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Modeling of activated sludge wastewater treatment processes

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Abstract The performance of an activated sludge wastewater treatment process consisting of an aeration tank and a secondary settler has been studied. A tanks-in-series model with backflow was used for mathematical modeling of the activated sludge wastewater treatment process. Nonlinear algebraic equations obtained from the material balances of MLSS (mixed liquor suspended solids or activated sludge), BOD (biological oxygen demand) and DO (dissolved oxygen) for the aeration tank and the settler and from the behavior of the settler were solved simultaneously using the modified Newton-Raphson technique. The concentration profiles of MLSS, BOD and DO in the aeration tank were obtained. The simulation results were examined from the viewpoints of mixing in the aeration tank and flow in the secondary settling tank. The relationships between the overall performance of the activated sludge process and the operating and design parameters such as hydraulic residence time, influent BOD, recycle ratio and waste sludge ratio were obtained.

List of symbols

A	m^2	cross-sectional area of the settler
b	l/h	microorganism decay coefficient
bf		backflow parameter
C^*	mg/l	saturated oxygen concentration
C	mg/l	dissolved oxygen concentration
G_L	g/m^2h	limiting solids flux
G_S	g/m^2h	solids settling flux due to gravity
K_La	l/h	overall volumetric mass transfer coefficient
K_o	mg/l	saturation constant for oxygen or DO
K_s	mg/l	saturation constant for activated sludge or MLSS
k	m^3/g	constant of settling characteristics
m		number of hypothetical stages in tanks-in-series model
Q	m^3/h	flow rate
r_1		mass of DO used/activated sludge produced
r_2		mass of DO used/activated sludge decay
S	mg/l	BOD
X	mg/l	MLSS (activated sludge concentration)
X_L	mg/l	MLSS corresponding to the limiting solids flux
$Y_{x/s}$	mg/mg	yield of growth rate
V	m^3	volume of aeration tank
V_0	m/h	constant of settling characteristics

Greeks symbols

α		sludge recycle ratio
τ	h	hydraulic residence time
μ_{max}	l/h	maximum specific growth rate of activate sludge
ω		sludge wastage ratio

Subscripts

out	effluent
in	influent
r	recycle

1 Introduction

In order to treat the domestic and industrial wastewater, the activated sludge process has been the most commonly used [1, 2]. It is considered to be the most cost-effective way to remove the organic materials from wastewater. Besides that, it is very flexible and can be adapted to almost any type of biological wastewater treatment problem. The design and operation of the treatment processes, however, have not been elucidated. They are highly empirical and accurate description of the performance of activated sludge wastewater treatment processes is still difficult. In most previous studies, an ideal mixing approximation, i.e. the perfect mixing model [3, 4] or the plug flow model [4, 5] has been used to model mixing in aeration tanks. Little work deals with imperfect or actual mixing in aeration tanks [e.g. 6–8]. In most of them, the mixing model used to represent imperfect and actual mixing is an axial dispersion model which contains one parameter, the axial dispersion coefficient, characterizing the deviations from ideal mixing. It should be noted that the axial dispersion model is a kind of modification of the plug flow model and therefore can represent satisfactorily only mixing which deviates not too largely from the plug flow mixing. Furthermore, a set of differential equations and boundary conditions obtained for the axial dispersion model has to be solved by rather complicated numerical techniques. The extension of the axial dispersion model to more complicated mixing is very difficult. On the other hand, a tanks-in-series model used in this work is appli-

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cable to the whole mixing extents including perfect mixing and plug flow mixing. Moreover, the tanks-in-series model provides a set of non-linear algebraic equations, which can be solved using rather simple numerical techniques. In the tanks-in-series model, a modification for the micro-mixing or back-mixing into the model can be accomplished simply by introducing backflow which causes no difficulty in solving the equations. Therefore, the tanks-in-series model is more rational and usable as compared with the axial dispersion model.

In order to design and operate an activated sludge wastewater treatment system efficiently, it is necessary to understand the role of the microorganisms to decompose the organic waste and to form a satisfactory floc, which is a prerequisite for the effective separation of the biological solids in the settler. Even though excellent floc formation is obtained, the effluent from the system could still be high in biological solids as a result of poor design of the secondary settler and poor operation of the aeration tank. The performance of the secondary settler is sometimes crucial for achieving the effluent quality required. When, therefore, we discuss the overall performance of a wastewater treatment process, not only an aeration tank but also a settler must be examined.

In this paper, the overall performance of the activated sludge wastewater treatment process consisting of an aeration tank and a secondary settling tank has been discussed from the viewpoints of the mixing in the aeration tank and the behavior of the secondary settler. A tanks-in-series model has been used to consider incomplete or actual mixing in the aeration tank besides the ideal mixing conditions, i.e. a complete mixing and a plug flow. The multi-nonlinear algebraic equations for the tanks-in-series model resulting from material balances on activated sludge or MLSS (mixed liquor suspended solids), BOD (biochemical oxygen demand) and DO (dissolved oxygen) and from the behavior of the secondary settler under the steady-state condition were solved using the modified Newton-Raphson technique.

2
Mathematical modeling

Figure 1 shows a schematic diagram of the tanks-in-series model for an activated sludge wastewater treatment process consisting of an aeration tank and a secondary settler. This is a typical configuration of the activated sludge wastewater treatment process. The tanks-in-series model for the aeration tank is composed of m hypothetical well-mixed stages connected in series with backflow.

The steady-state material balance of MLSS, BOD and DO for the aeration tank may be written as follows:

MLSS or activated sludge:

$$f_{x1} = QX_{\rm in} + \alpha QX_{\rm r} - (1+bf)(Q+\alpha Q)X_1 + bf(Q+\alpha Q)X_2 + \frac{V}{m} \cdot \left(\mu_{\max}\frac{S_1}{K_{\rm s}+S_1}\frac{C_1}{K_{\rm o}+C_1} - b\right)X_1 = 0 \ , \qquad (1)$$

$$f_{xi} = (Q+\alpha Q)(X_{i-1}-X_i)(1+bf) + bf(Q+\alpha Q)X_{i+1} - bf(Q+\alpha Q)X_i + \frac{V}{m}\cdot\left(\mu_{\max}\frac{S_i}{K_{\rm s}+S_i}\frac{C_i}{K_{\rm o}+C_i} - b\right)X_i = 0 \ , \qquad (2)$$

$$f_{xm} = (Q+\alpha Q)(X_{m-1}-X_m) - bf(Q+\alpha Q)X_m + \frac{V}{m}\cdot\left(\mu_{\max}\frac{S_m}{K_{\rm s}+S_m}\frac{C_m}{K_{\rm o}+C_m} - b\right)X_m = 0 \ , \qquad (3)$$

BOD:

$$f_{s1} = QS_{\rm in} + \alpha QS_{\rm r} - (Q+\alpha Q)(1+bf)S_1 + bf(Q+\alpha Q)S_2 - \frac{V}{m}\cdot\frac{\mu_{\max}}{Y_{x/s}}\frac{S_1}{K_{\rm s}+S_1}\frac{C_1}{K_{\rm o}+C_1}X_1 = 0 \ , \qquad (4)$$

$$f_{si} = (Q+\alpha Q)(S_{i-1}-S_i)(1+bf) + bf(Q+\alpha Q)S_{i+1} - bf(Q+\alpha Q)S_i - \frac{V}{m}\cdot\frac{\mu_{\max}}{Y_{x/s}}\frac{S_i}{K_{\rm S}+S_i}\frac{C_i}{K_{\rm o}+C_i}X_i = 0 \ , \qquad (5)$$

$$f_{sm} = (Q+\alpha Q)(S_{m-1}-S_m)(l+bf) - bf(Q+\alpha Q)S_m - \frac{V}{m}\frac{\mu_{\max}}{Y_{x/s}}\frac{S_m}{K_{\rm s}+S_m}\frac{C_m}{K_{\rm o}+C_m}X_m = 0 \ , \qquad (6)$$

DO:

$$f_{c1} = QC_{\rm in} + \alpha QC_{\rm r} - (Q+\alpha Q)(1+bf)C_1 + bf(Q+\alpha Q)C_2 + \frac{V}{m}\left[K_{\rm L}a(C^* - C_1) - \left(r_1\cdot\mu_{\max}\frac{S_1}{K_{\rm s}+S_1}\cdot\frac{C_1}{K_{\rm o}+C_1} + r_2\cdot b\right)X_1\right] = 0 \ , \qquad (7)$$

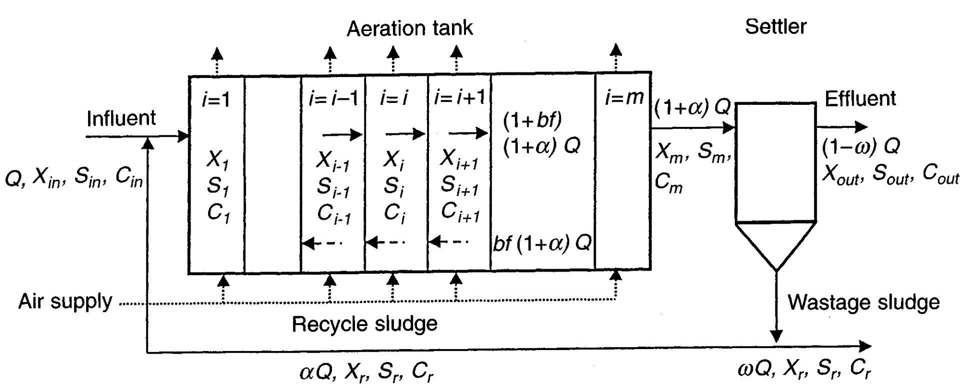

Fig. 1. Schematic diagram of the tanks-in-series model for an activated sludge wastewater treatment process

$$f_{ci} = (Q+\alpha Q)(C_{i-1}-C_i)(1+bf) + bf(Q-\alpha Q)C_{i+1} - bf(Q+\alpha Q)C_i + \frac{V}{m}\cdot\left[K_L a(C^* - C_i) - \left(r_1\cdot\mu_{\max}\frac{S_i}{K_s+S_i}\cdot\frac{C_i}{K_o+C_i} + r_2\cdot b\right)X_i\right] = 0 \ , \quad (8)$$

$$f_{cm} = (Q+\alpha Q)(C_{m-1}-C_m)(1+bf) - bf(Q+\alpha Q)C_m + \frac{V}{m}\cdot\left[K_L a(C^* - C_m) - \left(r_1\cdot\mu_{\max}\frac{S_m}{k_s+S_m}\cdot\frac{C_m}{K_o+C_m} + r_2\cdot b\right)X_m\right] = 0 \quad (9)$$

As a kinetic model, the Monod type equation including the effects of oxygen limitation [9] and the death of microorganisms [4, 5, 10] is used. It should be emphasized that, even if more complicated kinetic models are used, the simulation algorithm described below does not require any change and still is very useful. The material balances for DO, Eqs. (7)–(9), include the oxygen uptake by microorganisms [9].

The steady-state material balances for the settler may be written as follows:

MLSS:

$$f_{m+1} = (Q+\alpha Q)X_m - (\alpha Q+\omega Q)X_r - (Q+\omega Q)X_{out} = 0 \ ; \quad (10)$$

BOD:

$$f_{m+2} = S_m - S_r = 0 \ ; \quad (11)$$

DO:

$$f_{m+3} = C_m - C_r = 0 \ . \quad (12)$$

Eq. (11) for material balance of BOD indicates that no biological reaction occurs in the secondary settler. This relationship also indicates that BOD in the effluent S_{out} is also equal to BOD in the aeration tank outlet and recycle flow S_m and S_r. The DO balance for the settler, Eq. (12), implies no oxygen consumption in the settler. Therefore, the DO in the effluent C_{out} is equal to C_m and C_r.

The main functions of settler are clarification and thickening of sludge. They play an important role in the performance of the activated sludge wastewater treatment process. The settler has to be operated in either a critical loading or under-critical loading for steady state. The over-critical loading results in operation failure, since under this condition a portion of the solids flux beyond the limiting flux is lost in the effluent.

According to the limiting flux theory [11], the gravity settling flux of MLSS or activated sludge in the secondary settler is given as [12–14]:

$$G_S = X_L V_0 \exp(-kX_L) \ . \quad (13)$$

Here X_L denotes the suspended solids concentration corresponding to the limiting solids flux. Under the critical loading condition, G_S is written as:

$$G_S = G_L\left(1-\frac{X_L}{X_r}\right) \ , \quad (14)$$

where G_L is the limiting solids flux in the settler. This is equivalent to the tangent of Eq. (13) from $X = X_L$. From these two equations, we have:

$$G_L\left(1-\frac{X_L}{X_r}\right) = X_L V_0 \exp(-kX_L) \ . \quad (15)$$

Equating the derivatives of Eqs. (13) and (14) gives:

$$-\frac{G_L}{X_r} = V_0(1-kX_L)\exp(-kX_L) \ . \quad (16)$$

Using Eqs. (15) and (16), we obtain the expression for X_L:

$$X_L = \left(\frac{X_r}{2}\right)\left(1+\sqrt{1-\frac{4}{kX_r}}\right) \ . \quad (17)$$

This equation provides the following residual function:

$$f_{m+4} = X_L - \left(\frac{X_r}{2}\right)\left(1+\sqrt{1-\frac{4}{kX_r}}\right) = 0 \ . \quad (18)$$

From the material balance around the secondary settler, we have:

$$G_L = \frac{\alpha Q X_r + \omega Q X_r}{A} \ . \quad (19)$$

Arranging Eqs. (15) and (16) and eliminating X_r gives:

$$G_L = kV_0X_L^2\exp(-kX_L) \ . \quad (20)$$

Using the above two equations, we obtain the following residual function:

$$f_{m+5} = X_r\frac{(\alpha+\omega)Q}{A} - V_0X_L^2k\exp(-kX_L) = 0 \ . \quad (21)$$

The residual functions obtained for the aeration tank and the secondary settler were solved simultaneously using the modified Newton-Raphson technique [15]. It was modified to avoid the divergence of numerical calculations. Iterations required achieving the final solutions in numerical examples described below were less than 10.

Equations (4)–(6), (10)–(12) and (16)–(18) form a set of $(3m + 5)$ non-linear algebraic equations and there are $(3m + 5)$ dependent variables in the model. In order to solve the set of algebraic equations, all other parameters included in the model must be specified. The dependent variables, which are calculated by solving the residual functions, must equal in number to the residual functions.

The residual functions and dependent variables for the operating analysis are grouped as:

$$\{f_{xi}, f_{si}, f_{ci}, f_{m+1}, f_{m+2}, f_{m+3}, f_{m+4}, f_{m+5}\}(i=1,\ldots,m) \ , \quad (22)$$

and:

$$\{X_i, S_i, C_i, X_r, S_r, C_r, X_L, \omega\}(i=1,\ldots,m) \ , \quad (23)$$

respectively.

This calculation scheme is applicable to various types of wastewater treatment process calculations without any significant change in the computer program. Even if one or more variables from the unspecified variable list is exchanged with the same number from the list of specified variables, the procedure required is only a replacement of those variables. For example, in the case of the operating analysis in which the concentration profiles of MLSS, BOD and DO are calculated. When the outlet concentration of BOD, S_{out} (=S_r), is specified and the corresponding recycle ration, α, is estimated, the residual functions can be considered to contain ($3m + 5$) dependent variables $\{X_i(i = 1, \ldots, m), S_i(i = 1, \ldots, m - 1), \alpha, C_i\ (i = 1, \ldots, m), X_r, S_r, C_r, X_L, \omega\}$.

3 Results and discussion

Values of parameters specified in most of the numerical examples are summarized in Table 1.

3.1 Backmixing or backflow

Typical concentration profiles of MLSS (X_i) and BOD (S_i) in the aeration tank are illustrated in Fig. 2. BOD decreases monotonously from the inlet to the outlet. On the other hand, the change in MLSS is relatively small and near the center of the aeration tank the maximum MLSS can be found. It should be noted that the extent of mixing represented by 20 hypothetical well-mixed stages is almost equivalent to a plug flow mixing. The effect of backmixing described by backflow on MLSS and BOD profiles is also depicted in Fig. 2. Values of the effluent BOD with and without backflow are 0.036 and 0.030 [mg/l], respectively. It is seen that the backmixing causing a decrease in the concentration gradients results in higher BOD in the effluent or lower effluent quality.

3.2 Hydraulic residence time or aeration tank volume

As shown in Fig. 3, the relationship between hydraulic residence time, $\tau\ (= V/Q)$, which is equivalent to the aeration tank volume, V, at constant Q, and the effluent BOD, S_{out}. With decreasing effluent BOD or increasing effluent quality, τ or V increases. An increase in mixing extent or decrease in m causes longer residence time or larger aeration tank volume. Effect of effluent quality on residence time or aeration tank volume becomes more significant at more intensive mixing. In other words, the aeration tank volume required significantly increases with an increase in effluent quality when the mixing in the aeration tank is complete ($m = 1$). For the aeration tank the mixing of which is equivalent to $m > 5$, on the other hand, the residence time is insignificantly affected by the effluent BOD.

3.3 Recycle ratio

As shown in Fig. 4, BOD in the effluent, S_{out}, considerably decreases with an increase in recycle ration for $\alpha < 0.3$, whereas MLSS at the aeration tank outlet or settler loading, X_m, increases with recycle ration. This result indicates that the operation for $\alpha > 0.3$ is not efficient. It is also seen from Fig. 4 that MLSS is not affected by the mixing extent in the aeration tank or the change in m.

3.4 Residence time

Figure 5 depicts the effects of τ in the aeration tank on BOD in the effluent, S_{out}, MLSS in the aeration tank outlet,

Table 1. Values of specified variables

$\mu_{max} = 0.35$ l/h	$\alpha = 0.18$
$K_s = 100$ mg/l	$Q/A = 1$ m/h
$K_o = 2$ mg/l	$S_{in} = 200$ mg/l
$b = 0.0067$ l/h	$\tau = 5$ h
$Y_{x/s} = 0.5$ mg/mg	$bf = 0$
$K_La = 5$ l/h	
$r_1 = 1$	
$r_2 = 0.75$	
$k = 0.0048$ m^3/g	
$V_0 = 5.68$ g/m^{2}h	
$C^* = 10$ mg/l	
$X_{out} = X_{in} = 0$ mg/l	

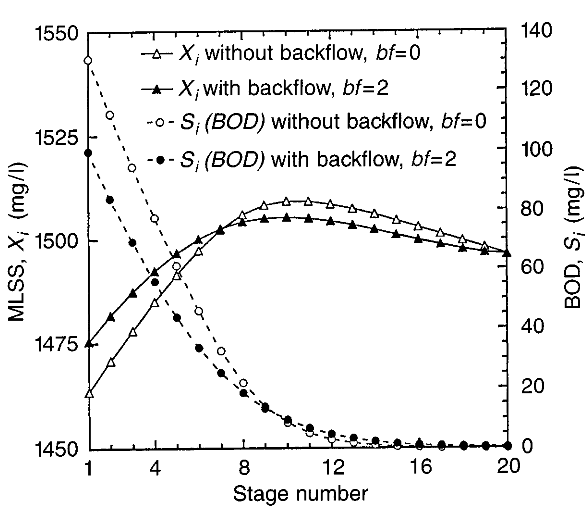

Fig. 2. Profiles of MLSS and BOD in the aeration tank

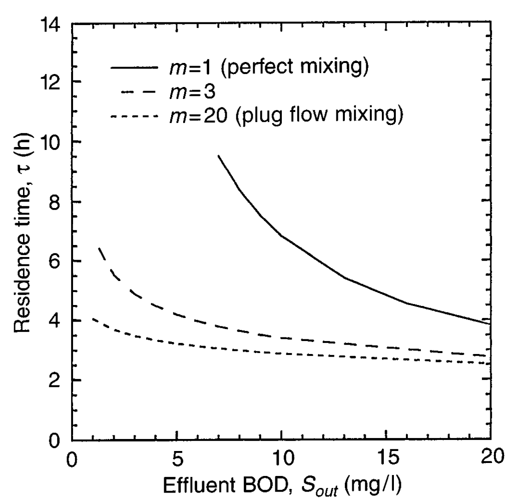

Fig. 3. Relationship between effluent quality, S_{out} and residence time, τ (or aeration tank volume, V)

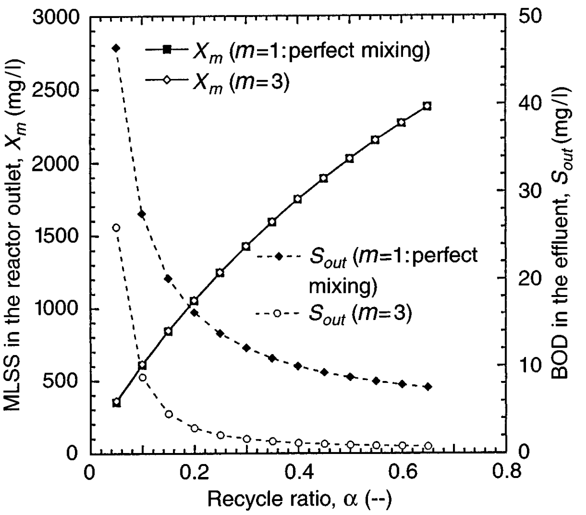

Fig. 4. Effects of recycle ratio, α, on MLSS at the aeration tank outlet, X_m, and effluent BOD, S_{out}

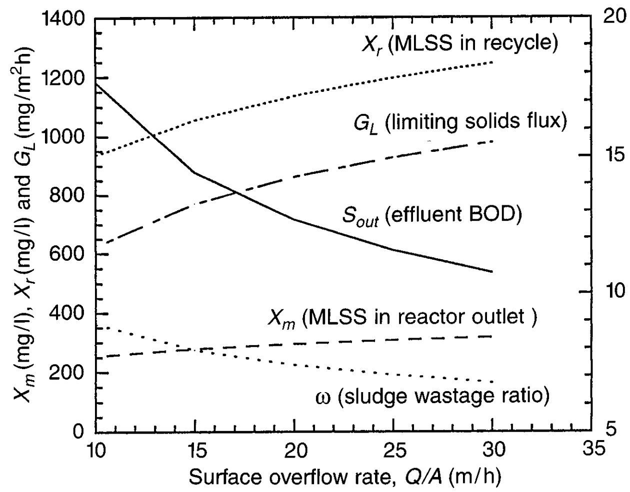

Fig. 6. Effects of surface overflow rate on effluent BOD, MLSS at the aeration tank outlet, MLSS in recycle flow and sludge wastage ratio

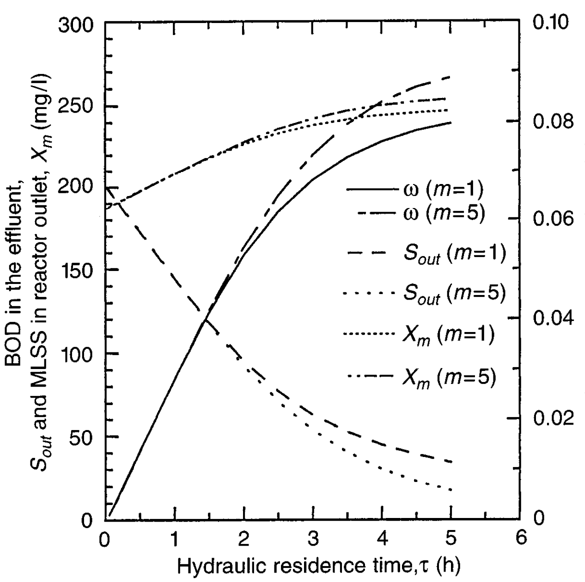

Fig. 5. Effects of residence time on S_{out}, X_m and ω

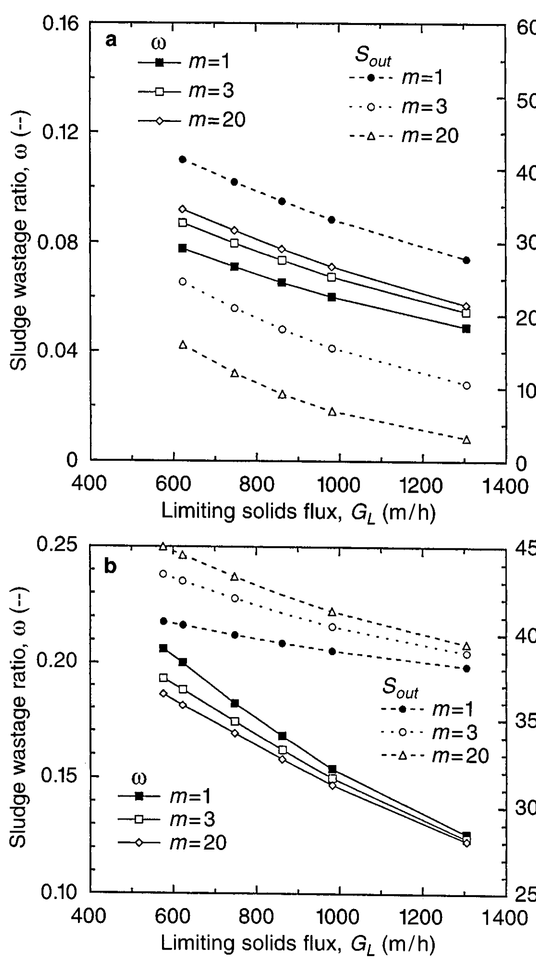

Fig. 7. Effects of solids flux on effluent BOD and sludge wastage ratio. a) $S_{in} = 200$ mg/l; b) $S_{in} = 800$ mg/l

X_m, and sludge wastage ratio, ω. The effluent BOD seems to decrease significantly as hydraulic residence time increases, while the MLSS and sludge wastage ratio increase. The increase in sludge wastage ratios is closely related to the increase in BOD concentration in the influent, will result in high excess sludge in the aerobic wastewater treatment process. At the hydraulic residence time over 4 hours, all parameters become almost constant. This means that the activity of the activated sludge to degrade the organic compounds contained in the wastewater is effective until the hydraulic residence time is less or equal to 4 hours. When the hydraulic residence time exceeds 3 hours, the influence of mixing extent in the aeration tank on S_{out}, X_m and ω is significant. The effluent BOD for incomplete mixing in the aeration tank ($m = 5$) is lower than that for complete mixing ($m = 1$). The sludge concentration or MLSS at the tank outlet for $m = 5$ is slightly higher than that for $m = 1$, the sludge wastage ratio for incomplete mixing is rather higher than that for complete mixing. The higher sludge wastage ratio implies that the excess sludge for incomplete mixing is higher than that for complete mixing. From the viewpoint of higher effluent quality, however, it is clear that the overall performance of an incomplete mixing aerator for activated sludge wastewater treatment process is better than that of a complete mixing aerator.

3.5
Performance of secondary settler
Figure 6 shows the influence of the surface overflow rate, Q/A, on effluent BOD, S_{out}, sludge concentration at the aeration tank outlet, X_m, and sludge recycle concentration, X_r. The design of sedimentation tanks is normally based on the surface overflow rate. The effluent BOD decreases with increasing surface overflow. Values of X_m are relatively constant whereas those of X_r increase.

In Fig. 7, the effects of limiting solids flux, G_L, on sludge wastage ratio, ω, and effluent BOD, S_{out} are shown . With increasing limiting solids flux or the performance of sedimentation, ω and S_{out} decrease. The higher performance of the secondary settler provides higher effluent quality and lower excess sludge. It is seen from Fig. 7a for S_{in} = 200 [mg/l] that the higher extent of mixing in the aceration tank or the smaller hypothetical stage number suppresses effluent quality and increases sludge wastage ratio. As shown in Fig. 7b for S_{in} = 800 [mg/l], on the contrary, the mixing in the aceration tank improves effluent quality and reduces excess sludge. The influence of mixing extent in the aeration tank is more significant at lower solids flux rates.

4
Conclusions
Using the computer simulation, the design and operation of the activated sludge wastewater treatment process have been discussed from the viewpoint of mixing in the aeration tank and performance of the secondary settler.

Performance of wastewater treatment processes consisting of an incomplete mixing reactor described by the tanks-in-series model is better than that of a completely mixed aeration tank. Mixing in wastewater treatment processes strongly and complicatedly influences BOD concentration, hydraulic residence time, sludge recycle ratio, sludge wastage ratio and settler performance. The model proposed in this work may be useful and usable for design, scale-ups and feasibility studies of the activated sludge wastewater treatment processes.

References

1. **Henze, M.; Harremoes, P.; Jansen, J.L.C.; Arvin, E.:** Wastewater Treatment. Springer, Berlin, 1995
2. Metcalf & Eddy, Inc.: Wastewater Engineering: Collection, Treatment, Disposal, McGrawHill. New York, 1972
3. **Chudoba, J.; Tucek, F.:** Production, degradation, and composition of activated sludge in aeration systems without primary sedimentation. J. Water Pollut. Control Fed. 57 (1985) 201–206
4. **Artan, N.; Orhon, D.:** The effect of reactor hydraulics on the performance of activated sludge system-II. The formation of microbial products. Water Res. 23 (1989) 1519–1525
5. **San, H.A.:** A kinetic model for ideal plug-flow reactors. Wat. Res. 23 (1989) 647–654
6. **Braha, A.; Hafner, F.:** Use of Monod kinetics on multi-stage bioreactors. Water Res. 19 (1985) 1217–1227
7. **Muslu, Y.:** Use of dimensionless parameters in design of cascade-connnected reactors. J. Chem. Tech. Biotechnol. 45 (1989) 123–134
8. **Orhon, D.; Soybay, S.; Tunay, O.; Artan, N.:** The effect of reactor hydraulics on the performance of activated sludge systems-I. The traditional modeling approach. Wat. Res. 23 (1989) 1511–1518
9. **Olsson, G.; Andrews, J.F.:** The dissolved oxygen profile-A valuable tool for control of the activated sludge process. Wat. Res. 12 (1978) 985–1004
10. **Nakhla, G.F.; Ahmed, A.M.; Farooq, S.:** Modeling of sequencing batch reactors treating inhibitory and noninhibitory wastewaters. Water Environment Reasearch 69 (1997) 6–13
11. **Dixon, D.C.:** Capacity and control of clarifiers and thickeners. J. Water Pollut. Control Fed. 57 (1985) 46–52
12. **Tyagi, R.D.; Du, Y.G.:** Modelling of waste treatment systems. in *Process Computations Biotechnology*, Ed. By T.K. Ghose, Tata McGraw-Hill Co., New Delhi 1995
13. **Roche, N.; Vaxelaire, J.; Prost, C.:** A simple empirical model for hindered settling in activated sludge clarifier. Water Environ. Res. 67 (1995) 775–780
14. **Krebs, P.:** Success and shortcomings of clarifier modeling. Wat. Sci. Tech. 31 (1995) 181–191
15. **Kanai, T.; Uzumaki, T.; Kawase, Y.:** Simulation of airlift bioreactors: Steady-state Performance of continuous culture processes. Computer Chem. Eng. 20 (1996) 1089–1099