# Nonlinearity in normal human EEG: cycles, temporal asymmetry, nonstationarity and randomness, not chaos

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**Abstract.** Two-hour vigilance and sleep electroencephalogram (EEG) recordings from five healthy volunteers were analyzed using a method for identifying nonlinearity and chaos which combines the redundancy–linear redundancy approach with the surrogate data technique. A nonlinear component in the EEG was detected, however, inconsistent with the hypothesis of low-dimensional chaos. A possibility that a temporally asymmetric process may underlie or influence the EEG dynamics was indicated. A process that merges nonstationary nonlinear deterministic oscillations with randomness is proposed for an explanation of observed properties of the analyzed EEG signals. Taking these results into consideration, the use of dimensional and related chaos-based algorithms in quantitative EEG analysis is critically discussed.

## 1 Introduction

During the last decade there has been a sustained interest in describing neural processes and brain signals, especially the electroencephalogram (EEG), within the context of nonlinear dynamics and theory of deterministic chaos (see, e.g., in this journal Soong and Stuart 1989; Gallez and Babloyantz 1991; Röschke and Aldenhoff 1991; Fell et al. 1993; and for comprehensive reviews Rapp et al. 1989; Başar 1990; Jansen 1991; Freeman 1992). If the nature of the analyzed signals was actually low-dimensional, the published results could be of immense importance for theoretical neuroscience and neurologic and psychiatric clinical practice. However, confidence in results obtained from experimental data, such as finite dimensions or positive Lyapunov exponents, and the reliability of chaos-based algorithms in general, have recently come into question, and alternative methods for identifying possible nonlinear determinism in experimental time series have been proposed (Weigend and Gershenfeld 1993; Paluš 1995; and references therein).

This paper presents results from applying a method for identifying nonlinearity and chaos which combines the

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redundancy–linear redundancy approach with the surrogate data technique, to vigilance and sleep EEG recordings of approximately 2 h duration from five healthy volunteers. The method consists of a test for nonlinearity in which a null hypothesis of a linear stochastic process is tested by statistics based on nonlinear and linear redundancies (Paluš 1995). The presence of a signature of chaos, positive Kolmogorov–Sinai entropy (KSE), is assessed using the marginal redundancy approach (Paluš 1993). Employing these methods (briefly introduced in Sect. 2; for details refer to Paluš 1993, 1994a, 1995), a nonlinear component was detected in all analyzed EEG records; however, signatures of low-dimensional chaos were absent. Similar results have been independently reported by Pritchard et al. (1995), Rombouts et al. (1995) and Stam et al. (1995). Theiler and Rapp (1996), Prichard and Theiler (1994), Theiler et al. (1992) and Casdagli (1992) also rejected low-dimensional chaos and confirmed nonlinearity in the EEG; however, they reported only weak evidence for nonlinearity in the normal EEG. In this study very long EEG recordings were analyzed, which could be the source of greater statistical power of the tests performed, and thus stronger evidence for nonlinearity, than in the other studies. We also indicate that the EEG may possess temporal asymmetry as a nonlinear property.

A process that merges nonstationary nonlinear deterministic oscillations with randomness is proposed for an explanation of the observed properties of the EEG signals analyzed. Taking these results into consideration, the use of dimensional and related chaos-based algorithms in quantitative EEG analysis is critically discussed.

## 2 Principles of the methods

Consider $n$ discrete random variables $X_1, \ldots, X_n$ with sets of values $\Xi_1, \ldots, \Xi_n$, respectively. The probability distribution for an individual $X_i$ is $p(x_i) = \Pr\{X_i = x_i\}$, $x_i \in \Xi_i$. We denote the probability distribution function by $p(x_i)$, rather than $p_{X_i}(x_i)$, for convenience. Analogously, the joint distribution for the $n$ variables $X_1, \ldots, X_n$ is $p(x_1, \ldots, x_n)$. The redundancy $R(X_1; \ldots; X_n)$, in the case of two variables also known as mutual information $I(X_1; X_2)$, quantifies the average amount of common information contained in the $n$

variables $X_1, \ldots, X_n$:

$$R(X_1; \ldots; X_n)$$
$$= \sum_{x_1 \in \Xi_1} \cdots \sum_{x_n \in \Xi_n} p(x_1, \ldots, x_n) \log \frac{p(x_1, \ldots, x_n)}{p(x_1) \ldots p(x_n)} \quad (1)$$

The marginal redundancy $\varrho(X_1, \ldots, X_{n-1}; X_n)$ quantifies the average amount of information about the variable $X_n$ contained in the $n-1$ variables $X_1, \ldots, X_{n-1}$:

$$\varrho(X_1, \ldots, X_{n-1}; X_n)$$
$$= \sum_{x_1 \in \Xi_1} \cdots \sum_{x_n \in \Xi_n} p(x_1, \ldots, x_n) \log \frac{p(x_1, \ldots, x_n)}{p(x_1, \ldots, x_{n-1})p(x_n)}$$
$$(2)$$

The relation

$$\varrho(X_1, \ldots, X_{n-1}; X_n)$$
$$= R(X_1; \ldots; X_n) - R(X_1; \ldots; X_{n-1}) \quad (3)$$

can be derived by simple manipulation.

When the discrete variables $X_1, \ldots, X_n$ are obtained from continuous variables on a continuous probability space, then the redundancies depend on a partition $\xi$ chosen to discretize the space. Various strategies have been proposed to define an optimal partition for estimating redundancies of continuous variables (see Paluš 1995; Weigend and Gershenfeld 1993; and references therein). Here we use the 'marginal equiquantization' method described in detail in Paluš (1993, 1995). Pompe (1993) also describes this kind of partitioning.

Now, let the $n$ variables $X_1, \ldots, X_n$ have zero means, unit variances and correlation matrix $\mathbf{C}$. Then, we define the *linear redundancy* $L(X_1; \ldots; X_n)$ of $X_1, X_2, \ldots, X_n$ as

$$L(X_1; \ldots; X_n) = -\frac{1}{2} \sum_{i=1}^{n} \log(\sigma_i) \quad (4)$$

where $\sigma_i$ are the eigenvalues of the $n \times n$ correlation matrix $\mathbf{C}$.

If $X_1, \ldots, X_n$ have an $n$-dimensional Gaussian distribution, then $L(X_1; \ldots; X_n)$ and $R(X_1; \ldots; X_n)$ are theoretically equivalent (Morgera 1985).

Based on (3) we define the *linear marginal redundancy* $\lambda(X_1, \ldots, X_{n-1}; X_n)$, quantifying linear dependence of $X_n$ on $X_1, \ldots, X_{n-1}$, as

$$\lambda(X_1, \ldots, X_{n-1}; X_n)$$
$$= L(X_1; \ldots; X_n) - L(X_1; \ldots; X_{n-1}) \quad (5)$$

In practical applications one deals with a time series $\{y(t)\}$, considered as a realization of a stochastic process $\{Y(t)\}$, which is stationary and ergodic. Then, due to ergodicity, all the subsequent information-theoretic functionals are estimated using time averages instead of ensemble averages, and the variables $X_i$ are substituted as

$$X_i = y(t + (i-1)\tau) \quad (6)$$

Due to stationarity the redundancies

$$R^n(\tau) \equiv R(y(t); y(t+\tau); \ldots; y(t+(n-1)\tau)) \quad (7)$$
$$\varrho^n(\tau)$$
$$\equiv \varrho(y(t), y(t+\tau), \ldots, y(t+(n-2)\tau); y(t+(n-1)\tau)) \quad (8)$$

$$L^n(\tau) \equiv L(y(t); y(t+\tau); \ldots; y(t+(n-1)\tau)) \quad (9)$$
$$\lambda^n(\tau)$$
$$\equiv \lambda(y(t), y(t+\tau), \ldots, y(t+(n-2)\tau); y(t+(n-1)\tau)) \,(10)$$

are functions of $n$ and $\tau$, independent of $t$.

The surrogate-data based nonlinearity tests (Theiler et al. 1992; Schiff and Chang 1992) consist of computing a *nonlinear* statistic from the data under study and from an ensemble of realizations of a linear stochastic process which mimics 'linear properties' of the studied data. If the computed statistic for the original data is significantly different from the values obtained for the surrogate set, one can infer that the data were not generated by a linear process; otherwise the null hypothesis, that a linear model fully explains the data, is accepted. For the purpose of such test the surrogate data must preserve the spectrum[1] and, consequently, the autocorrelation function of the series under study. An isospectral linear stochastic process to a series can be constructed by computing the Fourier transform (FT) of the series, keeping unchanged the magnitudes of the Fourier coefficients but randomizing their phases and computing the inverse FT into the time domain. Different realizations of the process are obtained using different sets of the random phases.

To evaluate the test, the test statistic is defined as the difference between the redundancy obtained for the original data and the mean redundancy of a set of surrogates, in the number of standard deviations (SDs) of the latter. Thus both the redundancies and redundancy-based statistics can be evaluated as functions of the lag $\tau$ and the embedding dimension $n$.

The redundancy $R^n(\tau)$ [or, $\varrho^n(\tau)$], based on probability distributions, measures general dependences among the series $\{y(t)\}$ and its lagged versions, whereas the linear redundancy $L^n(\tau)$ [or, $\lambda^n(\tau)$], based on correlations, reflects only their linear relations. Comparing the plots of $R^n(\tau)$ and $L^n(\tau)$ (or $\varrho^n(\tau)$ and $\lambda^n(\tau)$) can provide an informal test for important nonlinearities in the studied data (Paluš et al. 1993). This approach, in Paluš (1995) referred to as *qualitative testing*, or *qualitative comparison*, can bring some additional information to the results of the quantitative (surrogate-data-based) test. Moreover, one can not always construct good surrogate data. That is, despite theoretical expectations, in numerical practice linear properties of the surrogates may differ from those of the data under study. Changes in linear properties are reflected in nonlinear statistics as well, and thus may result in spurious detection of nonlinearity in linear data (Paluš 1995). Therefore, we also evaluate the statistic based on the linear redundancy $L^n(\tau)$, which specifically reflects changes in linear properties. Then, only those significant differences in the nonlinear statistic can reliably count for nonlinearity that are not detected in the linear statistic (Paluš 1995).

If a time series $\{y(t)\}$ was generated by an $m$-dimensional chaotic system, then for $n > m$ and under some conditions the marginal redundancy $\varrho^n(\tau)$ decreases with $\tau$ as

$$\varrho^n(\tau) \approx H_1 - |\tau|h \quad (11)$$

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[1] Also, preservation of histogram is usually required. A histogram transformation used for this purpose is described in Paluš (1995) and references within.

where $h$ is the KSE of the underlying dynamical system and $H_1$ is a constant related to precision (quantization) of data (Fraser 1989; Paluš 1993). Fulfilling the conditions for (11) can be limited by the amount of data available (Paluš 1994a, 1996). Therefore we propose a comparison of the slopes of the redundancy $\varrho^n(\tau)$ and the linear redundancy $\lambda^n(\tau)$ [or, the slopes of $\varrho^n(\tau)$ obtained from the data under study and $\varrho^n(\tau)$ from its surrogates] to find whether the decrease in $\varrho^n(\tau)$ is a nonlinear (possibly chaotic) phenomenon or can be explained by a linear stochastic process. The latter case renders a hypothesis of deterministic chaos improbable.

## 3 Material and methods

Five paid, physically and psychically healthy volunteers (Table 1), chosen from a larger group by passing standard EEG-study criteria, participated in the experiment giving written informed consent. The five subjects were recorded in two different states:

1. Relaxed vigilance with closed eyes. Vigilance was assured by the volunteers' participation in a simple mental task: subjects were asked to listen to a text recorded on a tape-recorder and to count occurrences of the conjunction 'but'. (For brevity, the term 'vigilant state' will be used henceforth to refer to this state.)
2. Sleep evoked by a 1 or 2 mg dose (according to individual sensitivity) of flunitrazepam (tablets). (Henceforth this will be referred to as 'sleep state'.)

Eight channels of EEG data, sampled by 256-Hz frequency and 12-bit precision, were recorded from the positions $O_1$, $O_2$, $T_5$, $T_6$, $F_7$, $F_8$, $P_z$ and $F_z$, using the common Goldman reference electrode. Each record consists of ninety 90-s segments (90 $\times$23 040 samples).

The marginal redundancies $\lambda^n(\tau)$ and $\varrho^n(\tau)$ and the related linear and nonlinear statistics were calculated from the redundancies $L^n(\tau)$ and $R^n(\tau)$, estimated from the data and the surrogates (30 realizations of the surrogates for each dataset). Due to the segmentation of the data, the n-tuples $y(t); y(t + \tau); \ldots; y(t + (n - 1)\tau)$ were constructed only inside the individual segments, whereas final probability distributions and the correlation matrices were averaged through all the segments. For the qualitative comparisons, the original–data redundancies $R^n(\tau)[o]$ and $L^n(\tau)[o]$ were computed from entire 23 040-sample segments, which gives the effective series length $N = 2\,037\,600$, given embedding dimensions $n = 2$–5, a lag range 1–100 and the segmentation. In the surrogate data tests 16 384 (16k)-sample subsegments were used, employing fast Fourier transform (Press et al. 1986), requiring a series length equal to a power of 2. This yields the effective series length $N = 1\,438\,560$ applicable for the redundancy estimations. The surrogate data were generated for each segment separately, i.e., using 16 384 samples. Gaussianization (Paluš 1995) was performed in the quantitative analysis; before the analysis, the data were filtered using a notch filter in the frequency domain to eliminate interference from the recording equipment.

All the redundancies were computed for embedding dimensions $n = 2$–5 using $Q = 4$–16 marginal equiquantal (equipopulated) bins. In the qualitative testing, the redundancies were also computed with $Q$ up to 64, but $n$ was only 2 and 3.

## 4 Results

Figure 1 presents the analysis results of the EEG of subject 2 recorded in position $O_1$ in the vigilant state. In the qualitative comparison there are practically no differences between the linear marginal redundancy $\lambda^n(\tau)$ (Fig. 1a) and the marginal redundancy $\varrho^n(\tau)$ (Fig. 1b). The result of quantitative analysis is different: there are several highly significant differences (about 10 SDs) detected in the statistic based on nonlinear redundancy (Fig. 1d). The possibility that these
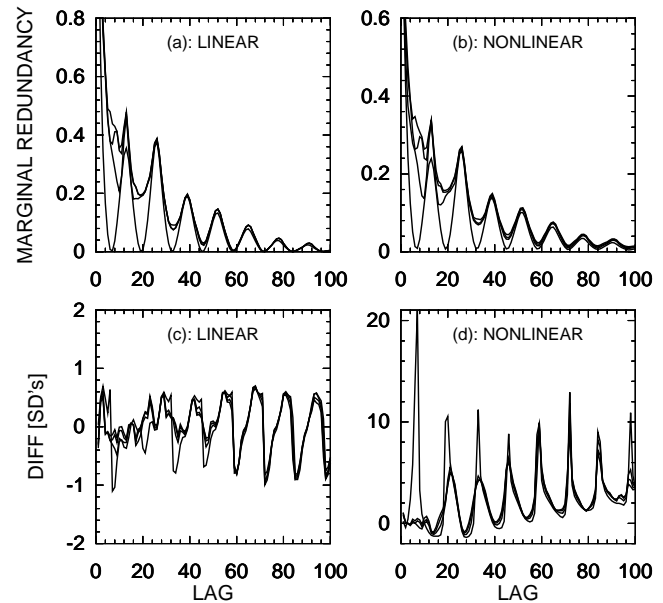


**Fig. 1. a** Linear marginal redundancy and **b** marginal redundancy, as functions of time lag, for the vigilance electroencephalogram (EEG) of subject 2 recorded from position $O_1$. The four different *curves* in each figure are the redundancies for different embedding dimensions, $n = 2$–5, reading from bottom to top. **c** Linear redundancy statistic, and **d** nonlinear redundancy statistic, as functions of time lag and for embedding dimensions $n = 2$–5, for the same EEG record

significances are spurious (caused by imperfect surrogates) is ruled out by the results in the statistic based on linear redundancy, where no significant differences (differences between –1 and 1 SD) were found (Fig. 1c). The hypothesis of a linear stochastic process was thus rejected.

Looking at the nonlinear redundancy statistic (Fig. 1d) in detail, one can see that the significant differences are periodically distributed and located in the lags in which $\varrho^n(\tau)$ has its minima. This means that $\varrho^n(\tau)$ is *significantly* greater than zero also in its minima, which is not apparent from Fig. 1b, especially in the case of $\varrho^2(\tau)$. Thus, $\varrho^2(\tau) > 0$ and the variables $x(t)$ and $x(t + \tau)$ are dynamically (deterministically) connected for the whole range of considered lags, i.e., up to 100 samples, or 390 ms. In the case of the surrogates, $\varrho^2(\tau)$ vanishes in its minima. For other lags the differences between the EEG data and the surrogates are not significant. This phenomenon, typical for *nonlinear deterministic oscillations* (Paluš 1995), will be called *periodic nonlinearity*.

The long-term decreasing trend in $\varrho^n(\tau)$ is exactly reflected also in linear $\lambda^n(\tau)$; and is fully explained by the isospectral linear stochastic process, because there is no indication of a decreasing pattern in the time-lag dependence of differences (Fig. 1d; cf. Fig. 3d in Paluš 1994a).

The results in Fig. 1b and d were obtained using $Q = 4$ marginal equiquantal bins. The tests using finer partitions, possible due to the large amount of data available, were also performed (see Sect. 3). Using finer partitions did not, however, bring any new information in either the qualitative comparison or the quantitative analysis (Paluš 1994b). This phenomenon is typical for all EEG data analyzed.

Results similar to those depicted in Fig. 1 and described above, were obtained from the EEG signals of all five sub-
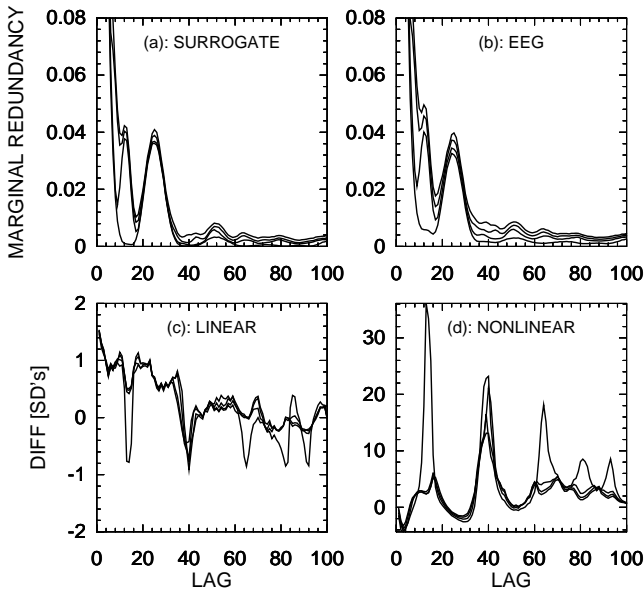
**Fig. 2.** Marginal redundancy for **b** the sleep EEG of subject 1 from position $O_1$ and **a** its surrogates, as functions of the time lag. The four different *curves* in each figure are the redundancies for different embedding dimensions, $n = 2$–5, reading from bottom to top. **c** Linear redundancy statistic, and **d** nonlinear redundancy statistic, as functions of the time lag and for embedding dimensions $n = 2$–5, for the same EEG record
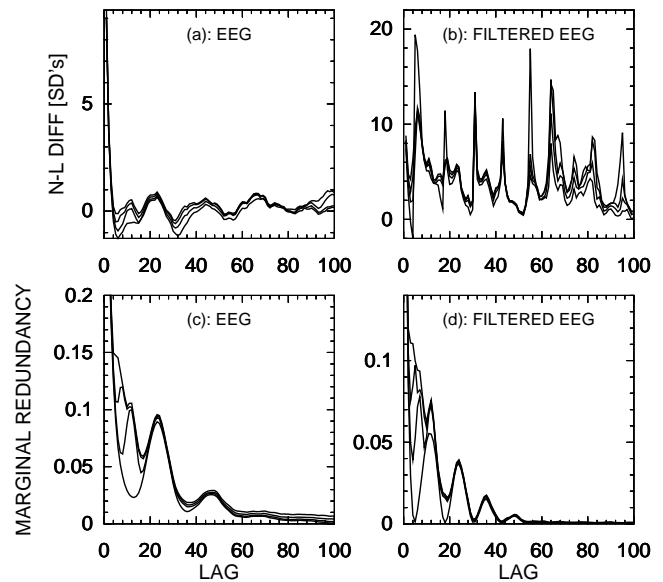
**Fig. 3. a, b** Nonlinear redundancy statistic and **c, d** marginal redundancy for the vigilance EEG of subject 1 from position $F_7$, before **a, c** and after **b, d** high-pass filtering, as functions of time lag and for embedding dimensions $n = 2$–5

jects recorded in the vigilant state from positions $O_1$, $O_2$, $T_5$, $T_6$, $P_z$ and $F_z$, i.e., from all channels except the two frontal ones ($F_7$ and $F_8$). For the sleep state and the above six channels we can also present one typical example. The analysis results obtained from the EEG of subject 1, recorded in the sleep state from position $O_1$, are presented in Fig. 2. In general, the sleep EEGs are less coherent than the vigilance ones, the values of the redundancies are about 10 times smaller and decrease faster, i.e., the coherence length[2] is shorter. The results of the nonlinearity tests, however, are the same as those for the vigilance EEG, described above: no qualitative differences between $\varrho^n(\tau)$ computed from the EEG and from the surrogates[3] were found (Fig. 2a,b). No significant differences (from –1 to 1.6 SDs) were found in the linear redundancy statistic (Fig. 2c); but several highly significant differences (tens of SDs) were detected in the nonlinear redundancy statistic (Fig. 2d). These significant differences are located in the lags in which $\varrho^n(\tau)$ has its minima.

Properties of the EEG signals recorded in the frontal loci ($F_7$ and $F_8$; note that $F_z$ was included in the above six channels) in both the vigilant and sleep states are similar to those obtained from other channels in the sleep state, with two differences:

1. In addition to the periodic nonlinearity, significant differences in 'short' lags, i.e., in the lags 1–2 samples (4–

8 ms) or 1–3 samples (4–12 ms) were detected (Fig. 3b). We will henceforth refer to this phenomenon as 'short-lag nonlinearity'.

2. In three subjects in both the vigilant and sleep states, this short-lag nonlinearity was the only significant difference detected (Fig. 3a). We observed, however, that the spectra of these records are dominated by slow activity.[4] When this slow activity was eliminated by a digital filter in spectral domain, a typical pattern of periodic nonlinearity emerged. The short-lag nonlinearity, however, was not eliminated.

An example of this phenomenon is presented in Fig. 3. The analysis results of the vigilant state EEG of subject 1, recorded in position $F_7$, are depicted in Fig. 3a,c. (We do not present $\varrho^n(\tau)$ from the surrogates, which is not qualitatively different from $\varrho^n(\tau)$ obtained from the EEG, nor the linear redundancy statistic, in which no significant differences were found.) In the nonlinear statistic the only significant differences are located in the lags 1–2 samples (Fig. 3a). The lag-dependence of $\varrho^n(\tau)$ (Fig. 3c), however, is similar to that presented in Fig. 2b, showing the typical sleep state result. After eliminating the slow activity from the signal, the lag dependence of $\varrho^n(\tau)$ is close to the typical vigilance pattern (Fig. 1), but the coherence length is shorter (Fig. 3d). Results from the quantitative test are also typical, i.e., significant differences in the minima of $\varrho^n(\tau)$ were detected, but with the addition of significant differences in the lags 1–2 samples (Fig. 3b).

In general, the properties of EEG signals recorded in frontal positions $F_7$ and $F_8$ are not different from the properties of other EEG signals described above. The only difference is the short-lag nonlinearity, which is typical for these channels but can also be found in other locations (Table 1).

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[2] Under the coherence length we understand here such time lag $\gamma$ that $\varrho^2(\tau) > 0$ significantly $\forall \tau \leq \gamma$, where $\gamma, \tau \geq 0$.

[3] The comparison of $\varrho^n(\tau)$ computed from the EEG data with $\varrho^n(\tau)$ from the surrogates is equivalent to the comparison of $\lambda^n(\tau)$ with $\varrho^n(\tau)$ from the EEG, and may be even more reliable (Paluš 1995). Therefore we will use the former in all remaining tests presented here. $\varrho^n(\tau)$ from the surrogates means the mean values of $\varrho^n(\tau)$ obtained from 30 realizations of the surrogates.

[4] Unfortunately we were not able to determine whether it was slow EEG activity or eye artifacts, but the latter are more probable.
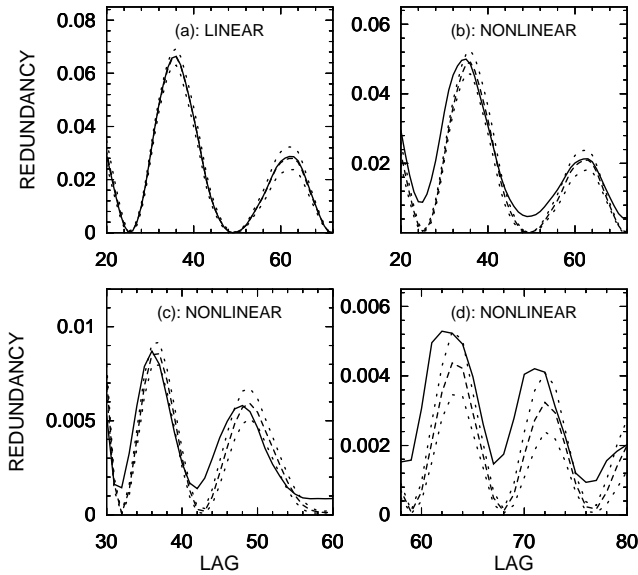
**Fig. 4. a** Linear $\lambda^2(\tau)$ and **b** nonlinear marginal redundancy $\varrho^2(\tau)$ for the sleep EEG of subject 5 from position $P_z$ (*continuous lines*) and its surrogates (mean value, *dashed lines*; mean $\pm$ SD, *dotted lines*). **c, d** The same as **b**, but **c** for the vigilance EEG of subject 1, position $O_1$ and **d** for the simulated data with gradually changing fast frequency during oscillatory episodes, discussed in Sect. 5

**Table 1.** Basic characteristics of the subjects and results of the analysis

| | Subject | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | | 4 | | 5 | |
| Age (years) | 43 | | 40 | | 44 | | 31 | | 20 | |
| Sex | F | | F | | M | | F | | M | |
| $O_1$ – vigil | P | | P | | P | | P | | P | |
| $O_1$ – sleep | P | | P | | P | | P | S | P | |
| $O_2$ – vigil | P | | P | | P | | P | | P | |
| $O_2$ – sleep | P | | P | | P | | P | S | P | |
| $T_5$ – vigil | P | S | P | S | P | | P | S | P | S |
| $T_5$ – sleep | P | | P | S | P | S | P | S | P | S |
| $T_6$ – vigil | P | S | P | S | P | | P | S | P | S |
| $T_6$ – sleep | P | | P | S | P | S | P | S | P | S |
| $P_z$ – vigil | P | | P | | P | | P | S | P | S |
| $P_z$ – sleep | P | | P | | P | | P | S | P | |
| $F_z$ – vigil | P | | P | | P | | P | S | P | S |
| $F_z$ – sleep | P | | P | | P | S | P | S | P | |
| $F_7$ – vigil | F | S | P | S | F | S | F | S | P | S |
| $F_7$ – sleep | F | S | P | S | F | S | F | S | P | S |
| $F_8$ – vigil | F | S | P | S | F | S | F | S | P | S |
| $F_8$ – sleep | F | S | P | S | F | S | F | S | P | S |

F, female; M, male; vigil, the vigilant state; sleep, the sleep state; P, periodic nonlinearity; F, periodic nonlinearity detected after high-pass filtering; S, short-lag nonlinearity

Defining qualitative differences between $\varrho^n(\tau)[\text{o}]$ from the original data and $\varrho^n(\tau)[\text{s}]$ from the surrogates is sometimes problematic. Above we have stated that no such differences were found in analysis of the EEG data studied. This is true in the sense of the definition of qualitative differences given in Paluš (1995), which emphasizes differences in ranges of lags in which $\varrho^n(\tau)$ is positive, locations of extrema and long-term trends of $\varrho^n(\tau)$. Looking at more subtle details, however, one can discover an interesting feature which differentiates the nonlinear oscillations in the EEG from trivial periodic nonlinearity, described in Paluš (1995). For such a detailed comparison, we have plotted in one figure (Fig. 4b) $\varrho^2(\tau)[\text{o}]$ from the sleep EEG data of subject 5, recorded in position $P_z$ (continuous line) and $\varrho^2(\tau)[\text{s}]$ from its surrogates (mean, dashed line; mean $\pm$ SD, dotted lines). We can observe an asymmetry in the peaks of $\varrho^n(\tau)[\text{o}]$ compared with those of $\varrho^n(\tau)[\text{s}]$. The fact that this is a nonlinear phenomenon and not an artifact of imperfect surrogates, is demonstrated by comparing $\lambda^2(\tau)[\text{o}]$ and $\lambda^2(\tau)[\text{s}]$ (Fig. 4a; line-type codes as in Fig. 4b): these coincide, thus linear properties of the surrogates are the same as those of the EEG data.

This phenomenon can be found also in other subjects in both the vigilant and sleep states, but in most cases the difference is very small and almost invisible. Another example, when this asymmetry is easily detectable, is the vigilant state EEG from position $O_1$ of subject 1 (Fig. 4c). In Fig. 4d this phenomenon is observed in numerically generated data, discussed in Sect. 5.

In summary, all the EEG records possess a nonlinearity. In particular, a type of periodic nonlinearity was detected in all the EEG records (although in some of the EEGs from frontal channels $F_7$ and $F_8$ it was not detected in the raw data but after high-pass filtering). EEGs from the frontal channels $F_7$ and $F_8$ and from certain other channels also possess the short-lag nonlinearity. These results are summarized in Table 1.

## 5 Discussion

*Periodic nonlinearity*, detected in practically all the EEG records analyzed, was characterized by significant differences located in the lags in which the redundancy $\varrho^n(\tau)$ has its minima, while the relations between $y(t)$ and $y(t+\tau)$ in other lags were well reproduced by the linear stochastic surrogates. Also, the linear stochastic surrogates mimic the EEG signals very well from the viewpoint of qualitative comparison. This kind of behavior was observed in periodic processes related to dynamics on limit cycles or tori, while 'specifically nonlinear dynamics', like evolution on strange attractors, was characterized by both qualitative differences and significant results of the quantitative tests in entire ranges of studied lags (Paluš 1994a, 1995). On the other hand, regular periodic processes have zero KSE and thus no long-term decreasing trend in $\varrho^n(\tau)$ (Paluš 1993). Dynamical information loss, measured by a long-term decrease in $\varrho^n(\tau)$, however, is not a phenomenon specific only to chaotic dynamics, but can be caused by any kind of dynamical noise, such as that in autoregressive processes (Paluš 1994a, 1996), or by a nonstationarity or fluctuation in some parameter of a dynamical system that might otherwise be regular, i.e., have zero KSE (Paluš 1993). The long-term decreasing trend in $\varrho^n(\tau)$ detected in all EEG records analyzed, has been found consistent with the linear stochastic hypothesis. Thus, studied EEG signals can be a mixture of nonlinear deterministic oscillations, linear stochasticity and/or nonstationarity. In particular, in a numerical study the following simulated time series were constructed: Episodes of two-frequency oscillations of the form

$$y(t) = A \sin(\omega_1 t) \sin(\omega_2 t) \tag{12}$$
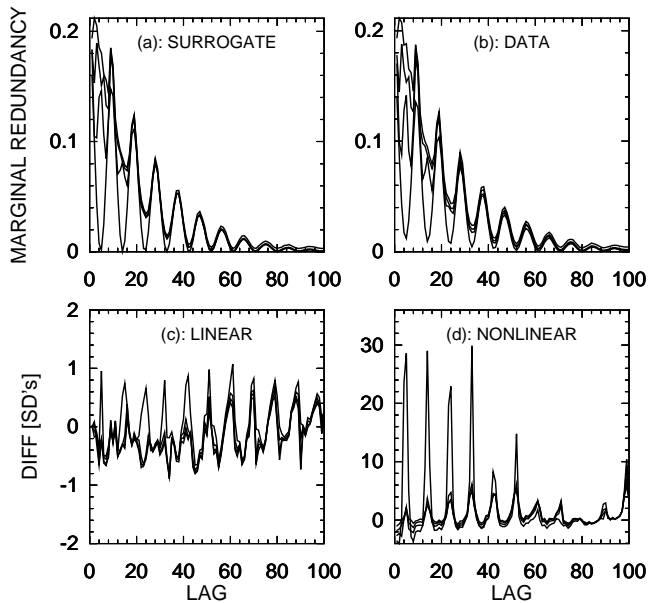
**Fig. 5.** Marginal redundancy for **b** the simulated time series discussed in Sect. 5 and **a** its surrogates, as functions of time lag. The four different *curves* in each figure are the redundancies for different embedding dimensions, $n = 2$–$5$, reading from bottom to top. **c** Linear redundancy statistic, and **d** nonlinear redundancy statistic, as functions of time lag and for embedding dimensions $n = 2$–$5$, for the same simulated series

where $\omega_1 = 2\pi/T_1$ represents fast dominant oscillations (e.g., $T_1 = 0.1$ s) and $\omega_2 = 2\pi/T_2$ represents a slower 'envelope' (e.g., $T_2 = 0.5$–$1.0$ s), were randomly distributed in a background of Gaussian noise. Durations of the episodes were random, varying from $0.8T_2$ to $2.2T_2$. Also other parameters were randomized: The amplitude $A$ as $A_t = A_0 + \alpha_t$, the argument of the 'envelope sine' as $\omega_2 t + \xi_t$ and the fast frequency $\omega_1$ as $\omega_1(t) = \omega_0 + o_t$, where $\alpha_t$, $\xi_t$ and $o_t$ are Gaussian deviates with zero means and variances of about 20% of $A_0$, $2\pi/3$ and 10% of $\omega_0$, respectively. Finally, the spectra of these artificial signals are more realistic (i.e., more closely resemble real EEG spectra), when the background noise is not white but filtered into a $1/f$ type. Tuning the above parameters (the variances of the random components, the durations of the episodes, the frequency of episode occurrence, and the type of the background noise) one can produce signals which, when analyzed, yield results practically indistinguishable from the results obtained from various EEG recordings. An example of results from such an artificial time series is presented in Fig. 5: $\varrho^n(\tau)$ of the simulated data (Fig. 5b) and of its surrogates (Fig. 5a), the linear redundancy statistic (Fig. 5c) and the nonlinear redundancy statistic (Fig. 5d). These results are practically the same as those obtained from the vigilance EEG recordings from all but the frontal channels (Fig. 1). By changing the background noise, in particular by increasing the spectral power of slow frequencies relatively to the power of the main $\omega_1$ oscillations, the 'vigilant state artificial EEG' can be changed into the 'sleep state artificial EEG'; i.e., the results become similar to those presented in Fig. 2.

When the fast oscillation frequency $\omega_1$ is randomized as described above (or constant) no asymmetries such as those in Fig. 4 are observed. In an additional study we found that such asymmetries may occur when the frequency $\omega_1$ changes gradually during the oscillatory episode. In particular, we used linearly changing period $T_1$ as $T_1(t) = T_0 + ct$, where $t$ is the time from the beginning of an oscillatory episode. The sign of the parameter $c$ determines on which side of a peak of $\varrho^2(\tau)$ the asymmetry occurs: when $c$ is negative the asymmetry is located on the left side of the peaks, as in the cases presented in Fig. 4; when $c$ is positive, the asymmetry is located on the right side. Thus, if the asymmetries detected in the real EEG data were indeed caused by this mechanism, related $c$ values are negative and $T_1$ values decrease, or the dominant frequency (alpha) increases during the oscillatory episodes (alpha spindles).

The process described above is obviously asymmetric with respect to time. Although the actual dynamical mechanisms underlying the EEG can be different,[5] the temporal asymmetry can be detected in the real EEG dynamics.[6] Recently, Chialvo and Millonas (1995) have pointed out the importance of temporally asymmetric fluctuations for biological energy transduction. The possible existence of and a role for temporally asymmetric processes in brain dynamics could be an interesting subject for research.

The simulated time series, which combine nonstationary deterministic oscillations with randomness, namely, in which oscillatory episodes with some randomized parameters are randomly distributed in a noisy background, can mimic all properties of the real EEG signals analyzed.[7] Of course, this is not evidence that this 'model'[8] explains real EEG dynamics, although it seems more probable than the hypothesis of low-dimensional chaos. Without insisting on a particular model or hypothesis, we can conclude that in the analyzed EEG recordings nonlinearity was detected, though not consistent with the hypothesis of low-dimensional chaos. Considering this result, applications in computerized EEG analysis of algorithms, designed for characterization of low-dimensional chaotic systems, should be critically reassessed.

Some authors (Mayer-Kress and Layne 1987; Koukkou et al. 1993; Wackermann et al. 1993; Lutzenberger et al. 1995) no longer insist on interpreting their results, such as finite dimension estimates, as evidence for underlying low-dimensional chaos, but propose these measures, particularly the correlation dimension (CD) (Grassberger and Procaccia 1983), as measures for relative characterization of different

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[5] A generalized Wiener process with a nonzero drift can be considered as a plausible stochastic alternative to the above deterministic drift in $T_1$ ($\omega_1$).

[6] The asymmetry in the peaks of $\varrho^2(\tau)$ is not necessarily equivalent to the temporal asymmetry of the underlying process (time series), and the redundancy $\varrho^2$ (mutual information $I$) is not a measure suitable for detecting the temporal asymmetry, due to its symmetry properties [$I(\xi;\eta) = I(\eta;\xi)$]. Considering $\xi = y(t)$ and $\eta = y(t+\tau)$, the temporal asymmetry of a series $\{y(t)\}$ can be detected by the significant difference between the probability distributions $p(\xi,\eta)$ and $p(\eta,\xi)$, quantified by the Kullback-Leibler information $KLI = \sum_{\xi}\sum_{\eta}[p(\xi,\eta) - p(\eta,\xi)]\log[p(\xi,\eta)/p(\eta,\xi)]$. Application of this approach led to positive detection of the temporal asymmetry in the EEG; however, only a part of the EEG data described in this paper was processed using the $KLI$ measure. Therefore the temporal asymmetry in the EEG is reported here only as a possibility, which requires further study before it is considered a general feature of the EEG.

[7] Except that of the short-lag nonlinearity, detected in a part of the datasets (Table 1).

[8] The process described is not considered as a model generating EEG, like, for example, that proposed by Jansen and Rit (1995), nor as a time-series model estimated from EEG data (e.g., Jimenez et al. 1995).

EEG recordings. Although these authors demonstrate that such CD estimates may have some discriminating power with respect to EEGs recorded in different physiologic or psychologic conditions, considering that the underlying processes can be stochastic, these low numbers, formally obtained from dimensional algorithms, are probably spurious and have no theoretically justified meaning and interpretation. Moreover, it can hardly be established how robust or sensitive measures such as CD are when they are used in the relative characterization of processes the dimensionality of which can be effectively infinite. For these reasons we do not consider the dimensional and Lyapunov exponents algorithms, based on the particular hypothesis of a low-dimensional attractor, to be promising in computerized EEG analysis. Nonlinear methods, which quantify the real existing properties of a process under study, should be used. For instance, differences in coherence lengths of various EEG records can induce differences in estimates of CD or Lyapunov exponents. The coherence length of a process is related to its entropy rate and thus can be directly quantified, e.g., by an 'approximate entropy' (Pincus et al. 1991) or by 'coarse-grained entropy rates' (Paluš 1996), which can be applied to both deterministic–chaotic and stochastic systems, and are designed specifically for noisy, finite-precision and finite-length experimental data.

## 6 Conclusion

Two-hour vigilance and sleep EEG recordings from five healthy volunteers were analyzed using a method for identifying nonlinearity and chaos which combines the redundancy – linear redundancy approach with the surrogate data technique. A nonlinear component in the EEG was detected, but was found inconsistent with the hypothesis of low-dimensional chaos. A possibility that a temporally asymmetric process may underlie or influence the EEG dynamics was indicated. A process which merges nonstationary nonlinear deterministic oscillations with randomness, namely in which deterministic oscillatory episodes are randomly distributed in a noisy background, has been proposed for an explanation of observed properties of the analyzed EEG signals. Considering these findings, the use of dimensional and related chaos-based algorithms in the quantitative EEG analysis was criticized, and measures applicable also to nonlinear stochastic processes were proposed instead. However, any EEG study in which authors apply nonlinear measures should also present results of elaborate analysis using linear methods, and/or test hypotheses[9] using isospectral surrogate data, in order to demonstrate that nonlinear measures are indeed necessary to quantify the phenomena under study.

[9]Let A and B be two groups of EEG recordings. Positive identification of nonlinearity in individual recordings is not sufficient evidence for assertions such as 'The groups A and B can be distinguished only by a nonlinear measure and not by linear methods'. We propose also the comparison of the surrogates of group A with the surrogates of group B using the same nonlinear measure as for the comparison of the original data.

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