Eur J Appl Physiol (2000) 82: 345–360

REVIEW ARTICLE

Pietro E. di Prampero

Cycling on Earth, in space, on the Moon

Accepted: 20 March 2000

Abstract The mechanical power for cycling (P_c) at constant ground speed (s), in the absence of wind on smooth hard terrain is the sum of the power dissipated against rolling resistance, gravity and air resistance: $P_c = a \cdot s + M \cdot g \cdot s \cdot \sin\gamma + b \cdot s^3$, where a and b are constants, M is the mass of the subject plus bike, g is the acceleration of gravity and γ is the angle of the terrain with the horizontal. The constant b depends upon the drag coefficient (C_d), the overall area projected on the frontal plane (A_f), and the air density (ρ_a): $b = 0.5 \cdot C_d \cdot A_f \cdot \rho a$. In turn, ρ_a depends on air pressure (P_B) and temperature (T): $\rho_a = \rho_0 \cdot 0.359 \cdot P_B \cdot T^{-1}$, where ρ_0 is the air density at 760 mmHg (101.3 kPa) and 273 K. The metabolic power developed by the cyclist ($\dot{E}_c$) is related to $P_c : \dot{E}_c = P_c \cdot \eta^{-1}$, where η is the mechanical efficiency of cycling. The experimental values of a, b and η are fairly well known so that, if the maximal metabolic power as a function of the performance time is known for a given cyclist, the following set of data can be individually calculated: (1) best performances over any given distance and for any given altitude above sea level, (2) the effects of posture and body size on maximal speeds, and (3) the maximal incline of the terrain that can be overcome at any given speed or coasting speed for any given downslope. The above set of information makes it possible also to calculate the characteristics of a "Twin Bikes System" (TBS) for preventing microgravity deconditioning during long-term space flight. The TBS consists of two bicycles that are mechanically coupled by a differential gearing, which move at the very same speed, but in the opposite sense, along the inner wall of a cylindrically shaped space module. The circular trajectories induce a centrifugal acceleration vector (a_c) oriented along the head-to-feet direction of each subject: $a_c = v_t^2 \cdot r^{-1}$ where v_t is the tangential velocity and r the radius of gyration, which is equal to the inner radius of the space module. So, any desired value of a_c can be achieved by appropriately selecting v_t, wherefrom the mechanical and metabolic powers that the astronauts must generate can be readily calculated. Experiments performed in a ground-based human centrifuge have shown that the discomfort derived from the rotating environment is reasonably low and well tolerated. If the appropriate atmospheric pressure is provided, cycling on circular or elliptical tracks may be useful to reduce cardiovascular deconditioning that occurs due to the reduced gravity in permanently manned lunar bases. Indeed, on the curved parts of the path a cyclist will generate a horizontal outward acceleration: $a_c = s^2 \cdot r^{-1}$, where s is the velocity along the track and r is the radius of curvature. To counterbalance a_c, the subject plus bike must lean inwards so that the vectorial sum of a_c plus the lunar gravity ($g_L = 1.62\ \mathrm{m \cdot s^{-2}}$) is applied along a straight line that includes the centre of mass of the system and the point of wheel contact with the ground. For values of s from 10 to 20 $\mathrm{m \cdot s^{-1}}$ and r from 50 to 200 m, this vectorial sum ranges from 1.05 to 5.03 g_L (0.17–0.83 g).

P. E. di Prampero (✉)
Department of Biomedical Sciences, University of Udine, Piazzale M. Kolbe 4, 33100 Udine, Italy
e-mail: pprampero@makek.dstb.uniud.it
Tel.: +39-0432-494330; Fax: +39-0432-494301

Foreword

The ever-increasing number of scientific publications makes it more and more difficult for the average scientist to keep abreast of his own field of interest, in spite of the electronic dissemination of information. The Editors and Publisher of the European Journal of Applied Physiology have recently decided to make things easier for the readers of the Journal, inviting well-known scientists to condense into concise reviews our present knowledge on a number of topics that were deemed of interest, providing the reader with a succinct, yet easily accessible picture of the matter at stake.

It is both a privilege and a burden for me to open this series with a brief review on the bioenergetic and biomechanical aspects of cycling. This choice was dictated, on the one side by my personal interest in the science of cycling, and on the other by the somewhat unique characteristics of cycling that will be described below. Finally, bicycles, as well as three- or four-wheeled human-powered vehicles, are cheap and easily available tools by which we may reduce environmental pollution in our cities and countryside, which are increasingly crowded by cars, motorcycles, lorries, all of which are propelled by internal combustion engines.

Definitions for the abbreviations used in this text may be found in Table 1.

Introduction

On 6 September 1996, in Manchester, UK, Chris Boardman set the present world distance record for 1 h of unaccompanied cycling of 56.375 km. This extraordinary performance, in addition to showing the outstanding athletic capacity of Boardman, also highlights the formidable characteristics of cycling as a means of locomotion on flat terrain. Because of the saddle, which stabilises the body in the vertical plane, and the pedals, which transform the alternate action of the limbs into a continuous forward motion, the amount of energy wasted against gravitational and inertial forces with each pedal cycle is minimal. This allows a cyclist to utilise nearly all of his metabolic energy against the air and rolling resistance. As a consequence, the effective energy expenditure for forward progression is less, and the speed attained for a given power is greater, than for any other form of human locomotion (Tucker 1975).

Since the second half of the 19th century, these unique characteristics of bicycling have prompted physiologists, engineers, and sportsmen, to investigate in detail essentially all aspects of the bicycle and of its engine, the cyclist (for references see Burke 1986; Burke and Newsom 1988; Sjøgaard et al. 1984).

The aim of this review is to present an updated and comprehensive review of the principal characteristics of cycling under a wide variety of conditions. The first sections will be devoted to mechanical work performance, to the energy expenditure and to the efficiency of bicycling. These will be followed by sections on the rolling and air resistance, and on the effects of altitude, body size and shape, and the incline of the road. I will then try to convince the reader that the analysis summarised in the preceding sections, together with our present understanding of the energetics of muscular exercise, allows us to predict performances with a fair degree of accuracy for any given subject, and under any given set of standardised conditions. The final two sections will be devoted to “science fiction” scenarios: it will be shown that cycling can be used in space or on the Moon to avoid the deconditioning of the muscular and circulatory systems that occur there due to the absence or reduction of gravity. Throughout these sections, I have tried to maintain a rigorous analytical approach, while at the same time avoiding cumbersome details. The readers will judge whether these ambitious aims have been achieved.

Work performance and the energy cost of cycling

When cycling at constant speed, on flat terrain, mechanical work is performed against: (1) the overall rolling resistance, as given by the sum of the rolling resistance of the tyres and of the frictional losses in hub bearings and drive train, and (2) the air resistance (e.g. see van Ingen Schenau and Cavanagh 1990):

$$W_c = a + b \cdot v^2 \quad (1)$$

where W_c is the mechanical work performed per unit of distance, v is the air speed and, for a given set of conditions, a and b are constants. The corresponding metabolic energy expenditure depends on the overall cycling efficiency (η):

$$C_c = W_c \cdot \eta^{-1} = (a + b \cdot v^2) \cdot \eta^{-1} \quad (2)$$

or, by setting $\alpha = a \cdot \eta^{-1}$ and $\beta = b \cdot \eta^{-1}$:

$$C_c = \alpha + \beta \cdot v^2 \quad (3)$$

where C_c is the energy cost of cycling (above resting) per unit of distance and, again for a given set of conditions, α and β are constants. The constant α (or a) is the metabolic energy (or mechanical work) spent per unit of distance to overcome the rolling resistance; it is independent of the speed and is set by the frictional losses in the hub bearings and drive train, and (mostly) by the size, type and inflation pressure of the tyres and by the characteristics of the terrain (see “The rolling resistance”, below).

The second term of Eqs. 1 and 3 is the metabolic energy (or mechanical work) spent per unit of distance against the aerodynamic drag force, where, for a given set of aerodynamic conditions (see “The air resistance”, below), β (or b) is constant.

The mechanical (P_c) or metabolic ($\dot{E}_c$) power required to proceed at constant ground speed (s) is given by the product of W_c or C_c and the speed itself. Thus, in the absence of wind, for which conditions $v = s$, as from Eqs. 1 and 3:

$$P_c = W_c \cdot s = a \cdot s + b \cdot s^3 \quad (4)$$

$$\dot{E}_c = C_c \cdot s = \alpha \cdot s + \beta \cdot s^3 \quad (5)$$

(if, according to S.I. units, C_c is expressed in $J \cdot m^{-1}$ and s in $m \cdot s^{-1}$, P_c and $\dot{E}_c$ will turn out in Watts. However, it is often convenient to use other units, such as $mlO_2 \cdot m^{-1}$ for C_c, in which case, if s is given in $m \cdot min^{-1}$, $\dot{E}_c$ will turn out in $mlO_2 \cdot min^{-1}$, the traditional physiological units for the metabolic power output. Throughout this paper, it will be assumed that the energy equivalent for O_2 consumption ($\dot{V}O_2$) is 20.9 $kJ \cdot l^{-1}$. The energy equivalent for O_2 depends upon the respiratory quotient

Table 1 Glossary

Symbol	Definition	Units	Table/Figure
a	[a]Constant relating the rolling resistance to the ground speed	$N \cdot s \cdot m^{-1}$	Table 1, Fig. 1
A	Fractional decrease of $\dot{V}O_{2\,max}$ with altitude		Fig. 5
A_{bs}	Body surface area	m^2	Fig. 4
a_c	Centrifugal acceleration	$m \cdot s^{-2}$	
A_f	Area projected on the frontal plane	m^2	Fig. 4
AMS	Acute motion sickness	–	
AnS	Maximal capacity of anaerobic energy stores	kJ	Fig. 7
α	[a]Constant relating the energy expenditure per unit of distance against the rolling resistance to the ground speed	$J \cdot s \cdot m^{-2}$	Fig. 2
b	[a]Constant relating the aerodynamic drag force to the square of the air velocity	$N \cdot s^2 \cdot m^{-2}$	Table 2, Fig. 1
β	[a]Constant relating the energy expenditure per unit of distance against the aerodynamic drag force to the air velocity squared	$J \cdot s^2 \cdot m^{-3}$	Fig. 2
C_c	[a]Metabolic energy cost of cycling (above resting) per unit of distance	$J \cdot m^{-1}$	Fig. 2
C_d	Drag coefficient		Table 2
d	Distance, horizontal	m	
$\dot{E}_c$	Metabolic power (i.e. rate of metabolic energy expenditure above resting) per unit of time	kW	Fig. 7
E_k	Kinetic energy per unit of mass	$J \cdot kg^{-1}$	
E_{ktot}	Overall kinetic energy	kJ	
$\dot{E}_{max}$	Maximal rate of metabolic energy expenditure	kW	Fig. 7
η	Efficiency of cycling		Fig. 3
F	Fraction of $\dot{V}O_{2\,max}$ sustained throughout the effort		
f_p	Pedal frequency	Hz	Fig. 3
g	Acceleration of gravity on Earth	$m \cdot s^{-2}$ (9.81)	
g'	Vectorial sum of the lunar acceleration of gravity and of the centrifugal acceleration	$m \cdot s^{-2}$	Figs. 11, 12, 13
g_L	Acceleration of gravity on the Moon	$m \cdot s^{-2}$ (1.62)	
γ	Angle from the horizontal	degrees	
h	Distance, vertical	m	
k	Fractional decrease of the air density with altitude		Fig. 5
M	Mass	kg	
P	Blood pressure	mmHg (kPa)	[a]Fig. 10
P_B	Barometric pressure	mmHg (kPa)	
P_c	Mechanical power output in cycling	W, kW	Tables 2, 3
r	Radius of gyration	m	
rpm	Revolutions per minute	min^{-1}	
R_T	[a]Tractive resistance	N	Fig. 1
ρ_a	Air density	$kg \cdot m^{-3}$	Fig. 5
ρ_b	Blood density	$kg \cdot m^{-3}$ (1.059)	
ρ_0	Air density for T = 273 K and P_B = 760 mmHg (101.3 kPa)	$kg \cdot m^{-3}$ (1.293)	
s	Ground speed	$m \cdot s^{-1}$	
s_a	Ground speed at altitude	$m \cdot s^{-1}$	Fig. 6
s_{sl}	Ground speed at sea level	$m \cdot s^{-1}$	Fig. 6
T	Temperature	K; C	
TBS	Twin Bikes System	–	Fig. 8, 9
t_e	Exhaustion time	s	Fig. 7
t_p	Performance time	s	Fig. 7
τ	Time constant for reaching $\dot{V}O_{2\,max}$ at the onset of supramaximal efforts	(10) s	
v	Air velocity	$m \cdot s^{-1}$	
$\dot{V}CO_2$	[b]Rate of CO_2 production	$ml \cdot min^{-1}$	
$\dot{V}O_2$	[b]Rate of O_2 consumption	$ml \cdot min^{-1}$	
$\dot{V}O_{2\,max}$	[b]Maximal rate of O_2 consumption	$ml \cdot min^{-1}$	
v_t	Tangential velocity	$m \cdot s^{-1}$	
W_c	[a]Mechanical work of cycling per unit of distance	$J \cdot m^{-1}$	Fig. 1
W_{cg}	[a]Mechanical work of cycling against gravity per unit of distance covered along the road	$J \cdot m^{-1}$	
ω	Angular velocity	$rad \cdot s^{-1}$	

[a] Dimensionally, mechanical work (or metabolic energy) per unit of distance and force are equal. Both notations have been used for mechanical variables (e.g. R_T in N, or W_c in $J \cdot m^{-1}$), whereas the metabolic variables are reported only in terms of energy per unit of distance (e.g. C_c, $J \cdot m^{-1}$)

[b] $\dot{V}CO_2$, $\dot{V}O_2$ and $\dot{V}O_{2\,max}$ are expressed at standard temperature and pressure [(T = 273K; P_B = 760 mmHg (101.3 kPa)], dry

(RQ; i.e. on the ratio of metabolic CO_2 output to $\dot{V}O_2$, $RQ = \dot{V}CO_2/\dot{V}O_2$). RQ is set by the type of fuel being oxidised, and varies from 1.00 for pure carbohydrates to 0.71 for pure lipids, the corresponding energy equivalent for O_2 ranging from 21.13 $kJ \cdot l^{-1}$ (for $RQ = 1.00$) to 19.62 $kJ \cdot l^{-1}$ (for $RQ = 0.71$). The value used throughout this paper (20.9 $kJ \cdot l^{-1}$) applies for $RQ = 0.96$, as can be expected to occur during heavy aerobic exercise.

The classical approach for assessing the constants a and b of Eq. 1 is that of determining the tractive resistance (R_T) as a function of the speed (Capelli et al. 1993; di Prampero et al. 1979; Pugh 1974). Indeed, R_T (a force) is obviously equal to the mechanical work performed per unit of distance ($R_T = W_c$). Thus, plotting R_T as a function of s^2, as from Eq. 1, the intercept on the y-axis yields a, and the slope ($\Delta W_c/\Delta s^2$) is b (see Fig. 1). For such an approach to yield meaningful results, the measurements must be taken at constant speed, on flat terrain, in the absence of wind, and under rigorously standardised conditions.

A similar approach can be used to obtain the constants α and β (Eq. 3; Capelli et al. 1993, 1998). In this case, the dependant variable is the energy expenditure per unit of distance (C_c; see Fig. 2). Thus, since C_c is generally obtained from the ratio of steady-state $\dot{V}O_2$ to speed, the range of speeds that can be meaningfully investigated is smaller than for W_c, its upper limit being obviously set by the subject's maximal $\dot{V}O_2$ ($\dot{V}O_{2max}$). In addition, the equipment used to assess $\dot{V}O_2$ during cycling must not interfere with the freedom of motion of the cyclist, nor must it increase in any significant way the area projected on the frontal plane (A_f) nor the drag coefficient (C_d). This can be achieved by using small, lightweight portable instruments such as the K2 or K4 (Cosmed, Rome, Italy; Kawakami et al. 1992; Lucia et al. 1993).

If the bicycle is instrumented to permit measuring the forces applied and the mechanical work performed on the pedals (Sargeant and Davies 1977; Sargeant et al. 1978), the biomechanical and bioenergetic approaches can be usefully combined, thus obtaining all four constants (a, α, b, β) in one set of measurements (Davies 1980). It should be pointed out here that the mechanical work determined by means of instrumented bicycles also includes the work performed on the drive train, which is not the case when using R_T methods. An ingenious combination of $\dot{V}O_2$ measurements during actual road cycling and on a mechanically braked cycle ergometer, proposed by Pugh (1974), can be considered as the forerunner of these combined approaches.

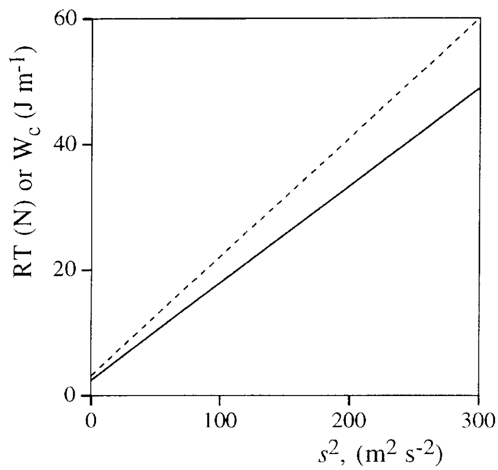

Fig. 1 Tractive resistance (R_T, N) as a function of the square of the ground speed (s^2, $m^2 \cdot s^{-2}$) for riding traditional (*upper broken line*; from di Prampero et al. 1979) or aerodynamic (*lower continuous line*; from Capelli et al. 1993) racing bicycles in the absence of wind in dropped posture. R_T is also equal to the mechanical work per unit of distance (W_c, $J \cdot m^{-1}$). The anthropometric data of the cyclists were: body mass 70 and 71 kg; stature 180 and 184 cm; body surface area 1.8 and 1.84 m^2, respectively. The masses of their bicycles were 7 and 14 kg, respectively. Air pressure (P_B) and temperature (T) were: $P_B = 755$ and 758 mmHg (100.7 and 101 kPa), respectively; $T = 15$ and 11 °C, respectively. Only regressions are shown. Traditional (*broken line*): $R_T = 3.2 + 0.19 \cdot s^2$ ($r^2 = 0.96$, $n = 33$); aerodynamic (*continuous line*): $R_T = 2.4 + 0.155 \cdot s^2$ ($r^2 = 0.96$, $n = 19$)

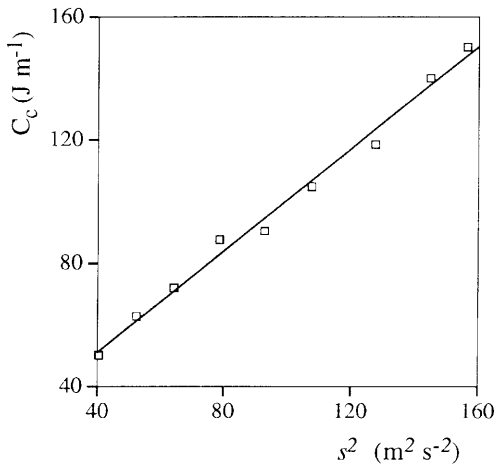

Fig. 2 Energy cost of cycling at constant speed (C_c, $J \cdot m^{-1}$) as a function of the square of the ground speed (s^2, $m^2 \cdot s^{-2}$) for a traditional racing bicycle in the absence of wind. Cyclist (mass 77.5 kg; stature 195 cm; body surface area 2.09 m^2) in dropped posture. Bicycle mass 10.6 kg. $P_B = 755$ mmHg (100.7 kPa); $T = 26$ °C. The regression is described by: $C_c = 16.7 + 0.84 \cdot s^2$; $r^2 = 0.99$, $n = 9$ (Taken from Capelli et al. 1998)

Other techniques for assessing a and b consist of measuring the coasting velocity down a constant-sloping terrain or the (negative) acceleration during a free run over flat terrain. The latter is somewhat more complicated analytically (Candau et al. 1999; de Groot et al. 1995); it has, however, the great practical advantage that it can be performed easily over rather short distances (a long enough laboratory corridor may suffice). More sophisticated approaches such as wind tunnels or treadmills in wind tunnels can also be applied profitably for determining the whole set of biomechanical and bioenergetic constants at stake.

Before illustrating the possible applications of Eqs. 1–5, a few paragraphs will be devoted to the illustration of the most relevant characteristics of the efficiency of cycling and of the rolling and air resistance.

The efficiency of cycling

The mechanical efficiency of cycloergometric exercise and of cycling (η) is probably the most widely studied of all efficiencies. It is commonplace to state that η is not far from 0.25, and that it depends upon the pedal frequency (f_p), being highest at $f_p \cong 1$ Hz. Indeed, since the original observations of Sylvia Dickinson (1929), the data from various sources (e.g. Banister and Jackson 1967; Coast and Welch 1985; Ericson 1988; Francescato et al. 1995; Gaesser and Brooks 1975; Luhtanen et al. 1987; Seabury et al. 1977; Zoladz et al. 1995, 1998) consistently show that the optimum f_p increases from 0.7 to 1 Hz as the P_c is increased from 50 to 300 W, and that the value of η corresponding to the optimum frequency is close to 0.25. These data also show that, for relatively large variations of f_p above or below its optimum value, η decreases only slightly below 0.25. Indeed, for a P_c of 100 W, at which the optimum f_p is 0.75 Hz, η is reduced to 0.24 for an f_p of 0.55 or of 0.95 Hz, and to 0.22 for an f_p of 0.45 or of 1.15 Hz (Fig. 3). Similarly, for a P_c of 300 W at which the optimum f_p is 1.0 Hz, the 0.24 (0.22) efficiency thresholds are reached at f_p values of 0.70 (0.45) Hz or of 1.6 (1.9) Hz (Fig. 3). So, at high values of P_c, the relationship between η and f_p is rather flat. This may help us to understand the well-known observation that, during competition cycling f_p is higher than that yielding the optimum η. For example when, in 1984, Francesco Moser established the world record for 1 h of unaccompanied cycling in Mexico with 51.151 km, his average f_p was 1.75 Hz, yielding an efficiency value, estimated from Fig. 3 (for $P_c \cong 380$ W, Tables 2, 3, 4 and Eq. 4, 6, 8, 9), lower than 0.25 but still slightly higher than 0.24. The reduction of the forces applied on the pedals resulting from this higher than optimum f_p is likely to lead to a reduced anaerobic energy release, thus compensating for the slight fall in efficiency. This bioenergetic explanation seems plausible. It is not the only one, however, since other factors related to muscle

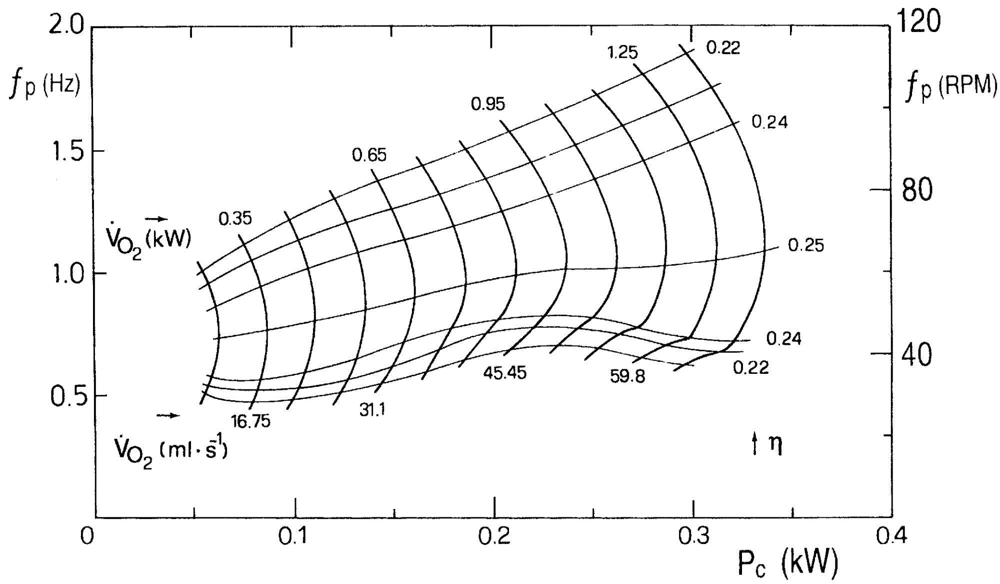

Fig. 3 Pedal frequency (f_p, Hz, *left ordinate*, or *rpm*, *right ordinate*) as a function of the mechanical power output (P_c, kW) in cycling. Iso-metabolic power lines (O_2 consumption, $\dot{V}O_2$, kW or ml · s^{-1}) are also drawn, thus allowing the assignation of a given efficiency value (η) to any given point of the plane. The points characterised by $\eta = 0.22$, $\eta = 0.24$ and $\eta = 0.25$ are joined by iso-efficiency functions. This Figure was compiled mainly from the data of Banister and Jackson (1967) and Seabury et al. (1977), and was modified after di Prampero 1986

Table 2 Overall mechanical power (P_c, W) and its relative utilisation against rolling or air resistance (%) are reported at the indicated speeds for a recreational bicycle (trunk leaning forward), for a standard racing bicycle (subject's trunk in dropped posture) on a smooth flat road, and for an aerodynamic bicycle with the subject in dropped posture on a linoleum track, in the absence of wind, at sea level, and at an air temperature of 20 °C. The constant *a* was assumed to be: 5.8, 2.8 and 1.1, respectively, and the constant *b* was assumed to be: 0.271, 0.193 and 0.155, respectively (see Tables 3 and 4)

Speed km · h^{-1}	(m · s^{-1})	Posture on bicycle								
		Recreational, leaning forward			Racing, standard			Racing, aerodynamic		
		P_c (W)	Rolling (%)	Air (%)	P_c (W)	Rolling (%)	Air (%)	P_c (W)	Rolling (%)	Air (%)
10	(2.78)	22.0	73.2	26.8	11.9	65.3	34.7	6.4	47.7	52.3
15	(4.17)	43.8	55.2	44.8	25.6	45.6	54.4	15.8	29.0	71.0
20	(5.55)	78.7	40.9	59.1	48.6	32.0	68.0	32.7	18.7	81.3
25	(6.94)	131.0	30.7	69.3	84.1	23.1	76.9	59.5	12.8	87.2
30	(8.33)	205.2	23.6	76.7	135.0	17.3	82.7	98.9	9.3	90.7
35	(9.72)	305.4	18.5	81.5	204.6	13.3	86.7	153.1	7.0	93.0
40	(11.11)	436.2	14.7	85.3	295.8	10.5	89.5	224.8	5.4	94.6
45	(12.50)	601.8	12.0	88.0	411.9	8.5	91.5	316.5	4.3	95.7
50	(13.89)	806.5	10.0	90.0	553.9	7.0	93.0	430.5	3.5	96.5
55	(15.28)	1054.8	8.4	91.6	730.9	5.8	94.2	569.5	2.9	97.1

Table 3 Rolling coefficient ($a \cdot M^{-1} \cdot g^{-1}$) and rolling resistance (a) for an overall moving mass (M = cyclist + bike) of 85 kg for different types of tyres on concrete or asphalt roads ($g = 9.81\ m \cdot s^{-2}$). Wheel diameter, together with tyre width (of the tyre contact patch) and inflation pressure (1 MPa = 9.87 atm = 7501 mmHg), are also indicated. Energy expenditure against the rolling resistance, per unit of distance, can be obtained easily by dividing the rolling coefficient by the overall efficiency of cycling (≅0.25, see "The efficiency of cycling") and multiplying it by $M \cdot g$. A to C, knobby (mountain bike type) or standard road tyres with tube; D, road tubular tyres (180 g); E, road tubular tyres (160 g); F, track tubular tyres (80 g). On linoleum or wooden tracks, the rolling coefficient is reduced to about 60% of the present values; on rough surfaces it can be much larger. Data from Capelli et al. (1993), di Prampero et al. (1979), Kyle (1986, 1988, 1989), Olds et al. (1993, 1995), and Pugh (1974)

Type of tyre	Tyre characteristics		Wheel diameter	Rolling coefficient	Rolling resistance
	Width (cm)	Pressure (MPa)	(cm)	$a \cdot M^{-1} \cdot g^{-1}$	for M = 85 kg: $a \cdot (J \cdot m^{-1})$
A knobby	5.7	0.32	50.8	0.017	14.2
B knobby	5.7	0.32	68.6	0.013	10.8
C road, standard	4.5	0.46	68.6	0.007	5.8
D road, tubular	3.2	0.85	50.8	0.0045	3.8
E road, tubular	1.8	0.85	68.6	0.0034	2.8
F track, tubular	1.8	0.85	68.6	0.0021	1.8

Table 4 Drag coefficient (C_d) and area projected on the frontal plane (A_f) for a cyclist of 70 kg body mass and 175 cm stature (body surface area, $A_{bs} = 1.85\ m^2$) at sea level (P_B = 760 mmHg = 101.3 kPa) and at 20 °C air temperature are reported for several conditions. The fourth column reports the constant relating the mechanical work per unit of distance to the square of air speed (b). This is also expressed per m² (A_{bs}; third column, $b \cdot A_{bs}^{-1}$). The corresponding values of the constant β can be easily obtained, since $\beta = b \cdot \eta^{-1}$ and $\eta \cong 0.23$ (see "The efficiency of cycling"). Data from Capelli et al. (1993), di Prampero et al. (1979), Kyle (1986, 1988, 1989), Olds (1993, 1995), and Pugh (1974). The fifth and sixth columns are the P_c as a percentage of that required when riding a standard racing bicycle in dropped posture at the same speed, and the speed attained on flat smooth terrain with a P_c of 0.735 kW. These speeds were calculated considering a value of the rolling resistance equal to that applying to tubular tyres for road conditions (see Table 3). It should also be noted that a P_c of 0.735 kW (1 HP) can easily be sustained for 30 s by well-trained subjects, and that elite athletes can maintain powers of 1.1–1.3 kW for 30 s. The reported speeds are therefore realistic for short-duration sprints

Bicycle	C_d	A_f (m²)	$b \cdot A_{bs}^{-1}$ ($N \cdot s^2 \cdot m^{-2} \cdot m^{-2}$)	b ($N \cdot s^2 \cdot m^{-2}$)	%P_c	s ($km \cdot h^{-1}$) for P_c = 0.735 kW
Traditional, sitting	1.1	0.51	0.182	0.337	175	45.4
Recreational, leaning forward	1.0	0.45	0.146	0.271	140	51.2
Touring (standard with fenders and flask), dropped	0.87	0.44	0.124	0.230	119	52.1
Racing (standard) dropped posture	0.80	0.40	0.104	0.193	100	55.3
Racing (special frame and wheels), dropped	0.65	0.40	0.085	0.155	80	60.6

activation strategies, and/or to perceived exertion (Marsh and Martin 1993, 1995, 1998) may play a non-negligible role in setting the self-selected f_p.

The rolling resistance

The frictional losses in the bearings and drive train of a good-quality bicycle are very small (Kyle 1986). So, the rolling resistance is set essentially by the size, type and inflation pressure of the tyres, and by the characteristics of the terrain. The rolling resistance is independent of the speed and is proportional to the overall weight (cyclist plus bicycle). It is therefore customary to report the rolling resistance as a "rolling coefficient" (i.e. as the ratio between the rolling resistance itself and the overall weight). The rolling coefficients for several sets of conditions are reported in Table 3. The constant a can be calculated from these values, once the overall weight is known. It should also be noted that when the rolling coefficient that yields the constant α is not available, it can be calculated from that yielding a (since $\alpha = a \cdot \eta^{-1}$), assuming an efficiency value of 0.23–0.25, without fear of committing large errors (see "The efficiency of cycling", and Fig. 3). Table 3 shows that the minimal values attained for tyres of small width (of the tyre contact patch) at high inflation pressures on smooth surfaces are about three times smaller that those for conventional tyres on standard concrete roads, and about ten times smaller that those for knobby tyres.

The air resistance

The constant relating the air resistance (b), or the corresponding energy expenditure per unit of distance (β) to the s^2 in Eqs. 1 and 3 is proportional to C_d, to A_f and to the air density (ρ_a):

$$b = 0.5C_d \cdot A_f \cdot \rho_a \quad (6)$$

$$\beta = b \cdot \eta^{-1} = 0.5C_d \cdot A_f \cdot \rho_a \cdot \eta^{-1} \quad (7)$$

In turn, ρ_a is a function of the air pressure (P_B) and temperature (T), as described by:

$$\rho_a = \rho_0 \cdot 0.359 P_B \cdot T^{-1} \tag{8}$$

where ρ_0 ($= 1.293$ kg $\cdot$ m^{-3}) is the air density at 760 mmHg (101.3 kPa) and 273 K (see Weast 1989), and $0.359 = 273/760$. It should be pointed out that ρ_a also depends upon the air humidity, the effect of which, however, is very small and will be neglected here. P_B decreases with altitude above sea level; as a first approximation, for $T = 273$ K, the decrease of P_B with altitude (km) is described by:

$$P_B(\mathrm{mmHg}) = 760 \cdot e^{-0.124\,\mathrm{km}} \tag{9}$$

$$P_B(\mathrm{kPa}) = 101.3 \cdot e^{-0.124\,\mathrm{km}} \tag{9'}$$

Equations 5–9 show that changes in A_f, C_d, or altitude, all lead to proportional changes in the C_c at a given velocity or to a different velocity for a given $\dot{E}_c$. These effects are discussed briefly below.

On size and shape

An immediately intuitive strategy for reducing the A_f is that of leaning forward. In racing bicycles, especially in the most recent ones, this strategy is pushed to the extreme so that the cyclist is forced to assume the position yielding the least possible value of A_f (Capelli et al. 1993; Grappe et al. 2000), even if this can lead to a slight, but significant fall of $\dot{V}O_{2\,max}$ (Welbeergen and Clijsen 1990) or of η (Gnehm et al. 1997). Forward lean also reduces the value of C_d, bringing about a further reduction of the air drag force (Capelli et al. 1993).

Obviously, the most appropriate strategy for reducing C_d is that of adopting streamlined fairing, in which case values as low 0.09–0.11 have been reported (Gross et al. 1983; Dal Monte 1997, personal communication). These considerations are summarised quantitatively in Table 4, where A_f and C_d are reported from literature data under several sets of conditions, together with the constant b.

Another intuitive strategy for reducing the effects of the air resistance is to move in the wake of another cyclist or of a vehicle proceeding at the same speed, in which case the cyclist takes advantage of the aerodynamic draft behind the lead rider or vehicle. At racing speeds, cyclists closely in the wake of one another can reduce their power output by about 30% (Kyle 1979; Olds 1998). The extreme example of this strategy is the incredible speed of 223.13 km $\cdot$ h^{-1} attained over a distance of 1.2 km by A. Abbott (USA) in 1973, cycling on a specially prepared bicycle in the wake of a specially prepared car. The power reduction also depends upon the position within a pace line. Indeed, as shown by Broker et al. (1999), who measured the mechanical power during a 4000-m team pursuit, at 60 km $\cdot$ h^{-1} the average power was 607 W in lead position. This was reduced to 430 W (70.8%) in second, and to 389 W (64%) in third and fourth positions.

As a first approximation, humans can be viewed as solids of similar shape and equal density; hence the body surface area (A_{bs}) increases with the square of the linear dimension, and the body mass with the cube of the same dimension. If the further assumption is made that, for a given body posture, A_f is a constant fraction of A_{bs} (which may not be so straightforward after all; e.g. see Capelli et al. 1998; Swain et al. 1987), it necessarily follows that larger subjects have a lesser A_f per unit body mass than smaller subjects. Hence, in larger cyclists the energy expenditure per unit body mass at a given speed will be less or, conversely, the speed attained with a given $\dot{E}_c$ per unit body mass will be greater. For cyclists of extremely different body sizes (50 kg, 150 cm vs 100 kg, 200 cm) the difference in $\dot{E}_c$ at any given speed reaches 18% (see Fig. 4). It should also be pointed out here that this type of analysis, as stated, applies only to the energy expenditure against air resistance; it therefore neglects the rolling resistance and, more importantly, the energy expenditure against gravity when cycling uphill (see "On sloping ground"), which are both proportional to the mass transported. Furthermore, since the $\dot{V}O_{2\,max}$ per unit of mass tends, on average, to be smaller in larger subjects (Åstrand and Rodahl 1986), these considerations also suggest that there is an optimum body mass for level cycling performances.

Altitude and performance

The present world record for 1 h of unaccompanied cycling (56.375 km) was set by Boardman at sea level,

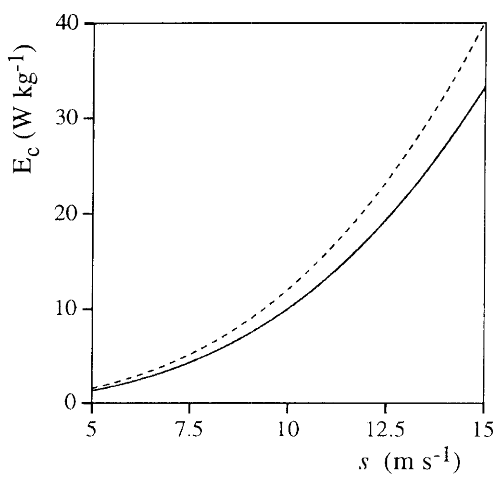

Fig. 4 Metabolic power output per kg body mass, for riding a traditional racing bicycle in dropped posture at constant speed in the absence of wind ($\dot{E}_c$, W $\cdot$ kg^{-1}) as a function of the ground speed (s, m $\cdot$ s^{-1}) for two cyclists of widely different body size. *Upper broken curve* mass 50 kg; stature 150 cm; body surface area 1.433 m^2; *lower continuous curve* mass 100 kg; stature 200 cm; body surface area 2.369 m^2. Bicycle mass 10 kg. Curves were calculated from the data given in Tables 3 and 4, for $P_B = 760$ mmHg (101.3 kPa) and $T = 20$ °C, and assuming $\eta = 0.25$. For each cyclist, the maximal aerobic speed is set by the intersection between the appropriate function and a *horizontal line* at the level of the subject's $\dot{V}O_{2\,max}$

as were the preceding records by Rominger (55.291 and 53.832 km), Indurain (53.040 km), Obree (52.713 km) and Boardman himself (52.270 km). However, since the constant β of Eq. 5 decreases at altitude (Eqs. 8 and 9), the $\dot{E}_c$ required to proceed at a given speed will be less (and conversely, the speed attained with a given power will be larger) at altitude. Furthermore, the effects of altitude on cycling performance can be calculated, provided that the cyclist's $\dot{V}O_{2\,max}$, together with its reduction at altitude are known (Bassett et al. 1999; Capelli and di Prampero 1995; di Prampero et al. 1979; Péronnet et al. 1989). It was also shown by di Prampero et al. (1979) that: (1) the ideal altitude ought to be in the order of 4 km above sea level, and (2) in an "evacuated" velodrome, an elite cyclist could cover about 580 km in 1 h, pedalling in a pressurised suit providing him with a partial pressure of O_2 in the inspired air of about 150 mmHg. Apart from these unrealistic scenarios, the aim of what follows is to calculate the maximal distance that could be covered in 1 h by a cyclist with the same body build and $\dot{V}O_{2\,max}$ as Chris Boardman, riding the same bike on a similar velodrome at altitude. Additional assumptions will be that the performance occurs in the absence of wind at an air temperature of 20 °C, and that the cycling posture is exactly the same as at sea level.

The maximal speed a cyclist can maintain over 1 h depends essentially on his $\dot{V}O_{2\,max}$ and on the fraction of $\dot{V}O_{2\,max}$ that he can sustain throughout the effort (F). Indeed, the average power that can be derived from anaerobic energy stores over 1 h amounts to about 1.3% of that obtained, over the same time, from $\dot{V}O_2$ (Capelli and di Prampero 1995). Thus, at maximal "1-h" speed at sea level, Eq. 5 becomes:

$$\dot{E}_c = \mathrm{F} \cdot \dot{V}O_{2\,max} = \alpha \cdot s_{sl} + \beta \cdot s_{sl}^3 \quad (10)$$

where $\dot{V}O_{2\,max}$ is expressed in Watts on the basis of a conversion factor of 20.9 kJ · l^{-1}, F($\leqslant$1) is the maximal fraction of $\dot{V}O_{2\,max}$ that the cyclist can maintain throughout the performance, and where the suffix "sl" denotes sea-level conditions. The decreased barometric pressure at altitude affects both $\dot{V}O_{2\,max}$ and the constant β. Therefore, at altitude:

$$\mathrm{A} \cdot \mathrm{F} \cdot \dot{V}O_{2\,max} = \alpha \cdot s_a + \mathrm{k} \cdot \beta \cdot s_a^3 \quad (11)$$

where: (1) A and k are the fractional decreases of $\dot{V}O_{2\,max}$ and of β, respectively, due to altitude, and (2) the suffix "a" denotes altitude conditions. Substituting Eq. 10 into Eq. 11:

$$\mathrm{A} \cdot (\alpha \cdot s_{sl} + \beta \cdot s_{sl}^3) = \alpha \cdot s_a + \mathrm{k} \cdot \beta \cdot s_a^3 \quad (12)$$

The metabolic power dissipated against non-aerodynamic forces at altitude and at sea level is rather close: $\mathrm{A} \cdot \alpha \cdot s_{sl} \cong \alpha \cdot s_a$. Indeed, the decline brought about by A (<1.0, see Fig. 5) is compensated for in part by the increase in s_a (see below). Furthermore, the metabolic power dissipated against non-aerodynamic forces at high speeds is less than 3% of the total (Table 2). Hence, as a first approximation these two terms can be omitted. Therefore, simplifying and rearranging Eq. 12:

$$s_a/s_{sl} = \sqrt[3]{(\mathrm{A}/\mathrm{k})} \quad (13)$$

the relative gain in speed at any given altitude can be obtained provided that A and k are known.

The decrease in $\dot{V}O_{2\,max}$ with altitude is rather well known from literature data on non-athletic subjects (for a review see Cerretelli 1981); in athletes it is substantially larger (Ferretti et al. 1997; Fig. 5). Since the calculations that follow refer to top performances by elite

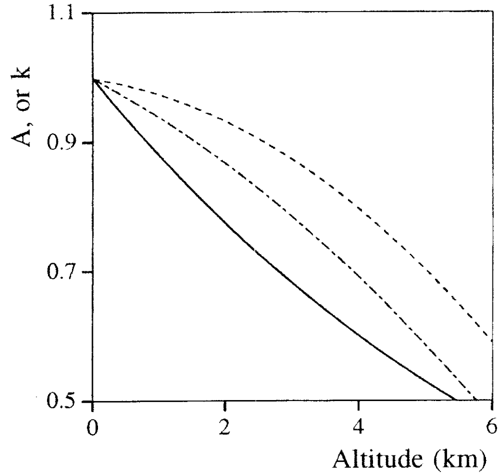

Fig. 5 Ratio of $\dot{V}O_{2\,max}$ at altitude to $\dot{V}O_{2\,max}$ at sea level (A) as a function of altitude above sea level. The *upper broken curve* was obtained in a study on trained non-athletic subjects (from Cerretelli 1981); the *middle dashed curve* was obtained in a study on trained athletes whose average $\dot{V}O_{2\,max}$ at sea level was 66.3 ml · kg^{-1} · min^{-1} (from Ferretti et al. 1997). The fall in air density at constant temperature (k) is shown by the *lowest*, *continuous curve*

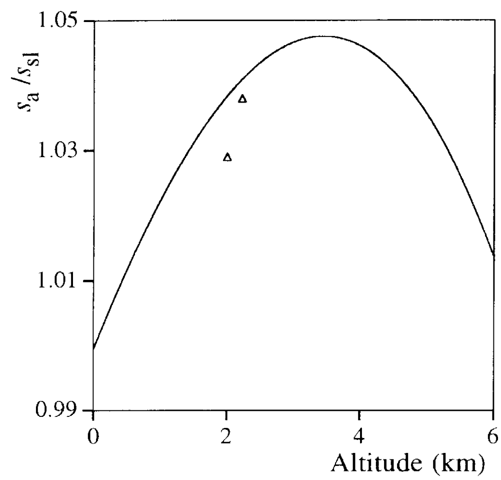

Fig. 6 The ratio between the speed for 1-h unaccompanied cycling at altitude and at sea level for an elite cyclist (s_a/s_{sl}) is shown as a function of altitude by the *continuous curve*. The two *triangles* indicate the relative speeds at Colorado Springs (2 km) and at Mexico City (2.22 km) of J. Longo and F. Moser. (see text for details.)

athletes, the factor A (i.e. the relative fall in $\dot{V}O_{2\,max}$ at altitude as compared to sea level) can be obtained from this last set of data. In addition, since it was assumed here that air temperature and body posture (and thus A_f and C_d) are the same at sea level and altitude; the constant k reduces to the ratio of the P_B at altitude and at sea level (see Eqs. 8 and 9 and Fig. 5). The ratio s_a/s_{sl} can therefore be calculated for any given altitude. Figure 6 shows that in Mexico (2230 m above sea level), s_a/s_{sl} amounts to 1.04, which would bring Boardman to cover 58.63 km in 1 h. Figure 6 also shows that, other things being equal, and as originally pointed out by di Prampero et al. (1979), the optimum altitude is in the order of 3.8 km, not far from that of the Alto Irpavi velodrome (La Paz, Bolivia), where the ratio s_a/s_{sl} reaches 1.046, and where, therefore, Boardman could cycle 58.97 km in 1 h. However, a much larger improvement could be obtained if Boardman could cycle in an air-tight velodrome wherein he could breath pure O_2 at a partial pressure of 98 mmHg (i.e. equal to the P_B prevailing at the "ideal" altitude of 3.8 km, and at which A = 0.75). Taking into account the different molecular masses of O_2 and of average air, this yields a value for k of 0.143. Hence, as from Eq. 13, $s_a/s_{sl} = 1.706$, which would allow Boardman to cycle the remarkable distance of 96.18 km in 1 h!

The calculations reported above are based on the implicit assumptions that: (1) the fall in $\dot{V}O_{2\,max}$ at altitude is the same in elite as in medium level athletes ($\dot{V}O_{2\,max}$ at sea level = 66.3 (3.2) ml · kg^{-1} · min^{-1}, see Ferretti et al. 1997), from which the factor A was obtained, and (2) the fraction of $\dot{V}O_{2\,max}$ that the cyclist can maintain at altitude (F, in Eqs. 10 and 11) is the same as at sea level. That this may not be far from the truth is suggested by the fact that the two cyclists (J. Longo and F. Moser) who, within a short period of time, attempted maximal 1-h performances at sea level and at altitude (Colorado Springs, 2.00 km and Mexico, 2.22 km) improved their performances by 2.9 and 3.8%, respectively (i.e. by a fraction close to that predicted from Eq. 13; see Fig. 6).

On sloping ground

The preceding sections were devoted to the analysis of cycling on flat terrain, a fact that is strictly applicable only to track conditions. To deal with the energetics of cycling on upsloping or downsloping roads, the additional mechanical work, or power, dissipated against (or made available by) gravity must be considered. The mechanical work performed against gravity when cycling uphill is given by the product of the overall moving mass (M), the acceleration due to gravity, g ($=9.81$ m · s^{-2}) and the vertical displacement (h): $M \cdot g \cdot h$. When expressed per unit of distance covered along the road (d), and since $h = d \cdot \sin\gamma$ (where γ is the angle between the terrain and the horizontal), the mechanical work performed against gravity becomes:

$$W_{cg} = M \cdot g \cdot h \cdot d^{-1} = M \cdot g \cdot d \cdot (\sin\gamma) \cdot d^{-1} = M \cdot g \cdot \sin\gamma \quad (14)$$

The quantity expressed by Eq. 14 can be added to Eq. 1 to obtain a comprehensive description of the mechanical work performed per unit of distance when cycling at constant ground speed in the absence of wind (i.e. $s = v$):

$$W_c = a + b \cdot s^2 + M \cdot g \cdot \sin\gamma \quad (15)$$

For downsloping terrain, the third term of Eq. 15 is negative: it represents the energy equivalent made available by gravity that can be utilised against the other forces opposing motion (air and ground friction). Equation 15 can then be used to calculate the speed attained during a free-wheel run down a known slope. For example, assume a 70-kg, 175-cm cyclist, riding a 10-kg standard racing bicycle in fully dropped posture down a smooth asphalt road that has a gradient with an angle $\gamma = 5°$. The corresponding values of a and b (as from Tables 3, 4) can then be inserted into Eq. 15. So, the constant free-wheel speed can be obtained by setting $W_c = 0$ and solving for s. It amounts to 18.44 m · s^{-1} (66.4 km · h^{-1}).

The overall C_c at constant speed in the absence of wind can be obtained easily by substituting a and b in Eq. 15, with α and β (see Eqs. 2 and 3) and dividing the last term by η (see "The efficiency of cycling"):

$$C_c = \alpha + \beta \cdot s^2 + M \cdot g \cdot (\sin\gamma) \cdot \eta^{-1} \quad (16)$$

The overall $\dot{E}_c$ then becomes:

$$\dot{E}_c = C_c \cdot s = \alpha \cdot s + \beta \cdot s^3 + M \cdot g \cdot s \cdot (\sin\gamma) \cdot \eta^{-1} \quad (17)$$

This equation can be utilised to calculate the maximal slope that a given cyclist can climb while riding a bike. To this aim, the speed must be assigned a minimal value below which equilibrium can not be maintained. In this case, if the subject's maximal metabolic power ($\dot{E}_c$) is known, and if s in Eq. 17 is given this minimal value, the incline of the terrain can be easily obtained. These calculations show that, if the minimal speed is tentatively given a value of 1 m · s^{-1} (3.6 km · h^{-1}), the maximal slope a subject can climb with a $\dot{V}O_{2\,max}$ above resting of 43 ml · kg^{-1} · min^{-1} (15 W kg^{-1}) is about 42% ($\gamma \cong 23°$); it increases to about 70% ($\gamma \cong 35°$) for a $\dot{V}O_{2\,max}$ of 63 ml · kg^{-1} · min^{-1} (22 W · kg^{-1}), where the power is expressed per kg overall weight (cyclist + bike). Obviously, these performances will be possible only on smooth terrain and with the use of an appropriate gear system allowing optimum f_p also at very slow speeds. These calculations can be easily performed algebraically or by simple computer programs; they can also be simplified by the use of appropriate nomograms (e.g. see di Prampero 1986; di Prampero et al. 1979).

Finally, it should be noted that the slope of the terrain is generally expressed as a percentage [i.e. as the tangent (×100) of the angle γ]. However, for $\gamma \leqslant 16°$, tan $\gamma \cong \sin \gamma$ within an error of 4% so that, within this range of slopes, replacing the sine of the angle with its tangent does not lead to any major errors. For larger angles, however, the difference between tangent and sine increases progressively, so that the two quantities should not be used interchangeably.

Top performances

The speed attained with any given $\dot{E}_c$ can be easily obtained by using Eq. 5 (or Eq. 17), provided that the actual cycling conditions are known, so that the appropriate coefficients can be used. Indeed, this makes it possible to calculate the speed corresponding to any preset value of $\dot{E}_c$, either graphically (e.g., see Fig. 4) or by numerical iteration.

This approach can be appropriately used only if $\dot{E}_c$ is constant, independent of the performance time (t_p), as is typically the case for long-duration events, such as the 1-h world record (see Air resistance).

The aim of this section is to extend this approach to include top performances over much shorter distances, in which case the $\dot{E}_c$ developed by the subject is greatly affected by the duration of the performance, and hence by the distance covered.

We will deal here only with track performances in the absence of wind, under which conditions the metabolic power requirement to cover the distance d in the t_p, and hence at the speed s (where $s = d \cdot t_p^{-1}$) is given by (see Eq. 5):

$$\dot{E}_c = C_c \cdot d \cdot t_p^{-1} = \alpha \cdot d \cdot t_p^{-1} + \beta \cdot d^3 \cdot t_p^{-3} \quad (18)$$

Track cycling events are often performed from a stationary start, in which case Eq. 18 must be modified to take into account the energy spent to accelerate the body from zero to the final speed (E_{ktot}). As a first approximation this can be viewed as:

$$E_{ktot} = 0.5 \cdot M \cdot d^2 \cdot t_p^{-2} \cdot \eta^{-1} \quad (19)$$

or, expressing E_{ktot} per unit of mass and of distance covered:

$$E_k = E_{ktot} \cdot M^{-1} \cdot d^{-1} = 0.5 \cdot d \cdot t_p^{-2} \cdot \eta^{-1} \quad (20)$$

It should be noted here that Eq. 20 is strictly correct if, and only if, the final speed attained during the event is equal to the average speed. This is certainly not the case, so that a more correct formulation of E_k should take this fact into account. However, as a first approximation, Eq. 20 yields a satisfactory quantitative estimate of E_k (Capelli et al. 1998). For a more detailed analysis of this point, the reader is referred to Olds et al. (1993, 1995). So, for events initiated from a stationary start, Eq. 18 becomes:

$$\begin{aligned}\dot{E}_c &= \alpha \cdot d \cdot t_p^{-1} + \beta \cdot d^3 \cdot t_p^{-3} + E_k \cdot d \cdot t_p^{-1} \\ &= \alpha \cdot d \cdot t_p^{-1} + \beta \cdot d^3 \cdot t_p^{-3} + 0.5 \cdot d^2 \cdot t_p^{-3} \cdot \eta^{-1}\end{aligned} \quad (21)$$

Equations 18 and 21 show that, for any given distance d, $\dot{E}_c$ increases with decreasing t_p. It also follows that the shortest time (fastest speed) over that distance will be achieved when $\dot{E}_c$ is equal to $\dot{E}_{max}$ (Fig. 7). Many studies have shown that $\dot{E}_{max}$ is a decreasing function of the duration of exercise to exhaustion (exhaustion time, t_e). According to Wilkie (1980):

$$\dot{E}_{max} = \dot{V}O_{2max} + AnS \cdot te^{-1} - \dot{V}O_{2max} \cdot (1 - e^{-te/\tau}) \cdot \tau \cdot t_e^{-1} \quad (22)$$

where $\dot{V}O_{2max}$ is expressed in Watts, AnS is the amount of energy derived from the complete exploitation of the anaerobic sources (maximal lactic acid formation and maximal phosphocreatine breakdown), and where the third term takes into account the fact that, at the onset of exercise, $\dot{V}O_{2max}$ is not reached instantaneously, but with a time constant τ. Eq. 22 applies for 50 s $\leqslant t_e \leqslant$ 15 min (i.e. in a time range sufficient for full utilisation of AnS, but within which $\dot{V}O_{2max}$ can be fully maintained throughout the exercise. Thus, if τ, $\dot{V}O_{2max}$ and AnS are known, $\dot{E}_{max}$ can be described analytically as a function of t_e.

On the other hand, if C_c is known, $\dot{E}_c$ can be described analytically as a function of t_p, over any given distance d, by means of Eqs. 18 or 21. Furthermore, it can also be assumed that the best t_p, over a given dis-

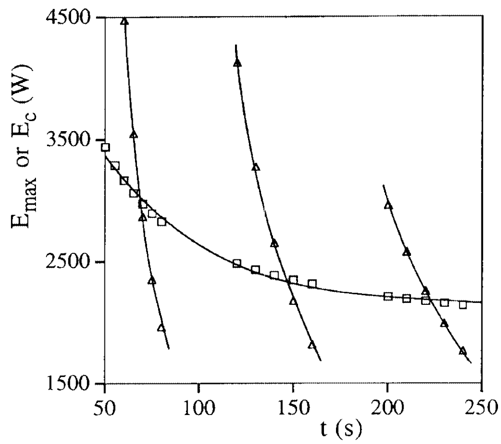

Fig. 7 The metabolic power ($\dot{E}_c$, W) required in track cycling to cover the distances of 1, 2, or 3 km from a stationary start, in the time reported on the abscissa, is indicated by the three steep functions (*triangles*). The maximal metabolic power ($\dot{E}_{max}$, W) that a top athlete can maintain at a constant level during an all-out effort of the duration reported on the same time axis is also indicated (*empty squares*). The best performance time over the three distances for the athlete in question is given by the time value at which the appropriate $\dot{E}_c$ function crosses the $\dot{E}_{max}$ function. $\dot{E}_c$ was calculated by inserting into Eq. 21 the appropriate distances and the values of a and b for a 70-kg, 175-cm athlete riding a standard racing bicycle on a linoleum-covered track in the absence of wind at $P_B = 760$ mmHg (101.3 kPa) and $T = 20$ °C (see Tables 3, 4), and assuming further $\eta = 0.25$. $\dot{E}_{max}$ was obtained from Eq. 22, assuming $\tau = 10$ s (Wilkie 1980), $\dot{V}O_{2max} = 2.0$ kW (76.5 $mlO_2 \cdot kg^{-1} \cdot min^{-1}$ above resting) and the maximal capacity of the anaerobic energy stores (AnS) = 140 kJ (corresponding to 90 $mlO_2 \cdot kg^{-1}$). See text for details

tance for a given cyclist, is the time value for which exhaustion and performance coincide (i.e. $t_e = t_p$). If this is so, the best t_p (for a given cyclist and distance) can be obtained (graphically or by numerical iteration) as the time value for which $\dot{E}_{max}$ (Eq. 22) and $\dot{E}_c$ (Eqs. 18 or 21) become equal (see Fig. 8).

This approach was originally proposed by di Prampero (1986, 1989) and applied to track running by di Prampero et al. (1993) and by Pérronnet and Thibault (1989). Recently, Capelli et al. (1998) have applied it to track cycling from a stationary start in a group of amateur cyclists of medium level. They have compared the theoretical best performance times, calculated as described above, to the actual best performances, obtained during coeval maximal runs over the same distances. The results, summarised in Table 5, show that actual times and theoretical times are rather close, their average (SD) ratio amounting to 1.035 (0.058) (Capelli et al. 1998).

A similar approach, based on overall energy expenditure, rather than on power output, was applied to cycling by Olds et al. (1993, 1995). Also, in this case the agreement between facts and theories is rather good, thus showing that our understanding of the biomechanics and of the bioenergetics of bicycling is sufficiently good as to allow us to realistically predict performances (Capelli 1999).

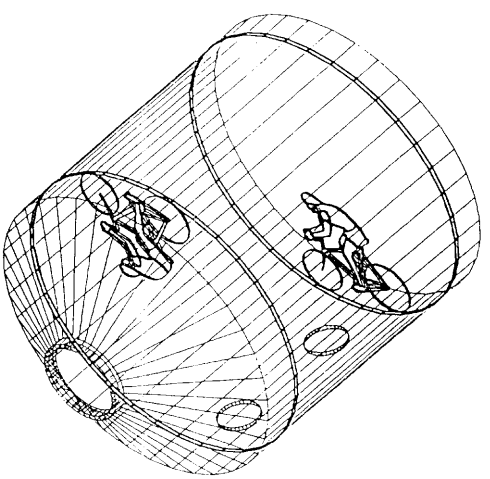

Fig. 8 Cyclists moving along the inner wall of a cylindrically shaped space module generate an acceleration vector that mimics gravity

Table 5 Ratios between actual and theoretical times (t_{act}/t_{theor}) are reported for the four indicated distances. Values are means (SD). With the exception of the shortest distance, all average ratios were significantly different from 1. Even so, the agreement between theoretical and actual times of performance seems remarkably good (see text for details). From Capelli et al. (1998)

Distance (km)	t_{act}/t_{theor} (SD)	n
1.02	0.980 (0.055)	10
2.05	1.035 (0.027)	10
5.12	1.056 (0.043)	10
10.24	1.070 (0.061)	10
Grand average	1.035 (0.058)	40

Cycling in space

The preceding sections were devoted to traditional forms of cycling. I will try here to convince the reader that cycling can also be very useful in less conventional situations.

Exposure to microgravity leads to a reduced exercise capacity and tolerance. Appropriate countermeasures are therefore necessary for long-term space flights. These countermeasures are generally based on: (1) exercise training programmes, (2) lower-body negative pressure suits, (3) elastic cords pulling the subject's body towards the floor of the cabin, and (4) artificial gravity obtained by rotation of the spacecraft, or parts thereof.

In a previous paper (Antonutto et al. 1991), we proposed to simulate gravity on a space vehicle by using two mechanically coupled counter-rotating bicycles (a Twin Bikes System, TBS) that move along appropriately constructed structures. The obvious advantage of this approach is that of combining exercise and simulated gravity, with the aim of preventing at one and the same time muscle atrophy, bone demineralisation and cardiovascular deconditioning.

The TBS consists of two bicycles that move at the very same speed, but in the opposite sense, along the inner wall of a cylindrically shaped space module. Two adjustable masses, applied to an axle mounted in opposition to each bike, prevent the repetitive yaws that would otherwise occur when the two bicycles are on the same side of the space module (Figs. 8, 9). The wheels run on two parallel rails that provide the necessary initial friction. The circular trajectories induce centrifugal acceleration vectors (a_c) oriented along the head-to-feet direction of each subject. Since a_c is given by the ratio of the square of the tangential velocity (v_t) to the radius of gyration (r):

$$a_c = v_t^2 \cdot r^{-1} \tag{23}$$

the v_t yielding $a_c = 1\ g$ at the feet level can be calculated for any given value of r (e.g. for $r = 2$ m, as is generally the case for a conventional space module, $v_t = 4.5\ \text{m} \cdot \text{s}^{-1}$).

The mechanical and metabolic powers yielding a given v_t, and hence a given acceleration, depend upon the air density (and hence P_B and T) prevailing inside the space module and, to a lesser extent, on the wheels rail friction (see Eqs. 4 and 5). Assuming $P_B = 760$ mmHg (101.3 kPa) and $T = 293$ K and a rolling resistance applying to a knobby-tyred bike on a concrete surface on Earth (see Tables 3 and 4), and assuming for simplicity that the air in the space module remains still in spite of the two counter-rotating cyclists, the mechanical and metabolic powers yielding 1 g at the feet level, for a 70-kg, 175-cm astronaut can be calculated. For $r = 2$ m,

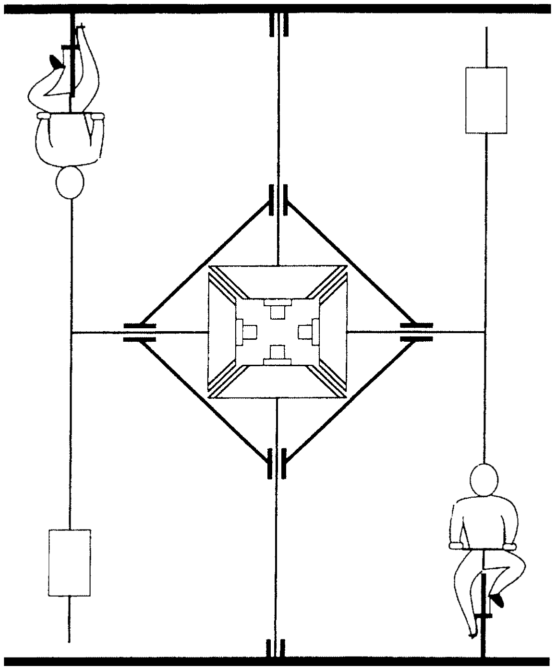

Fig. 9 Schematic view of the Twin Bikes System (TBS). *Thick lines* indicate the space module walls. The differential gear coupling the two cyclists is drawn on a larger scale. Adjustable masses are also shown. See text for details

they amount to 75 W and 1.2 $lO_2 \cdot min^{-1}$, respectively. If $r = 6$ m, then $v_t = 7.8$ m $\cdot$ s^{-1}, and the mechanical and metabolic powers increase to 240 W and 3.05 l $\cdot$ min^{-1}, respectively. The value of r is substantially smaller at the level of the head than at the feet, so a_c will also be smaller at the head, the ratio of a_c at the feet to a_c at the head being 5 for $r = 2$ m, and 1.5 for $r = 6$ m.

Since the subject is sitting on the saddle, the centrifugal force due to the upper part of the body is supported by the frame of the bike, whereas the lower limbs sustain only a force equal to their own mass times a_c. This situation is similar to that applying to a subject pedalling on Earth, where the only part of the subject's weight-bearing skeleton directly supporting gravity is the spine. The femurs and tibiae support only their own weight plus the forces generated by the muscle action during the pushing phases of each pedal revolution.

The above considerations do not apply to the circulatory system, wherein the continuity of the blood column within the vessels mimics to a closer extent the condition of a subject standing on Earth. Indeed, at any level in the circulatory system, the prevailing pressure is given by the sum of the pressure generated by the heart plus (or minus) the hydrostatic component (ΔP) equal to the weight of the column of blood from the heart to the point in question. In turn, ΔP is given by:

$$\Delta P = \rho_b \cdot a_c \cdot h \quad (24)$$

where ρ_b is the blood density, a_c is the average centrifugal acceleration, and h the distance between the heart and the point in question. As stated above, a_c depends upon the v_t ($a_c = v_t^2/r$), itself a function of the angular velocity (ω):

$$v_t = \omega \cdot r \quad (25)$$

Hence:

$$a_c = v_t^2/r = \omega^2 \cdot r \quad (26)$$

For a rigid body rotating at a constant rate, the angular velocity is also constant and independent of r. It therefore follows that the average a_c value between any two points at distances $r1$ and $r2$ from the centre of gyration is given by:

$$a_c = (\omega^2 \cdot r1 + \omega^2 \cdot r2)/2 = 0.5 \cdot \omega^2(r1 + r2) \quad (27)$$

Since $h = r2 - r1$, from Eq. 27 and Eq. 24, and rearranging:

$$\Delta P = \rho_b \cdot \omega^2 \cdot 0.5 \cdot (r2^2 - r1^2) \quad (28)$$

where $r1$ and $r2$ are the distances from the gyration centre of the two points considered. Taking $r1$ at the level of the heart, Eq. 28 can be used to calculate the arterial pressure at any given point in the circulatory system, provided that the pressure at heart level is known. For $a_c = 1$ g at the subject's feet, and an average pressure of 100 mmHg in the aortic bulbus, the average arterial pressures at the head and feet levels, for a subject riding a conventional bike, are 95/150 mmHg (12.7, 20 kPa) for $r = 2$ m, and 80/170 mmHg (10.7, 22.7 kPa) for $r = 6$ m (Fig. 10).

When riding the TBS described above, slight head movements may lead to vestibular disturbances induced by the Coriolis cross-coupled angular accelerations of the semicircular canals, which are attributable to simultaneous rotation about more than one axis. Indeed, it is generally believed that the cross-coupled stimulation of two canals and the resulting sensorial conflict are the major determinants of acute motion sickness (AMS). To test this possibility, six healthy male subjects were tested on the human centrifuge at the Karolinska Institute in Stockholm, Sweden (Antonutto et al. 1993). A cycloergometer was fixed to one arm of the centrifuge, at a distance of 2.2 m from the rotation centre, essentially equal to the radius of a conventional space module. The cycloergometer was inclined by 45°, so that the subject's head was closer to the rotation axis, and the centrifuge rotation rate was set at 21 rpm (0.35 Hz), yielding an angular velocity close to that required in the TBS to attain 1 g at the subject's feet. This resulted in a horizontal outward acceleration vector of 1 g at the level of the inner ear. The sum of this vector with the Earth's gravity resulted in a vector of 1.41 g being applied to the subject's inner ear and aligned along his body axis.

The experiment consisted of 20 min of pedalling at 50 W ($f_p = 1$ Hz) during centrifuge rotation. The subject was asked to keep the head still or to move it according to a protocol involving various degrees of rolling, pitching or yawing. The protocol was repeated with eyes open or closed and was assumed to maximise cross-coupled angular accelerations of the semicircular canals.

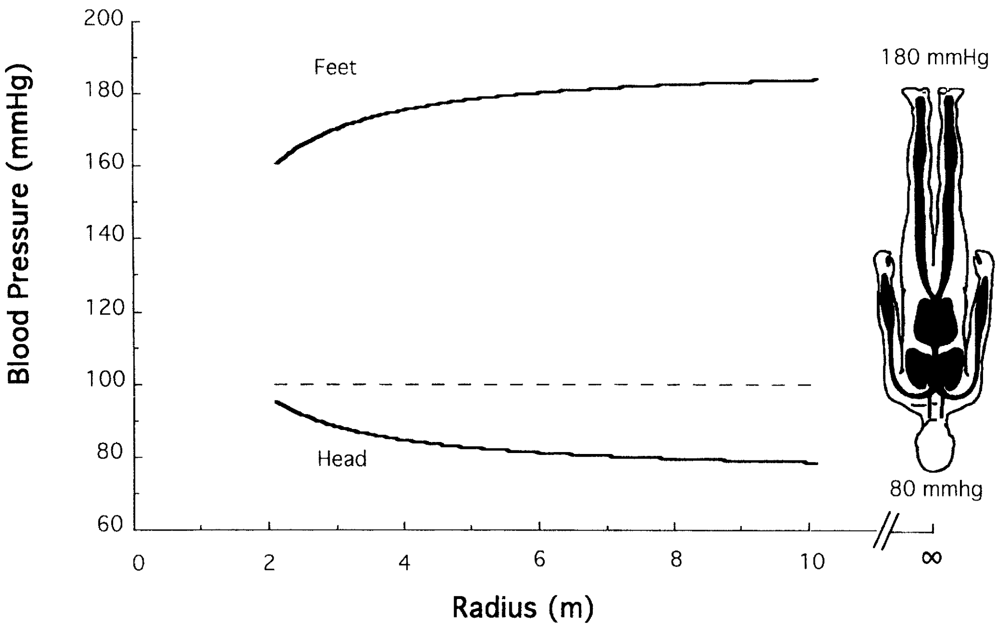

Fig. 10 For a cyclist pedalling in the TBS, the weight of a given column of blood (e.g. 0.01 m) depends upon its distance from the centre of gyration, being zero at the centre itself and increasing progressively less as the distance is increased. The mean arterial pressures prevailing at the feet and head levels, for a peripheral velocity yielding a centrifugal acceleration (a_c) of 9.81 m · s^{-2} at the feet is indicated here as a function of the gyration radius. The mean arterial pressure generated in the aortic bulbus by the left ventricle is assumed to be 100 mmHg (13.3 kPa; *dotted line*). It is apparent that the canonical values applying at the feet and head levels on Earth are attained for gyration radii $\geqslant$ 10 m. For calculations see text

The subjects were interviewed during the centrifuge runs and asked to rate their AMS symptoms according to the diagnostic categorisation proposed by Lackner and Graybiel (1986). In short, only one subject out of six suffered a mild level of AMS (score = 3, out of a maximum of 16), corresponding to a subjective definition of "moderate malaise". The symptoms worsened with eyes open and disappeared rapidly after the end of the run.

The above results demonstrate that the discomfort derived from the rotating environment necessary to generate artificial gravity is reasonably low and well tolerated. Thus, the TBS may indeed prove to be a useful tool for maintaining the astronauts' physical fitness and cardiovascular conditioning. In addition, at variance with other systems that have been proposed for mimicking gravity, the TBS does not need any external power, being operated by the subjects themselves. Finally, a gravity threshold lower than 9.8 m · s^{-2} may be sufficient for reducing microgravity deconditioning to acceptable levels during long-term space flights (Keller et al. 1992). However, the impact loads generated by the TBS may not be sufficient to maintain skeletal bone mass and density, a fact that was not considered or discussed in the preceding paragraphs, and nor will it be discussed in the section that follows. Therefore, it may well become necessary to implement additional countermeasures for maintaining skeletal bone mass and density in space (Nicogossian et al. 1994).

Cycling on the Moon

Cycling on appropriately constructed tracks may be useful also for maintaining physical fitness and cardiovascular conditioning of crews living in permanently manned lunar bases. Indeed, a subject riding a bicycle along a curved path generates a centrifugal acceleration vector given by Eq. 25: $a_c = s^2/r$, where s is the ground speed and r is the radius of curvature of the cyclist's path. Since a_c is applied horizontally outwards, to compensate for it the cyclist must lean inwards so that the vectorial sum of a_c and the constant acceleration of gravity lies in the plane that includes the centre of mass of the system and the points of contact between wheels and terrain (Fig. 11). Thus, the resulting vector (g') can be calculated by simple geometry as:

$$g' = \sqrt{\left(g_L^2 + a_c^2\right)} \tag{29}$$

where g_L (= 1.62 m · s^{-2}) is the acceleration of gravity on the surface of the Moon.

It can therefore be readily calculated that for values of s ranging from 10 to 20 m · s^{-1} (36–72 km · h^{-1}) and r ranging from 50 to 200 m, g' ranges from 1.05 to 5.03 times g_L (i.e. from 0.17 to 0.83 of the Earth's gravity; Fig. 12). Therefore, a cyclist riding a bicycle on a circular or elliptical track, will generate in its curved parts a force acting in the head-to-feet direction that depends upon the radius of the track and on the ground speed, and that can be expected to mimic, to a certain extent, the effects of gravity. As discussed in the previous section dedicated to "space cycling", this state of affairs is likely to counteract atleast partially muscle atrophy, bone demineralisation and cardiovascular deconditioning that may result from long-duration permanence in lunar bases.

It goes without saying that, for the above-described tracks to be operational on the Moon, they must be enclosed in appropriate structures within which the air is maintained at a pre-determined P_B and T. In view of the biomechanical characteristics of cycling discussed in previous sections of this review, the speeds necessary to achieve sufficiently large values of the vector simulating

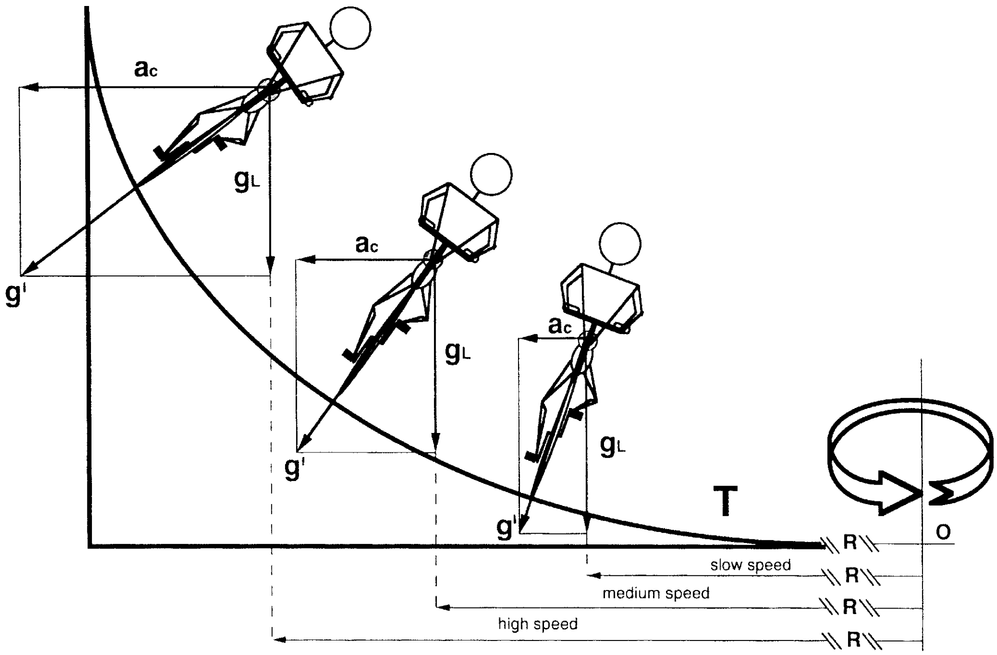

Fig. 11 Schematic frontal view of a cyclist pedalling on the curved path of a "lunar track" (*T*). To compensate for the outwards acceleration (a_c), itself a function of the radius of gyration (r) and of the ground speed (s), $a_c = s^2r^{-1}$, the cyclist leans inwards so that the vectorial sum (g') of a_c and the lunar gravity (g_L) lies in the plane that includes the centre of mass (*grey circles*) and the points of contact between the wheels and the terrain. The three values for s indicated result in progressively larger a_c (and hence g') values. In addition, the angle between g' and the vertical increases with a_c (Fig. 13), so that the track must be appropriately constructed to avoid skidding

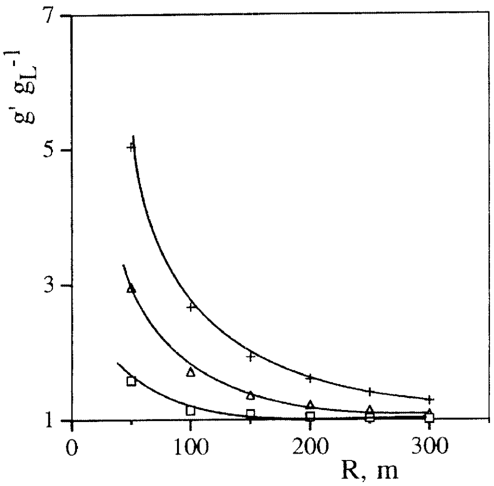

Fig. 12 Vectorial sum (g') of the outward acceleration and of the lunar gravity ($g_L = 1.62\ \mathrm{m \cdot s^{-2}}$), divided by the lunar gravity, as a function of the radius of curvature of the track (r, m), for velocities of 10 m · s^{-1} (*open squares*), 15 m · s^{-1} (*triangles*), and 20 m · s^{-1} (*crosses*)

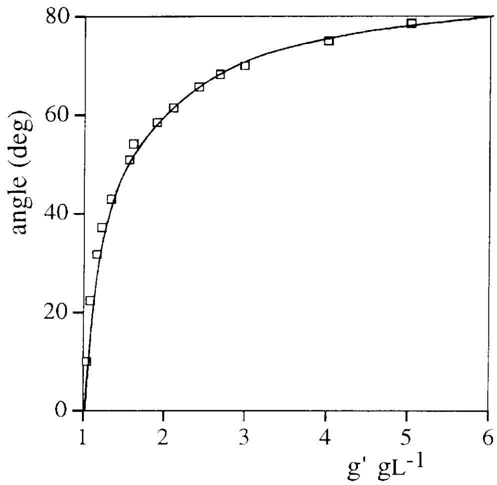

Fig. 13 The angle between the vertical and the vector g' is plotted as a function of the ratio of g' to the lunar gravity ($g_L = 1.62\ \mathrm{m \cdot s^{-2}}$)

gravity (g') can be achieved without surpassing the subjects' $\dot{V}O_{2\,max}$ only if the air density in the track tunnel is approximately equal to that corresponding to a P_B of about 250 mmHg (33.3 kPa) and to a T of about 20 °C. Thus, the gas contained in the "track tunnel" should be appropriately enriched in O_2, so as to bring its inspiratory fraction to about 0.50. Finally, the angles with the vertical of the vectorial sum of a_c plus g_L (g'), in the range of speed and radiuses mentioned above, will vary from 10° to 78.6°, thus showing that the curved parts of the track should be appropriately constructed (see Figs. 11, 13).

Conclusions

The preceding sections are devoted to a brief analysis of the biomechanics and bioenergetics of cycling under a large variety of conditions, ranging from track or uphill cycling to admittedly unrealistic scenarios, such as cycling on the Moon. On the other hand, several realistic, but complicated aspects, such as the effects of head, tail or lateral winds, have been omitted entirely. In addition, no attempt has been made to review extensively the vast number of papers dealing with other everyday aspects, such as cycling in the wake of a preceding cyclist, or vehicle, or the effects of pedal frequency.

In closing, and in spite of the inevitable limits of this brief review on cycling, I do hope that the readers may consider it a stimulus and a challenge to study further this fascinating form of human locomotion.

Acknowledgements Parts of this paper were presented as a Keynote Lecture at the International Symposium of Biomechanics held in Tokyo (Japan) in August 1997.

References

Antonutto G, Capelli C, di Prampero PE (1991) Pedalling in space as a countermeasure to microgravity deconditioning. Microgravity Q 1: 93–101
Antonutto G, Linnarsson D, di Prampero PE (1993) On Earth evaluation of neurovestibular tolerance to centrifuge simulated artificial gravity in humans. Physiologist 36 [Suppl 1]: S85–S87
Åstrand PO, Rodahl K (1986) Textbook of work physiology. McGraw-Hill, New York, pp 391–411
Banister EW, Jackson RC (1967) The effect of speed and load changes on oxygen intake for equivalent power outputs during bicycle ergometry. Arbeitsphysiol 24: 284–290
Bassett DR Jr, Kyle CR, Passfield L, Broker JR, Burke ER (1999) Comparing cycling world hour records 1967–1996: modelling with empirical data. Med Sci Sports Exerc 31: 1665–1676
Broker JR, Kyle CR, Burke ER (1999) Racing cyclist power requirements in the 4000-m individual and team pursuits. Med Sci Sports Exerc 31: 1677–1685
Burke ER (ed) (1986) Science of cycling. Human Kinetics, Champaign, Illinois, pp 215 + viii
Burke ER, Newsom MM (eds) (1988) Medical and scientific aspects of cycling. Human Kinetics, Champaign, Illinois, pp 266 + iv
Candau RB, Grappe F, Ménard M, Barbier B, Millet GY, Hoffman MD, Belli AR, Rouillon JD (1999) Simplified deceleration method for assessment of resistive forces in cycling. Med Sci Sports Exerc 31: 1441–1447
Capelli C (1999) Physiological determinants of best performances in human locomotion. Eur J Appl Physiol 80: 298–307
Capelli C, di Prampero PE (1995) Effects of altitude on top speeds during 1 h unaccompanied cycling. Eur J Appl Physiol 71: 469–471
Capelli C, Rosa G, Butti F, Ferretti G, Veicsteinas A, di Prampero PE (1993) Energy cost and efficiency of riding aerodynamic bicycles. Eur J Appl Physiol 67: 144–149
Capelli C, Schena F, Zamparo P, Dal Monte A, Faina M, di Prampero PE (1998) Energetics of best performances in track cycling. Med Sci Sports Exerc 30: 614–624
Cerretelli P (1981) Energy metabolism during exercise at altitude. Medicine Sport, Karger, Basel, pp 175–190
Coast JR, Welch HJ (1985) Linear increase in optimal pedal rate with increased power output in cycle ergometry. Eur J Appl Physiol 53: 339–342
Davies CTM (1980) Effect of air resistance on the metabolic cost and performance of cycling. Eur J Appl Physiol 45: 245–254
de Groot G, Sargeant A, Geysel J (1995) Air friction and rolling resistance during cycling. Med Sci Sports Exerc 27: 1090–1095
Dickinson S (1929) The efficiency of bicycle-pedalling, as affected by speed and load. J Physiol (Lond) 67: 242–255
di Prampero PE (1986) The energy cost of human locomotion on land and in water. Int J Sports Med 7: 55–72
di Prampero PE (1989) Energetics of world records in human locomotion. In Wieser W, Gnaiger E (eds) Energy transformations in cells and organisms. Thieme, Stuttgart, pp 248–253
di Prampero PE, Cortili G, Mognoni P, Saibene F (1979) Equation of motion of a cyclist. J Appl Physiol 47: 201–206
di Prampero PE, Capelli C, Pagliaro P, Antonutto G, Girardis M, Zamparo P, Soule RG (1993) Energetics of best performances in middle-distance running. J Appl Physiol 74: 2318–2324
Ericson MO (1988) Mechanical muscular power output and work during ergometer cycling at different work loads and speeds. Eur J Appl Physiol 57: 382–387
Ferretti G, Moia Ch, Thomet J-M, Kayser B (1997) The decrease of maximal oxygen consumption during hypoxia in man: a mirror image of the oxygen equilibrium curve. J Physiol (Lond) 498: 231–237
Francescato MP, Girardis M, di Prampero PE (1995) Oxygen cost of internal work during cycling. Eur J Appl Physiol 72: 51–57
Gaesser GA, Brooks GA (1975) Muscular efficiency during steady-rate exercise: effects of speed and work rate. J Appl Physiol 38: 1132–1139
Gnehm P, Reichenbach S, Altpeter E, Widmer H, Hoppeler H (1997) Influence of different racing positions on metabolic cost in elite cyclists. Med Sci Sports Exerc 29: 818–823
Grappe F, Candau R, Belli A, Rouillon JD (2000) Aerodynamic drag in field cycling with special reference to the Obree's position. Ergonomics (in press)
Gross AC, Kyle CR, Malewicki DJ (1983) The aerodynamics of human-powered land vehicles. Sci Am 249: 124–134
Kawakami Y, Nozaki D, Matsuo A, Fukunaga T (1992) Reliability of measurement of oxygen uptake by a portable telemetric system. Eur J Appl Physiol 65: 409–414
Keller TS, Strauss AM, Szpalski M (1992) Prevention of bone loss and muscle atrophy during manned space flight. Microgravity Q 2: 89–102
Kyle CR (1979) Reduction of wind resistance and power output of racing cyclists and runners travelling in groups. Ergonomics 22: 387–397
Kyle CR (1986) Mechanical factors affecting the speed of a cycle. In: Burke RE (ed) Science of cycling. Human Kinetics, Champaign, Illinois, pp 133–135
Kyle CR (1988) The mechanics and aerodynamics of cycling. In: Burke RE, Newsom MM (eds) Medical and scientific aspects of cycling. Human Kinetics, Champaign, Illinois pp 235–251
Kyle CR (1989) The aerodynamics of helmets and handlebars. Cycling Sci 1: 22–26
Lackner JR, Graybiel A (1986) The effective intensity of Coriolis cross-coupling stimulation is gravitoinertial force dependent: implication for space motion sickness. Aviat Space Environ Med 57: 229–235
Lucia A, Fleck SJ, Gotshall RW, Kearney JT (1993) Validity and reliability of the Cosmed K2 instrument. Int J Sports Med 14: 380–386
Luhtanen P, Rahkila P, Rusko H, Viitasalo JT (1987) Mechanical work and efficiency in ergometer bicycling at aerobic and anaerobic thresholds. Acta Physiol Scand 131: 331–337
Marsh AP, Martin PE (1993) The association between cycling experience and preferred and most economical cadences. Med Sci Sports Exerc 25: 1269–1274
Marsh AP, Martin PE (1995) The relationship between cadence and lower extremity EMG in cyclists and non-cyclists. Med Sci Sports Exerc 27: 217–225
Marsh AP, Martin PE (1998) Perceived exertion and the preferred cycling cadence. Med Sci Sports Exerc 30: 942–948
Nicogossian AE, Sawin CF, Grigoriev AI (1994) Countermeasures to space deconditioning. In: Nicogossian AE, Leach Huntoon C, Pool SL (eds) Space physiology and medicine. Lea and Febiger, Philadelphia, pp 447–467
Olds T (1998) The mathematics of breaking away and chasing in cycling. Eur J Appl Physiol 77: 492–497
Olds TS, Norton KI, Craig NP (1993) Mathematical model of cycling performance. J Appl Physiol 75: 730–737
Olds TS, Norton KI, Lowe LA, Olive S, Reay F, Ly S (1995) Modelling road-cycling performance. J Appl Physiol 78: 1596–1611
Péronnet F, Thibault G (1989) Mathematical analysis of running performance and world running records. J Appl Physiol 67: 453–465
Péronnet F, Bouissou Ph, Perrault H, Ricci J (1989) Comparaison du record de l'heure cycliste selon l'altitude et le matériel utilisé. Can J Sport Sci 14: 93–98

Pugh LGCE (1974) The relation of oxygen intake and speed in competition cycling and comparative observations on the bicycle ergometer. J Physiol (Lond) 241: 795–808

Sargeant AJ, Davies CTM (1977) Forces applied to cranks of a bicycle ergometer during one- and two-leg cycling. J Appl Physiol 42: 514–518

Sargeant AJ, Charters A, Davies CTM, Reeves ES (1978) Measurement of forces applied and work performed in pedalling a stationary bicycle ergometer. Ergonomics 21: 49–53

Seabury JJ, Adams WC, Ramey MR (1977) Influence of pedalling rate and power output on energy expenditure during cycling ergometry. Ergonomics 20: 491–498

Sjøgaard G, Nielsen B, Mikkelsen F, Saltin B, Burke ER (1984) Physiology in bicycling. Movement Publication, Ithaca, New York, pp 1–110

Swain DP, Coast JR, Clifford PS, Miliken MC, Stray-Gundersen J (1987) Influence of body size on oxygen consumption during cycling. J Appl Physiol 62: 668–672

Tucker VA (1975) Energetic cost of moving about. Sci Am 63: 413–419

van Ingen Schenau GJ, Cavanagh P (1990) Power equations in endurance sports. J Biomech 23: 865–881

Weast RC (ed) (1989) Handbook of chemistry and physics, 69th edn (1988–1989). CRC, Boca Raton, Florida, USA, pp F8, F9, F149

Welbeergen E, Clijsen LPVM (1990) The influence of body position on maximal performance in cycling. Eur J Appl Physiol 61: 138–142

Wilkie DR (1980) Equations describing power input by humans as a function of duration of exercise. In: Cerretelli P, Whipp BJ (eds) Exercise bioenergetics and gas exchange. Elsevier, Amsterdam, pp 75–80

Zoladz JA, Rademaker ACHJ, Sargeant AJ (1995) Non-linear relationship between O_2 uptake and power output at high intensities of exercise in humans. J Physiol (Lond) 448: 211–217

Zoladz JA, Duda K, Majerczak J (1998) Oxygen uptake does not increase linearly at high power outputs during incremental exercise tests in humans. Eur J Appl Physiol 77: 445–451