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ORIGINAL ARTICLE

Dynamic mental number line in simple arithmetic

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Abstract Studies have found that spatial-numerical associations could extend to arithmetic. Addition leads to rightward shift in spatial attention while subtraction leads to leftward shift (e.g., Knops et al. 2009; McCrink et al. 2007; Pinhas & Fischer 2008), which is consistent with the hypothesis of static mental number line (MNL) for arithmetic. The current investigation tested the hypothesis of dynamic mental number line which was shaped by the relative magnitudes of two operands in simple arithmetic. Horizontal and vertical electrooculograms (HEOG and VEOG) during simple arithmetic were recorded. Results showed that the direction of eye movements was dependent on the relative magnitudes of two operands. Subtraction was associated with larger rightward eye movements than addition (Experiment 1), and smaller-operand-first addition (e.g., 2+9) was associated with larger rightward eye movement than larger-operand-first addition (e.g., 9+2) only when the difference of two operands was large (Experiment 2). The results suggest that the direction of the mental number line could be dynamic during simple arithmetic, and that the eyes move along the dynamic mental number line to search for solutions.

Introduction

Numerical and spatial representations have been shown to be closely linked since Galton (1880a, 1880b) first mentioned the relationship between number and space more than 130 years ago. He reported that some people had the power of mentally "seeing" numerals and manipulating them by mental imagery in the same form as doing arithmetic with pens and paper, which indicates that numbers may have spatial properties. Decades later, researchers found that when people compared the magnitude of a pair of two numbers, reaction time was longer for number pairs with smaller numerical differences than for those with larger numerical differences, which suggests that numbers may be sequentially encoded on a virtual line (i.e., the number line) (e.g., Moyer & Landauer, 1967; Restle, 1970). Recent studies further showed a left-to-right alignment of number representations in mind (Dehaene, Bossini, & Giraux, 1993; Fischer, Castel, Dodd, & Pratt, 2003; Hartmann, Mast, & Fischer, in press; Ranzini, Lisi, & Zorzi, in press; Schwarz & Keus, 2004).

The relation between number and spatial representations has been extended to arithmetic. Several studies suggest that an oriented mental number line is used autonomously in arithmetic calculations (e.g., Hartmann, Mast, & Fischer, 2015; Klein, Huber, Nuerk, & Moeller, 2014; Knops, Dehaene, Berteletti, & Zorzi, 2014; Knops et al., 2009; Knops, Zitzmann, & McCrink, 2013; Masson & Pesenti, 2014; McCrink et al., 2007; Pinhas & Fischer, 2008). One of the evidence for the involvement of mental number line in arithmetic is the "Operational Momentum" (OM) effect.

McCrink et al. (2007) first reported the OM effect in non-symbolic arithmetic. They asked participants to watch short videos which presented a set of additions and subtractions based on numerosity. Participants were required

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to answer whether the outcome sets were correct or incorrect. The OM effect was characterized by overestimation of addition and underestimation of subtraction outcomes: participants were more likely to choose larger answers in addition problem and smaller answers in subtraction problem compared with the correct solutions. This OM effect was also found in symbolic arithmetic in a number localization task (Pinhas & Fischer, 2008). In each trial of this study, an addition or subtraction problem and a horizontal line flanked by digits 0 and 10 were presented at the center of the screen. Participants were instructed to solve the addition and subtraction problems and to locate the correct answers on the horizontal line. The results replicated the OM effect that response to addition problems (e.g., 2+4) shifted rightward and response to subtraction problems (e.g., 8−2) shifted leftward, even if the correct solutions were identical for the addition and subtraction problems. This OM effect persisted even when the second operand was zero (e.g., 2+0, 2−0).

The OM effect could be linked to the eye movement during arithmetic (Hartmann, 2015; Myachykov, Ellis, Changelosi, & Fischer, in press). Knops et al. (2009) used fMRI and machine learning to investigate the association between arithmetic and eye movement. A linear classifier was trained to discriminate between leftward and rightward eye movements using BOLD (blood-oxygen-level dependent) signal in the posterior parietal cortex in an eye movement experiment. In another fMRI experiment, the participants were instructed to solve addition and subtraction problems with matched operands (e.g., 48+29, 48−29). The two operands of each problem were presented serially. Participants were asked to estimate the solution and to choose the closest number from seven candidate answers. The authors showed that the classifier from the eye movement experiment could successfully classify addition and subtraction problems, which suggested that participants could equate addition and subtraction with rightward and leftward eye movements, respectively. Thus, addition was associated with a rightward eye movement, whereas subtraction was associated with a leftward eye movement, presumably along the mental number line. Hartmann et al. (2015) studied spontaneous eye movements when participants were solving verbally presented arithmetic questions. Addition and subtraction did not show any difference in horizontal gaze position, whereas addition showed a more upward trend compared to subtraction in vertical gaze position.

Studies on the OM effect indicate that eye movements during arithmetic can be mediated by operation (addition vs. subtraction), which may be associated with the nature of operations and the involvement of mental number line during arithmetic (Knops et al., 2009; McCrink & Wynn, 2009; McCrink et al., 2007). That is, the OM effect is likely caused by over-shifting of spatial attention along the mental number line in arithmetic calculations: outcomes of addition problems are typically larger than the operands and thus shift spatial attention to larger numbers on the mental number line, whereas outcomes of subtractions are typically smaller than the first operand and shift spatial attention to smaller numbers (e.g., Hartmann et al., 2015; Knops et al., 2009; Knops et al., 2013; Masson & Pesenti, 2014; McCrink & Wynn, 2009; McCrink et al., 2007; see Fischer & Shaki, 2014, for a review).

The findings from the studies on the OM effect suggest that the mental number line involved in arithmetic is organized in conventional left-to-right mode in which smaller numbers are located at the left side and larger numbers are located at the right side (e.g., Dehaene et al., 1993; Zorzi, Priftis, & Umiltà, 2002). This conventional left-to-right mental number line involved in arithmetic is referred to as the static mental number line. The direction of the mental number line, however, does not appear to be always static in the left-to-right mode.

A series of studies have shown that the direction of the mental number line in numerical processing (other than cognitive arithmetic) could be influenced by environment, culture, or language (e.g., Bächtold, Baumüller, & Brugger, 1998; Dehaene et al., 1993; Fischer & Fias, 2005; Ristic, Wright, & Kingstone, 2006; Zebian, 2005). For example, Bächtold, Baumüller and Brugger (1998) found that the form of the mental number line was shaped by experimental instructions: when the participants were instructed to imagine digits as times on a clock, the orientation of mental number line was reversed. Zebian(2005) was the first to show a right-to-left directionality for Arabic monoliterates due to their right-to-left language system. This reversed mental number line was replicated in Palestinians who also have right-to-left reading and writing habits (Shaki, Fischer, & Petrusic, 2009). Hung et al., (2008) asked Chinese participants to perform parity judgment on numerals expressed with Arabic digits and Chinese number words. Arabic digits are read and written from left to right routinely, but Chinese number words are used from top to bottom for a long time in Chinese culture. Results showed a conventional mental number line for Arabic numbers but an up-to-down mental number line for Chinese number words.

The direction of the mental number line can also be reshaped by short-term experience (e.g., Fischer, Mills, & Shaki, 2010; Shaki & Fischer, 2008). Dehaene et al. (1993) found that participants responded more quickly to smaller numbers than to larger numbers with left hand and more quickly to larger numbers than to smaller numbers with right hand, which was presumably due to the activation of left-to-right mental number line. The effect was termed as the effect of spatial–numerical association of response codes (SNARC). Shaki and Fischer (2008) found that bilingual Russian-Hebrew readers had regular SNARC effect after reading Cyrillic script (from left-to-right) but had significantly reduced SNARC effect after reading Hebrew script (from right-to-left) in an experiment of about 45 min. Fischer et al., (2010) further showed that a short-term reading practice could even induce a reverse SNARC effect. Therefore, all the evidence suggests that the mental number line is of high flexibility, and that its dynamic reflects the influence of number input. The notion is consistent with the conclusion that "SNARC reflects any recently experienced spatial–numerical mappings" by Fischer et al. (2010, p. 335). These results suggest that the mental number line could be easily adapted to varied situations.

Although the direction of the mental number line has been proved to be influenced by various environmental, cultural, and language factors during numerical processing, it remains unclear whether the mental number line may dynamically change directions in simple arithmetic. Arithmetic involves multiple strategies and processing stages, which are more complex than those involved in basic numerical processing (e.g., parity judgment). The complex nature of arithmetic may affect the orientation of the mental number line. For example, when spatial layout of two operands (e.g., 7+4) is inconsistent with the conventional left-to-right mental number line, people may choose to re-orient the direction of the mental number line to right-to-left, which may take less cognitive resources than re-aligning the two operands on the conventional left-to-right number line. The type of mental number line whose direction is affected by arithmetic expression and operation is referred to as the dynamic mental number line.

There has been little evidence for the dynamic mental number line in arithmetic. A recent eye movement study on simple arithmetic (Zhou, Zhao, Chen, & Zhou, 2012) showed that larger-operand-first additions (e.g., 9+5) induced leftward eye movements, and that smaller-operand-first additions (e.g., 5+9) induced rightward eye movements. Importantly, this effect was found in the research for arithmetic solution phase and thus cannot be explained by eye movements during the encoding of two operands on a static left-to-right mental number line. This difference in eye movement might reflect the dynamics of the mental number line in different addition operations. In larger-operand-first additions (e.g., 9+5), the first operand is larger than the second one, which might induce a right-to-left oriented (9, 5) mental number line. The participants therefore might make leftward eye movements to locate the solution (e.g., 14). On the other hand, in smaller-operand-first additions (e.g., 5+9), the second operand is larger than the first one, which might induce a left-to-right oriented (5, 9) mental number line. The participants then might make rightward eye movements to locate the answer (e.g., 14). Thus, in simple arithmetic, the direction of the mental number line appears to depend on the relative magnitudes of two operands and the solution.

In this study, we tested the hypothesis of dynamic mental number line in simple arithmetic. In Experiment 1, eye movements during larger-operand-first additions and subtractions (e.g., 7+5 and 7−5) were compared. According to the hypothesis of dynamic mental number line, given the relative magnitudes of operands, both additions and subtractions will induce a right-to-left oriented number line, which may lead eye movements toward left for additions and toward right for subtractions when locating solutions on the mental number line. For example, the mental number line for "7+5" might be "12, 7, 5", and the mental number for "7−5" might be "7, 5, 2". In contrast, according to the hypothesis of static mental number line, both additions and subtractions induce a conventional left-to-right mental number line. Thus, addition relative to subtraction would elicit greater rightward eye movement due to their larger result or solution. In Experiment 2, we compared eye movements in larger-operand-first and smaller-operand-first additions with the same operands and solution. Different from Zhou et al. (2012), we manipulated the magnitude of distance between operands in each addition operation: in half of the additions, differences between operands were less than 3 (e.g., 6+5, 5+6); in the other half, differences between operands were larger than or equal to 3 (e.g., 9+2, 2+9). Eye movement along the mental number line built on the two operands of the arithmetic problems would be more salient for the problems with large distance between two operands. Thus, the hypothesis of dynamic mental number line predicts salient differences in eye movements between larger-operand-first and smaller-operand-first additions when differences between operands were larger than or equal to 3, but not when differences were smaller. According to the hypothesis of static mental number line, all four types of arithmetic problems would induce a conventional left-to-right number line. Given that all arithmetic problems share the same operands and result, they may be associated with similar eye movements.

Experiment 1

Methods

Participants

Thirty-six healthy right-handed university students (18 males) from Beijing Normal University participated in this experiment. The average age of the subjects was 22.8 ± 2.0 years old. They self-reported to have normal or corrected-to-normal eyesight. They had not participated in any experiment similar to the present one (i.e., addition and subtraction) for the previous six months. Informed written consent was obtained from each participant after procedures had been fully explained. The experiment was approved by IRBs of the State Key Laboratory of Cognitive Neuroscience and Learning at Beijing Normal University.

Materials

The materials included 21 single-digit larger-operand-first addition problems and 21 single-digit larger-operand-first subtraction problems. The difference between two operands of all subtraction problems is equal to or more than 2. Additions and subtractions have matched operands (i.e., 4+2, 5+2, 5+3, 6+2, 6+3, 6+4, 7+2, 7+3, 7+4, 7+5, 8+2, 8+3, 8+4, 8+5, 8+6, 9+2, 9+3, 9+4, 9+5, 9+6, 9+7 for additions, and changing the "+" to "−" to become subtractions). Due to the limited number of problems, each problem was presented twice in different blocks to ensure enough trials for averaging of EEG data.

Procedure

Participants were seated 105 cm away from the computer screen in a dimly lit and sound-attenuated room. All stimuli were presented visually in white against black background at the center of the computer screen. Addition and subtraction problems were presented in different blocks to reduce additional attention resource of recognizing operation types. The order of blocks was counterbalanced between participants. Before each block, participants were told the type of operation they would perform in this block. Participants were asked to orally report the answer for each problem through a microphone. Before each arithmetic problem was presented, a fixation sign "*" was presented in the center of the screen. The position of the fixation sign was the position of the operation sign of the arithmetic problems. Participants were asked to focus on the fixation sign for 500 ms. Each trial includes one arithmetic problem which consisted of two operands and an operation sign ("+" for addition and "−" for subtraction). The arithmetic problem remained on the center of the screen until the participants responded. Following the response, a 2000-ms blank screen was presented. Throughout the experiment, an experimenter sat beside the participant to record what he or she reported. Before formal tests, participants were trained to respond orally so that their oral response could activate the voice-controlled switch. Participants were also asked to keep their head steady and avoid eye blinking before an oral response. Eye blinking was allowed in the 2000-ms blank interval after each oral response.

HEOG and VEOG recording and preprocressing

Horizontal electrooculogram (HEOG) and scalp voltages for the arithmetic tasks were recorded using a SCAN system (Neurosoft, Inc., Sterling, USA) with a 64-channel Quickcap. Linked ears served as reference, and the middle of the forehead served as ground. Two channels were placed at the outer canthi of both eyes to record the HEOG, and another two channels above and below the left eye for the vertical electrooculogram (VEOG). The default algorithms were used, in which HEOG was calculated as the right eye minus the left eye, and VEOG was calculated as the upper side minus the lower side. Therefore, the wave was negative-going when the eyes moved towards the left, and positive-going when they moved towards the right. Meanwhile, the wave was negative-going when eyes moved towards the lower side, and positive-going when eyes moved towards the upper side (Zhou et al. 2012). The electroencephalogram (EEG) was amplified online with a low-pass frequency filter of 30 Hz. The sampling rate was 1000 Hz. The impedance of all electrodes was kept below 5 kΩ. The scalp EEG was not analyzed in the current study. HEOG and VEOG were processed in NeuroScan EDIT (Version 4.3). A direct current (DC) correction was first applied. Then eye blinks on the VEOG were removed by using an in-house computer program built on Matlab. The continuous EEG data were segmented into epochs starting from 200 ms before the onset of the arithmetic problems and continuing for 1500 ms. The 200-ms prestimulus served as the baseline. All the epochs were baseline corrected. Epochs exceeding the range of –150 ~ 150 μV were excluded as artifacts for HEOG and VEOG. The HEOG and VEOG have minor influence on one another (e.g., Zhou et al., 2012), and thus were separately trimmed. That is, the epochs with large amplitudes that were excluded for the VEOG analysis could still be kept for the HEOG analysis, and vice versa. A total of 95.7 ± 2.0 % of trials for all participants were kept for HEOG analysis, and a total of 72.3 ± 22.8 % of trials were kept for VEOG analysis. The corrected data were averaged for each participant by conditions.

Statistical analysis

For behavioral data, any trial with a reaction time of more than 2 s was first discarded. Then trials with a reaction time three standard deviations above or below the mean were also discarded. Reaction times and error rates were then compared between addition and subtraction conditions using paired sample t tests.

The mean HEOG and VEOG data for each condition of the eye movement tasks was used to demonstrate the relation between deflection directions of HEOG and

VEOG. Average HEOG and VEOG waveforms of each condition were first plotted. Previous studies have shown that mean difference potentials between two arithmetic operations (multiplication and addition) mainly occurred in the interval 296–444 ms poststimulus (Zhou et al., 2006). The mean reaction time was 600–700 ms for Chinese adult samples to perform simple arithmetic in previous studies (e.g., Zhou et al., 2006, 2007). Thus, the time window 300–700 ms poststimulus that could cover typical cognitive arithmetic processing was selected to compare differences between conditions. This time window was then divided into small time windows, which were subjected to further ANOVA analysis.

Results

Behavioral results

Mean error rates across all participants were 2.39 % for addition and 2.22 % for subtraction. The error rates were low and thus were not further analyzed. Reaction time showed no difference between addition (853 ± 138 ms) and subtraction (869 ± 151 ms).

HEOG and VEOG results

Figure 1 shows the HEOGs for simple addition and subtraction. The time window of 300–700 ms poststimulus was divided into six 50-ms bins. A repeated-measure 2 × 8 ANOVA using HEOG as the dependent variable and arithmetic type (addition vs. subtraction) and time bins (300–350 vs. 350–400 vs. 400–450 vs. 450–500 vs. 500–550 vs. 550–600 vs. 600–650 vs. 650–700 ms) as within-subject factors showed a significant main effect of arithmetic type, $F(1,35) = 5.81$, $p < 0.05$, and a main effect of time bins, $F(7245) = 6.58$, $p < 0.001$. Interaction effect was not significant, $F(7245) = 0.49$, $p = 0.64$. The main effect of arithmetic type is that subtraction induced greater positive HEOG than addition, which suggests that subtraction was associated with greater rightward eye movements compared to addition.

Figure 2 shows the VEOGs for simple addition and subtraction. Vertical eye movements in the same time windows (300–700 ms) were also analyzed separately. For each time window, repeated-measure ANOVA using VEOG as the dependent variable and arithmetic type and time bins (50 ms interval) as within-subject factors showed no significant main effect of operation or interaction effect.

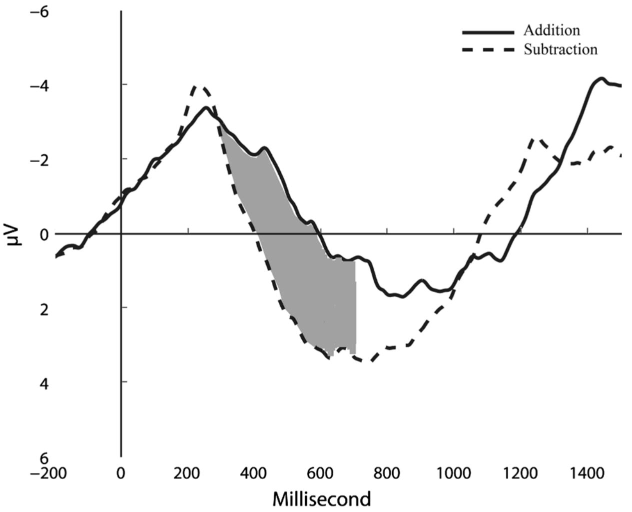

Fig. 1 Single-digit subtraction relative to single-digit addition leads to greater rightward eye movement as shown by the greater positive deflection of horizontal electrooculograph (HEOG) for subtraction in time window 300–700 ms poststimulus (Experiment 1)

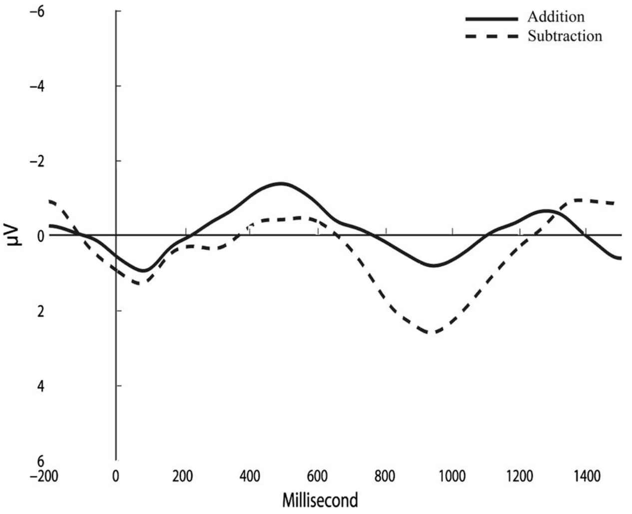

Fig. 2 Vertical electrooculograph (VEOG) for single-digit addition and subtraction (Experiment 1)

Discussion

The HEOG results were consistent with the dynamic mental number line hypothesis. The larger-operand-first addition operations (e.g., 7+5) produced a right-to-left oriented mental number line (e.g., 7, 5), which may be associated with leftward eye movements to the answer (e.g., 12) on the left side. The subtraction operations (e.g., 7−5) also produced a right-to-left oriented mental number line (e.g., 7, 5), which may be associated with rightward eye movements to the answer (e.g., 2) on the right side.

The comparison of addition and subtraction with the same operands might be confounded by their different solutions. To eliminate this potential confound, we further tested the dynamic mental number line hypothesis with only addition in Experiment 2. To produce the same solution across conditions, each pair of operands were used to generate two problems: larger-operand-first and smaller-operand-first problems (e.g., 9+2 vs. 2+9). We also manipulated the distance between two operands to be either small (smaller than 3) or large (larger than or equal to 3) to investigate its influence on eye movements.

Experiment 2

Methods

Participants

Forty healthy right-handed university students (nineteen male) from Beijing Normal University participated in this experiment. They had not participated in any experiment similar to the present one (i.e., addition and subtraction) for the previous six months. The average age of the participants was 22.7 ± 2.1 years old. Informed written consent was obtained from each participant after experimental procedures had been fully explained. The experiment was approved by IRBs of the State Key Laboratory of Cognitive Neuroscience and Learning at Beijing Normal University.

Materials

The experimental stimuli were addition problems. All materials were single digit addition problems with operands ranging from 2 to 9. A 2 × 2 within subject design was used. One independent factor was operand order, the relative position of two operands of the addition problems: either the larger or the smaller operand was placed before the operation sign. Another independent factor was the distance between two operands. The difference between two operands was either small (smaller than 3) or large (larger than or equal to 3). Thus, Experiment 2 included four conditions: large-distance smaller-operand-first (e.g., 2+9), large-distance larger-operand-first (e.g., 9+2), small-distance smaller-operand-first (e.g., 5+6), and small-distance larger-operand-first (e.g., 6+5).

Addition problems with two identical operands were excluded. We selected 28 smaller-operand-first addition problems (i.e., 2+3, 2+4, 2+5, 2+6, 2+7, 2+8, 2+9, 3+4, 3+5, 3+6, 3+7, 3+8, 3+9, 4+5, 4+6, 4+7, 4+8, 4+9, 5+6, 5+7, 5+8, 5+9, 6+7, 6+8, 6+9, 7+8, 7+9, 8+9) and 28 larger-operand-first addition problems by switching the order of two operands of the smaller-operand-first addition problems. Each block included all 56 trials, and the experiment included two blocks. Trials were presented randomly in each block.

Procedure

The procedure of this experiment was exactly the same with that of Experiment 1.

HEOG and VEOG recording and preprocessing

HEOG and VEOG recording and analysis were exactly the same with those in Experiment 1. A total of 86.5 ± 8.9 % of trials for all participants were kept for HEOG analysis, and a total of 86.7 ± 9.9 % of trials were kept for VEOG analysis.

Statistical analysis

The standard and procedure of statistical analysis were exactly the same with those in Experiment 1.

Results

Behavioral results

Mean error rates were 2.40, 2.18, 1.15, and 1.97 % for large-distance smaller-operand-first additions, large-distance larger-operand-first additions, small-distance smaller-operand-first additions, and small-distance larger-operand-additions, respectively. The error rates were low and thus were not further analyzed. Reaction time were 860 ± 156, 840 ± 142, 824 ± 151 and 840 ± 143 ms for the four types of addition problems, respectively. A repeated measure ANOVA of reaction time showed a significant main effect of distance, $F(1, 39) = 6.36$, $p < 0.05$. The main effect of operands order was not significant, $F(1, 39) = 0.10$, $p = 0.75$. There was also a significant interaction between distance and order, $F(1, 39) = 9.58$, $p < 0.01$. Simple effect analyses showed that in small-distance conditions, difference between smaller- and larger-operand-first problems was not significant, $F(1, 39) = 3.67$, $p = 0.06$; in large-distance conditions, difference was significant, $F(1, 39) = 6.19$, $p < 0.05$, suggesting that when distance between operands was large, smaller-operand-first problems took longer to solve.

HEOG and VEOG results

Figure 3 shows the HEOGs for all four conditions. We selected addition problems with the same solutions (e.g., 6+5 vs. 5+6) to eliminate the confounding factor of solution between conditions. We selected the time window of 300–600 ms to further investigate the effects. This time window was divided into six 50-ms bins. A repeated measure 2 × 2 × 8 ANOVA using the HEOGs as the

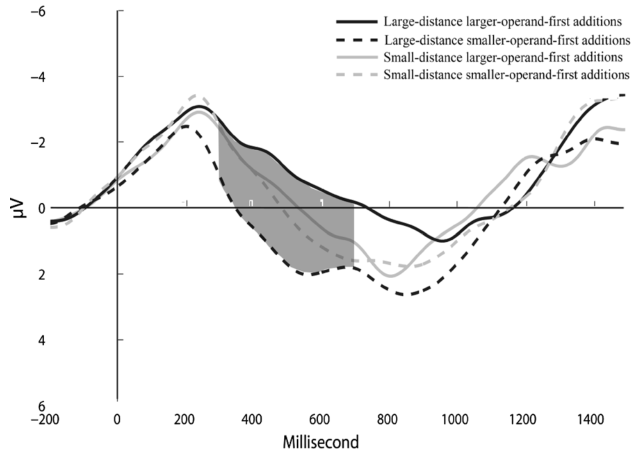

Fig. 3 Large-distance small-operand-first addition relative to large-distance large-operand-first addition leads to greater rightward eye movement as shown by the greater positive deflection of horizontal electrooculograph (HEOG) for large-distance small-operand-first addition in time window 300–700 ms poststimulus (Experiment 2)

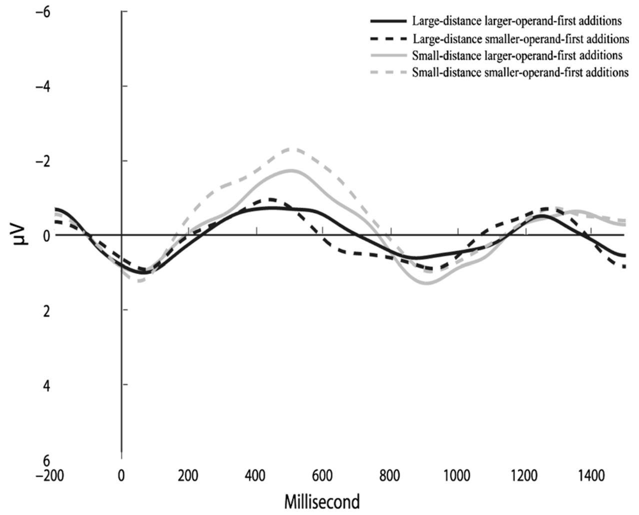

Fig. 4 Vertical electrooculograph (VEOG) by order and distance of operands for single-digit addition (Experiment 2)

dependent variable and operand distance (small distance vs., large distance), operand order (smaller-operand-first vs., larger-operand-first), and time bins (300–350 vs. 350–400 vs. 400–450 vs. 450–500 vs. 500–550 vs. 550–600 vs. 600–650 vs. 650–700 ms) as within-subject factors showed no interaction effect between time and the other two variables, $F(7245) = 0.83$, $p = 0.44$. The main effect of operand order was significant, $F(1, 39) = 5.83$, $p < 0.05$, with larger-operand-first problems showing greater negative HEOG than smaller-operand-first problems. The interaction between operand order and distance was significant, $F(1, 39) = 4.32$, $p < 0.05$. Simple effect analyses showed that in small-distance conditions, the difference between smaller- and larger-operand-first problems was not significant, $F(1, 39) = 0.02$, $p = 0.89$; in large-distance condition, the difference was significant, $F(1, 39) = 6.25$, $p < 0.05$, that is, larger-operand-first problems showed greater negative HEOG than smaller-operand-first problems did only when the difference between operands were large.

Figure 4 shows the VEOGs for all four conditions. The VEOGs in the same time windows (300–700 ms) were also analyzed separately. For each time window, there was no significant main effect of operand distance or operand distance and no interaction effect of operand distance and operand order.

Discussion

This experiment showed that horizontal eye movement exhibited different patterns for additions with the same solution but different distances and different orders of the operands. Larger-operand-first problems showed greater negative HEOG than smaller-operand-first problems when the distance between two operands were relatively large. In contrast, when the distance of two operands was relatively small, there was no significant difference between larger- and smaller-operand-first problems. These results are consistent with the dynamic mental number line hypothesis. Larger-operand-first additions may induce a right-to-left mental number line, whereas smaller-operand-first additions may induce a left-to-right mental number line. To navigate to the correct answer, the eyes should move leftward in the larger-operand-first additions and rightward in the smaller-operand-first additions. These results also demonstrate that operand distance may determine the magnitude of distance between adjacent labels on the mental number line. When two operands are far apart from each other, the mental number line may show large gaps between adjacent digits and thus may lead to large eye movements. In contrast, when two operands are close, the mental number line may show small gaps and would hardly induce any eye movements.

The larger-operand-first problems might have greater cognitive load due to inconsistency with the conventional left-to-right mental number line. However, RT analysis showed no effect of operand order, suggesting that the larger-operand-first problems were not more difficult than the smaller-operand-first problems. This result can be explained by better memory of arithmetic facts for the larger-operand-first problems in the context of the min model during acquisition of arithmetic facts. The min model refers to a counting strategy to solve addition problems, that is, the counter in the min model is set to the maximum digit and is then incremented by the minimum digit. Groen and Parkman (1972) have shown that the min model is the best model to account for solution time on addition problems. The application of min model might lead to the preferred max + min memory for addition facts (Butterworth, Zorzi, Girelli, & Jonckheere, 2001). Thus, the advantage in memory for the larger-operand-first problems might counterbalance the effect of switching the direction of conventional mental number line.

An alternative explanation for the effect of operand order on eye movement is that differences in eye movements between smaller- and larger-operand-first additions were associated with shifts of spatial attention along a static left-to-right mental number line during the encoding phase of arithmetic problems. When smaller-operand-first (e.g., 2+6) problems were presented, participants' eyes would move to the right on the conventional mental number line to locate the second operand and then move further right to locate the result. In contrast, when larger-operand-first (e.g., 6+2) problems were presented, participants' eyes would move to the left to locate the second, smaller operand and then to the right to locate the result. Hence, relatively more rightward movements were observed for smaller-operand-first problems. This explanation, however, does not appear to be consistent with the time window in which the effect of operand order were observed. The effect of operand order was observed in the 300–700 ms time window. Previous event-related potential studies showed that adult samples could start to retrieve or calculate answers for simple arithmetic around 300 ms (Zhou et al., 2007, 2009). Thus, eye movements in the 300–700 ms time window likely reflected substantial arithmetic processing, that is, solution retrieval or execution of arithmetic procedure, instead of encoding of two addition operands.

The results in our study is consistent with results from a recent study (Shaki, Sery, & Fischer, 2015) in which the participants were presented with smaller- or larger-operand-first single digit addition problems and were instructed to produce the sum by changing the length of a horizontal line. The results showed that participants produced longer lines for smaller-operand-first additions than for larger-operand-first additions. The authors interpreted the results under the conventional mental number line hypothesis. Larger-operand-first problems (e.g., 5+1) first induced rightward attention to the sum and them leftward attention to the larger operand on the left side, whereas smaller-operand-first problems (e.g., 1+5) were associated with consistent rightward attention. Thus, compared to larger-operand-first problems, smaller-operand-first problems would have greater operational momentum, which may be the cause for the production of longer lines. The authors also proposed a sequential activation hypothesis in which the localization was typically affected by the second digit and thus a larger second digit (e.g., 1+5) would lead to a more rightward shift than a smaller second digit (e.g., 5+1). The alternative explanation is still based on the conventional mental number line.

The results of Shaki et al. (2015) can also be explained under the dynamic mental number line hypothesis. During arithmetic calculation, larger-operand-first additions (e.g., 5+1) induced a right to left mental number line, and answers were located on the left; smaller-operand-first additions (e.g., 1+5) induced a left-to-right mental number line, and answers were located on the right. When the participants finished arithmetic processing (i.e., the search for solution) and started to adjust line length, the participants likely relied on the conventional left-to-right mental number line. Importantly, the dynamic mental number line activated during the solution searching stage (within 600 ms) may still have sustained effect on the latter, line-length adjustment stage. Thus, the right-to-left oriented mental number line associated with larger-operand-first addition conflicted with the conventional mental number line and thus may lead to production of shortened lines.

General discussion

The aim of the current investigation was to use measures of eye movement to test the dynamic mental number line hypothesis during arithmetic. According to the dynamic mental number line hypothesis, the direction of the mental number line (left-to-right or right-to-left) during arithmetic is determined by the comparative magnitudes of operands and the result. This hypothesis is distinct from the traditional static mental number line hypothesis, which posits that the direction of the mental number line is fixed and does not vary during arithmetic. Experiment 1 showed that subtraction lead to greater rightward eye movements compared with addition. Experiment 2 showed that smaller-operand-first additions were associated with greater rightward eye movements than larger-operand-first additions when the distance of two operands was relatively large (larger than or equal to 3). Both results were consistent with predictions of the dynamic mental number line hypothesis.

Static mental number line and operational momentum effect

Previous researches have shown that a left-to-right mental number line is automatically activated when numbers are presented to people (e.g., Dehaene et al., 1993; Wood, Willmes, Nuerk, & Fischer, 2008; Yang et al., 2014). Several recent studies showed that the mental number line could be recruited for solving arithmetic problems (e.g., McCrink et al., 2007; Pinhas & Fischer, 2008). McCrink et al. (2007) used non-symbolic arithmetic to study mental spatial–numerical representations of numerical magnitudes and found that solutions for addition problems were overestimated and those for subtraction problems were underestimated (i.e., the OM effect). The authors argued that numerical operations are analogous to movements on an internal continuum which is analogous to the mental number line. Addition produces large results which lead rightward shift of spatial attention, whereas subtraction produces small results which lead leftward shift of spatial attention. Pinhas and Fischer (2008) replicated these effects in symbolic arithmetic. These studies suggest that the left-to-right mental number line could impact arithmetic processing. The OM effect might reflect the intuitive knowledge towards addition and subtraction, that is, addition produces large result and subtraction produces small result.

The studies typically sequentially presented arithmetic problems and focused on the answer reporting stage (e.g., Klein et al., 2014; Knops et al., 2009). Thus, eye movements were typically directed by the answer other than by the substantial arithmetic processing (i.e., the search for arithmetic solution). The answer as a single number could easily activate the conventional static mental number line. During the arithmetic processing, participants have to process more than one number, that is, operands and their candidate answers. The static mental number line that typically serves to single number might not be applicable. Thus, the dynamic mental number line that can reflect the spatial layout of operands and answers during arithmetic processing might be activated.

Dynamic mental number line during arithmetic processing

The direction of the mental number line used for arithmetic is traditionally hypothesized to be invariant to arithmetic problems. This hypothesis, however, was not supported by the results in the current study, which instead showed in two experiments that the direction of the mental number line was dependent on relative magnitudes of two operands in the arithmetic problems. Experiment 1 showed that larger-operand-first subtraction (e.g., 7−5) was associated with greater rightward eye movements by comparison with larger-operand-first addition (e.g., 7+5). These results might be explained by a right-to-left mental number line other than a conventional left-to-right line. Both larger-operand-first addition and subtraction problems may activate a right-to-left oriented mental number line, and the eyes move more rightward for subtraction than for addition in the basis of their results along the mental number line. For example, the mental number line might be 12, 7, 5 for addition "7+5", and the mental number line might be 7, 5, 2 for subtraction "7−5".

Experiment 2 used addition problems with the same solution to eliminate the confounding factor of different solutions. It showed that the direction of the mental number line still varied even when the solution was the same for a larger-operand-first addition and a smaller-operand-first addition with the same operands. This result demonstrates that the direction of the mental number line is determined by the relative differences of two operands and their results. For example, the mental number line for problem "9+2" might be 11, 9, 2, and the mental number line for problem "2+9" might be 2, 9, 11. This experiment also showed that eye movements were not significantly different between two types of additions when the distance between operands was small (less than 3), which further confirms that the relative magnitude of two operands is critical for the mental number line. When two operands are too close together, the magnitude of distance between adjacent labels on the mental number line may become too small to induce any eye movement effect in arithmetic. These results are consistent with predictions from the dynamic number line hypothesis in arithmetic.

Hartmann et al. (2015) did not show the effect of operation on horizontal eye movement, but showed a more upward eye movement for addition compared to subtraction when single-digit arithmetic problems were presented verbally. They also found that the effect on vertical eye movement could even be found during verbal presentation of operation sign (i.e., + or −) before the second operand was presented. The authors proposed that verbally presented arithmetic operations are associated with a vertical mental number line, and that addition is associated with upper space and subtraction with lower space (Lugli, Baroni, Anelli, Borghi, & Nicoletti, 2013; Wiemers, Bekkering, & Lindemann, 2014). The operation effect is different from the one found in the current investigation, suggesting that visual input might be a cue to activate the horizontal dynamic mental number line. Future study could test if visual vertical presentation of arithmetic problems could elicit similar operation effect of vertical eye movement as the one found in verbal presentation of arithmetic problems.

Klein and colleagues (2014) physically manipulated the horizontal direction of the number line by presenting either 0-100 or 100-0 on the screen, and they instructed the participants to verbally report their answers to arithmetic problems (e.g., 28 + 15 = 43 or 67 − 24 = 43) and then to click a mouse to locate their answers on the corresponding position of the number line. They found regular OM effects even when the direction of the physical (other than mental) number line was reversed: addition still led eye movement to larger magnitudes, and subtraction still led eye movement to smaller magnitudes, regardless of the direction of the physical number line. The OM effect first disclosed intuitive knowledge about addition and subtraction, that is, addition produces large result and subtraction produces small result. Meanwhile, as discussed earlier, the OM effect might also be associated with the mental number line that was activated by single solution other than by arithmetic processing. Importantly, the manipulation of number line (either 0–100 or 100–0) showed that the left-to-right mental number line for single number could also be re-oriented to right-to-left one. The current study further shows that even if no physical number line was presented, the direction of the mental number line was still influenced by the relative magnitudes of two operands.

Models for simple arithmetic and dynamic mental number line hypothesis

Several models have been proposed for simple arithmetic, including the min model for addition (Groen & Parkman, 1972) and the smaller-count model for subtraction (Woods, Resnick, & Groen, 1975). The two models are regarded as counting strategies, and also affect the memory of arithmetic facts.

The min model can be viewed as a specific application of dynamic mental number line in arithmetic. The min model is a counting strategy for young children to solve addition problems: the children first choose the larger operand to initialize the counter and then increment the counter by the smaller operand. Thus, the min model leads to the smallest number of counting and the shortest distance between the larger operand and solution for all addition problems. Under the min model, the counter can be initialized to either the first or the second operand. Thus, the min model is consistent with the dynamic mental number line hypothesis. Moreover, the application of the min model could result in preferred memory of arithmetic facts, such as, max + min as proposed by Butterworth et al. (2001) in which the larger operand could be as the center of memory. Even if the participants with fluent arithmetic do not explicitly use the min model, access and retrieval of arithmetic memory could lead to more attention to the larger operand and thus result in dissociated eye movements between larger-operand-first and smaller-operand-first addition problems.

The min model does not provide an account for the effect of operand distance on eye movement that only the addition problems with two operands that have larger distance (e.g., larger than or equal to 3) showed clear dissociation of eye movements between smaller- and larger-operand-first additions. The operand distance effect can be explained under the context of the dynamic mental number line hypothesis. When operand distance is small, the distance between adjacent labels on the mental number line may be too small to show any effect; when operand distance is large, spatial shifts along the mental number line may span larger space and thus lead to significant difference between conditions.

Besides the min model in addition, the smaller-count model was often used in subtraction (Woods et al., 1975). Subtraction is typically expressed with the larger number (i.e., minuend) on the left and the smaller number (i.e., subtrahend) on the right (e.g., 9−3, 9−6). Thus, the right-to-left mental number line could be involved in the subtraction. The smaller-count model proposes that children could execute one of two counting strategies, whichever can be executed with fewer counts. The first counting strategy (counting down) is to count down along the number sequence from the minuend (first number), and stop when the number of counts is equal to the subtrahend (second number). The stopping point is the answer. For example, for the problem 9−3, children could count 8→7→6, stop at 6 because the number of counts is 3, and take 6 as the answer. The second counting strategy (counting up) is to count up along the number sequence from the subtrahend to minuend and take the number of counts as the answer. For example, for problem 9–6, children could count 9←8←7 (7 as the starting point), and take 3 as the answer.

If the conventional left-to-right mental number line is applied, the counting sequence for the counting down strategy on the example problem 9−3 could be 6←7←8 (8 as the starting point), and the counting sequence for the counting up strategy on the example problem 9−6 could be 7→8→9 (7 as the starting point). The counting sequence for counting down and counting up strategies is not consistent with the spatial layout of operands in subtraction problems. For example, 9 is on the left side of the problem 9−6, but it is on the right end of the counting sequence 7→8→9. Thus, children might have to re-orient the conventional left-to-right mental number line to the right-to-left one to be consistent with the input from subtraction problems.

The application of counting down and counting up strategies might lead to the involvement of right-to-left oriented mental number line in the memory of subtraction facts. Since the minuend is larger than both the subtrahend and the solution, the eyes might typically focus on the right side of the minuend along the right-to-left mental number line. On the other hand, for the larger-operand-first addition problems, the eyes might typically focus on the left side of the first number along the right-to-left mental number line, because the sum is larger than the first number. Thus, as the finding in Experiment 1 showed, addition was associated with more rightward eye movements compared with subtraction.

Another eye movement study showed that smaller-operand-first problems (e.g., 2+9), relative to larger-operand-first problems (e.g., 9+2), lead to greater rightward eye movement (Zhou et al. 2012), which is consistent with the results in Experiment 2 in the current study. Zhou et al. (2012) interpreted this eye movement pattern in the context of long-term memory of arithmetic facts and argued that this pattern supports the hypothesis that the mental representations of addition facts are operand-order-specific single-representations, meaning that people develop memory of larger operand as the center for addition facts. When solving an addition problem, people would first compare the two operands to find the larger one and then access the memory of facts or alternatively add the smaller one to the larger one as a backup strategy. A min model for addition was proposed to explain the formation of memory of arithmetic facts: the counter in the min model is set to the maximum digit and is then incremented by the minimum digit. Although Zhou et al. (2012) did not use the mental number line to theoretically account for the eye movement, its results were consistent with results in the current study.

Conclusion

The current study tested the dynamic mental number line hypothesis in arithmetic. Two experiments showed that addition relative to subtraction was associated with greater leftward eye movement, and that larger-operand-first addition relative to smaller-operand-first addition was associated with greater leftward eye movement only when the difference of two operands is large (larger than or equal to 3). The results suggest that the direction of mental number line during simple arithmetic is dynamic, and that eyes move along the dynamic mental number line to search for the solution. Future studies should explore the relation between the involvement of mental number line in simple arithmetic and children's arithmetic development. The dynamic mental number line might be applicable to be treated as a knowledge scaffold for children to learn arithmetic.

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