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Unraveling the finding of $1/f^{\beta}$ noise in self-paced and synchronized tapping: a unifying mechanistic model

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Abstract $1/f^{\beta}$ noise has been revealed in both self-paced and synchronized tapping sequences, without being consistently taken into consideration for the modeling of underlying timing mechanisms. In this study we characterize variability, short-range, and long-range correlation properties of asynchronies and inter-tap intervals collected in a synchronization tapping experiment, attesting statistically the presence of $1/f^{\beta}$ noise in asynchronies. We verify that the linear phase correction model of synchronization tapping in its original formulation cannot account for the empirical long-range correlation properties. On the basis of previous accounts of $1/f^{\beta}$ noise in the literature on self-paced tapping, we propose an extension of the original synchronization model by modeling the timekeeping process as a source of $1/f^{\beta}$ fluctuations. Simulations show that this '$1/f$-AR synchronization model' accounts for the statistical properties of empirical series, including long-range correlations, and provides an unifying mechanistic account of $1/f^{\beta}$ noise in self-paced and synchronization tapping. This account opens the original synchronization framework to further investigations of timing mechanisms with regard to the serial correlation properties in performed time intervals.

Keywords Linear phase correction · Long-range correlation · Self-pacing · Synchronization · Tapping · Timing

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1 Introduction

Biological rhythms are the outcome of complex systems with multiple interacting processes. Analyzing the variability of rhythms generated by diverse functions in the human organism has revealed the presence of persistent long-range correlation, or $1/f^{\beta}$ noise. This recurrent result was evidenced notably in heart rate (e.g., Peng et al. 1995, 1999; West et al. 1999), respiratory cycles (Fadel et al. 2004; Peng et al. 2002), stride intervals in gait (Hausdorff et al. 1995, 1996; West and Scafetta 2003), brain activity (e.g., Bédard et al. 2006; Novikov et al. 1997), or in the production of rhythmic movements (Gilden et al. 1995; Gilden 2001; Delignières et al. 2004; Delignières et al. 2008; Madison 2004; Pressing and Jolley-Rogers 1997; Yamada 1995). $1/f^{\beta}$ noise denotes a very specific kind of variability defined by two main features: First, series of successive measures present persistent long-range correlation. Taking rhythmic movement as an example, the current period is positively correlated with a large set of previous values, and the series is said to possess long-term memory as opposed to the short-term dependence typical of ARMA processes. Second, series of successive measures are self-similar: their statistical properties remain unchanged whatever the scale of measurement, so that the process has no characteristic time scale. These features are notably revealed by the auto-correlation function of series that exhibits a slow power-law decay, and the power spectrum in bi-logarithmic coordinates that shows a linear shape with a slope close to -1.

In the present article we focus on timing mechanisms, and the finding of $1/f^{\beta}$ noise in rhythmic self-paced and externally paced finger tapping. Self-paced tapping requires an internal regulation of timing, which has been supposed to involve a central timekeeping process (Wing and Kristofferson 1973). Numerous studies have shown that series of inter-tap intervals (ITI) in self-paced tapping contained $1/f^\beta$ noise (Chen et al. 2002; Delignières et al. 2004; Gilden et al. 1995; Gilden 2001; Madison 2004; Yamada 1995), which was assumed to represent the variability inherent to the internal timekeeping process (Gilden et al. 1995; Gilden 2001; Delignières et al. 2004; Delignières et al. 2008). In this view, Delignières et al. (2008) proposed to provide the timekeeper with $1/f$ properties using the so-called *shifting strategy model*, and showed that a combination of this fractal timekeeper and the original Wing and Kristofferson (1973)'s model allowed to account for the statistical properties typically observed in self-paced tapping.

In externally paced tapping, the goal is to maintain a constant phase relationship between the taps and the metronome signals. In particular, synchronization consists in tapping in-phase with the metronome. Analyses focus on series of asynchronies (ASYN), defined as the time intervals between the taps and the metronome. In this condition, $1/f^\beta$ noise has been evidenced in ASYN series; ITI series, in contrast, present anti-persistent (negative) correlations (Chen et al. 1997, 2001, 2002; Ding et al. 2002; Pressing and Jolley-Rogers 1997). These two results are consistent, as ITI series correspond to the differentiation of ASYN series, plus the constant period (τ) imposed by the metronome:

$$\mathrm{ITI}_n = \mathrm{ASYN}_{n+1} - \mathrm{ASYN}_n + \tau \tag{1}$$

Vorberg and collaborators (Vorberg and Schulze 2002; Vorberg and Wing 1996) proposed a linear phase correction model for synchronization tapping, as an extension of the Wing and Kristofferson model. According to its simplest formulation, asynchronies are locally corrected by a first-order auto-regressive process, without modifying the functioning of the timekeeper:

$$\mathrm{ASYN}_{n+1} = (1-\alpha)\mathrm{ASYN}_n + K_n - \tau \tag{2}$$

In this model τ represents the constant time intervals imposed by the metronome, and K_n the inter-tap intervals predicted by the Wing and Kristofferson model for self-paced tapping, as limit case without any effective correction process ($\alpha = 0$):

$$K_n = C_n + M_{n+1} - M_n \tag{3}$$

Here, C_n are the time intervals prescribed by the central timekeeper, delimited by the consecutive discrete events (n and $n+1$), and M_n represents the motor delays that affect each tap associated to a central event. C_n and M_n are defined as two independent white noise processes (Wing and Kristofferson 1973; Vorberg and Wing 1996; Vorberg and Schulze 2002).

Despite of an important amount of experimental studies and analytical works based on this model, the way the assumed timing processes involved in synchronization tapping could account for the observed $1/f^\beta$ noise has been largely disregarded so far (for an exception, see Pressing and Jolley-Rogers 1997). One reason for this might be a still hesitant and doubtful approach of the $1/f^\beta$ phenomenon, and what it represents. For instance, following the idea that $1/f^\beta$ noise represents simple background noise inherent to the timing system, one might indeed question 'Why modeling the errors of a clock'? Another, more insidious reason may be an unclear vision of how to deal with $1/f^\beta$ noise, which seems actually related to the existence of controversial perspectives on the $1/f^\beta$ phenomenon (see Kello et al. 2007; Torre and Wagenmakers 2008; Van Orden et al. 2003; Wagenmakers et al. 2005). On the one hand, a *nomothetic* perspective seeks generic principles accounting for $1/f^\beta$ noise, given its ubiquity. In this view, $1/f^\beta$ noise has often been associated with concepts such as complex systems, emergent dynamics, metastability, and self-organized criticality (Van Orden et al. 2003; Kello et al. 2007). On the other hand, a *mechanistic* perspective seeks domain-specific and experimentally testable (and thus falsifiable) models of the processes that generate $1/f^\beta$ noise (Delignières et al. 2008; West and Scafetta 2003). Beyond the theoretical issue raised by these contrasted perspectives, they also affect the way to deal with $1/f^\beta$ noise in research, and the actual impact on the modeling of psycho-biological phenomena (Torre and Wagenmakers 2008).

Also with regard to our present issue, following a nomothetic or a mechanistic perspective to account for the finding of $1/f^\beta$ noise in series of inter-tap intervals produced in self-paced tapping and series of asynchronies in synchronization tapping would bring about divergent conclusions. In the nomothetic perspective it would be assumed that both the way the system globally organizes to perform self-paced taps and the way it (re)organizes to synchronize the taps with an external stimulus answered some generic form of organization that caused $1/f^\beta$ noise in the resulting inter-tap intervals and asynchronies, respectively. Following this perspective, modeling synchronization through a simple auto-regressive correction process appears clearly inconsistent with the finding of $1/f^\beta$ noise in asynchronies, since simple auto-regressive mechanisms can only produce short-range dependence in series, featuring an exponential decay in the auto-correlation function and a plateau in the low-frequency region of the power spectrum, as opposed to the typical signatures of $1/f^\beta$ noise we outlined above (see Thornton and Gilden 2005; Wagenmakers et al. 2004).

The second perspective consists in assuming that there may be specific processes and/or localizable entities in the system that generate $1/f^\beta$ noise, and which are common to the performance of self-paced and synchronization tapping. In this case, following the results of previous studies on self-paced tapping, the internal timekeeping process can be considered the source of $1/f^\beta$ correlation (Delignières et al. 2004; Delignières et al. 2008; Gilden et al. 1995). This approach implies a specific mechanistic model, where the correlation properties in ASYN series would result from the combination of correlations generated by the timekeeping process, the auto-regressive synchronization process, and motor variability (Eqs. 2 and 3).

The aim of this article was to examine how the established linear phase correction framework for synchronization tapping could be reassessed for proposing an experimentally testable and plausible model that accounts for the experimentally observed long-range correlations. Therefore we organized the article as follows: In the first part, we characterize the serial correlation properties of asynchronies and inter-tap interval series collected in a synchronization tapping experiment. Experimental results are briefly discussed. In the second part, we show that the synchronization tapping model in its original formulation (Vorberg and Schulze 2002) does not allow accounting for the empirical long-range correlation. In the third part we propose to extend the original synchronization model by considering the internal timekeeping process as a source of $1/f^{\beta}$ noise, and modeling it using the shifting strategy model. We show that this $1/f$ synchronization model accounts for the empirical serial correlation. Notably, as the auto-regressive correction parameter varies, the model accounts for the typical correlation structures of inter-tap intervals in self-paced tapping, and asynchronies in synchronization tapping. We discuss the implications and the theoretical meaning of this model.

2 Synchronization tapping data

2.1 Method

Data used here were obtained as part of a larger study addressing the timing mechanism engaged in unimanual and bimanual tasks (Torre and Delignières 2008); here we only present the part that is of interest for our present concern.

Twelve participants (eight males and four females, mean age 29 ± 7.2) took part in the experiment. Ten participants declared themselves right-handed, and two left-handed. None of them had extensive practice in music. They declared no particular competence involving specific coordination between the upper limbs, and no neurological injury or recent upper limb injury.

The task consisted in tapping in synchrony with an auditory signal delivered at a constant frequency of 2 Hz. Participants performed series of about 600 taps, corresponding to 5 min trials. Participants were seated comfortably, their forearm, hand palm and other fingers resting on the table so that only the index finger of the dominant hand moved. They were instructed to keep the taps on the beep and to minimize the contact duration on the surface.

The auditory signals were generated by a PC-driven metronome. The taps were performed on a flat rectangular (4 cm × 4 cm) pressure sensor fixed on a table and adjusted to the participants' comfort. The pressure data and metronome sequences were recorded with a sampling frequency of 300 Hz, using LabJack U12 device.

2.2 Data analysis

We analyzed ITI and ASYN series. The times of the taps (t_{T}) and the auditory signals (t_{M}) were identified as the reaching of a threshold at each signal onset. ASYN were defined as the difference $t_{\mathrm{T}} - t_{\mathrm{M}}$ between the tap and the corresponding auditory signal.

For examining the short-range correlation properties of ITI and ASYN, we computed the auto-correlation functions (ACF) of series from lag 1 to 20. For assessing the long-range correlation structure, we first examined the series spectral properties using $^{\mathrm{low}}\mathrm{PSD}_{\mathrm{we}}$ (Eke et al. 2000), an improved version of the classical spectral analysis, including some preprocessing operations before the application of the Fast Fourier Transform (for details, see Eke et al. 2000). The spectral exponent β was estimated by the negative of the linear regression slope in the log-log power spectrum. As proposed by Eke et al. (2000) we excluded in the fitting of β the high-frequency power estimates ($f > 1/8$ of maximal frequency) for obtaining more accurate estimates. Spectral analysis allows classifying series as fractional Gaussian noise (fGn), i.e., stationary series ($\beta < 1$), or fractional Brownian motion (fBm), i.e., non-stationary series ($\beta > 1$; Eke et al. 2000).

So far, the presence of $1/f^{\beta}$ noise in ASYN series has been reported on the basis of the linear regression slope in log-log power spectra. However, combinations of different short-range processes are known to be likely to mimic the characteristic $1/f^{\beta}$ power spectra (see Thornton and Gilden 2005; Wagenmakers et al. 2004), and the presence of 'genuine' long-range correlation in ASYN series has not been attested statistically. In the present study we used ARFIMA/ARMA modeling (*Auto-regressive Fractionally Integrated Moving Average*, Wagenmakers et al. 2004; Torre et al. 2007) in addition to spectral analysis, in order to evaluate the statistical evidence for the presence of genuine long-range correlation in experimental and simulated ASYN series. This method consists in fitting 18 models to the studied series: nine are ARMA (p, q) models, p and q varying systematically from 0 to 2, and the other nine are the corresponding ARFIMA (p, d, q) models, where d is the fractional integration parameter. The best model is selected using a goodness-of-fit statistic that is based on a trade-off between accuracy and parsimony. We used the Bayes Information Criterion (BIC) that was proven to give the best results in the detection of long-range dependence (Torre et al. 2007). The ARFIMA/ARMA procedure provides two complementary criteria. The first one is the

Table 1 Results obtained from experimental series, and from simulated series generated by the original synchronization model (white noise assumption), and by the proposed fractal model

		Experimental data	Original synchronization model	$1/f$ synchronization model
SD (ms)	ASYN	35	33	31
	ITI	32	35	34
Auto-correlation lag1	ASYN	0.39 (0.21)	0.45 (0.04)	0.39 (0.04)
	ITI	−0.33 (0.13)	−0.38 (0.04)	−0.037 (0.04)
Asymptotic auto-correlation	ASYN	**0.12 (0.10)**	**−0.01 (0.04)**	**0.08 (0.04)**
	ITI	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Spectral index (β)	ASYN	**0.69 (0.48)**	**0.20 (0.36)**	**0.69 (0.29)**
	ITI	**−1.11 (0.43)**	**−1.86 (0.31)**	**−1.27 (0.29)**
% ARFIMA sum of weights	ASYN	**83.3%**	**65%**	**94%**
		0.94	**0.76**	**0.94**

Experimental ASYN series contain $1/f^{\beta}$ noise, as indicated in bold by the positive asymptote value in the auto-correlation function, the spectral index, and the results of ARFIMA modeling. Accordingly, experimental ITI series contain anti-persistent noise. While the two simulations gave SD and lag 1 auto-correlations similar to experimental results, only the $1/f$ synchronization model allowed to reproduce the empirical long-range correlation properties assessed by the spectral index and ARFIMA modeling

percentage of series that are better fitted by an ARFIMA model. The second is based on a transformation of the raw BIC values into weights (i.e., the probability that this model is the best over the set of candidate models; see Wagenmakers et al. 2004). We then computed the sum of the weights captured by the nine ARFIMA models, considering that the weights of all tested model sum to one.

2.3 Statistical properties of experimental series

The results for experimental series are summarized in Table 1 (left column). The mean experimental ASYN was −62 ms, with a mean within-trial standard deviation of 35 ms. The mean ITI was 499 ms, with a mean within-trial standard deviation of 32 ms.

The average autocorrelation functions of ASYN and ITI series are presented in Fig. 1 (upper panel). Mean lag 1 autocorrelation of ASYN series was 0.39 (SD = 0.21), and the ACF exhibited an asymptote approaching 0.12 (SD = 0.10, mean over lag 10 to lag 20). The mean lag 1 auto-correlation of ITI series was −0.33 (SD = 0.13), and the ACF exhibited an asymptote approaching 0.00 (SD = 0.00, mean over lag 10–20).

Regarding the long-range correlation structure, the mean log-log power spectra of ASYN and ITI series are presented in Fig. 1 (upper panel). The mean spectral indexes were $\beta = 0.69$ (SD = 0.48) for ASYN, and $\beta = -1.11$ (SD = 0.43) for ITI. Importantly, the mean power spectrum for ASYN did not present any plateau in the low-frequency region, but a linear negative slope along the whole range of frequencies.

ARFIMA/ARMA modeling provided statistical evidence for the presence of long-range correlation in 10 out of 12 (83.3%) experimental ASYN series, with a mean sum of ARFIMA weights captured by ARFIMA models of 0.94.

2.4 Discussion

Our series presented Gaussian statistics consistent with those reported in similar experiments, especially reproducing the typical anticipation tendency (negative mean asynchronies) that has usually been reported in synchronization tapping (for a review see Repp 2005). Regarding short-term correlation, our data notably reproduced the characteristic negative lag 1 auto-correlation that is expected in ITI series, in both self-paced and synchronization conditions (Vorberg and Wing 1996).

Regarding long-range correlation, our results showed that ITI and ASYN series could be characterized as fGn (i.e., stationary series). This is not so surprising, since the metronome imposes a constant timing reference that allows controlling the mean time intervals over the trial and then to avoid drifts, such as those often reported in self-paced ITI (Ogden and Collier 1999).

Our analyses gave consistent statistical evidence for the presence of genuine long-range correlation in ASYN series. Mean spectral indexes indicated that series were $1/f^{\beta}$ noise ($0.5 < \beta < 1.5$), and ARFIMA/ARMA modeling tended to attest this result statistically. Although the percentage of series recognized as ARFIMA processes remained slightly lower than the indicative threshold of 90% recommended by Torre et al. (2007), this result could be considered with some confidence, given the low number of analyzed series and their relative ‘shortness’ (the performance of ARFIMA/ARMA modeling as other methods for analyzing long-range correlation are known to be sensitive to the length of series, see

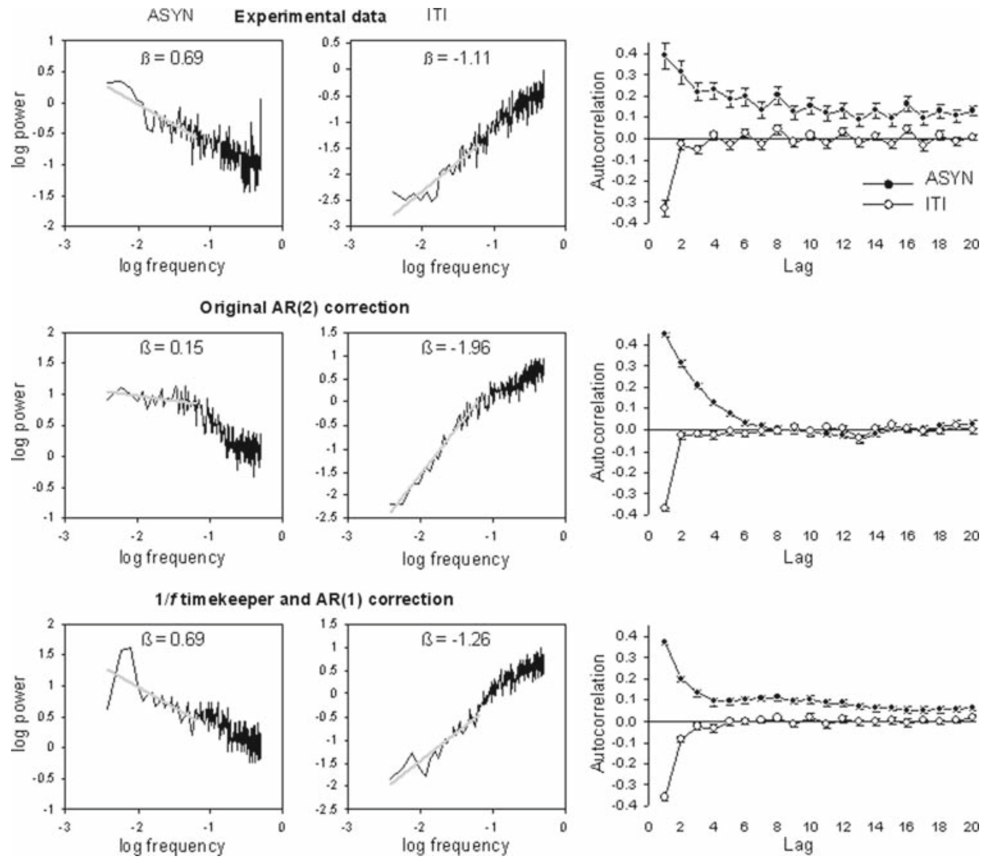

Fig. 1 Average log-log power spectra and auto-correlation functions of asynchronies and inter-tap intervals collected experimentally, simulated by the original synchronization model, and simulated by the $1/f$ synchronization model. For the auto-correlation functions, error bars represent standard errors. The graphs for simulated series were determined from 12 over 100 simulated series chosen randomly

Delignières et al. 2006; Torre et al. 2007). Moreover, this result was reinforced by the mean sum of weights captured by ARFIMA models (0.94) that was close to the maximal obtainable sum.

The analysis of experimental ITI series gave a completely different pattern of results, consistent with the fact that those series are mathematically the differentiation of ASYN series. ITI series were anti-persistent fGn ($\beta < 0$), and the auto-correlation function presented the expected shape for this kind of processes, with a negative value at lag 1, and an extinction of correlation at higher lags.

On the basis of this characterization of serial correlations in experimental ASYN and ITI data, the aim of the following sections was to test the capability of two candidate models in accounting for the evidenced statistical features in experimental synchronization tapping series, in terms of variability, short-range, and long-range correlation. First, we focus on the model proposed by Vorberg and Schulze (2002) in its original formulation. Second, we propose an amended version of the model, considering the central timekeeper as generating $1/f^{\beta}$ noise.

3 Simulation 1: the original synchronization model

Vorberg and Schulze (2002) conducted an analytical study of the stochastic behavior of the above presented AR(1) synchronization model (Eq. 1; Vorberg and Wing 1996). This framework is based on the *linear phase correction assumption*: in the case where the metronome delivers constant periods, participants are supposed to sufficiently know the requested periods, so that correction of periods at the central timekeeper level can be neglected, and local phase corrections are sufficient to keep synchronized (Semjen et al. 1998, 2000; Vorberg and Wing 1996; Vorberg and Schulze 2002). Vorberg and Schulze (2002) further analyzed an extended version of the initial model including a second-order auto-regressive correction (AR(2)) on asynchronies, so that corrections of the last and the next-to-last asynchronies were combined. The expression of asynchronies (Eq. 2) becomes

$$\mathrm{ASYN}_{n+1} = (1-\alpha)\mathrm{ASYN}_n - \gamma \mathrm{ASYN}_{n-1} + K_n - \tau \quad (4)$$

Note that according to Eq. 3, the error correction is based on the 'real' asynchronies between the effective tap and the metronome. Considering that real asynchronies are actually perceived with some feedback delay, Vorberg and Schulze (2002) further proposed to apply the auto-regressive correction on perceived asynchronies by including two distinct delay terms into the model, one depending on the physical properties of the metronome and the other related to the internal registration of the produced motor responses. However, for conciseness with regard to the present focus on serial correlation, and because the inclusion of the delay terms influences the mean asynchrony without modifying the correlation properties substantially (Flach 2005), we limited ourselves to examine the AR(2) synchronization model (Eq. 4).

Vorberg and Schulze (2002) showed analytically that this model, based on the assumption that both the timekeeper (C) and the motor components (M) were sources of uncorrelated white noise, was able to account for the Gaussian properties and typical short-range correlation in synchronization tapping series. The consistency of this model with regard to the typical long-range correlation properties has not been evaluated. Because the only sources of serial correlation in this model are the term of differenced white noise which affects each produced ITI and the AR(2) correction of asynchronies, one may predict, nevertheless, that the synchronization model in its original formulation would be confined to account for short-range correlations up to the second lag and would provide no account for long-range correlation properties.

We verified this assumption by simulating 100 series (512 points) obeying Vorberg and Schulze's AR(2) synchronization model (Eq. 4). The timekeeper periods C_n and motor delays M_n were considered white noise processes with standard deviations σ_C and σ_M, respectively. We estimated the parameters θ, γ, σ_C and σ_M on the basis of the auto-covariance function of experimental synchronization series averaged over participants, following the equations proposed by Vorberg and Schulze (2002, p.75, Eq.17). Accordingly, we set $\alpha = 0.5$, $\gamma = -0.1$, $\sigma_C = 26$, and $\sigma_M = 10$.

The main results are summarized in Table 1 (middle column). Simulations allowed the reproduction of the experimental variability of ASYN series, with a mean within-series standard deviation of 33 ms. The mean of simulated ITI was 500 ms, with a mean within-series standard-deviation of 35 ms.

The average auto-correlation functions of simulated ASYN and ITI series are presented in Fig. 1 (middle panel). The mean lag 1 auto-correlation of ASYN series was 0.45 (SD = 0.04), and the ACF exhibited an asymptote approaching −0.01 (SD = 0.04, mean over lag 10–20), that contrasted with the persistent, slightly positive correlation obtained in experimental asynchronies. The mean lag 1 auto-correlation of IRI series was −0.38 (SD = 0.04), and the ACF exhibited an asymptote approaching 0.00 (SD = 0.00, mean over lag 10–20).

Regarding long-range correlation, the average power spectra of simulated asynchronies and inter-response interval series are presented in Fig. 1 (middle panel). The power spectrum of ASYN series presented an obvious inflexion between a mean slope of 0.20 (SD = 0.36) in the low-frequency region (frequencies lower than 1/8th of the maximal frequency), and a mean slope of −0.72 (SD = 0.15) in the high-frequency region. Such a spectrum is typical of pure auto-regressive series (see Pressing and Jolley-Rogers 1997; Wagenmakers et al. 2004). Accordingly ARFIMA/ARMA modeling detected long-range correlation in only 65% of ASYN simulated series, and the mean sum of weights captured by ARFIMA models was 0.76. For simulated ITI series, the mean spectral index was $\beta = -1.86$ (SD = 0.31).

In sum, the Vorberg and Schulze (2002)'s AR(2) model gave a satisfying account for experimental variability, as well as for the lag 1 autocorrelation of ASYN and ITI series. However, assuming that the timekeeper variability was white noise, this model was not able to account for the long-range correlation structure evidenced in experimental ASYN and ITI series. Recently, Delignières et al. 2008 proposed to re-assess the Wing and Kristofferson model for self-paced tapping by providing the timekeeper periods (C_n) with $1/f^\beta$ variability. In the following section, we propose to extend Vorberg and collaborators' synchronization model by including the timekeeper as a source of $1/f^\beta$ noise. We test for the capability of this extended model to account for the empirical correlations observed in synchronization tapping.

4 Simulation 2: $1/f$-AR synchronization model

4.1 A fractal model for timekeeper periods

The central timekeeper of the Wing and Kristofferson model for self-paced tapping is assumed to generate regularly spaced cognitive events, and the fluctuations in the resultant successive time intervals are assumed to follow a random distribution. Such a timer can be represented by a threshold/activation mechanism (Ivry 1996; Schöner 2002), where an activation process (a) increases linearly over time until the reaching of a given threshold level (T) that determines a particular 'event'. Such an event triggers the motor response and simultaneously resets the activation process. Thus, the iteration of this model generates regular inter-event intervals whose duration is entirely determined by the ratio between threshold level and activation growth rates. For constant threshold and activation growth rate, the process produces periodic events. Assuming that the threshold and the activation growth rate fluctuate randomly around their baseline levels T_0 and a_0, the resulting inter-event interval series would be uncorrelated white noise.

In order to allow the Wing and Kristofferson model to account for $1/f^\beta$ properties previously evidenced in ITI series produced in self-paced tapping, Delignières et al. (2008) proposed to model the timekeeper using an amended version of this classical threshold/activation mechanism: the *shifting strategy model*, initially developed by Wagenmakers et al. (2004). This model rests on the assumptions that (i) the threshold presents non-stationarity over time, characterized by a plateau-like evolution around the baseline level T_0, and (ii) the growth rate of activation varies over successive iterations according to an auto-regressive mechanism around a baseline rate a_0.

The amplitudes T'_n of the threshold deviations from the baseline level are sampled from a uniform distribution of range R. Each deviation T'_n is maintained for a duration d_n sampled uniformly from a range $[d_{\min}; d_{\max}]$ of possible state durations. For each iteration, the current threshold is then given by

$$T_n = T_0 + T'_n, \qquad (5)$$

and the current activation rate by

$$a_n = a_0 + \varphi(a_{n-1} - a_0) + \mu\varepsilon_n, \qquad (6)$$

where φ is the auto-regressive parameter, and ε_n a centered white noise with unit variance. In agreement with the Wing and Kristofferson model, the motor responses are then triggered by an internal timer generating inter-event intervals C_n given by

$$C_n = T_n/a_n, \qquad (7)$$

Incorporating the shifting strategy model at the timekeeper level into the original Wing and Kristofferson model (Eq. 2) was shown to give a satisfying account for both the short- and long-range correlation properties of ITI series (notably the negative lag 1 autocorrelation and $1/f^{\beta}$ noise) in self-paced tapping (Delignières et al. 2008).

4.2 A fractal model for synchronization tapping

We propose to account for the empirical properties of synchronization tapping by incorporating the shifting strategy model at the timekeeper level of the AR(2) synchronization model (Vorberg and Schulze 2002).

As for the simulation of the original synchronization model, we used the estimations of timekeeper and motor delay standard deviations $\sigma_C = 26$ and $\sigma_M = 10$. For simulating the timekeeper variability, the parameters of the shifting strategy model were set in order to provide C_n with reproducible $1/f^{\beta}$ structure, centered around 500 (corresponding to the experimental 2 Hz tapping frequency) and with the requested variance σ_C. We used linear activation parameters $a_0 = 2$, $\varphi = 0.3$, $\mu = 0.09$, and threshold parameters $T_0 = 1,000$, $R = 30$, $d_{\min} = 1$, $d_{\max} = 100$. We checked that these parameters produced the expected timekeeper series by simulating 100 series of 512 points: The mean of simulated C_n was 501 ms, with a mean standard deviation of 25 ms. Series were $1/f^{\beta}$ noise, with a mean spectral index β of about 0.68, and ARFIMA/ARMA modeling provided evidence for long-range correlation in 89% of C_n series with a mean sum of weights captured by ARFIMA models of about 0.91. On this basis, the synchronization parameters α and γ were set in order to provide the best fit (in terms of Gaussian statistics and serial correlation properties) for our experimental ASYN and ITI series. The best solution was obtained for $\alpha = 0.8$ and $\gamma = 0$, and we performed 100 simulations (512 points) using these parameters.

Simulation results are summarized in Table 1 (right column) for comparison with experimental results. The mean within-series standard deviation of simulated ASYN series was 30 ms. The mean of simulated ITI was 500 ms, with a mean within-series standard deviation of 33 ms.

The average auto-correlation functions of simulated ASYN and ITI series are presented in Fig. 1 (bottom panel). The mean lag 1 auto-correlation of ASYN series was 0.39 (SD = 0.04), and the ACF exhibited an asymptote approaching 0.08 (SD = 0.04, mean over lag 10–20). The mean lag 1 auto-correlation of ITI series was −0.37 (SD = 0.04), and the ACF exhibited an asymptote approaching 0.00 (SD = 0.00, mean over lag 10–20).

Regarding long-range correlation, the average power spectra of simulated ASYN and IRI series are presented in Fig. 1 (bottom panel). The mean spectral index was β=0.69 (SD= 0.29) for ASYN, and $\beta = -1.27$ (SD = 0.29) for ITI. ARFIMA/ARMA modeling provided statistical evidence for the presence of long-range correlation in 94% of ASYN series, with a mean sum of weights captured by ARFIMA models of 0.94.

In sum, simulation results showed that this '$1/f$-AR synchronization model' accounted for both the empirical short- and long-range correlations, reproducing the negative lag 1 auto-correlation in ITI series, the global shapes of auto-correlation functions of ASYN and ITI, and the spectral indexes showing that ASYN and ITI series contained $1/f^{\beta}$ noise and anti-persistent noise, respectively[1]. ARFIMA/ARMA modeling attested statistically for the presence of long-range correlation in simulated ASYN series. Note that we retained an AR(1) correction process for the $1/f$-synchronization model. This result is consistent with the observations of Pressing and Jolley-Rogers (1997) showing that in this range of prescribed periods, a first-order process was sufficient.

[1] As it becomes obvious in Table 1, autocorrelations and spectral indexes exhibited higher variability in experimental than in simulated series. With regard to this discrepancy one might wonder whether the number of empirical series was sufficient and what could be the meaning of simulating such variable results. However, the main purpose of the present study was to perform a qualitative characterization of empirical correlation properties and to propose a model that accounted for these properties with respect to the average empirical behavior. Despite the large inter-individual variability in autocorrelations and spectral indexes the present results were qualitatively consistent, allowing notably to discriminate unambiguously between $1/f^{\beta}$ noise in ASYN series and anti-persistent noise in ITI series, as well as between the characteristic autocorrelation functions. The comparatively low variability observed for simulated series is related to the fact that simulations were performed using a single set of parameters, aiming to fit the average empirical behavior.

5 General discussion

Following the current synchronization tapping framework, the correlation structure in produced asynchronies results from the association of three sources of variability: the timekeeping process, the synchronization process, and the motor implementation. The original formulation of Vorberg and collaborators' synchronization model assumes that the variabilities inherent to motor implementation and to the timekeeping process are both uncorrelated white noises, and that synchronization is achieved by an auto-regressive (short-range) mechanism. Accordingly, this model accounts for short-range correlations, but not for the empirical long-range correlations.

Two alternative approaches to this issue were possible. On the one hand, one could have assumed that the correlation structure of asynchronies arises from the processes that are specific to synchronization tasks. In this perspective, for instance, Chen et al. (1997) argued that the finding of $1/f^{\beta}$ noise in asynchronies opposed Vorberg and collaborators' formalization of synchronization processes as auto-regressive error correction. Similarly, Thaut and collaborators (Thaut and Miller 1994; Thaut et al. 1998) argued that the finding of a negative lag 1 auto-correlation in inter-tap intervals but a positive lag 1 auto-correlation in asynchronies indicated different synchronization strategies for phase and period corrections, and opposed a simple auto-regressive correction.

In contrast, in the present study we assume that serial correlation properties including the finding of $1/f^{\beta}$ noise in asynchronies do not arise from synchronization processes themselves, but from the internal timekeeping processes which is common to self-paced and synchronized rhythmic tapping. Therefore, we incorporated the shifting strategy model which has previously been used for modeling self-paced tapping, at the timekeeper level into Vorberg and collaborators' (Vorberg and Wing 1996; Vorberg and Schulze 2002) original synchronization model. We showed that provided that the timekeeper is considered a source of $1/f^{\beta}$ noise, the auto-regressive synchronization process assumed in Vorberg and collaborators' framework remains consistent with the empirical correlation properties. In this regard, the present '$1/f$-AR synchronization model' provides an unifying account for the typical long-range correlation properties evidenced in self-paced and synchronized tapping.

Note that the finding of $1/f^{\beta}$ correlations in self-paced tapping has sometimes been considered spurious, as it was assumed to be an experimental artefact due to the collection of very long tapping sequences, rather than being inherent to the timekeeping process (Pressing and Jolley-Rogers 1997). Factors such as fatigue and boredom were supposed to induce artifactual non-stationarity, participants being unable to maintain a constant timing reference over the whole trial, so that the observed long-range correlations in self-paced tapping were attributed to mechanisms that are extraneous to the actual timing processes. In the case of synchronization, however, the external pacing provides a continual reference that prevents such artifactual drifts. Consequently, if the assumption of an artifactual origin of long-range correlations was true, such dependencies should not appear in asynchronies. The present results show, on the contrary, that the origin of $1/f^{\beta}$ correlation is not trivial, and our model suggests that the timekeeping process is the source of $1/f^{\beta}$ noise in both self-paced and synchronisation conditions.

5.1 Implications and experimental testing of the '$1/f$-AR synchronization model'

We argue that the $1/f^{\beta}$ noise in asynchronies arises from the variability inherent to the timekeeping process. Taking account of the long-range correlation caused by the timing mechanisms might probably allow to explain a source of residuals of the fit processes of different synchronization models proposed in literature. Let us consider the implications of the present '$1/f$-AR synchronization model' for the estimation of model parameters based on experimental series, in the current synchronization framework.

The asynchrony at time n is given by the difference between the summation of completed inter-tap intervals and the sum of completed metronome periods:

$$\mathrm{ASYN}_n = \sum_{1}^{n-1} \mathrm{ITI}_n - (n-1)\tau + \varepsilon_n \quad (8)$$

From the moment that noise affects the timekeeper periods, 'correction' is needed for preventing a divergence between the tapping sequence and the metronome sequence, and obtaining stationary ASYN series. The auto-regressive correction of asynchronies assumed in Vorberg and collaborators' synchronization framework allows stationarizing the differences between the two sequences, without modifying the properties of timekeeper periods. However, consider the 'virtual' asynchronies that would be obtained without any effective synchronization process. According to the nature of variability at the timekeeper level, the virtual asynchronies have different correlation properties: Assuming that the timekeeper variability is white noise, the virtual asynchronies would be Brownian motion, corresponding to the integration of white noise. Assuming that the timekeeper variability is $1/f^{\beta}$ noise, in contrast, virtual asynchronies would be persistent fractional Brownian motion, corresponding to the integration of $1/f^{\beta}$ noise. Consequently, the nonstationarity of asynchronies should be stronger in the case of a $1/f^{\beta}$ timekeeper (see Fig. 2), and a stronger correction should be needed with a $1/f^{\beta}$ timekeeper than with a white noise

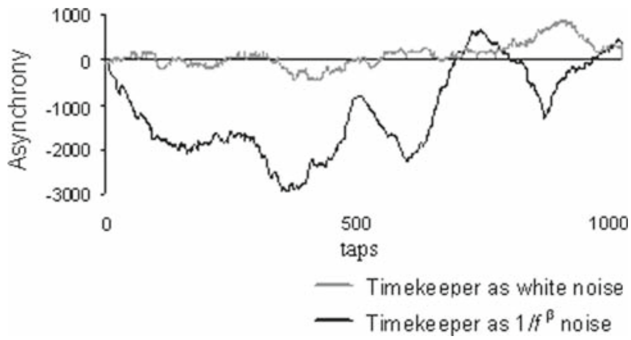

Fig. 2 Evolution of asynchronies without any correction ($\alpha = 0$), simulated with the timekeeper generating $1/f^{\beta}$ noise (*black trait*), and the timekeeper generating white noise (*gray trait*). The curve structure results from the addition of persistent fractional Brownian motion and white noise, minus a linear function in the first case, and from the addition of Brownian motion plus white noise, minus a linear function in the second case. 'Virtual' asynchronies diverge more when the timekeeper variability is $1/f^{\beta}$ noise

timekeeper for obtaining similar variance and correlation properties in ASYN series.

The estimation of auto-regressive parameters (α and γ) have usually been performed on the basis of the covariance function of asynchronies (Pressing and Jolley-Rogers 1997; Vorberg and Schulze 2002; Vorberg and Wing 1996). As we showed here, co-variation in the successive asynchronies does not only arise from the error correction process, but also from the initial persistent long-range correlations induced by the timekeeper. Thus, one implication of the present model is that analytical approaches yielding parameter estimation should in some way take the non-random structure of series into account to prevent underestimation of the strength of correction. For instance, in the present study the estimation of auto-regressive parameters on the basis of experimental series gave $\alpha = 0.5$ and $\gamma = -0.1$; however we needed to use $\alpha = 0.8$ in the '$1/f$-AR synchronization model' to approach the statistical properties of empirical asynchronies.

Similarly, estimations of the relative contributions of timekeeper variance and motor variance have usually been based on the negative lag 1 auto-correlation in ITI series. However, because ITIs were shown to be anti-persistent noise (i.e., negatively correlated) corresponding to the differentiation of $1/f^{\beta}$ noise in asynchronies, the negative lag 1 autocorrelation does not only come from the differenced white noise corresponding to motor variability, but also from the long-range correlation induced by the timekeeper. Accordingly, estimations based on the assumption that the timekeeper variability is white noise might over-estimate the ratio between motor and timekeeper variances.

Following the original linear phase correction model (Vorberg and Schulze 2002; Vorberg and Wing 1996) we assumed that synchronization was achieved through a local correction of asynchronies that did not modify the timekeeper periods. Because of this independence between the timekeeping process and the error correction processes, as a function of the strength of correction starting from the limit case of self-paced tapping ($\alpha = 0$) and increasing progressively, the long-range correlation structure in asynchronies turns gradually from persistent Brownian motion (i.e., non-stationary series) into the $1/f^{\beta}$ noise of empirical asynchronies (here for $\alpha = 0.8$; see Fig. 3). At the same time, the correlation structure of inter-tap-intervals naturally turns from $1/f^{\beta}$ noise into anti-persistent noise. Figure 3 makes obvious that the '$1/f$-AR synchronization model' allows accounting for the whole range of correlation properties, in particular the typical correlations of self-paced and synchronization tapping, by modulating the auto-regressive parameter only. As such, this model—and the underlying assumption that the timekeeper causes the $1/f^{\beta}$ fluctuations—provides a unifying account of the finding of $1/f^{\beta}$ noise in inter-tap intervals in self-paced tapping and in asynchronies in synchronized tapping.

In this regard, the model offers perspectives for further empirical testing. Notably, the model predicts an increase of persistent correlations in asynchronies in conditions where synchronization is known to be less efficient and error correction can be assumed to be less accurate. This prediction seems indeed consistent with results obtained by Chen et al. (2002), showing stronger persistent correlations with use of a visual metronome than with an auditory metronome. As well, this prediction could account for the results of Chen et al. (2001), evidencing stronger persistent correlations in syncopation than in synchronization tapping.

5.2 A mechanistic account of $1/f^{\beta}$ noise in synchronization tapping: theoretical meanings

The present '$1/f$-AR synchronization model' is a mechanistic model for $1/f^{\beta}$ noise that clearly assumes that the timekeeping process is the source of the observed long-range correlations. It might by discussed in two respects.

First, different theories have alternatively accounted for the timing of rhythmic movements in terms of central timekeeping processes that are responsible for the representation of the required time intervals, or in terms of temporal properties of movements that emerge from a complex (self-organizing) system. In particular, the information processing approach essentially deals with synchronization in terms of 'error correction', while the dynamic systems approach formalizes synchronization in terms of 'coupling' between an oscillator and an external stimulus (see Pressing 1999; Repp 2005, for a review). Even though the linear phase correction model we investigated in the present study is typically in line with the first approach, we did not wish to argue in favor of one or another theoretical perspective. Our approach actually relies on previous studies of the neural basis and

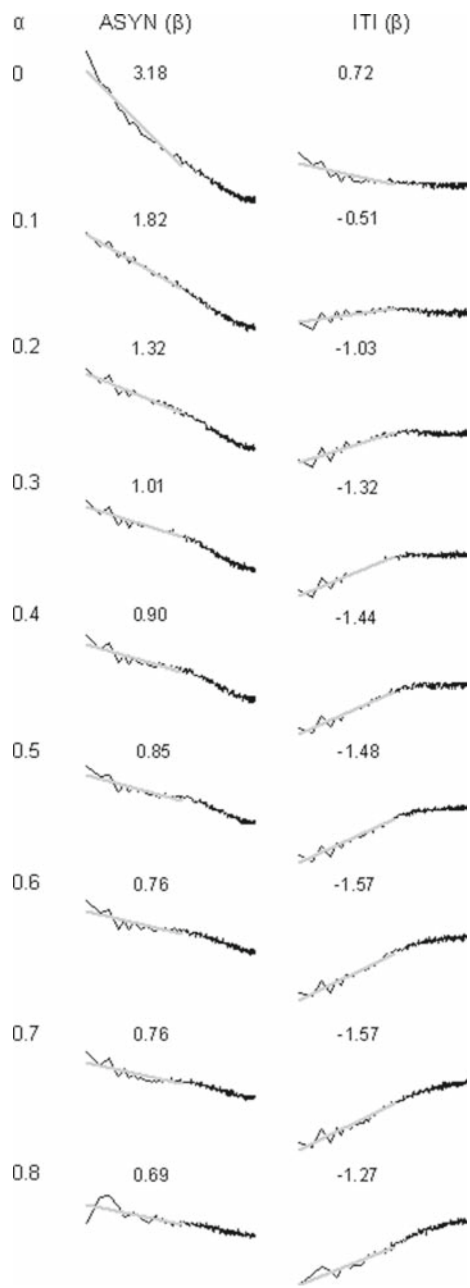

Fig. 3 Average power spectra ($N = 100$) of ASYN series and the corresponding ITIs simulated by the $1/f$-AR(1) synchronization model. All parameters except the auto-regressive parameter α are constant ($a_0 = 2, \varphi = 0.3, \varepsilon = 0.09, T_0 = 1,000, T'_{\max} = 4, d_{\min} = 1, d_{\max} = 100$). α varies from 0 to 0.8 by steps of 0.1, so that the top graphs show the results for self-paced tapping, as a limit case. This graph illustrates the fact that the model provides a unifying account of the finding of $1/f^\beta$ noise in self-paced and synchronized tapping since the empirical correlation properties showed in the two situations can be obtained by simply adjusting the synchronization parameter α

the variability in performance of rhythmic movement timing (Delignières et al. 2004; Delignières et al. 2008; Robertson et al. 1999; Ivry et al. 2002; Spencer and Ivry 2005; Spencer et al. 2003; Zelaznik et al. 2000, 2002; see also Huys et al. 2008) that have supported the distinction between two modes of temporal control: *event-based* and *emergent* timing, associated with the performance of discontinuous movements like tapping, and continuous movements like oscillations or circle drawing, respectively. While emergent timing is assumed to arise from the continuous regulation of non-temporal parameters (as oscillator stiffness) that determine movement frequency without needing any explicit representation of time, event-based timing is thought to involve an internal, explicit representation of temporal goals that is prescribed to the effectors independently of the motor execution itself.

Second, the assumption that the timekeeping process is the source of the observed long-range correlations might be discussed with regard to current controversial perspectives on the $1/f^\beta$ phenomenon (see Van Orden et al. 2003; Wagenmakers et al. 2005). The nomothetic perspective aims at uncovering the universal principles that explain the ubiquitous occurrence of $1/f^\beta$ noise. Following this perspective, $1/f^\beta$ noise has notably been related to self-organized criticality (Bak et al. 1987) and considered as emerging from the coordinated interactions between the functional elements of a complex system (e.g., Kello et al. 2007). Accordingly, nomothetic accounts are hardly compatible with the idea of identifying specific processes or entities in a particular system that may cause $1/f^\beta$ noise. Here, in contrast, we adopt a mechanistic perspective that aims at identifying the specific sources of $1/f^\beta$ noise in a specific phenomenon instead of providing a universal account of $1/f^\beta$ noise. One main reason for going for a mechanistic approach, we argue, is that domain-specific models may be more directly 'useful' for research since they allow to bridge the gap between accounts of $1/f^\beta$ noise and currently existing theories and models of a given object of research. Also, because such mechanistic models allow to establish relationships between model parameters and different experimental factors, they may predict specific variations in the correlation structure that are experimentally testable. In this perspective, our present model represents a way of summarizing the functioning of the system by a few simple mechanisms that are psychologically and biologically plausible. Such a mechanistic approach does not oppose theories of self-organizing complex systems. It just proposes a simplified view of the system that constitutes a relevant entry point for unraveling the finding of $1/f^\beta$ noise in synchronized and self-paced tapping.

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