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ORIGINAL ARTICLE

A novel equation that incorporates the linear and hyperbolic nature of the force–velocity relationship in lower and upper limb exercises

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Abstract

The purpose of this study is to provide a force–velocity (F–V) equation that combines a linear and a hyperbolic region, and to compare its derived results to those obtained from linear equations. A total of 10 cross-training athletes and 14 recreationally resistance-trained young men were assessed in the unilateral leg press (LP) and bilateral bench press (BP) exercises, respectively. F–V data were recorded using a force plate and a linear encoder. Estimated maximum isometric force (F_0), maximum muscle power (P_{max}), and maximum unloaded velocity (V_0) were calculated using a hybrid (linear and hyperbolic) equation and three different linear equations: one derived from the hybrid equation (linear$_{hyb}$), one applied to data from 0 to 100% of F_0 (linear$_{0–100}$), and one applied to data from 45 to 100% of F_0 (linear$_{45–100}$). The hybrid equation presented the best fit to the recorded data ($R^2 = 0.996$ and 0.998). Compared to the results derived from the hybrid equation in the LP, significant differences were observed in F_0 derived from linear$_{0-100}$; V_0 derived from linear$_{hyb}$, linear$_{0–100}$ and linear$_{45–100}$; and P_{max} derived from linear$_{hyb}$ and linear$_{45–100}$ (all $p < 0.05$). For the BP, compared to the hybrid equation, significant differences were found in F_0 derived from linear$_{0–100}$; and V_0 and P_{max} derived from linear$_{hyb}$, linear$_{0–100}$ and linear$_{45–100}$ (all $p < 0.05$). An F–V equation combining a linear and a hyperbolic region showed to fit adequately recorded F–V data from ~20 to 100% of F_0, and overcame the limitations shown by linear equations while providing relevant results.

Keywords Torque–velocity · Muscle power · Muscle mechanics · Maximum unloaded velocity · Maximum isometric force · Load–velocity

Abbreviations

F–V	Force–velocity
F_0	Estimated maximum isometric force
F_{opt}	Optimal force
Linear$_{hyb}$	Linear equation included in the hybrid equation
Linear$_{0–100}$	Linear equation applied to all force–velocity data
Linear$_{45–100}$	Linear equation applied to force–velocity data above 45% of maximum isometric force
P_{max}	Maximum muscle power
R^2	Coefficient of determination
SEE	Standard error of the estimate
S_{FV}	Slope of the linear force–velocity relationship
V_0	Estimated maximum unloaded velocity
V_{opt}	Optimal velocity

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Introduction

The assessment of the force–velocity (F–V) relationship has gained increasing attention in recent years due to its association to physical performance in populations ranging from young athletes to mobility-limited older people (Dorel et al. 2005; Cross et al. 2015; Giroux et al. 2017; Jimenez-Reyes et al. 2016; Alcazar et al. 2018). The F–V relationship illustrates the ability to produce force at different movement or muscle contraction velocities, which relate to each other in an inverse fashion. In addition, the higher the force produced at a given movement velocity, the higher the amount of mechanical work that can be generated per unit of time (i.e., mechanical power). Therefore, its assessment provides relevant information on the individual's ability to yield mechanical work effectively at several movement velocities that may be of interest for specific sport- or daily life-related activities. In this sense, the *F*–*V* relationship can be used as a diagnosing tool by identifying weaknesses or strengths at different force or velocity levels, instead of the traditional one-repetition maximum assessment (Jimenez-Reyes et al. 2016; Alcazar et al. 2018). Then, interventions addressing those weaknesses or strengths can be applied to ultimately influence positively over the sport- or daily life-related activity of interest.

The evaluation of the *F*–*V* relationship is usually conducted by collecting force and velocity data from several differently loaded trials and fitting an equation to the collected data. The latter avoids the necessity of collecting too many trials, while providing information on the whole *F*–*V* relationship through the interpolation and extrapolation of non-registered data. Therefore, the type of equation fitted to the *F*–*V* data is paramount, since different equations yield different results (Alcazar et al. 2021a, b; Iglesias-Soler et al. 2019). The main *F*–*V* equations used in the literature have been reviewed elsewhere (Alcazar et al. 2019). Briefly, first studies on the topic observed a linear *F*–*V* relationship (Hill 1922), but it was shortly after reported that the relation was curvilinear when low enough forces were assessed (Fenn and Marsh 1935), and Hill introduced the so popular hyperbolic *F*–*V* equation that assembles a rectangular hyperbola (Hill 1938). However, not unfrequently, studies observed that *F*–*V* data in the high-force, low-velocity region (above ~ 90% of maximum isometric force) deviated downwards from the rectangular hyperbola (Alcazar et al. 2019), and a double-hyperbolic equation introduced by Edman demonstrated a better fit to that region of the *F*–*V* relationship (Edman et al. 1976). Of note, that shape noted at very high forces makes linear equations fit very well with *F*–*V* data collected at moderate-to-high forces (above ~ 45% of maximum isometric force) (Alcazar et al. 2021b). Altogether, the *F*–*V* relationship seems to behave quasi-linearly above a certain level of force, and in a curvilinear fashion below. This would explain the early observations of a linear *F*–*V* relationship, the curvilinear nature noted at high velocities, and deviations from the hyperbolic equation at low velocities (Alcazar et al. 2019). However, most of the very recent studies evaluating the *F*–*V* relationship in multi-joint exercises have used linear equations regardless of the region of the *F*–*V* relationship being measured. In addition, currently, there are no *F*–*V* equations that consider the linear and hyperbolic nature of the *F*–*V* relationship together. For that reason, the main goal of the present study was to provide an *F*–*V* equation that can be adequately fitted to both the linear and hyperbolic regions of the *F*–*V* relationship, and to compare its derived results to those obtained from linear equations.

Materials and methods

The present investigation is an extension of a previous study (Alcazar et al. 2021b). However, the data analysis and results reported in this investigation are original and have not been presented before.

Subjects

A total of 10 young men (regional and national cross-training athletes; 25.8 ± 5.4 years old; height: 1.77 ± 0.04 m and body mass: 78.4 ± 3.2 kg) and 14 young men (recreationally resistance-trained; 24.0 ± 4.3 years old; height: 1.74 ± 0.06 m and body mass: 73.7 ± 9.3 kg) were assessed on the leg press (unilateral) and bench press (bilateral) exercises, respectively. All the participants gave their written informed consent. The study was performed in accordance with the Helsinki Declaration and approved by the local ethical committee.

Experimental setting

Unilateral leg press

A leg press machine (Selection MED, Technogym, Italy) instrumented with a force plate (Type 9286BA, Kister, Switzerland) and a linear encoder (Chronojump Bosco System, Spain) was used to assess external force and velocity, respectively. The force plate was installed on the feet platform of the leg press device, while the linear encoder was attached to the weights of the apparatus. Force data were acquired using a specialized software at 1500 Hz (MyoResearch 3.10, Noraxon, USA), and velocity was acquired at 1019 Hz using the manufacturer's software. Both signals were synchronized using an external custom-built trigger (USB-6501, National Instruments, USA) associated with a software (LabView, National Instruments, USA).

Bilateral bench press

A Smith machine with no counterweight mechanism (Multipower Fitness Line, Peroga, Spain) instrumented with a force plate and a linear encoder (both *T*-Force System, Ergotech, Spain) was employed to collect external force and velocity data, respectively. The force plate was placed under a bench specifically built to be used over the force plate and the linear encoder was attached to the bar used during the bench press execution. Force and velocity data were acquired at 1000 Hz and synchronized by the manufacturer's software (*T*-Force System v.3.65.1, Ergotech, Spain).

Experimental protocol

For both exercises, after a standardized 15-min warm-up, the maximum isometric force was assessed at several joint angles in a randomized order with 2 min resting periods in between (knee joint: 97.4° ± 4.5°, 110.2° ± 4.9°, 121.8° ± 3.8°, 131.3° ± 3.7°, and 141.0 ° ± 3.6°; elbow joint: 39.6° ± 6.7°, 58.7° ± 6.6°, 77.3° ± 8.0°, 100.8° ± 8.0°, 127.4° ± 8.6°, 141.9° ± 8.6°, and 179.0° ± 3.0°; full extension = 180°). Joint angles were determined by video analysis (HD Pro Webcam C920 1080p, 30 Hz, Logitech, Switzerland) and superficial anatomical markers placed on the participants' skin (greater trochanter, lateral condyle, and lateral malleolus in the lower limb; and acromion, epicondyle and midpoint between the radial and ulnar styloids in the upper limb). The participants performed 2–3 attempts of 4 s maximal isometric contractions per joint angle, and the highest registered peak force among all the trials was selected for further analysis. Then, the participants were asked to perform maximal concentric contractions as fast as possible, first against increasing loads (above 140 kg and 80 kg in the leg press and bench press exercises, respectively) until reaching the one-repetition maximum, and then against decreasing loads (from 140 to 2.5 kg and from 80 to 0.5 kg in the leg press and bench press exercises, respectively), with 2 attempts per load. A third attempt per load was performed in the reverse order to discard a fatigue bias. The exercises were performed using a partial range of movement (from 117.0° ± 4.9° of knee flexion and 127.4° ± 8.6° of elbow flexion to full extension) near to the optimal angle to record *F*–*V* data as close as possible to the individuals' maximum isometric force (see further details elsewhere (Alcazar et al. 2021b)). An adequate resting period (1–3 min) was allowed between attempts and loads. Peak force and velocity at peak force were recorded from each trial, and the best attempt (highest power) with each load was selected for further analysis.

Force–velocity equations

Hybrid (i.e., linear and hyperbolic) equation

A hybrid equation (Eq. 1) that combines a linear and a curvilinear (Hill-type) region was created. This equation was inspired by the equation introduced by Edman (Edman 1988), which combines two different rectangular hyperbolas (double-hyperbolic equation). Briefly, our equation includes the linear (Jaric 2016) and hyperbolic (Hill 1938) equations, as well as associated coefficients to each of the two equations [c_1 (Eq. 2) and c_2 (Eq. 3), respectively] that provide different albeit complementary weights ($c_1 + c_2 = 1$) to each of them as a function of relative intensity (i.e., in this case force relative to maximum isometric force)

$$V = c_1\left[\frac{F - F_0}{S_{FV}}\right] + c_2\left[\frac{(F_0^* - F)b}{F + a}\right] \tag{1}$$

$$c_1 = \frac{1}{1 + e^{(k - F/F_0)^S}} \tag{2}$$

$$c_2 = \frac{1}{1 + e^{(F/F_0 - k)^S}}, \tag{3}$$

where V is velocity, c_1 is the coefficient associated to the linear equation, c_2 is the coefficient associated to the hyperbolic equation, F is force, F_0 is estimated maximum isometric force, F_0^* is estimated maximum isometric force from the hyperbolic equation, S_{FV} is the slope of the linear region, a and b are Hill-type constants, k is a constant that determines the point of transition from the linear to the hyperbolic equation (i.e., when c_1 and c_2 are both 0.5), and s (set at 20) is a constant that determines how smooth the model runs from the linear to the hyperbolic region. In addition, estimated maximal unloaded velocity (V_0) was calculated as the intercept of the velocity axis, and S_{FV}/F_0 and a/F_0 were calculated to describe the decrease in force as a function of contraction velocity in the linear (i.e., slope) and hyperbolic (i.e., curvature) regions, respectively, of the *F*–*V* relationship. Then, power was calculated as the product of force and velocity, and maximum muscle power (P_{max}) was identified at the apex of the power–velocity relationship, as well as optimal force (F_{opt}) and optimal velocity (V_{opt}) (i.e., those produced at P_{max}). Thus, provided the force and velocity values measured in the current experiments, a commercial software package (Solver VBA, Microsoft Excel, USA) was used to calculate the constants of the *F*–*V* equation (non-linear least-squares method; 20,000 iterations with pre-established limits based on observed data). The constants were calculated to provide the best possible fit between the measured data and the resulting: (a) equation, (b) linear part of the equation at relative $F > k$, and c) hyperbolic part of the equation at relative $F < 0.9$. Nevertheless, to ensure an optimal assessment of the *F*–*V* relationship, and based on basic physiological principles, the following automatized instructions were implemented in the software for the selection of suitable force and velocity data before running the analyses (Fig. 1): maximum isometric force values lower than any of the registered dynamic force values were discarded; dynamic trials showing a lower force value for a corresponding velocity compared to a faster contraction were discarded; and dynamic trials showing a lower power for a corresponding

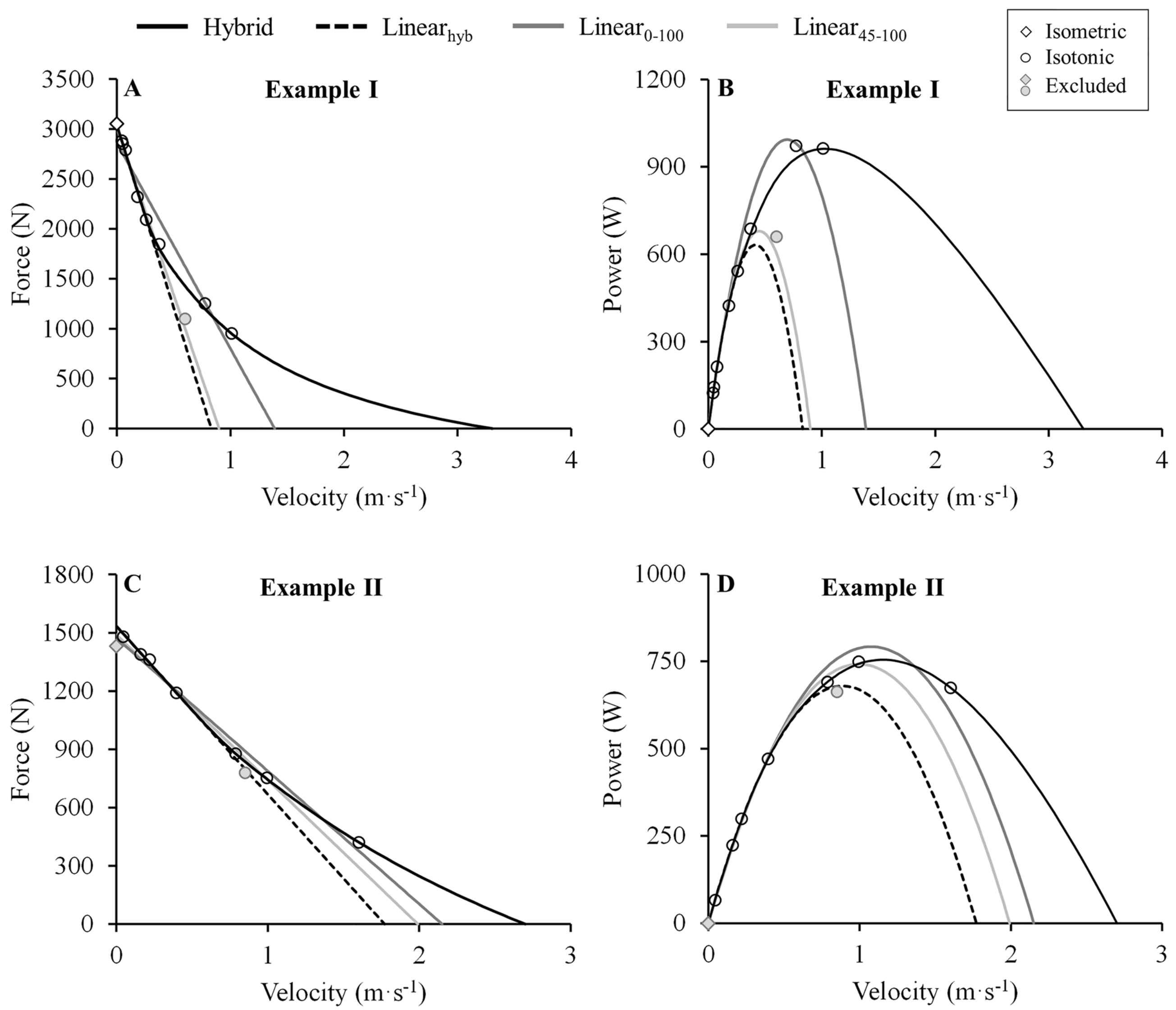

Fig. 1 Analysis of the force–velocity (**A**, **C**) and power–velocity (**B**, **D**) relationships. **A**, **B** (Example I: leg press) and **C, D** (Example II: bench press) correspond to the same set of data, respectively. The equation was applied to measured data after excluding those data that fulfilled the following criteria: maximum isometric torque values lower than any of the registered dynamic torque values (in **C**); dynamic trials showing a lower torque value for a corresponding velocity compared to a faster contraction (in **A**); and dynamic trials showing a lower power for a corresponding velocity compared to both a slower and a faster contraction (in **B**, **D**). Note: hybrid equation: combines a linear and a hyperbolic equation. $\text{Linear}_{\text{hyb}}$ equation: represents the linear part of the hybrid equation. $\text{Linear}_{0\text{-}100}$ equation: a linear equation is applied to data from 0 to 100% of F_0. $\text{Linear}_{45\text{-}100}$ equation: a linear equation is applied to data from 45 to 100% of F_0. F_0: estimated maximum isometric force

velocity compared to both a slower and a faster contraction were discarded. In addition, those either isometric or dynamic trials that by visual inspection showed diminished force compared to the one that would be expected according to the rest of the trials were discarded only if after excluding those trials the difference between measured and estimated values reached ≥ 10%. Finally, the average number of *F*–*V* data points included per participant was 8.0 ± 1.3 and 8.2 ± 1.3 for the leg press and bench press exercises, respectively, and ranged from 34.2 ± 4.1 to 100% of the maximum isometric force in the leg press and from 18.6 ± 4.4 to 100% of the maximum isometric force in the bench press. The average number of discarded *F*–*V* data points based on the filtering process was 0.5 ± 0.7 and 0.4 ± 0.6 for the leg press and bench press exercises, respectively.

Linear equation

A linear equation (Eq. 4) (Alcazar et al. 2021b) was fitted to the same *F*–*V* data used for the hybrid equation following three different strategies: (a) the one included in the hybrid equation ($linear_{hyb}$), (b) applied to all F–V data regardless the region of the F–V relationship (i.e., from 0 to 100% of maximum isometric force) ($linear_{0–100}$); and (c) applied to F–V data above 45% of maximum isometric force ($linear_{45–100}$)

$$F = S_{FV} V + F_0, \quad (4)$$

where F is force, S_{FV} is the slope of the linear F–V relationship, V is velocity, and F_0 is estimated maximum isometric force. S_{FV}/F_0, P_{max}, F_{opt}, V_{opt}, and V_0 were also calculated from the resulting equations.

Statistical analyses

Data are presented as mean and standard deviation unless otherwise stated. Coefficient of determination (R^2) and standard error of the estimate (SEE) values were calculated for each equation and used to assess the fitting of the models to the measured F–V data. The main parameters and constants derived from each equation were compared by one-way ANOVA tests with the type of equation as a within-subject factor (i.e., hybrid, $linear_{hyb}$, $linear_{0–100}$ and $linear_{45–100}$). Velocities and power values obtained from the equations were also compared at 5% force intervals (i.e., 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, and 100% of maximum isometric force). When sphericity was violated, the Greenhouse–Geisser correction was applied. Fisher's F and partial eta-squared (η_p^2) values were reported. Pairwise comparisons were conducted using Bonferroni's correction. Statistical analyses were performed using SPSS v24 (SPSS Inc., USA), and the level of significance was set at $\alpha = 0.05$.

Results

The main parameters and constants derived from the different equations are presented in Tables 1, 2 for the leg press and bench press exercises, respectively.

For the leg press exercise, there were significant main effects of equation regarding F_0 ($F = 15.352$, $p = 0.002$, $\eta_p^2 = 0.630$), V_0 ($F = 28.650$, $p < 0.001$, $\eta_p^2 = 0.761$), S_{FV} ($F = 15.232$, $p = 0.002$, $\eta_p^2 = 0.629$), S_{FV}/F_0 ($F = 17.810$, $p = 0.001$, $\eta_p^2 = 0.664$), P_{max} ($F = 29.262$, $p < 0.001$, $\eta_p^2 = 0.765$), F_{opt} ($F = 48.239$, $p < 0.001$, $\eta_p^2 = 0.843$), and V_{opt} ($F = 36.919$, $p < 0.001$, $\eta_p^2 = 0.804$). For the bench press exercise, significant main effects of equation were observed for F_0 ($F = 17.443$, $p < 0.001$, $\eta_p^2 = 0.573$), V_0 ($F = 31.318$, $p < 0.001$, $\eta_p^2 = 0.707$), S_{FV} ($F = 30.120$, $p < 0.001$, $\eta_p^2 = 0.699$), S_{FV}/F_0 ($F = 43.829$, $p < 0.001$, $\eta_p^2 = 0.771$), P_{max} ($F = 43.674$, $p < 0.001$, $\eta_p^2 = 0.771$), F_{opt} ($F = 57.809$, $p < 0.001$, $\eta_p^2 = 0.816$), and V_{opt} ($F = 39.486$, $p < 0.001$, $\eta_p^2 = 0.752$). The results yielded by pairwise comparisons are reported in Tables 1, 2.

There were also significant differences among equations for both the leg press and bench press exercises in terms of R_2 ($F = 10.856$ and 12.692, $p = 0.005$ and < 0.001, $\eta_p^2 = 0.547$ and 0.494, respectively) and SEE ($F = 32.210$ and 26.997, both $p < 0.001$, $\eta_p^2 = 0.782$ and 0.675, respectively) values.

Finally, compared to the hybrid equation in the leg press exercise (Fig. 2), significant differences in velocity

Table 1 Main parameters derived from the different force–velocity equations in the unilateral leg press exercise

	A. Hybrid Mean ± SD	B. $Linear_{hyb}$ Mean ± SD	C. $Linear_{0–100}$ Mean ± SD	D. $Linear_{45–100}$ Mean ± SD	Comparison
F_0 (N)	2887.9 ± 299.2	2887.9 ± 299.2	2753.9 ± 214.4	2851.5 ± 268.1	ABD > C
V_0 (m s^{-1})	2.39 ± 0.70	1.13 ± 0.37	1.46 ± 0.33	1.20 ± 0.38	A > C > BD
S_{FV}	2878.7 ± 1189.0	2878.7 ± 1189.0	1999.5 ± 564.5	2649.4 ± 1030.4	AB > D > C
S_{FV}/F_0	0.97 ± 0.31	0.97 ± 0.31	0.72 ± 0.16	0.91 ± 0.29	AB > D > C
a/F_0	0.82 ± 0.52	–	–	–	–
k	0.54 ± 0.05	–	–	–	–
P_{max} (W)	971.6 ± 176.5	796.6 ± 187.4	991.0 ± 161.7	855.5 ± 197.8	AC > BD
F_{opt} (N)	1068.4 ± 113.4	1443.9 ± 149.6	1376.9 ± 107.2	1425.7 ± 134.1	A < C < BD
V_{opt} (m·s^{-1})	0.92 ± 0.22	0.57 ± 0.18	0.73 ± 0.16	0.60 ± 0.19	A > C > BD
R^2	0.996 ± 0.002	0.996 ± 0.002	0.969 ± 0.021	0.967 ± 0.018	AB > CD
SEE	0.020 ± 0.005	0.020 ± 0.005	0.057 ± 0.022	0.034 ± 0.013	AB < D < C

a/F_0 hyperbolic curvature of the force–velocity relationship, F_0 estimated maximal isometric force, F_{opt} optimal force, P_{max} maximal muscle power, R^2 coefficient of determination, *SD* standard deviation, *SEE* standard error of the estimate, S_{FV} linear slope of the force–velocity relationship, V_0 estimated maximal unloaded shortening velocity, V_{opt} optimal velocity, Symbols > and < denote significant differences between A (hybrid equation), B (linear equation from k to 100% of F_0), C (linear equation from 0 to 100% of F_0), or D (linear equation from 45 to 100% of F_0) ($p < 0.05$)

Table 2 Main parameters derived from the different force–velocity equations in the bilateral bench press exercise

	A. Hybrid Mean ± SD	B. $Linear_{hyb}$ Mean ± SD	C. $Linear_{0–100}$ Mean ± SD	D. $Linear_{45–100}$ Mean ± SD	Comparison
F_0 (N)	1646.8 ± 269.1	1646.8 ± 269.1	1538.3 ± 217.8	1607.1 ± 254.7	ABD > C
V_0 (m s^{-1})	2.68 ± 0.93	1.33 ± 0.27	1.95 ± 0.37	1.42 ± 0.28	A > C > BD
S_{FV}	1294.8 ± 376.0	1294.8 ± 376.0	813.9 ± 183.4	1156.5 ± 316.4	ABD > C
S_{FV}/F_0	0.78 ± 0.15	0.78 ± 0.15	0.53 ± 0.08	0.71 ± 0.13	ABD > C
a/F_0	0.86 ± 0.38	–	–	–	–
k	0.52 ± 0.08	–	–	–	–
P_{max} (W)	646.4 ± 128.4	543.4 ± 115.7	747.3 ± 167.5	570.9 ± 113.3	C > A > BD
F_{opt} (N)	625.9 ± 121.8	823.4 ± 134.5	769.1 ± 108.9	803.6 ± 127.4	A < C < BD
V_{opt} (m s^{-1})	1.06 ± 0.27	0.67 ± 0.14	0.97 ± 0.18	0.71 ± 0.15	AC > D > B
R^2	0.998 ± 0.002	0.998 ± 0.002	0.954 ± 0.029	0.947 ± 0.041	AB > CD
SEE	0.022 ± 0.010	0.022 ± 0.010	0.125 ± 0.067	0.087 ± 0.048	AB < D < C

a/F_0 hyperbolic curvature of the force–velocity relationship, F_0 estimated maximal isometric force, F_{opt} optimal forcé, P_{max} maximal muscle power, R^2 coefficient of determination, *SD* standard deviation, *SEE* standard error of the estimate, S_{FV} linear slope of the force–velocity relationship, V_0 estimated maximal unloaded shortening velocity, V_{opt} optimal velocity, Symbols > and < denote significant differences between A (hybrid equation), B (linear equation from k to 100% of F_0), C (linear equation from 0 to 100% of F_0), or D (linear equation from 45 to 100% of F_0) ($p < 0.05$)

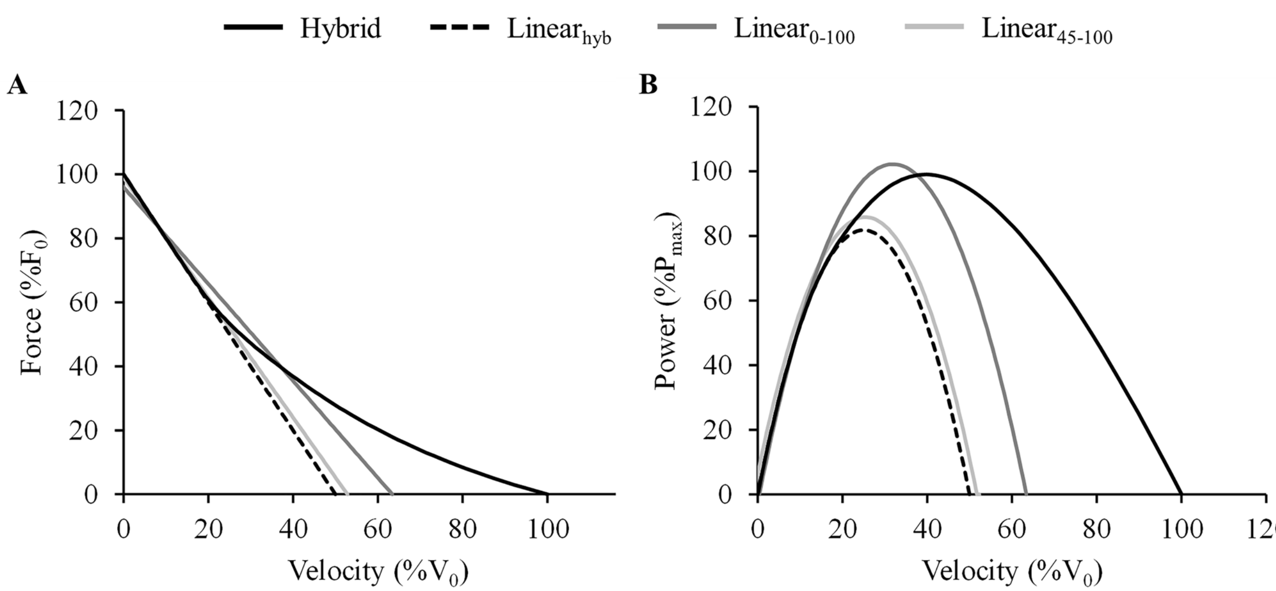

Fig. 2 Comparison of the force–velocity equations for the force–velocity relationship **A** and power-velocity relationship **B** in the unilateral leg press exercise. Data are presented normalized to the results derived from the hybrid equation. Note: hybrid equation: combines a linear and a hyperbolic equation. $Linear_{hyb}$ equation: represents the linear part of the hybrid equation. $Linear_{0-100}$ equation: a linear equation is applied to data from 0 to 100% of F_0. $Linear_{45-100}$ equation: a linear equation is applied to data from 45 to 100% of F_0. F_0: estimated maximum isometric force. V_0: estimated maximum unloaded contraction velocity. P_{max}: estimated maximum muscle power

and power were reported for the $linear_{hyb}$ equation from 0 to 55% of maximum isometric force; for the $linear_{0-100}$ equation from 0 to 30%, from 45 to 75%, and from 95 to 100% of maximum isometric force; and for the $linear_{45-100}$ equation from 0 to 50% of maximum isometric force (all $p < 0.05$). While compared to the hybrid equation in the bench press exercise (Fig. 3), significant differences regarding velocity and power were reported for the $linear_{hyb}$ equation from 0 to 55% of maximum isometric force; for the $linear_{0–100}$ equation from 0 to 20%, from 35 to 75%, and from 90 to 100% of maximum isometric force; and for the $linear_{45-100}$ equation from 0 to 45% of maximum isometric force (all $p < 0.05$).

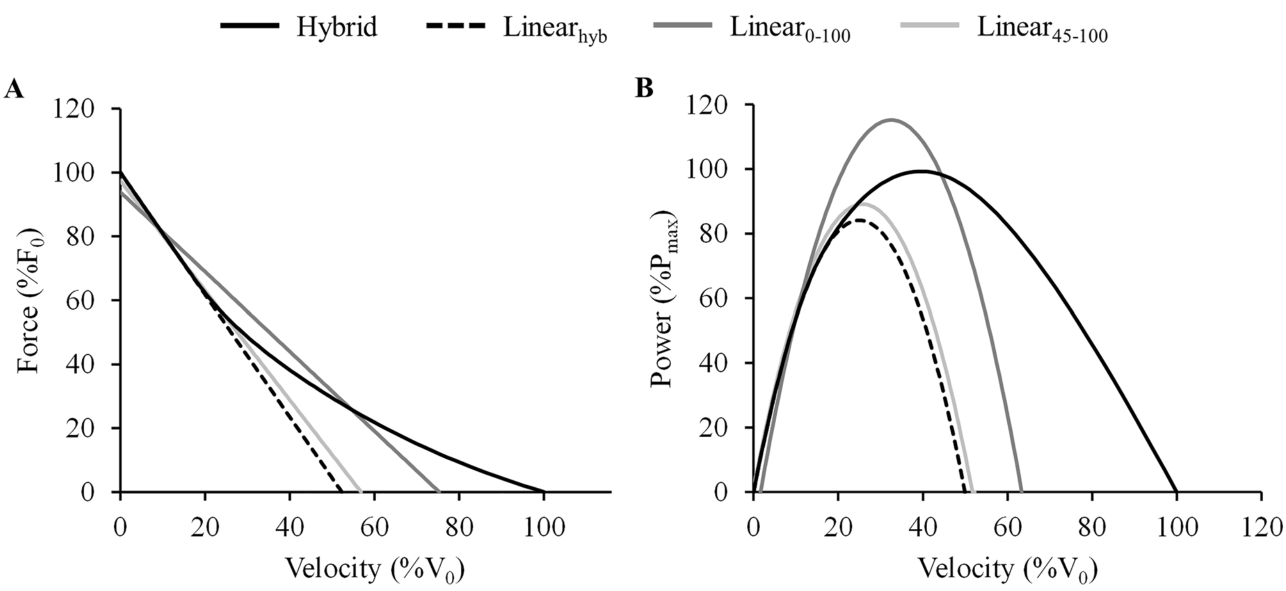

Fig. 3 Comparison of the force–velocity equations for the force–velocity relationship **A** and power–velocity relationship **B** in the bilateral bench press exercise. Data are presented normalized to the results derived from the hybrid equation. Note: hybrid equation: combines a linear and a hyperbolic equation. Linear$_{hyb}$ equation: represents the linear part of the hybrid equation. Linear$_{0\text{-}100}$ equation: a linear equation is applied to data from 0 to 100% of F_0. Linear$_{45\text{-}100}$ equation: a linear equation is applied to data from 45 to 100% of F_0. F_0: estimated maximum isometric force. V_0: estimated maximum unloaded contraction velocity. P_{max}: estimated maximum muscle power

Discussion

This is the first study to provide an *F–V* equation that assembles the linear and hyperbolic regions of the *F–V* relationship. This novel equation fitted almost perfectly measured data ($R^2 = 0.996$ and 0.998; SEE = 0.020 and 0.022, for the LP and the BP exercises, respectively), in this specific study collected from ~ 20 to 100% of maximum isometric force in young men. In addition, the hybrid equation showed significant differences in velocity and power when compared to the linear$_{hyb}$ and linear$_{45\text{-}100}$ equations below a certain level of force, demonstrating the curvilinear behavior of the *F–V* relationship. Finally, applying a linear equation to all the measured data (i.e., linear$_{0\text{–}100}$) yielded significant differences in velocity and power across the whole *F–V* relationship, with some minor exceptions, denoting the curvilinear nature of the *F–V* relationship below a certain level of relative force.

Since the first experiments on the *F–V* relationship during the first half of the nineteenth century (Hill 1922, 1938; Fenn and Marsh 1935), the assessment of the *F–V* relationship has been utilized to investigate diverse aspects such as the molecular mechanisms of muscle contraction (Piazzesi et al. 2007), the pathogenesis of some myopathies (Mansson 2014), or the development of prosthetic applications exhibiting muscle-like characteristics (Schmitt et al. 2012). In addition, one of the main recent applications of the assessment of the *F–V* relationship has been the identification of potential deficits in force, velocity, or both, for their subsequent treatment to improve physical performance (Jimenez-Reyes et al. 2016). In this sense, the higher accessibility to measuring instruments and the appearance of simplified protocols to evaluate the *F–V* relationship (Jaric 2016) have contributed to the exponential increment observed during the last 5 years in the number of records in the literature on applications of the *F–V* relationship. Basically, the observation made by several studies in regard to the linearity of the *F–V* relationship during moderately-to-heavily loaded multi-joint exercises in humans made much easier and faster its evaluation (Janicijevic et al. 2019). This occurred because of the use of a linear equation on the recorded *F–V* data, and the extrapolation of several outcomes such as F_0, V_0, and P_{max}, which can be accomplished even after collecting only two *F–V* data (Janicijevic et al. 2019), in comparison to the use of a hyperbolic equation that requires the recording of a greater amount of *F–V* data.

Nevertheless, evidence from studies assessing muscle function as well against very light loads has shown that the *F–V* relationship is not linear in the full range of *F–V* data, neither in single-joint nor multi-joint exercises (Hahn et al. 2014; Alcazar et al. 2021a, b; Armstrong et al. 2022). In fact, the *F–V* relationship seems to exhibit a quasi-linear shape from moderate to heavy forces, and a curvilinear (hyperbolic) shape from moderate to null forces. Of note, many studies in the literature justify the use of a linear *F–V* model in multi-joint exercises based on a biomechanical theory that supports such phenomenon (Bobbert 2012). Indeed, segmental dynamics observed in multi-joint exercises may decrease the magnitude of the curvature of the *F–V* relationship when compared to single-joint exercises, but that influence is not enough to transform the whole *F–V* relationship into a linear function (Hahn et al. 2014; Alcazar et al. 2021a, b; Armstrong et al. 2022). Nonetheless, this fact is not incompatible with the other fact that the practical application and relevance of linear fittings may overcome their limitations in some contexts, especially when all collected *F–V* data are above 45% of F_0. Still, the limitations of that procedure should be always acknowledged. For example, in the present study, we found that applying a linear equation to data above 45% of F_0 provides invalid V_0 data and slightly underestimated (by 12%) P_{max} values. Greater discrepancies can be found when applying a linear equation to data including values below 45% of F_0, which provided invalid V_0 data, underestimated F_0 values by 5−7% and S_{FV} values by 31−33%, as well as differences distributed at low-, moderate-, and high-force levels. Of note, V_0 values extrapolated from linear equations applied to moderate-to-heavy loads are a representation of velocity levels exhibited against those specific loads, but cannot be regarded as the maximal velocity produced under no load or force.

Another important point is the filtering of the recorded *F–V* data (exclusion of some attempts) based on some basic physiological principles: force must be lower with increasing velocity; and power must be higher with increasing velocity until one point (apex) after which power must be lower with increasing velocity. Despite the limitation that reliability was not assessed in the current investigation, this filtering has previously shown to improve reliability of results derived from the *F–V* relationship (Alcazar et al. 2017). Perhaps, the use of simplified methods lacking this data exclusion procedure in studies on the *F–V* relationship may be behind the low between-day reliability found by other studies (Valenzuela et al. 2021; Lindberg et al. 2021a), which may lead to participant misclassification. The latter may in turn be related to the absence of benefit of using an individualized resistance training approach based on *F–V* testing reported by other study (Lindberg et al. 2021b). Another important source of poor reliability has been shown to be the evaluation of a limited range of *F–V* data (García-Ramos et al. 2021). In this sense, we have also reported that the use of a simplified protocol (linear equation) may hide some of the actual adaptations achieved by a resistance training program, which, in turn, were observed when a traditional method (hyperbolic equation) was applied (Alcazar et al. 2021a). These limitations would be avoided by following the filtering process and using the hybrid equation reported in the current manuscript. In any case, it is important to note that V_0 values provided by any of the equations present a certain degree of uncertainty given the region of the *F–V* relationship that remains unexplored (below 34% of F_0 in the leg press and 18% of F_0 in the bench press exercises). Thus, future studies should focus on developing adequate procedures that can capture *F–V* data as close as possible to the actual V_0. In that scenario, only a curvilinear function (e.g., hybrid equation) would provide an adequate fit. Again, there are some scenarios where using a linear equation might be preferred: collected values correspond to *F–V* data above 45% of F_0; and collected values correspond to data above and below 45% of F_0, but the linear equation is applied only to data above 45% of F_0. In any of these scenarios, the limitations of linear *F–V* equations should be known and acknowledged.

Conclusion

An *F–V* equation combining a linear and a hyperbolic part showed to fit adequately recorded *F–V* data from ~ 20 to 100% of F_0. Importantly, this equation overcomes the limitations shown by linear equations while providing relevant data that may be used for different purposes, such as talent or deficit identification, as well as for the prescription of individual resistance training programs. Finally, establishing objective exclusion criteria for the recorded data based on physiological principles may provide more valid and reliable information on the actual *F–V* relationship.

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Author contributions JA, IA, and LMA: conceived and designed research. JA, FP, CR, HG, JS, and PJC: conducted experiments. JA, FP, CR, and PJC: analyzed data. JA, FP, IA, and LMA: wrote the manuscript. All authors read and approved the manuscript.

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Data availability The data analyzed during this study are included in this published article (Supplementary material).

Declarations

Conflict of interest The authors report no conflict of interest.

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