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Bidirectional links in the network of multiplication facts

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Abstract In three experiments, we tested the hypothesis that activation of multiplication operand nodes (e.g., 3 and 8) can occur through presentation of their product (e.g., 24). In Experiments 1 and 2 we found activation of the operands when the product was presented as a cue in a number-matching task. In Experiment 3, activation also occurred in a parity-matching task, where the product (24) was not relevant to the parity matching on its operands (3 and 8). We concluded that bidirectional links exist among the operands and their product for multiplication problems and these links can be activated in a purely stimulus-driven manner. We suggest this may constitute the basis for the solution of simple divisions by mediation through the complementary multiplication facts.

Introduction

Models of numbers and calculation (McCloskey, 1992; Dehaene, Piazza, Pinel, & Cohen, 2003) postulate the existence of a long-term memory store for simple arithmetic facts. For example, when an adult is required to solve the problem *6×8*, very likely he/she will reply *48*, without actually adding 6 to itself 8 times or 8 to itself 6 times. The repertory of arithmetic facts for each individual may depend on his/her familiarity with mathematics. Not everybody knows by heart the result of 25^2; however, the majority of adults possess at least a mnestic representation of multiplication tables, which allows them to retrieve rather than calculate the product of two single-digit numbers.

Recent research, however, has shown that educated adults do not seem to handle the four basic arithmetic operations in the same way. Additions are generally assumed to be solved mainly by retrieving the result from a dedicated network-like memory store (e.g., Ashcraft, 1992; McCloskey, 1992), although fast counting strategies or semantic manipulation would also be available and adopted more often than it was thought in the past (e.g., Thevenot, Barrouillet, & Fayol, 2001; Dehaene et al., 2003). Subtraction, on the other hand, would rely on semantic and procedural knowledge based on the manipulation of quantities (see Dehaene et al. 2003). As for multiplication, for which there is general agreement on a dominant retrieval strategy (e.g., Campbell, 1995; Dehaene et al., 2003), it has been proposed that simple facts are stored in an associative network that is similar to the networks for word representation, in which the nodes are numbers instead of words and the links between them represent arithmetical relations (Ashcraft, 1987). Upon presentation of a multiplication problem, activation would automatically spread from activated operands to linked nodes, such as the product.

The way in which educated adults prefer to deal with simple divisions has recently been the focus of much experimental work, with several studies (e.g., Campbell, 1999; Mauro, LeFevre, & Morris, 2003) suggesting a strong link between division and multiplication. So far, three hypotheses have been proposed to characterize the relation between division and multiplication. According to the first hypothesis, multiplication and division are functionally independent and possibly rely on two separate arithmetic networks. This position has been advocated on the grounds of the results coming from a neuropsychological study (Cipolotti & de Lacy Costello, 1995) reporting the case of an acalculic patient who showed a largely unimpaired ability to perform multiplication but heavily impaired division performance. In addition, Rickard, Healy, and Bourne (1994), who tested

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the effects of practice transfer between corresponding multiplication and division problems, found little evidence that practice transfers in either direction. Under the assumption that transfer measures the strengthening of the retrieval processes, this result points to the idea of separated retrieval networks for multiplication and division facts.

According to the second hypothesis, multiplication and division rely on a single memory network whose functioning is governed by the principle of multiplicative relations among numbers. Arguments consistent with this hypothesis come from the observation that multiplication is usually learned before division at school (Thornton, 1978), and children are often encouraged to learn divisions by retrieving the corresponding multiplication problems (Mulligan & Mitchelmore, 1997; Zhang & Zhou, 2003). Moreover, a patient described by Hittmair-Delazer, Semenza, and Denes (1994) showed a pattern of impaired corresponding multiplication and division problems. Crucially, the subsequent retraining of the impaired multiplication facts resulted in the rehabilitation of the corresponding division problems.

The third hypothesis, more cautiously, argues that simple divisions generally have only weak memory representations. According to this view, supported by several recent studies (e.g., Campbell, 1999; LeFevre & Morris, 1999; Mauro et al., 2003), many educated adults would solve divisions through the retrieval of multiplication facts. First, Campbell (1999), in two experiments addressing item-specific transfer in simple multiplications and divisions, found significant multiplication-to-division transfer effects for large division problems. In addition, self-report studies (e.g., Campbell & Xue, 2001; LeFevre & Morris, 1999) showed that participants often reported accessing multiplication facts to solve division problems. Finally, Mauro et al. (2003) conducted a study in which participants were to solve division problems presented in either standard division-based formats (e.g., $72 \div 8 = \_$) or multiplication-based formats (e.g., $8 \times \_ = 72$), and multiplication problems presented in a division-based format (e.g., $\_ \div 8 = 72$). The results showed that participants were far more accurate and faster at solving divisions in a multiplication-based format than in the standard division format. Moreover, the pattern was reversed when participants had to solve multiplications, which demonstrated that multiplication is the primary mode of representation for solving both multiplication and division problems. One way of reconciling the findings supporting the claim that division is often solved via multiplication with the data reported by Rickard et al. (1994) is to interpret their null transfer of practice as due to the extensive training on division problems that was required by the study itself, which presumably resulted in strengthening the initially weak division network.

In light of this analysis, multiplication is, out of the four basic arithmetic operations, the one that most strongly relies on memory and retrieval mechanisms. Two sources of evidence show associative processes and point to retrieval automaticity for multiplication facts. One is from direct arithmetical tasks, such as production and verification, and the other is from indirect tasks, such as number matching. The analysis of error patterns in production tasks (e.g., Campbell, 1994) shows that operation errors (i.e., correct answers to an operation different from the proposed one, e.g., "$9 + 3 = ?$" Answer: "27") are rather common. In addition, the analysis of reaction times (RTs) in verification tasks (e.g., Zbrodoff & Logan, 1986) reveals an associative confusion effect, that is a significant slowing in the rejection of a false answer, when it happens to be the correct result for an alternative operation on the same operands (e.g., "$9 + 3 = 27$; True? False?"). Rusconi, Galfano, Speriani, and Umiltà (in press) demonstrated the presence of obligatory activation within multiplication facts in the absence of a multiplication sign in a non-arithmetical task. They asked their participants to perform a number-matching task (LeFevre, Bisanz, & Mrkonjic, 1988; Thibodeau, LeFevre, & Bisanz, 1996). In this paradigm, pairs of numbers (cues) are presented (e.g., 3 and 9). After a variable delay, a target number substitutes the initial pair. If the target is one of the previously presented numbers (e.g., 3), then the participant answers "yes" by pressing a response key. If the target is not one of the previously presented numbers (e.g., 7), the participant answers "no" by pressing another key. Interestingly, a "no" answer is slower when the target happens to be the product of the presented pair (e.g., 3, 9, and 27) than when it is unrelated (e.g., 3, 9, and 25). It has been hypothesized that, at the onset of the initial pair, activation spreads automatically from the units corresponding to those numbers along the links in the network of multiplication facts, thus pre-activating their product node. When the product appears subsequently as a target, participants need more time to decide that it was not present in the initial pair, because it has recently received activation (interference effect). In summary, even in the absence of an explicit arithmetic context and of a multiplication sign, adults cannot avoid accessing arithmetic knowledge and retrieve the product (Galfano, Rusconi, & Umiltà, 2003; Rusconi et al., in press).

In the present study we tested whether automaticity of retrieval also occurs in the opposite direction, as might be expected in light of the results showing that division is often mediated via multiplication (e.g., Mauro et al., 2003). Several previous studies have demonstrated that the presentation of a product can facilitate or interfere with subsequent processing of its operands (e.g., Campbell, 1987; Meagher & Campbell, 1995; Zbrodoff & Logan, 2000). However, in those cases, participants were always explicitly asked to perform a direct arithmetic task (production or verification). For example, Campbell (1987; Experiment 2), presented a prime 300 ms before the appearance of a multiplication problem. Participants were required to produce as fast and accurately as possible the answer to that problem, irrespective of the preceding prime, which could be the correct result, a table-related product (i.e., a multiple of one of the operands), an unrelated product or could consist of two non-numerical characters (##, neutral prime). He found that trials preceded by a correct result were faster than trials preceded by a neutral prime, and that related primes were more interfering than neutral or unrelated primes. It was claimed that these facilitating and inhibiting priming effects were consistent with a response competition account, as performance depended on relative activations of candidate responses (see Campbell, 1995, for a formal model). In other terms, the presence of information competing or matching with the correct answer would exert its influence at the stage of answer generation. The possibility of facilitation or interference with the processing of operands instead of the answer generation was not considered. In fact, it would be difficult to test the presence of bidirectional links between primes and operands by a direct arithmetic paradigm, where retrieval is inferred from latency and accuracy of simple arithmetic problem solving. The presence of interference or facilitation by related, unrelated or correct primes may indeed have different sources, such as repetition priming, candidate answers priming, and/or operand priming. Moreover, the strategic use of priming information (e.g., plausibility check or "name-the-prime strategy"; see Meagher & Campbell, 1995) cannot be excluded with certainty. We tried to overcome this problem by adopting indirect tasks that required a non-arithmetical judgment on digit pairs instead of their arithmetical processing. By eliminating the stage of competition between retrieved answers in the network of multiplication (as there was no product answer to select), we could test directly whether the presence of a product affected the processing of its operands through backward spreading activation. By assuming the presence of bidirectional links between nodes in the network, presentation of a product should indeed elicit backward activation of its operands.

In the first two experiments, we tested the hypothesis of bidirectional links through a number-matching paradigm (LeFevre et al., 1988; Rusconi et al., in press). Crucially, unlike previous studies using the number-matching paradigm, we presented single numbers as cues and pairs of numbers as targets. Our participants were asked to make yes/no decisions about whether one of two target numbers matched the preceding cue number. "No" decisions were slower when the cue was the product of the target pair of digits than when it was arithmetically unrelated, which would demonstrate that operand representations were already active during product presentation. With Experiment 3, we showed that parity judgments about pairs of digits are interfered with by the previous presentation of their product. By rendering the product irrelevant to the task, we could demonstrate that these interference effects were due to activation spreading from a product to its operands rather than the opposite (i.e., backward priming; Kiger & Glass, 1983).

Experiment 1

The number-matching paradigm was first introduced to assess the automaticity of fact retrieval in simple arithmetic (e.g., LeFevre et al., 1988). The presence of interference, when the target number was the result of an arithmetic operation on the preceding pair of digits (cue), was assumed to demonstrate obligatory activation spreading from the operands to their associated results, such as the product. If the link between operands and result in the multiplication network were bidirectional, in a number-matching task we would expect interference when a product precedes its operands. At the onset of a product, activation would indeed spread from its unit along the links in the network, thus pre-activating the units corresponding to its operands. When the operands appear successively as target numbers, participants would require more time to decide that they were not present in the initial display, because they had recently received activation. In the present experiment, we reversed the order of stimuli with respect to our previous work (Rusconi et al., in press), and the pair of numbers was presented as a target. Just as in a typical number-matching task, we manipulated Trial Type, and compared the latency for "no" responses in the related condition (e.g., 24 followed by 8 and 3) to matched controls (e.g., 26 followed by 8 and 3). A short, an intermediate, and a long stimulus onset asynchrony (SOA) were included in the design in analogy with previous studies. LeFevre et al. (1988) and Thibodeau et al. (1996) argued that, in the number-matching paradigm, the presence of different SOAs can reveal onset and decay of spreading activation. For example, LeFevre et al. (1988) found interference at brief SOAs only (shorter than 180 ms) and claimed that activation reaching a sum was fast decaying; Thibodeau et al. (1996) found significant interference effects for products at short SOAs only (100 and 120 ms), although the Trial Type × SOA interaction was only marginally significant, and there was still a (nonsignificant) difference (13 ms) in the direction predicted by the interference effect at their longest SOA (350 ms). In addition, both Galfano et al. (2003) and Rusconi et al. (in press) could not find any significant Trial Type × SOA interaction, which indicated that product interference was still reliable at their longest SOA (400 ms). Thus, sum interference and product interference showed a different time course. As for the present experiment, we may expect the same pattern of results as we found in our previous studies (Galfano et al., 2003; Rusconi et al., in press), that is, a significant main effect of SOA and a null interaction between SOA and Trial Type. In the presence of a significant main effect of Trial Type, this would indicate close similarity between forward and backward links in the network of multiplication facts.

Method

Participants

Seventeen undergraduates (5 men and 12 women with a mean age of 24, range: 21–30) participated in the experiment as volunteers. All had normal or corrected-to-normal vision and were naïve as to the purpose of the experiment.

Apparatus, stimuli, and procedure

Each trial included an initial cue and a subsequent target pair. There were 6 types of stimuli, 3 belonging to the non-matching category and 3 belonging to the matching category. Non-matching/product stimuli consisted of cues that were correct products of the target digits (e.g., 24 and 8 and 3). Non-matching/unrelated stimuli had the same target as the product stimuli, but the cue was unrelated to either digit in the target (e.g., 26 and 8 and 3). For both product and unrelated stimuli, the cue neither matched nor included either number of the target. Non-matching/filler stimuli had a double-digit number in the target and a single-digit number in the cue, so that participants saw non-matching trials that included double-digit targets and also single-digit cues that required a "no" response (e.g., 9 and 48 and 7). Matching/cue-balancing stimuli had the same cues as those used in the non-matching/product trials, so that participants saw cases in which the double-digit number in the cue matched the double-digit number in the target (e.g., 53 and 7 and 53). Matching/target-balancing stimuli had the same targets as non-matching product stimuli. This condition was included so that participants saw matching trials consisting of two single-digit numbers in the target (e.g., 3 and 4 and 3). Matching/filler stimuli had a double-digit number in the target (e.g., 3 and 97 and 3), so that participants saw trials in which the single digit in the target matched the cue (the opposite happening for the cue-balancing stimuli).

The experimental list contained 11 stimuli of each type, presented at three different SOAs. In total, there were 198 stimuli presented in a randomized order. None of the numbers in the cue and in the target was 0 or 1, in order not to elicit back-up strategies instead of direct access to a result (e.g., Baroody, 1983). Combinations of cues and targets that may evoke activation on the basis of some relation among the items other than multiplication (e.g., addition) were discarded. Ties were excluded, because they appear to have an easier access to the memory store in comparison with other problems (e.g., Graham & Campbell, 1992). Participants were required to respond "no" if none of the target numbers matched the cue number (e.g., 54 and 6 and 9, or 8 and 6 and 75) and to respond "yes" if one of the target numbers matched the cue number (e.g., 6 and 6 and 9, or 89 and 2 and 89).

An IBM-compatible 486 computer connected to a 15-inch color VGA monitor controlled timing of events, generated stimuli, and recorded responses. The display background was black and the stimuli appeared in white. Each trial began with a 100-ms 500-Hz tone as warning signal. At the same time, a fixation hash sign appeared for 400 ms (Fig. 1). Then the cue appeared at the center of the screen and was visible for 30 ms. It was followed by a blank screen, which lasted for 35 ms, 90 ms or 370 ms, producing three different SOAs (65 ms, 120 ms, and 400 ms) occurring with the same frequency. The target appeared in the center of the screen and remained visible for 2,500 ms. Participants were required to decide whether one of the numbers in the target matched the number in the cue and to press the appropriate key (e.g., "p" for "yes" and "q" for "no") as fast as possible. RT and accuracy were recorded and, at the end of each trial, participants were given visual feedback on accuracy. They sat at an approximate distance of 60 cm from the screen. Each single digit measured 50 mm in height and 30 mm in width and the two digits in the target were always separated by three spaces. There was no multiplication sign between the two target numbers. Half the participants responded "yes" by pressing a key with their right index finger and "no" by pressing another key with their left index finger; the other half received the opposite instructions. The presence of interference was indexed by significantly slower RTs for non-matching/product trials than for non-matching/unrelated trials. We hypothesized that participants, upon presentation of the cue (e.g., 24), involuntarily activate a set of possible targets that includes the actual number presented, as well as associated numbers (e.g., 24, 8 and 3). Therefore, they should be slower at responding "no" when the cue is the product than when the cue is unrelated to the target (e.g., 26 and 8 and 3).

Results and discussion

All analyses reported here and in Experiment 2 refer to non-matching/product and non-matching/unrelated tri-

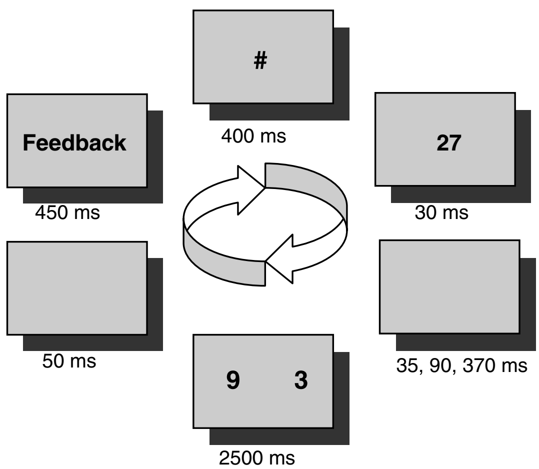

Fig. 1 Cycle of events (clockwise, starting from the top display) in each trial in Experiment 1

als, because these were the critical conditions for assessing our hypothesis (also see Rusconi et al., in press; Thibodeau et al., 1996). Mean correct RTs were submitted to a repeated measures analysis of variance (ANOVA) with two factors: SOA (65 ms, 120 ms or 400 ms) and Trial Type (product vs. unrelated). The ANOVA revealed a significant main effect of SOA, $F(2,32) = 109.47$, $MSE = 1{,}954$, $p < .001$, with RT decreasing as SOA increased (65-ms SOA: $M = 741$ ms, $SE = 33$; 120-ms SOA: $M = 662$ ms, $SE = 28$; 400-ms SOA: $M = 582$ ms, $SE = 29$). This presumably reflects a temporal warning effect (e.g., Sanders, 1975) and will not be discussed further. Also, the two-way SOA × Trial Type interaction resulted significant, $F(2,32) = 4.09$, $MSE = 1{,}770$, $p = .026$. A series of planned comparisons was then performed to test the difference between means on product and unrelated trials at each level of SOA (Fig. 2). At the shortest SOA, a significant interference was present, product trials being 32 ms slower than unrelated trials (product: $M = 757$ ms, $SE = 34$; unrelated: $M = 725$ ms, $SE = 33$; $t_{(16)} = 2.15$, $p = .047$); at the intermediate SOA the difference between means was in the direction of interference but did not reach significance (product: $M = 674$ ms, $SE = 27$; unrelated: $M = 649$ ms, $SE = 31$; $t_{(16)} = 1.69$, $p = .109$); at the longest SOA, the difference approached significance and revealed a 21-ms advantage for product over unrelated trials (product: $M = 572$ ms, $SE = 28$; unrelated: $M = 593$ ms, $SE = 31$; $t_{(16)} = 1.92$, $p = .072$). The same ANOVA was performed on the number of errors, which were 3.6% of total trials, and revealed only a significant main effect of SOA, $F(2,32) = 8.57$, $MSE = 1.655$, $p = .001$, in the same direction as latency data (i.e., the number of errors decreased as SOA increased; shortest SOA: $M = 1.47$, $SE = 0.39$; intermediate SOA: $M = .36$, $SE = .11$; slowest SOA: $M = .32$, $SE = .10$).

In summary, an interference effect appeared in spite of the reversed order of stimuli. When the cue (e.g., 24) was the product of target numbers (e.g., 8 and 3), participants' matching decision was slowed relative to an unrelated cue (e.g., 26). Thus, when the cue was an answer to a memorized multiplication problem, it activated its operands and this irrelevant activation slowed the "no match" decision. This effect was different from that previously found by Rusconi et al. (in press) for its time course. It was significant (32 ms) at the shortest SOA (65 ms), nonsignificant but in the same direction (25 ms) at the intermediate SOA (120 ms), and tended to reverse (−21 ms) at the longest SOA (400 ms). In contrast, the interference found by Rusconi et al. and attributed to activation spreading from the operands to the product persisted at the 400-ms SOA. This suggests that activation runs in both directions along the link between operands and results in the network of multiplication facts, but the effect of activation spreading forward lasts longer than the effect of activation spreading backward. Nevertheless, if the pre-activation of two operands by their product were shorter-lived, decreasing or disappearance of interference at the longest SOA might be expected, but not reversal. However, controlled processes may succeed to automatic priming at longer prime-target delays (Neely, 1991). By eliminating backward masking after the cue from the original procedure (Rusconi et al., in press), we may have favored the rise of controlled processes. Therefore, our second experiment was aimed at replicating the interference effect found in Experiment 1, while reproducing exactly the same procedure as that employed in Rusconi et al. (in press).

Experiment 2

Besides the crucial reversal of the stimulus sequence, Experiment 1 contained other minor procedural variations relative to the task employed by Rusconi et al. (in press). A very short SOA (65 ms) replaced the intermediate SOA (270 ms) and no masking was presented after the cue, which lasted for 30 ms instead of 60 ms. In order to provide additional evidence supporting the existence of associative links from the product to its operands, we performed an experiment with the same task as in Experiment 1 but with identical temporal and masking parameters to our previous studies (Galfano et al. 2003; Rusconi et al. in press). In those studies, the interference effect produced by activation spreading from the operands to the product did not interact significantly with SOA. Three levels of SOA were included in the design (120, 270, and 400 ms) and visual masking interrupted the processing of the cue. As reported earlier, Thibodeau et al. (1996) reported a trend toward significance for the interference × SOA interaction. In

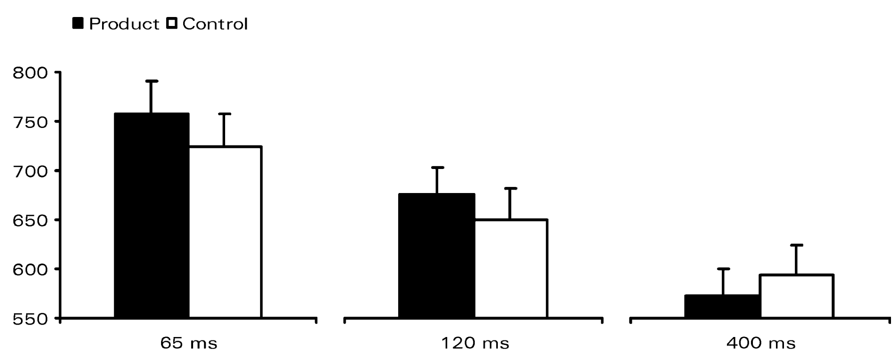

Fig. 2 The interference effect in Experiment 1 as a function of SOA. The *black bars* indicate mean response times (RTs) in product trials, the *white bars* indicate mean RTs in control trials. *Error bars* in this and all subsequent figures denote standard error of the mean

that study, planned comparisons revealed a significant interference effect at 100-ms and 120-ms SOAs, but not at 220-ms and 350-ms SOAs. However, visual inspection of their data clearly shows that interference decreased but not reversed, as at the 350-ms SOA there was still a (nonsignificant) difference in the direction predicted by the interference effect (13 ms). It is therefore possible that the trend toward reversal at the longest SOA of the interference effect observed in Experiment 1 was not the consequence of the main experimental manipulation (i.e., reversal of cue-target sequence), but resulted from procedural changes relative to previously reported studies instead.

Method

Participants

Eighteen undergraduates (9 men and 9 women with a mean age of 23, range 18–26) participated in the experiment as volunteers. All had normal or corrected-to-normal vision, and were naïve as to the purpose of the experiment. None had participated in the previous experiment.

Apparatus, stimuli, and procedure

These were the same as in Experiment 1. The only exceptions were substitution of the 65-ms SOA with a 270-ms SOA, presentation of the cue for 60 ms instead of 30 ms, and insertion of backward masking (two hash signs replaced the digits) for 40 ms after cue offset.

Results and discussion

Mean correct RTs were submitted to a two-way ANOVA with SOA (120 ms, 270 ms or 400 ms) and Trial Type (product vs. unrelated) as within-participants factors. It revealed significant main effects of SOA, $F(2,34) = 89.36$, $MSE = 1,048$, $p < .001$, and Trial Type, $F(1,17) = 9.29$, $MSE = 1,836$, $p = .007$, and a significant two-way interaction, $F(2,34) = 3.86$, $MSE = 1,106$, $p = .030$. RT decreased as SOA increased (120-ms SOA: $M = 677$ ms, $SE = 30$; 270-ms SOA: $M = 604$ ms, $SE = 28$; 400-ms SOA: $M = 579$ ms, $SE = 28$), and product trials were on average 25-ms slower than unrelated trials (product: $M = 633$ ms, $SE = 31$; unrelated: $M = 608$ ms, $SE = 27$). A series of planned comparisons was performed to test the interference effect at each SOA (Fig. 3). Interference had a significant result at the first SOA only (47 ms of interference; $t_{(17)} = 3.18$, $p = .005$); it approached significance at the intermediate SOA (27 ms; $t_{(17)} = 1.85$, $p = .082$), and disappeared at the longest SOA (3 ms; $t < 1$). The same ANOVA performed on the number of errors, which were 1.51% of total trials, revealed only a significant main effect of SOA, $F(2,34) = 4.02$, $MSE = .340$ $p = .027$, in the same direction as for RTs (i.e., errors decreased as SOA increased; shortest SOA: $M = .64$, $SE = .16$; intermediate SOA: $M = .47$, $SE = .14$; longest SOA: $M = 0.25$, $SE = .10$).

Whereas interference attributable to the pre-activation of a product from its operands was present and significant at both the shortest and the longest SOAs in the Rusconi et al. study, the effect of backward pre-activation (from a product to its operands) in the present experiment was significant at the shortest SOA only, decayed for the 270-ms SOA, and totally disappeared at the 400-ms SOA. Although in Experiment 2 procedural parameters were identical to the original study by Rusconi et al. (in press), and thus provided a more direct comparison with the data from our previous study testing associative links from operands to product, the interference with a reversed sequence of stimuli appears shorter-lasting. In Experiment 1, the most important procedural difference consisted of the absence of visual masking after the cue. Backward masking after the cue causes the interruption of its visual processing at a relatively early stage (it prevents elaboration at the level of extrastriate cortex or of its back projections to V1; Amassian et al., 1993; Hupe et al., 2001; Pascual-Leone & Walsh, 2001). In the absence of backward masking, the early visual processing of the cue might still receive the contribution of extrastriate back projections, as long as the target does not appear; as a consequence, with increasing SOA, the information remains available for longer and the participant is more likely to detect the

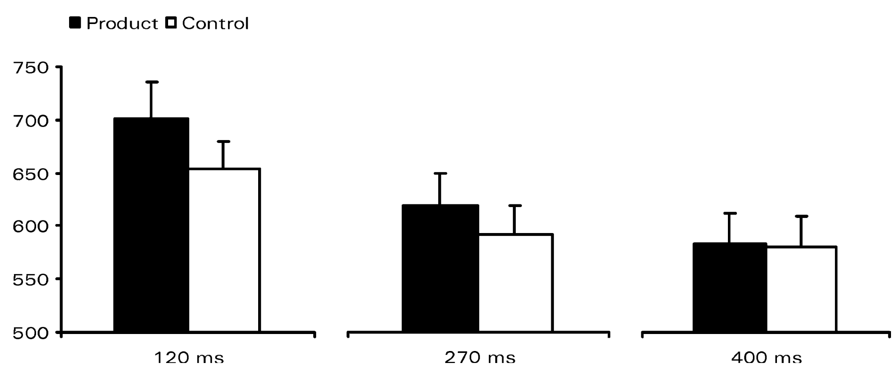

Fig. 3 The interference effect in Experiment 2 as a function of SOA. Conventions as in Fig. 2

arithmetical relation occurring among certain cue-target combinations. One of the crucial differences between aware and unaware processing consists of the intervention of late inhibitory mechanisms. For example, Fuentes and Humphreys (1996) showed that, in a patient with a right parietal lesion, extinguished (i.e., not available to awareness) stimuli produced positive priming and non-extinguished stimuli produced negative priming in the following trial. It is therefore possible that, when the arithmetical relation is detected by a participant, it is treated as distracting information and actively inhibited. This may explain why, in Experiment 1, interference tended to reverse at the longest SOA. In the presence of backward masking, however, early processing of the cue would be blocked at the same time interval from its appearance for all SOAs, and the intervention of controlled processes would become less likely, even at the longest SOA.

Given that procedural details in Experiment 2 were identical to those of previous experiments on activation spreading from operands to product (Rusconi et al., in press), the present results indicate that activation spreads both forward (i.e., from the operands) and backward (i.e., from the product). However, because, consistently with Experiment 1, interference was present at the shortest SOAs only (in Experiment 1, at the 120-ms SOA, a 25-ms difference in the direction predicted by the interference effect was observed), the data suggest that forward-spreading activation is stronger than backward-spreading activation, as only the former results in a long-lasting interference effect.

With the following experiment we tested whether priming from a product to its operands might also be detected in more restrictive conditions, that is, not only when the arithmetical relation between stimuli is task irrelevant, but even when processing the stimulus that contains the product *is not necessary* for the task at hand.

Experiment 3

As in previous studies on the effects of simple arithmetic in number matching (e.g., LeFevre et al., 1988; Rusconi et al., in press), we ascribed interference in Experiments 1 and 2 to the pre-activation of the second stimulus (in the present study, the operands) by the first (the product). Indeed, as required in the number-matching task participants responded "yes" to the second display if one of the numbers in it matched the number in the first display. They responded "no" if there was no matching, but responding "no" was more difficult when two single-digit targets were preceded by their product. This would demonstrate that the product (first display) already contained (or activated) its operands (second display), and therefore participants were slower at deciding that neither target digit matched the cue. However, the interference effect due to activation spreading backward from a product to its operands (Experiment 2) seems to fade in time, whereas the interference due to activation spreading forward from two operands to their product is still present at the 400-ms SOA (Rusconi et al., in press). It might be concluded that activation runs in both directions along the link between operands and results in the network of multiplication facts. The effect of activation spreading forward lasts longer than the effect of activation spreading backward.

However, given that a fully significant interference effect was found only at the 65-ms (Experiment 1) and 120-ms (Experiment 2) SOAs, a tentative alternative interpretation might be formulated on the basis of a study by Kiger and Glass (1983). They found facilitation in a lexical decision task when a related prime word, relative to an unrelated prime word, was presented 65 ms after the target word; the effect was attributed to activation spreading backward from the prime to the target. In order to be effective, activation would have to reach the target representation before a response was selected, and for that reason backward priming appeared only at very brief SOAs between target and prime. In other terms, as long as the participant was engaged in response selection, the presence of a related but task-irrelevant prime affected lexical decision time, even when the prime appeared after the target.

Also in our Experiments 1 and 2, two different stimuli were presented in sequence (a cue followed by a target pair); however, we used numbers instead of words and both the first and the second stimulus were relevant to the task. For this reason, response selection could be engaged only after both stimuli had appeared, and the interference effect may also derive from activation spreading backward from the operands (second stimulus) to their product (first stimulus). In non-matching related trials, response selection would be more difficult than in unrelated trials, because target digits would enhance the activation level of the cue (their product), thus rendering more plausible its presence in the target set of numbers. This kind of interference may disappear at longer SOAs because a response can be selected before activation reaches the product (indeed, at longer SOAs, RTs were significantly faster).

In Experiment 3 we introduced various modifications. First, we changed task requests and asked participants to perform parity matching on a pair of single-digit numbers (e.g., a "yes" response was required to the pairs 4 and 6 and 9 and 7 and a "no" response to the pair 6 and 9), while ignoring the two-digit number that preceded them. Therefore, only the operands were relevant to the task at hand and the product or matched control was task irrelevant. Because more even than odd numbers are divided, we carefully matched the parity of each control to the product parity. As only operand processing would be necessary to select an answer, there would be no reason to explain a difference between related and unrelated primes in terms of product activation by its operands (indeed, the product is irrelevant to the task; also see "Results and discussion"). Odd/even status of numbers influences arithmetic performance both in production (Siegler, 1988) and verification tasks (Krueger, 1986; Lemaire & Fayol, 1995). Moreover, according to associative network models (e.g., Ashcraft, 1992; Sokol, McCloskey, Cohen, & Aliminosa, 1991), number nodes in the network of arithmetic knowledge embed semantic information, such as magnitude and parity (i.e., the same information required to perform a parity-matching task). If it is assumed, as we do, that activation can spread from a product to the semantic representation of its operands, parity matching should be influenced by arithmetical relatedness between target and prime, regardless of the required response. We decided to use an SOA range with a larger step in between levels, as pilot work showed that very brief SOAs led to very poor accuracy in the parity-matching task. Furthermore, this allowed us to ascertain whether participants could strategically use the prime to anticipate the target digits. If this were the case, we would expect better performance in product trials than in control trials, at least at the longest SOA.

Method

Participants

Twenty-two undergraduates (13 men and 9 women, with a mean age of 24, range 22–30) participated as volunteers. All had normal or corrected-to-normal vision, and were naïve as to the purpose of the experiment. None had participated in the previous experiments.

Apparatus, stimuli and procedure

Stimuli were presented in the same format and with the same apparatus as in the previous experiments. Responses were made by pressing one of the two response keys on the keyboard. Participants fixated a central hash (Fig. 4) for 500 ms and then a two-digit number (prime) for 100 ms centered on the screen. After variable SOAs (150 ms, 350 ms, 650 ms), a pair of digits separated by four spaces was displayed at the center of the screen for 1,300 ms. After the execution of a response or after target offset, the screen remained empty for 100 ms and then a visual feedback on accuracy appeared for 500 ms. Then the next trial began.

The set of stimuli comprised 58 pairs (36 critical pairs and 22 fillers) of digits having the same parity and 58 (36 critical and 22 fillers) pairs of digits having different parity for each SOA. In half of the critical trials, the two-digit prime was the product of the target pair (e.g., 36 appeared before 9 and 4), and the pair of digits in the target never included identical (ties) digits, a 1 or a 5; in the other half (control trials) it was the product + or −2 (e.g., 38 appeared before 9 and 4). In filler trials, there was no arithmetical relation between prime and targets. On average, the controls had two-digit primes with the same magnitude as the product trials, and every control trial had a prime having the same parity as the matched product. Each experimental session comprised a total of 348 trials (216 critical and 132 fillers). Participants were instructed to ignore the primes as much as possible, because they were completely task irrelevant.

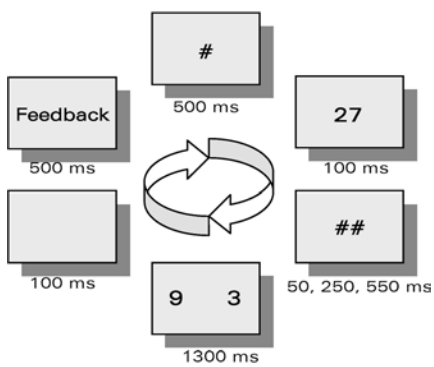

Fig. 4 Cycle of events (clockwise, starting from the top display) in each trial in Experiment 3

Results and discussion

Correct RTs for critical trials were submitted to a repeated measures ANOVA with three factors: SOA (150 ms, 350 ms or 650 ms) and Trial Type (product vs. control). The analysis revealed a significant main effect of SOA, $F(2,42) = 16.35$, $MSE = 1{,}323$, $p < .001$, with the typical advantage for longer over shorter SOAs (shortest SOA: $M = 778$ ms, $SE = 18$; intermediate SOA: $M = 759$ ms, $SE = 17$; longest SOA: $M = 734$ ms, $SE = 16$), and a trend toward significance for the main effect of Trial Type, $F(1,21) = 3.48$, $MSE = 458$, $p = .076$, with product slower than control trials. Moreover, the SOA × Trial Type interaction was significant, $F(2,42) = 3.40$, $MSE = 436$, $p = .043$, and subsequent planned comparisons revealed that the effect of Trial Type was present and significant at the 150-ms SOA only (21 ms; $t_{(21)} = 3.01$, $p < .001$. The same ANOVA was performed on the number of errors, which were 14.86% of total trials, and did not reveal any significant effects (Fs < 1).

As predicted by the main hypothesis, performance in the parity-matching task was modified by the presence of a product, in comparison to a two-digit number close in magnitude and identical for parity. Unlike in Experiments 1 and 2, in the present experiment only operands were relevant to the task, as participants were to decide whether they had the same parity or not, while ignoring the second event. This finding provides strong evidence that activation (or inhibition) spreads along associative links in the multiplication network in a purely stimulus-driven manner. The presence of a significant Trial Type × SOA interaction, showing that interference was reliable at the shortest SOA only, indicates that the associative link between the product and its operands is weaker than the opposite link, because activation spreading in the latter direction lasts longer (Fig. 5; Rusconi et al., in press).

General discussion

Multiplication tables are assumed to be the prototype of arithmetical facts and to be stored in long-term memory as associative networks. Both direct (production: Campbell, 1994; verification: Zbrodoff & Logan, 1986) and indirect (number matching: Rusconi et al., in press) tasks demonstrated that the presence of the multiplication sign and an explicit arithmetic processing mode do not represent necessary conditions to retrieve the product of two single-digit numbers. With three experiments we tested the hypothesis that activation can also spread from a product to its operands in the associative network. Also, a task-irrelevant product was shown to interfere with the processing of its operands during the execution of a parity-matching task.

In Experiments 1 and 2, the cue contained a single number and the target contained two numbers. The task was to indicate whether either number in the target matched the number in the cue. The key conditions were those in which the target contained two numbers whose product was represented by the number in the cue, and those in which the target contained two numbers that were both unrelated to the number in the cue. In Experiment 1 the cue was not masked and the SOA between the cue and the target was 65, 120, and 400 ms. In Experiment 2, the cue was followed by a visual mask and the SOA between the cue and the target was 120, 270 or 400 ms. In Experiment 1, participants were significantly slower in product trials than in unrelated trials at the shortest SOA. There was little difference between the two conditions at the intermediate SOA and a nonsignificant reversal of the interference effect at the longest SOA. In Experiment 2, participants were significantly slower in product trials at the shortest SOA, whereas a trend in the same direction was observed at the intermediate SOA, but not at the longest SOA.

Recall that the aim of Experiment 2 was to test the presence of interference by the product, as obtained in Experiment 1, by using experimental parameters—involving masking and SOA—directly comparable to those previously employed to test interference by the operands (Rusconi et al., in press). The apparently inconsistent pattern of interference across SOAs observed in the two experiments can be reasonably attributed to the intervention of controlled processing in Experiment 1, possibly favored by the absence of visual masking after the cue. Indeed, in the absence of a mask, participants may have become aware of the manipulation of arithmetic relatedness, at least when enough time was available for cue processing (i.e., at the longest SOA). This, in turn, may have resulted in participants actively trying to suppress the interfering information, which possibly led to a trend toward a reversal of interference. In Experiment 2, the presence of visual masking after the cue ensured that the possibility of aware processing was constant across SOAs. In Experiment 2, the interference effect was reliable at the shortest SOA, showed a trend toward significance at the intermediate SOA, and disappeared without reversing at the longest SOA.

The time course of this product-to-operands interference is different from that of the operands-to-product interference (Rusconi et al., in press). It might be concluded that product-to-operands-spreading activation is short-lived in comparison to operands-to-product activation, which is likely given that multiplication tables are far more practiced than operand decomposition of the products. However, an alternative explanation may also be evoked for accounting for the interference effect in Experiments 1 and 2 on the basis of the Kiger and Glass (1983) study. They found that, by presenting two stimuli in a sequence, the second stimulus influenced performance even when the task had to be performed on the first stimulus only, and that happened at very short SOAs. Following this line of reasoning, it might be suggested that interference in Experiments 1 and 2 resulted from activation spreading from the operands (second stimulus) to their product (first stimulus) and not vice versa. This interpretation is suggested by the observation that the number-matching paradigm, while possessing the advantage of not requiring arithmetic processing, has the disadvantage of necessarily requiring the processing of both the first and the second events, as participants are to compare these two stimuli before a response can be executed. In the light of this analysis, the results of Experiments 1 and 2 alone cannot be

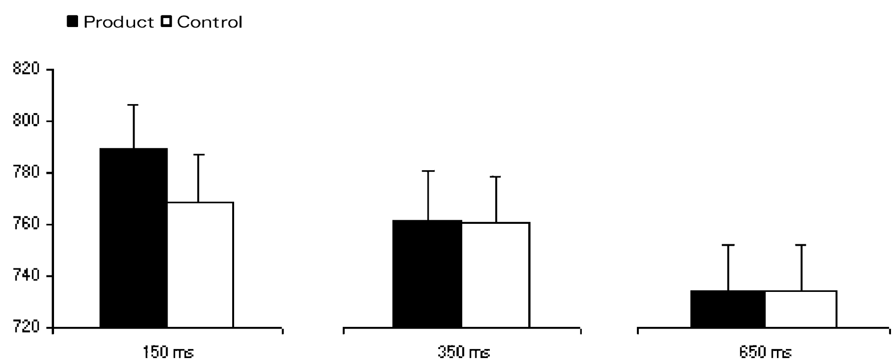

Fig. 5 The interference effect in Experiment 3 as a function of SOA. Conventions as in Fig. 2

interpreted as clear-cut evidence supporting the existence of an associative path running from the product to its operands.

In Experiment 3, only operands processing was necessary to perform the non-arithmetical task (parity matching), which represents a more stringent criterion for the assessment of a product-driven effect during processing of the operands. Moreover, we eliminated the possibility that activation spreading from the operands to their product affected performance, as the product was completely irrelevant to the task. Thus, finding any effect on performance due to the presence of arithmetical relatedness between the target operands and the preceding product prime could not be attributed to operands-to-product associative links.

Experiment 3 showed that parity matching was interfered with by the subsequent presentation of the product compared with an unrelated prime. This evidence would unequivocally demonstrate the existence of bidirectional links in the network of multiplication facts. In this last experiment, we replicated the same pattern of results as in Experiments 1 and 2 (i.e., lower performance in product trials than in control trials). Also, this pattern was present at the shortest SOA only, which resulted in a significant Trial Type × SOA interaction. Three levels of SOA were maintained, but their span covered longer durations. This allowed us to test whether, with the new paradigm, participants could strategically use the prime to anticipate the target digits. If this were the case, then we should have found performance to be higher in product trials than in control trials, at least at the longest SOA. However, no evidence of facilitation from the product was found in the new experiment. On the whole, the interference effect across the three experiments was reliable in the range between 65 and 150 ms.

As already pointed out in the introduction, an interesting characteristic of arithmetic facts is that a relation may exist between complementary operations, like multiplication and division. Thus, there can be problems that share all the number elements, but, depending on the operation, the same number element is either an operand or the result. The question arises, then, whether simple multiplications and divisions are represented independently or by means of the same nodes. Hittmair-Delazer et al. (1994) reported the case of a patient who showed similar impairment in simple multiplications and the complementary division problems. Successive rehabilitation of multiplication facts generalized to the complementary divisions. Also, evidence showing that division problem solving is often achieved through multiplication fact retrieval has been accumulating (e.g., Campbell, 1999; LeFevre & Morris, 1999; Mauro et al., 2003). For example, Campbell (1999) measured item-specific transfer between simple multiplication and division problems in a production task, and found significant facilitation of divisions when they were preceded by the complementary multiplication problem. Instead, division-to-multiplication transfer was absent. He hypothesized the presence of a strategy or a process, which would allow skilled adults to solve divisions by multiplications, at least under conditions in which potential mediators are readily accessible. Our data suggest that this putative mediation process may occur without the strategic use of conceptual knowledge and, rather, be based on bidirectional associative links in the multiplication network.

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