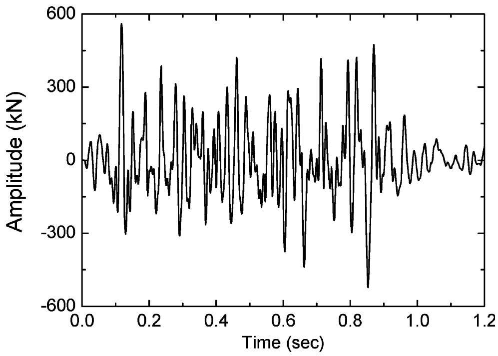


Fig. 8. Convergence curve of damping coefficients

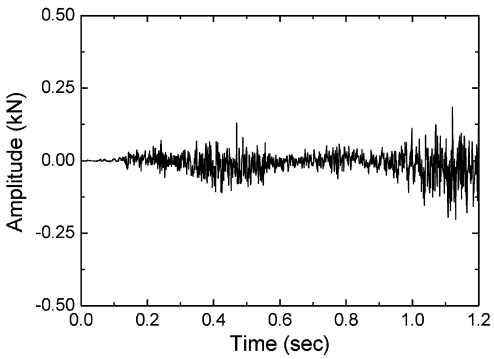
(2) The modification procedure is independent from the least-square identification technique involved in the method, which implies that other advanced identification techniques can easily be adopted to deal with problems such as ill-conditioning and measurement noise.
(3) For situation of complete and noise-free output measurements, numerical studies show that the proposed method can reliably and efficiently identify both the structural parameters and the input time history using a short duration of measurements. Moreover, the accuracy and convergence of the method is robust to the initial values selected for the unknown parameters.
(4) For situation of incomplete and noise-polluted measurements, the stiffness parameters as well as the input excitation can be identified satisfactorily even with high noise level. The identification accuracy of the damping coefficients is broadly acceptable at low noise level, and become relative poor at high noise level, which needs further improvements.
(5) The direct identification of the parameters and the input excitation using output-only measurements is quite challenging for real structures. The investigation



(a) Inversed input force at Node4 in y-direction



(b) Inversed input force at Node5 in y-direction



(c) Inversed input forces at Node1 in y-direction

Fig. 9. Time–history of the inversed force at different nodes. **a** Inversed input force at Node 4 in *y*-direction. **b** Inversed input force at Node 5 in *y*-direction. **c** Inversed input forces at Node 1 in *y*-direction

in this study brings up one possible solution of using the spatial information of the external force as additional optimal criterion for parameter identification. The identification results based on the proposed approach for numerical examples are quite accurate or reasonable. Nevertheless, experimental studies are needed and being made to thoroughly evaluate its potential for practical application on real structure.

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