



REVIEW

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Modelling and analysis of a cracked rotor: a review of the literature and its implications

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Abstract Timely detection of fatigue cracks is necessary to avoid catastrophic failure of rotating machines which can lead to economical losses and accidental risk. This paper presents an exhaustive literature survey on theoretical and experimental vibration analysis of the rotating shaft in the presence of crack. Various non-destructive methods adopted by several researchers for crack detection in rotating machinery have been discussed. The vibration-based crack detection methods such as vibration-based diagnostics methods and vibration-based signal processing techniques have been broadly categorized along with their advantages and disadvantages. In general, various methodologies such as breathing mechanism, finite-element method, Hilbert–Huang transform, artificial intelligence techniques, wavelet transform and wavelet finite-element transform have been applied to investigate the presence of crack in the rotating shaft. The parameters such as natural frequencies, $1\times$, $2\times$ and $3\times$ harmonic components of dynamic response, critical speeds and whirl orbits have been significantly influenced due to the presence of crack in the rotating shaft. Several studies have been carried out to study variations in these parameters. Still, there is a need of more reliable and accurate modelling approach to detect variations in these parameters. In this paper, an attempt has been made to deliver all the modelling approaches implemented by the various researchers to detect crack within the rotating shaft. The modelling approaches are categorized based on the methodologies adopted by the various researchers to detect crack. Moreover, the critical observations made from the proposed modelling approaches are summarized and presented.

Keywords Vibration-based methods (VBM) · Crack detection · Breathing mechanism (BM) · Artificial intelligence (AI) techniques · Wavelet finite-element method (WFEM) · Hilbert–Huang transform (HHT)

1 Introduction

The rotor-bearing system is an integral part (or heart) of any rotordynamic system. These systems have been extensively used in power generation, pumps, compressors, manufacturing industries, aircraft engines and many other miscellaneous industrial applications. Continuous operation of these rotordynamic systems in a heavy-load environment expected to produce unpredicted fatigue cracks or failure.

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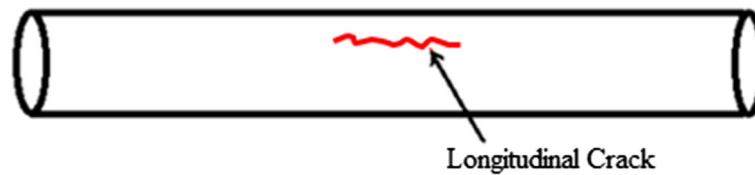


Fig. 1 Geometry of longitudinal crack

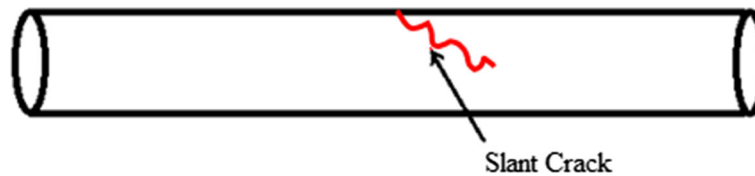


Fig. 2 Geometry of slant crack

Any unintentional discontinuities present in the shaft geometry are defined as cracks. The presence of this kind of unintentional discontinuities in the shaft geometry may lead to catastrophic and dangerous failure of rotating machines if not detected properly and on time. Since the catastrophic and dangerous failure of shaft may cause severe accidents, it intrudes smooth, effective and efficient operation and also increases overall cost due to expensive repairs. The geometry of crack has a great influence on the dynamics and characteristics of the cracked rotor. Therefore, the cracks are classified mainly into four groups as mentioned in the following subsections.

1.1 Longitudinal crack

Geometry of this kind of crack is relatively uncommon, appears parallel to the axis of the shaft (shown in Fig. 1) and has very less influence on the dynamic behaviour and characteristics of the cracked rotor. This kind of crack is generated when the rotor system is subjected to cyclic torsion. The torsional dynamic analysis of shaft with longitudinal and circumferential cracks has been carried out by Abdi et al. [1] and Nabian [2]. It has been observed that the presence of longitudinal crack in the shaft has very less influence on the resonance frequency.

1.2 Slant crack

Geometry of this kind of crack appears at an angle to the axis of the shaft as shown in Fig. 2. This type of crack appears less frequently compared to transverse cracks and has lesser effect on the dynamic behaviour and characteristics of the cracked rotor than the transverse cracks. Slant crack appears similar to the helicoidal cracks and has a significant influence on the rotor behaviour when torsional stresses are dominant. These kinds of cracks are generated in rotors primarily due to the torsional loading of shaft. Several authors [3–6] mainly focussed their work on finite element (FE) modelling and analysis of rotor system with slant crack. But the complete rotor dynamics of Jeffcott rotor considering a slant crack has been presented by Darpe [7]. Moreover, the differential equations of motion of a simple rotor system with slant crack have been derived and solved by Ichimonji and Watanabe [8] assuming the bending stiffness of the shaft changes synchronously with the opening/closing behaviour of crack. Finally, they concluded that by increasing the crack depth, the system becomes unstable.

1.3 Transverse crack

Transverse cracks are most common and serious [9]. The geometry of transverse crack appears perpendicular to the axis of the shaft (shown in Fig. 3) and has large influence on the dynamic behaviour and characteristics of the cracked rotor. This kind of crack reduces the cross-sectional area near to the crack surface of shaft and produces serious damage to the shaft. These cracks are generated when the rotor system is subjected to

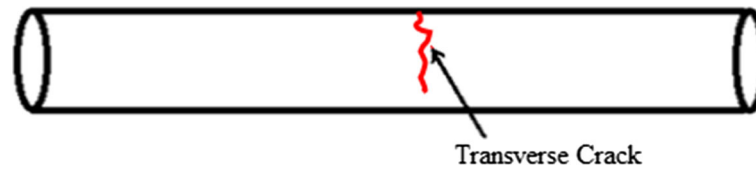


Fig. 3 Geometry of transverse crack

fluctuating bending stresses (i.e. cyclic stresses). Several researchers [10,11] have investigated the dynamic behaviour of rotor system considering the transverse crack. In addition, the stability analysis of a Laval rotor (one disc on an elastic shaft) having a transverse crack has been carried out by Gasch [12] to understand the complicated dynamic phenomena. They observed that when the crack grows, amplitudes at 1Ω , 2Ω and 3Ω grows, which is a clear indication of crack.

1.4 Breathing crack

Another type of crack which is frequently mentioned in research papers is breathing crack. Breathing crack opens and closes throughout the rotation of shaft, and hence, stresses are generated around the crack surface. The crack is termed as open when the surface of the shaft is under tensile stresses and termed as closed when the surface of the shaft is under compressive stresses during the rotation of shaft [13]. The equations of motion for a simple rotor with breathing crack have been derived and presented by Jun et al. [14]. In addition, the unique behaviour of the whirl orbits of a cracked rotor system with breathing transverse crack has been studied by Al-Shudeifat [15] considering the following two main aspects: (1) changes in these orbits nearby $1/2$ of the first critical speed and (2) sensitivity of these orbits to the unbalance force direction.

The presence of unintentional discontinuities (or cracks) in rotating rotor system will generate the typical vibration signals; therefore, vibration monitoring can be adopted as crack detection and identification techniques in the rotating machinery. The effort in the area of crack detection and identification in rotating shaft system is still continuing since 1960s. During this period, various review papers were also published to explore the investigations and studies carried out on modelling and analysis of cracked rotors. A comprehensive literature review on the various crack detection techniques applied for cracked shaft detection and diagnostics has been published by Sabnavis et al. [9]. They classified the reviewed literature based on three categories: (1) vibration-based methods, (2) modal testing and (3) non-traditional methods. In addition, Gasch [16], Dimarogonas [17] and Wauer [18] have presented an exhaustive literature survey on modelling and stability analysis of cracked rotating shaft for cracked shaft detection. More recently, Kumar and Rastogi [19] published a detailed literature review on modelling approaches adopted by the various researchers to investigate the presence of crack in the rotating shaft. Various modelling approaches based on finite-element method (FEM), breathing mechanism, wavelet transform (WT), wavelet finite-element method (WFEM), Hilbert–Huang transform (HHT) and so forth have been employed to study the dynamics of cracked rotor. But there is a need of more detailed analysis of these approaches to understand the procedure of crack diagnosis in the rotating shafts systematically. The work proposed in this paper will provide an in-depth understanding of various modelling approaches applied to investigate cracked rotor.

The purpose of the present review article is to provide the recent developments and advances in the field of crack diagnosis. The scope of the paper is limited to global methods that are applied to deduce crack from the deviations in vibration patterns of the structural systems. The vibration-based methods for crack diagnosis are classified in detail along with their advantages and disadvantages to study the characteristics of structural systems with cracks. The modelling approaches are classified on the basis of methodologies adopted by the various researchers to investigate the presence of crack in the rotating shaft. The fundamental concepts of mathematical formulation of these modelling approaches have been presented to understand the procedure of crack diagnosis in the rotating shaft systematically. Furthermore, the critical observations made from the proposed modelling approaches have also been presented.

The paper is organized as follows: Sect. 2 presents the classification of various crack detection methods along with their advantages and disadvantages. Section 3 presents the review of various modelling approaches adopted by the researchers to detect crack in the rotating shaft. Section 4 presents critical observations made from proposed modelling approaches. Section 5, finally, presents conclusion of the present review article.

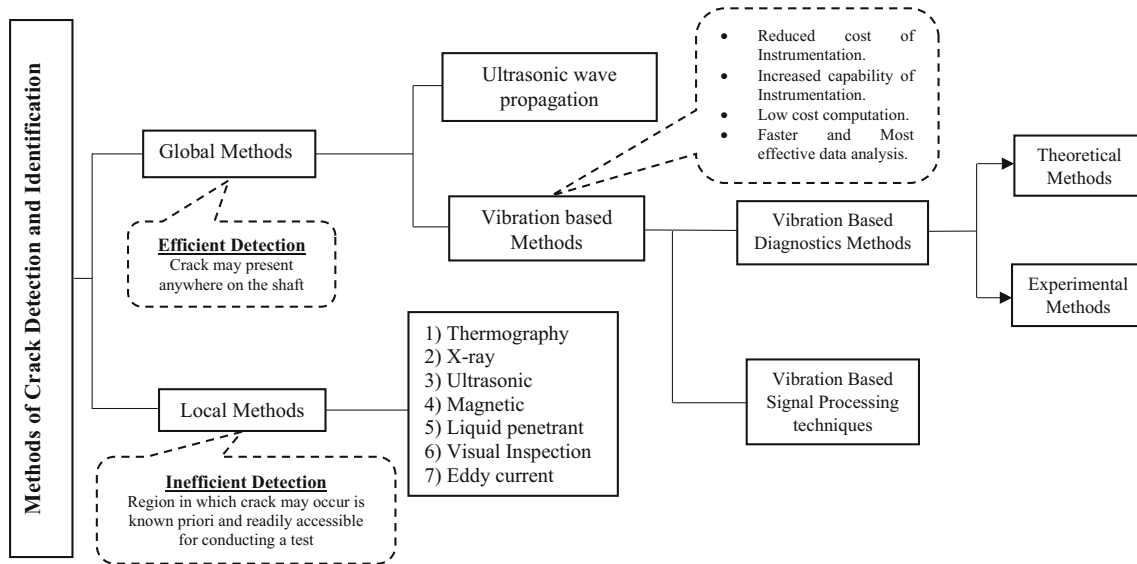


Fig. 4 Classification of methods used for crack detection and identification

2 Methods of crack detection and identification

Non-destructive testing (NDT) methods play a crucial role in crack fault diagnosis in the rotating machines. It is possible to detect crack fault in the rotating shaft without destroying their product integrity, surface texture and future usefulness. These methods can be performed on metals, ceramics, cermets, plastics, composites and coatings in order to detect cracks, surface cavities, internal voids, delamination and any type of flaw that could lead to premature failure. There are a number of non-destructive methods such as radiography, ultrasonic, magnetic particle and eddy current, which are used for crack detection and identification in rotating machinery. These methods can be divided into two parts, i.e. global methods and local methods. The flow diagram shown in Fig. 4 exhibits most commonly used non-destructive methods for crack detection and identification. The local crack detection techniques such as ultrasonic and X-ray methods require that the region in which a crack may occur is known a priori and readily accessible for conducting a test. However, most of the times these conditions cannot be guaranteed for most applications in mechanical engineering. To overcome these limitations, therefore, the vibration-based crack identification methods were developed as a global damage identification technique [20]. A detailed literature review on the vibration-based damage identification methods has been presented by Doebling and Farrar [21]. They categorized vibration-based damage identification methods as (1) linear versus nonlinear methods, (2) model-based versus non-model-based methods and (3) level of damage detection provided.

2.1 Vibration-based methods for crack detection

To study the features of structural systems with cracks, vibration-based crack detection methods are more powerful and preferable non-destructive methods than the other non-destructive crack identification methods. According to Doebling [21], the vibration-based damage detection technology can be effectively applied in the field of rotating machinery. Pattern recognition applied to time spectra or frequency spectra is the main base of vibration-based damage detection technology. With the help of measured vibration signatures, particular types of damage can be recognized effortlessly suggested by Carden and Fanning [22]. The central idea behind the vibration-based damage identification methods is that whenever the damage induced in rotating machinery, the variations within the physical properties such as stiffness, mass and damping occur which will cause significant variations in modal properties such as natural frequencies, modal damping and mode shapes, for example reduction in stiffness results due to the presence of cracks in the rotating machinery. Therefore, it is instinctive that damage can be easily recognized by studying the variations in vibration pattern of the rotating machinery. The vibration-based damage detection methods can be divided into the following two subgroups, namely vibration-based diagnostic methods and vibration-based signal processing techniques.

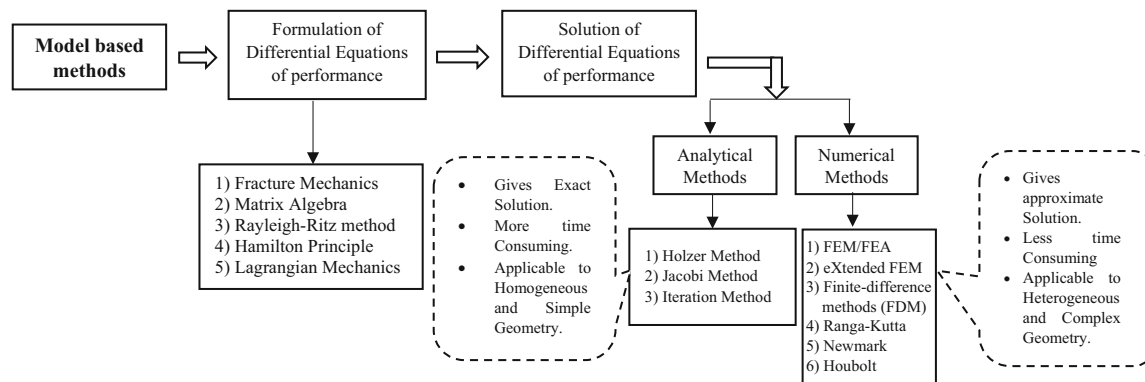


Fig. 5 Flow chart of model-based methods

2.1.1 Vibration-based diagnostic methods

The vibration-based diagnostic methods have been adopted by the numerous investigators for crack identification in the rotating machinery. These methods are subclassified as model-based methods and non-model-based methods.

(I) Model-based methods (theoretical methods)

These methods are fundamentally based on analytical or numerical models to simulate the dynamic behaviour of cracked rotors during operation. In model-based identification, the fault-induced change of the rotor system is taken into account by equivalent loads in the mathematical model. These equivalent loads are virtual forces and moments acting on the linear undamaged system to generate a dynamic behaviour identical to that measured in the damaged system [23]. Several researchers have developed analytical or numerical models to study vibrations generated by healthy and defective rotors. The accuracy of crack identification depends on degree of freedom and parameters considered in analytical or numerical models. A model-based transverse crack identification procedure has been developed by Pennacchi et al. [11]. The $1\times$, $2\times$ and $3\times$ rev. vibration components of response of transversely cracked shaft have been analysed to detect crack depth as well as crack position. In addition, a model-based technique based on FEM has been proposed by Sekhar [23] for online identification of more than one fault (i.e. unbalance and crack) present in the rotating shaft. The results show that $1\times$ harmonic component of response clearly indicates the presence of unbalance, while $2\times$ and $3\times$ harmonic components of response are characteristics of a crack fault. The flow diagram shown in Fig. 5 exposes the most commonly used model-based methods for crack detection in the rotating shaft.

(II) Non-model-based methods (experimental methods)

Some researchers have developed prototype or experimental test rig to study the vibration generated by rotor system. The vibration data had been captured using piezoelectric sensors/accelerometers. The captured vibration signals have been analysed in time domain and frequency domain to study the variations in the parameters such as eigenfrequencies, mode shapes and vibration response of healthy and defective rotors for crack identification. An experimental test rig of disc-rotor-bearing system has been developed by Guo et al. [24] to study the dynamic response of rotor with breathing crack. The results show that the appearance of inner loops when it passes through its $1/2$, $1/3$, $1/4$ and $1/5$ subcritical speed zones can be a good indicator for crack identification in a cracked rotor system. Figure 6 shows various experimental methods used for crack diagnosis in the rotating shaft.

2.1.2 Vibration-based signal processing techniques

Vibration signals captured from rotating machinery always carry large amount of dynamic information of the system. Therefore, effective and efficient processing of these vibration signals is necessary to extract the optimal features of a signal for fault detection. Various signal processing techniques such as WT, Wigner–Ville distribution (WVD), fast Fourier transform (FFT) and Hilbert–Huang transform (HHT) have been applied for this purpose. An exhaustive literature survey on various kinds of signal processing techniques to extract the optimal features of a signal for fault detection has been published by Yang et al. [25]. Figure 7 shows various vibration-based signal processing techniques along with their advantages, disadvantages and applications.

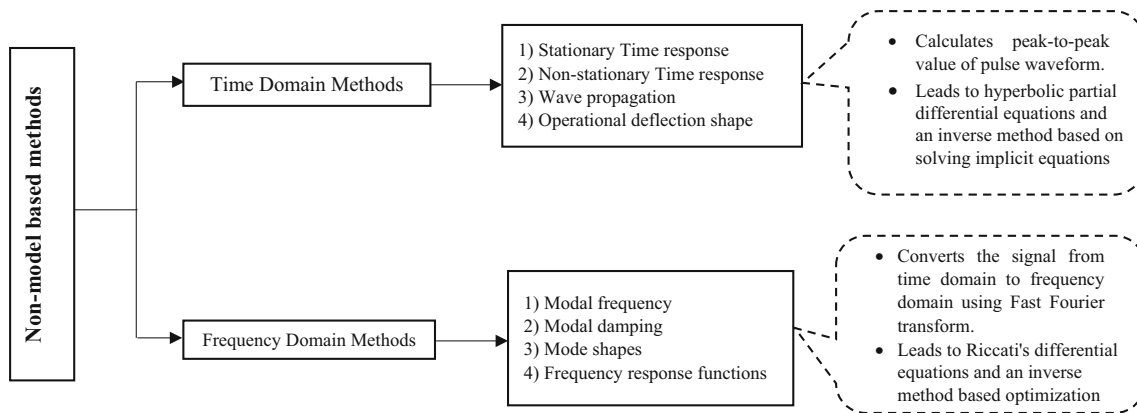


Fig. 6 Flow chart of non-model-based methods

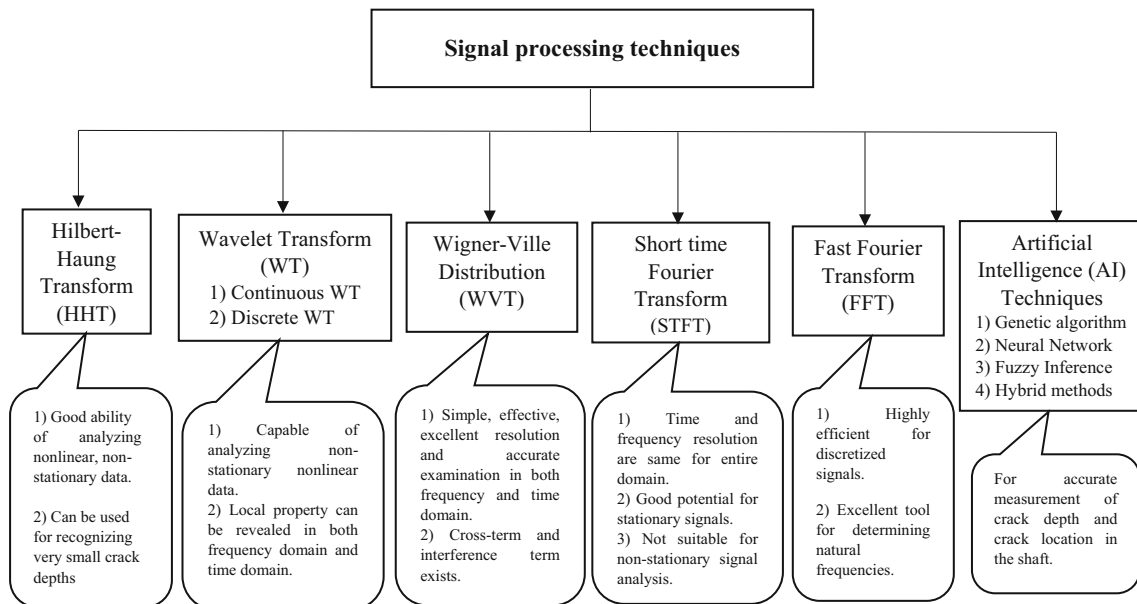


Fig. 7 Vibration-based signal processing techniques

3 Modelling approaches applied to investigate the presence of crack in the rotating shaft

Significant attention has been received in the area of crack diagnosis in rotating shaft from 1960 to till date. During this period, various methodologies have been applied by researchers for crack detection in the rotating shaft. These methodologies have some specific merits over the other. Therefore, comprehensive studies of all the modelling approaches are necessary to investigate the presence of crack in the rotating shaft. In this section, modelling approaches adopted by the various investigators have been categorized in detail. In preparing this review article, the authors decided to follow closely, but not identically, the modelling approaches were applied to investigate the presence of crack in the rotating shaft proposed by Kumar and Rastogi [19]. Figure 8 exposes various modelling approaches applied by the researchers to investigate cracked rotating shaft. These modelling approaches are further subclassified on the basis of model used to study the proposed modelling approaches.

3.1 Breathing mechanism (BM)-based approach

Breathing mechanism-based approach has been employed by many researchers [14,26–36] to study the dynamic behaviour of cracked rotating shaft. When the crack breaths (i.e. opens and closes), during each rotation of shaft it affects the dynamic response and characteristics of cracked rotor by changing time-varying

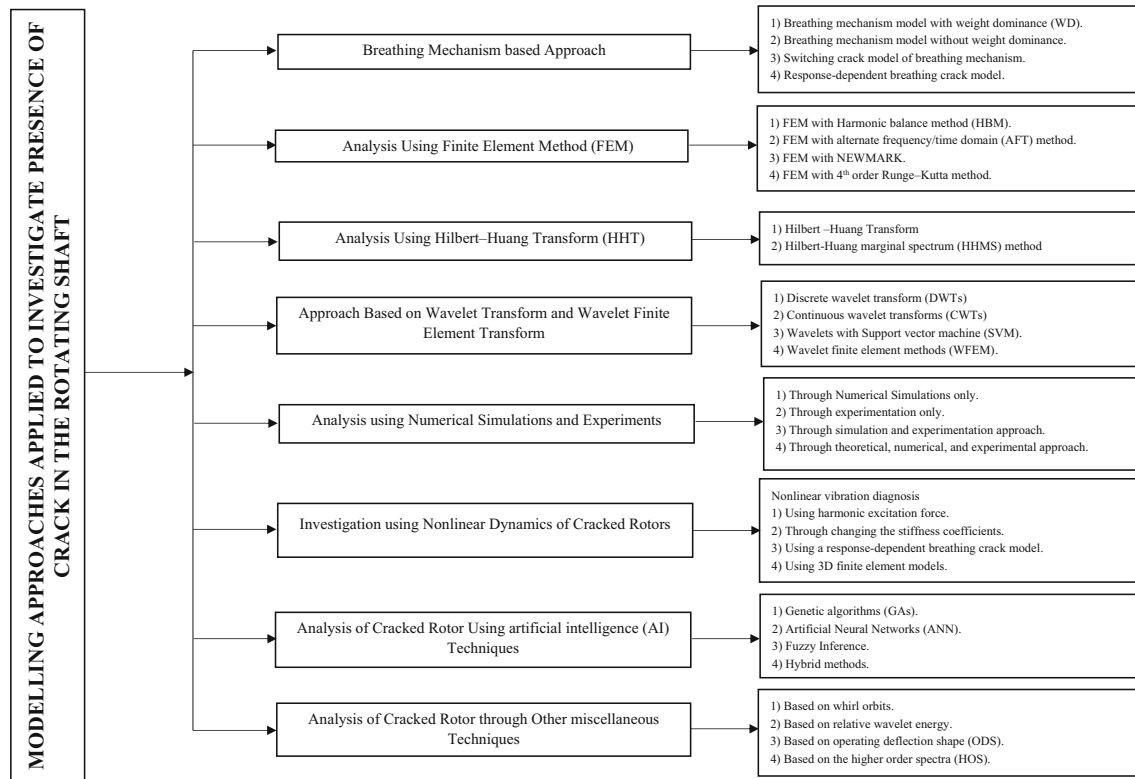


Fig. 8 Flow chart of modelling approaches applied to investigate cracked rotor

area moment of inertia (M.O.I.) of the cracked element. Hence, there is a need of more powerful and reliable models for the analysis of the presence of crack in rotating shaft. The proposed breathing mechanism-based approach can accurately model the presence of crack in rotating shaft, and hence, more accurate prediction of the dynamic response and characteristics of a cracked rotating shaft can be achieved.

3.1.1 Modelling of breathing mechanism

When the shaft begins to revolve, the locations of the axes of the cracked element change with respect to time during rotation. Consequently, tensile and compressive stresses are generated below and above the neutral axis, which tend to maintain the opening and closing of the crack, respectively. The breathing phenomena of the crack throughout the rotation of shaft can be described as shown in Fig. 9, and the breathing crack states for full rotational angle are given in Table 1.

From Fig. 9a, the location of centroid (e) and the cross-sectional (c/s) area (A_1) of cracked element on the fixed Y axis at $t = 0$ are given by

$$A_1 = R^2 (\pi - \cos^{-1} (1 - \mu) + (1 - \mu) \gamma, \tag{1}$$

$$e = \frac{2R^3}{3A_1} (\mu(2 - \mu))^{\frac{3}{2}} = \frac{2R^3}{3A_1} \gamma^3, \tag{2}$$

where $\mu = \frac{h}{R}$ represents non-dimensional crack depth, h is the crack depth along the radial direction, R is the radius of shaft and $\gamma = [\mu(2 - \mu)]^{\frac{1}{2}}$.

When the crack begins to close at $\Omega t = \theta_1$ as shown in Fig. 9d and turn into fully closed at $\Omega t = \theta_2$ shown in Fig. 9h, these two angles, θ_1 and θ_2 , are given by

$$\theta_1 = \tan^{-1} \left(\frac{e + R(1 - \mu)}{R\sqrt{\mu(2 - \mu)}} \right), \tag{3}$$

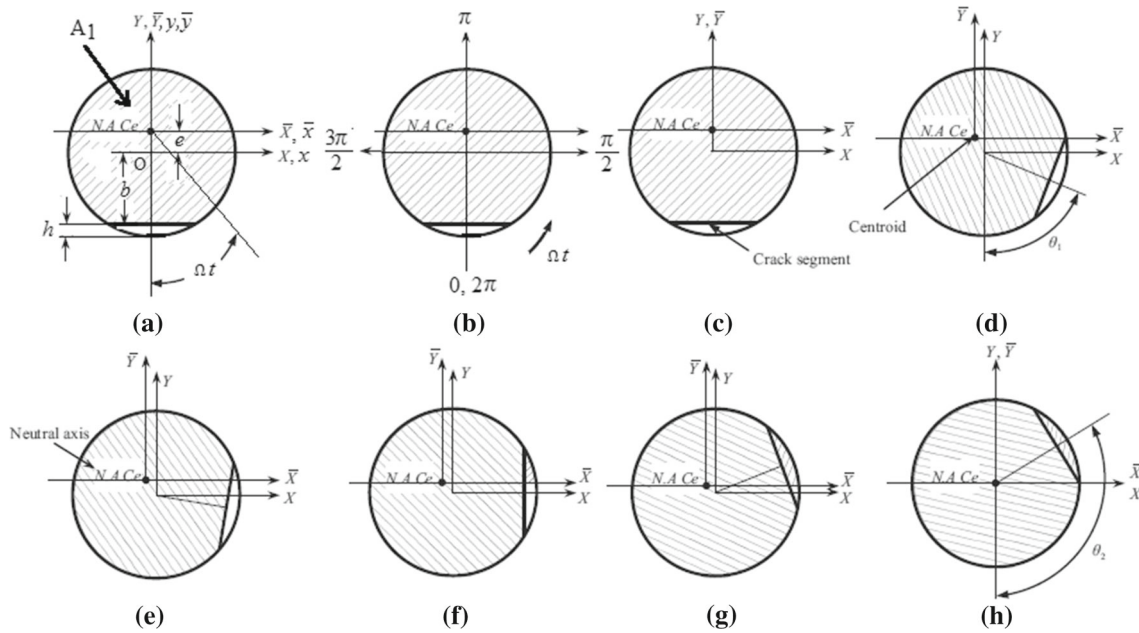


Fig. 9 Coordinate systems, breathing crack states and locations of centroid and neutral axis of the cracked rotor system [26]

Table 1 Breathing crack states for full rotational angle (0–2π)

| Rotational angle ($0 \leq \Omega t \leq 2\pi$) | Breathing crack states |
|---|------------------------|
| $0 \leq \Omega t < \theta_1, 2\pi - \theta_1 \leq \Omega t < 2\pi$ | Fully open |
| $\theta_1 \leq \Omega t < (\pi + \alpha)/2, \pi - \alpha/2 \leq \Omega t < 2\pi - \theta_1$ | Partially open |
| $(\pi + \alpha)/2 \leq \Omega t < (3\pi - \alpha)/2$ | Fully closed |

$$\theta_2 = \frac{\pi}{2} + \cos^{-1}(1 - \mu), \tag{4}$$

where $A_1 = c/s$ is the area of the cracked element for $0 \leq \Omega t < \theta_1, 2\pi - \theta_1 \leq \Omega t < 2\pi$ when the crack is fully open. Let I_1 and I_2 denote the area M.O.I. of A_1 about the fixed X and Y axes at $t = 0$. These are given by

$$I_1 = I - I_x^c, \tag{5}$$

$$I_2 = I - I_y^c, \tag{6}$$

where $I = \pi R^4/4$ represents the area M.O.I. of the shaft c/s for $(\pi + \alpha)/2 \leq \Omega t < (3\pi - \alpha)/2$ when the crack is fully closed and I_x^c and I_y^c denote the area M.O.I. of the crack segment about the fixed X and Y axes at $t = 0$. These are given by

$$I_x^c = \frac{\pi R^4}{8} - \frac{R^4}{4} [(1 - \mu)(2\mu^2 - 4\mu + 1)\gamma + \sin^{-1}(1 - \mu)], \tag{7}$$

$$I_y^c = \frac{R^4}{12} [(1 - \mu)(2\mu^2 - 4\mu - 3)\gamma + 3\sin^{-1}(\gamma)]. \tag{8}$$

The area M.O.I. of A_1 about the centroidal \bar{X} and \bar{Y} axes at $t = 0$ can be obtained by using

$$\bar{I}_1 = I_1 - A_1 e^2, \quad \bar{I}_2 = I_2. \tag{9}$$

As proposed by Pilkey [27], $I_X^{A_1}(t)$, $I_Y^{A_1}(t)$ and $I_{XY}^{A_1}(t)$ denote the area M.O.I. of A_1 about the fixed X and Y axes. These parameters are varying with time during shaft rotation and can be obtained in terms I_1 and I_2 as

$$I_X^{A_1}(t) = \frac{I_1 + I_2}{2} + \frac{I_1 - I_2}{2} \cos(2\Omega t) + I_{12} \sin(2\Omega t), \tag{10}$$

$$I_Y^{A_1}(t) = \frac{I_1 + I_2}{2} + \frac{I_1 - I_2}{2} \cos(2\Omega t) - I_{12} \sin(2\Omega t), \tag{11}$$

$$I_{XY}^{A_1}(t) = -\frac{I_1 - I_2}{2} \sin(2\Omega t) + I_{12} \cos(2\Omega t). \tag{12}$$

Considering the cross section of the cracked element is symmetrical about the Y axis, therefore, $I_{12} = 0$. The complete c/s area of the cracked element during revolution which is a time-varying quantity is given by

$$A_{ce}(t) = A_1 + A_2(t), \tag{13}$$

where $A_2(t)$ is a time-varying quantity which denotes the area of the closed portion of the crack as the crack starts to close at $\Omega t = \theta_1$. Let $I_X^{A_2}(t)$ and $I_Y^{A_2}(t)$ denote M.O.I. of the new formed area $A_2(t)$ about the X and Y axes. Therefore, the M.O.I. of the area $A_{ce}(t)$ about the X and Y axes is given by

$$I_X(t) = I_X^{A_1}(t) + I_X^{A_2}(t), \tag{14}$$

$$I_Y(t) = I_Y^{A_1}(t) + I_Y^{A_2}(t). \tag{15}$$

The area M.O.I. of $A_{ce}(t)$ about the \bar{X} and \bar{Y} axes which are parallel to fixed X and Y axes during rotation (Ref. Fig. 9) can be found by applying the parallel axis theorem

$$I_{\bar{X}}(t) = I_X(t) - Y_{ce}^2(t) A_{ce}(t), \tag{16}$$

$$I_{\bar{Y}}(t) = I_Y(t) - X_{ce}^2(t) A_{ce}(t), \tag{17}$$

where $X_{ce}(t)$ and $Y_{ce}(t)$ denote the instantaneous centroid coordinates of the area $A_{ce}(t)$ about the fixed X and Y axes. The functional relationship for $I_{\bar{X}}(t)$ was obtained by using Fourier series expansion and given by

$$I_{\bar{X}}(t) \cong \hat{I}_{\bar{X}}(t) = I - (I - \bar{I}_1) f_1(t), \tag{18}$$

$$f_1(t) = \left(\cos\left(\frac{\Omega t}{2}\right) \right)^n = \frac{1}{2^n} \left[\binom{n}{n/2} + 2 \sum_{j=0}^{\left(\frac{n}{2}\right)-1} \binom{n}{j} \cos(n-2j) \left(\frac{\Omega t}{2}\right) \right], \tag{19}$$

where n denotes a positive even number. The approximate formulas can be obtained as

$$I_{\bar{Y}}(t) \cong \hat{I}_{\bar{Y}}(t) = I + (I - \bar{I}_1) f_1(t) - (2I - \bar{I}_1 - \bar{I}_2) f_2(t), \tag{20}$$

$$f_2(t) = \frac{1}{\pi} \left[\frac{\theta_1 + \theta_2}{2} - \frac{2}{(\theta_2 - \theta_1)} \sum_{i=1}^p \frac{\cos(i\theta_2) - \cos(i\theta_1)}{i^2} \cos(i\Omega t) \right]. \tag{21}$$

For $I_{11} = I - \bar{I}_1$ and $I_{22} = -(2I - \bar{I}_1 - \bar{I}_2)$, the approximations for the area M.O.I. in Eqs. (18) and (20) can be redrafted as

$$\hat{I}_{\bar{X}}(t) = I - f_1(t) I_{11}, \tag{22}$$

$$\hat{I}_{\bar{Y}}(t) = I + f_1(t) I_{11} - f_2(t) I_{22}. \tag{23}$$

According to Pilkey [27], $I_{\bar{X}\bar{Y}}(t)$ is designed in terms of the centroid of the cracked element c/s area and $\hat{I}_{\bar{X}\bar{Y}}(t)$ is the Fourier series approximation of $I_{\bar{X}\bar{Y}}(t)$. An accurate functional relationship for approximation $\hat{I}_{\bar{X}\bar{Y}}(t)$ is given by

$$I_{\bar{X}\bar{Y}}(t) \cong \hat{I}_{\bar{X}\bar{Y}}(t) = I_{\bar{X}\bar{Y}}^{A_1} \sum_{k=1}^p \frac{2\bar{\theta}_2 \sin(\pi - k\bar{\theta}_2)}{\pi^2 - k^2\bar{\theta}_2^2} \sin(k\Omega t), \tag{24}$$

where $\bar{\theta}_2 = 0.8\theta_2$, p is a positive integer and the area product of inertia $I_{\bar{X}\bar{Y}}^{A_1}$ about the \bar{X} and \bar{Y} axes is calculated from Eq. (25)

$$I_{\bar{X}\bar{Y}}^{A_1} \left(t = \frac{\pi}{4} \right) = \left[\frac{I_1 - I_2}{2} \sin(-2\Omega t) + \frac{A_1 e^2 \sin(2\Omega t)}{2} \right]_{t=\frac{\pi}{4}} = -\frac{I_1 - I_2}{2} + \frac{A_1 e^2}{2}. \tag{25}$$

Hence based on the shifts in the centroid and neutral axis, the complete stiffness matrix of the Jeffcott rotor with a breathing crack is approximated by

$$K(t) = \begin{bmatrix} k_1(t) & k_{12}(t) \\ k_{21}(t) & k_2(t) \end{bmatrix} = \begin{bmatrix} \hat{I}_{\bar{X}}(t) & \hat{I}_{\bar{X}\bar{Y}}(t) \\ \hat{I}_{\bar{Y}\bar{X}}(t) & \hat{I}_{\bar{Y}}(t) \end{bmatrix}. \quad (26)$$

The breathing phenomenon of crack results in time-varying direct stiffnesses $k_1(t)$, $k_2(t)$ and the cross-coupled stiffnesses $k_{12}(t)$ and $k_{21}(t)$ within the governing equations of the cracked rotor system. These governing equations of the cracked rotor system are formulated by considering these breathing phenomena and given by [26,33]

$$m\ddot{u} + c\dot{u} + k_1(t)u + k_{12}(t)v = m_{ed}\Omega^2 \sin(\Omega t + \beta), \quad (27)$$

$$m\ddot{v} + c\dot{v} + k_2(t)v + k_{21}(t)u = m_{ed}\Omega^2 \cos(\Omega t + \beta) - mg. \quad (28)$$

Jun et al. [14] have derived the direct and cross-coupled stiffnesses of a rotor with a breathing crack based on fracture mechanics. The opening/closing phenomena of breathing crack have been simulated and simplified by means of switching crack model. The nonlinear dynamic characteristics of cracked rotating shaft have been studied by Patel and Darpe [28] through two well-known crack models: (1) switching crack model and (2) response-dependent breathing crack model. It is worth to mention here that in switching crack model crack remains either fully open or fully closed while in response-dependent breathing crack model considers partial opening and closing of the crack during shaft rotation. However, an analytical model (unbalanced one) has been developed by Mubarak et al. [29] to study the actual BM of crack in terms of crack location along shaft length, unbalance force orientation, shaft rotational angle and the ratio of gravitational force to unbalance force. The results obtained were compared with those of the balanced shaft and validated by Abaqus simulation to prove accuracy and consistency of their derived model. In addition, a restoring force-modified model has been proposed by Xie et al. [30] to investigate the crack BM of Jeffcott rotors without weight dominance. The characteristics of dynamic response of cracked Jeffcott rotors have been analysed by using waterfall plot method that can easily recognize the frequency components with feeble amplitudes. The dual resonant peak features found in the simulation signals as well as in the experimental results agreed well which validate the effectiveness of the proposed model.

Energy principles have been used to analyse time-variant flexibility of the cracked shaft by Georgantzinos and Anifantis [31]. Numerically achieved values of direct and cross-coupled flexibility coefficients are validated with published results in the literature to prove accuracy and consistency of their derived model.

The time-varying finite-element stiffness matrix of the cracked element has been developed by Al-Shudeifat and Butcher [32] through two new breathing functions. These breathing functions have been adopted to approximate the actual BM of the crack, and system's stability is examined by Floquet theory considering the depth of crack and rotating speed are presented by Guo et al. [33]. The same approach has been also used to derive time-varying finite-element stiffness matrix of the cracked element by Oliveira and Melo [34]. The derived set of equations has been solved by using two approaches, i.e. (1) Newmark method and (2) solution of discrete-time state equations. The dynamic response of two approaches was compared and found the processing time is much smaller in Newmark's method than the discrete-time state equations. In addition, a three-dimensional FEM consisting of a two-disc rotor with a transverse crack has been considered by Hossain and Wu [35] to study the effects of various parameters such as angular position of crack, ratio of crack depth to the shaft radius, axial position of crack, ratio of unbalanced force and angular position of unbalanced force on the crack BM. Moreover, Zhang and Li [36] have formulated the six DOFs coupled equations of motion of a cracked Jeffcott rotor, and the effect of breathing behaviour of the cracked rotor is studied considering various parameters such as depth of crack, shaft slenderness ratio and rotating speed ratio.

3.2 Analysis using finite-element method (FEM)

FEM has gained increasing attention from various researchers as an effective investigating tool for the analysis of cracked shaft. Due to the ability of this tool in three-dimensional (3D) modelling and easiness in simulation, several investigators [4,37–53] have employed FEM for modelling and analysis of the presence of crack in a rotating shaft. Figure 10 shows the steps for formulation of equations of motion for a disc-rotor-bearing system with crack by using FEM.

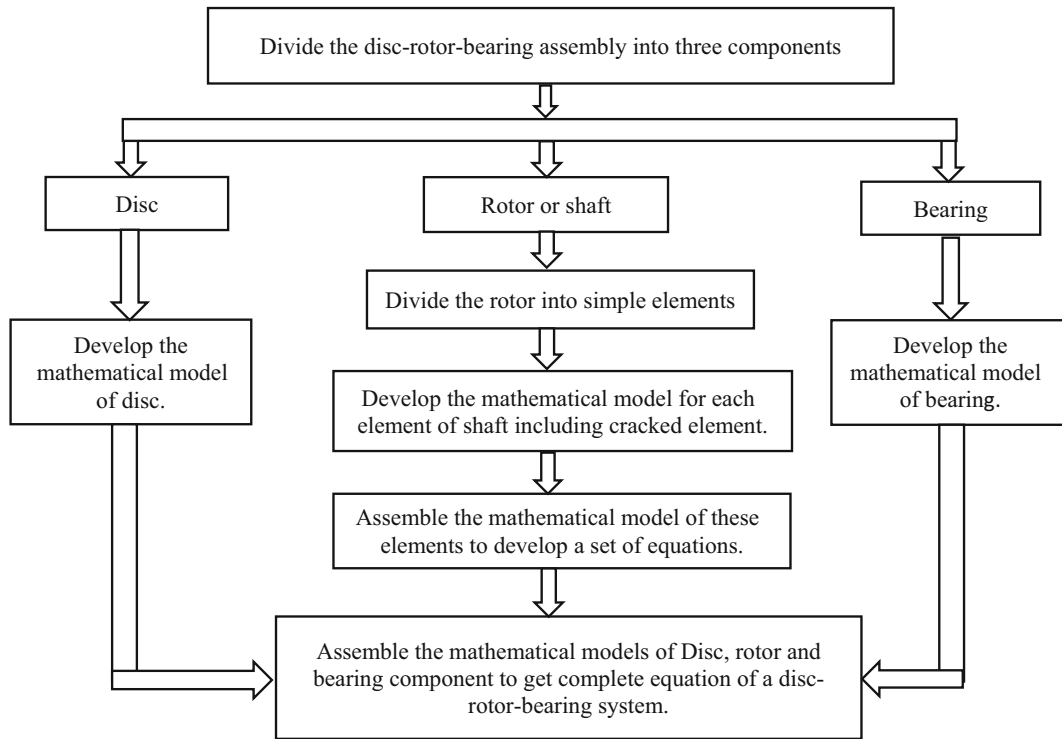


Fig. 10 Steps for formulation of the equations of motion for a cracked rotating shaft by using FEM

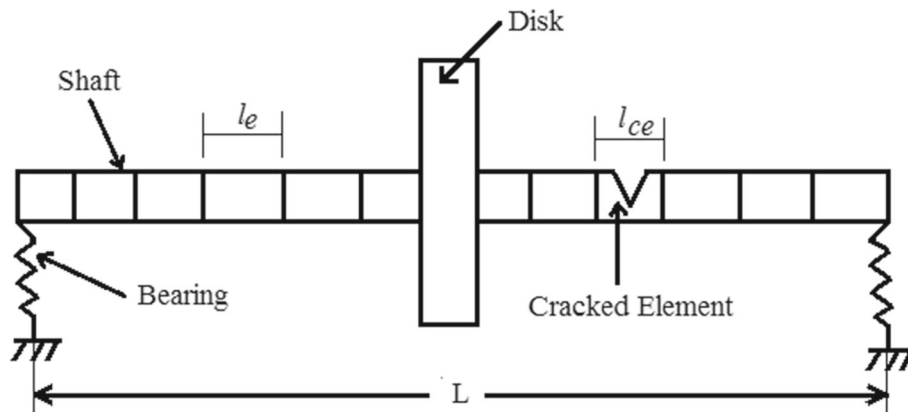


Fig. 11 Disc-rotor-bearing system with crack model cross section

3.2.1 Mathematical modelling of disc-rotor-bearing system with crack using FEM

Researchers have considered various components of disc-rotor-bearing system such as disc, shaft and bearing for modelling using FEM.

- (i) **Modelling of shaft** The rotor consists of a shaft with single disc at the middle of the shaft as shown in Fig. 11. The rotor shaft is modelled and discretized into 12 Timoshenko beam finite elements with four degrees of freedom (4 DOF) at every node. The element matrices are derived on the basis of the kinetic and strain energies within the beam element according to the lateral displacement $u_e(\xi, t)$ of the beam's neutral plane. The expression of the mass matrix M_e , the stiffness matrix K_e , the skew-symmetrical gyroscopic matrix G_e and the damping matrix C_e for the two bending planes neglecting the shear effect is generated based on the local coordinates vector $[u_1 \ v_1 \ \theta_1 \ \psi_1 \ u_2 \ v_2 \ \theta_2 \ \psi_2]^T$ given by

$$M_e = \frac{\rho_e A_e I_e}{420} \begin{bmatrix} 156 & 0 & 0 & 22l_e & 54 & 0 & 0 & -13l_e \\ 0 & 156 & -22l_e & 0 & 0 & 54 & 13l_e & 0 \\ 0 & -22l_e & 4l_e^2 & 0 & 0 & -13l_e & -3l_e^2 & 0 \\ 22l_e & 0 & 0 & 4l_e^2 & 13l_e & 0 & 0 & -3l_e^2 \\ 54 & 0 & 0 & 13l_e & 156 & 0 & 0 & -22l_e \\ 0 & 54 & -13l_e & 0 & 0 & 156 & 22l_e & 0 \\ 0 & 13l_e & -3l_e^2 & 0 & 0 & 22l_e & 4l_e^2 & 0 \\ -13l_e & 0 & 0 & -3l_e^2 & -22l_e & 0 & 0 & 4l_e^2 \end{bmatrix}, \quad (29)$$

$$K_e = \frac{E_e I_e}{l_e^3} \begin{bmatrix} 12 & 0 & 0 & 6l_e & -12 & 0 & 0 & 6l_e \\ 0 & 12 & -6l_e & 0 & 0 & -12 & -6l_e & 0 \\ 0 & -6l_e & 4l_e^2 & 0 & 0 & 6l_e & 2l_e^2 & 0 \\ 6l_e & 0 & 0 & 4l_e^2 & -6l_e & 0 & 0 & 2l_e^2 \\ -12 & 0 & 0 & -6l_e & 12 & 0 & 0 & -6l_e \\ 0 & -12 & 6l_e & 0 & 0 & 12 & 6l_e & 0 \\ 0 & -6l_e & 2l_e^2 & 0 & 0 & 6l_e & 4l_e^2 & 0 \\ 6l_e & 0 & 0 & 2l_e^2 & -6l_e & 0 & 0 & 4l_e^2 \end{bmatrix}, \quad (30)$$

$$G_e = \frac{\rho_e I_e}{15l_e} \begin{bmatrix} 0 & 36 & 3l_e & 0 & 0 & 36 & -3l_e & 0 \\ -36 & 0 & 0 & -3l_e & 36 & 0 & 0 & -3l_e \\ 3l_e & 0 & 0 & 4l_e^2 & -3l_e & 0 & 0 & -l_e^2 \\ 0 & 3l_e & -4l_e^2 & 0 & 0 & -3l_e & l_e^2 & 0 \\ 0 & -36 & 3l_e & 0 & 0 & 36 & 3l_e & 0 \\ 36 & 0 & 0 & 3l_e & -36 & 0 & 0 & 3l_e \\ 3l_e & 0 & 0 & -l_e^2 & -3l_e & 0 & 0 & 4l_e^2 \\ 0 & 3l_e & l_e^2 & 0 & 0 & -3l_e & -4l_e^2 & 0 \end{bmatrix}. \quad (31)$$

$C_e = \alpha K_e$, where α is a proportional factor to stiffness and describes the coefficient of damping.

- (ii) **Modelling of cracked element** The stiffness matrix (K_{crack}) of cracked element due to the transverse crack can be found at the location of crack by neglecting the shear effect using standard finite elements.

$$K_{crack} = \frac{E_e}{l_{ce}^3} \begin{bmatrix} 12I_X & 0 & 0 & 6l_{ce}I_X & -12I_X & 0 & 0 & 6l_{ce}I_X \\ 0 & 12I_Y & -6l_{ce}I_Y & 0 & 0 & -12I_Y & -6l_{ce}I_Y & 0 \\ 0 & -6l_{ce}I_Y & 4l_{ce}^2I_Y & 0 & 0 & 6l_{ce}I_Y & 2l_{ce}^2I_Y & 0 \\ 6l_{ce}I_X & 0 & 0 & 4l_{ce}^2I_X & -6l_{ce}I_X & 0 & 0 & 2l_{ce}^2I_X \\ -12I_X & 0 & 0 & -6l_{ce}I_X & 12I_X & 0 & 0 & -6l_{ce}I_X \\ 0 & -12I_Y & 6l_{ce}I_Y & 0 & 0 & 12I_Y & 6l_{ce}I_Y & 0 \\ 0 & -6l_{ce}I_Y & 2l_{ce}^2I_Y & 0 & 0 & 6l_{ce}I_Y & 4l_{ce}^2I_Y & 0 \\ 6l_{ce}I_X & 0 & 0 & 2l_{ce}^2I_X & -6l_{ce}I_X & 0 & 0 & 4l_{ce}^2I_X \end{bmatrix}. \quad (32)$$

Mayes and Davies [37] have developed a well-known model of a crack that has been adopted by several researchers. In this model, a cosine function was used to describe the opening and closing phenomena of the crack by assuming that the gravity force is much larger than the imbalance force. Table 2 describes the breathing function and the states of breathing crack.

- (iii) **Modelling of bearings** The proposed bearings in this work are assumed to be linear and obey the following governing equation which correlates the forces acting on the shaft because of the bearings with resultant displacements and velocities of the shaft.

Table 2 Breathing function and different states of breathing crack

| Breathing function describing breathing phenomena | Fully closed state of breathing crack | Fully open state of breathing crack |
|---|---------------------------------------|-------------------------------------|
| $f(t) = 1/2(1 - \cos(\omega t))$ where ω represents the angular speed of the shaft | $f(t) = 0$ | $f(t) = 1$ |

$$\begin{pmatrix} f_x \\ f_y \end{pmatrix} = - \begin{bmatrix} k_{uu} & k_{uv} \\ k_{vu} & k_{vv} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} - \begin{bmatrix} c_{uu} & c_{uv} \\ c_{vu} & c_{vv} \end{bmatrix} \dot{\begin{pmatrix} u \\ v \end{pmatrix}} \tag{33}$$

In vector notation, $Q_s = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$ and $q_b = \begin{pmatrix} u \\ v \end{pmatrix}$

$$Q_s = -K_b q - C_b \dot{q} \tag{34}$$

where $K_b = \begin{bmatrix} k_{uu} & k_{uv} \\ k_{vu} & k_{vv} \end{bmatrix}$ is the stiffness matrix of the bearings and $C_b = \begin{bmatrix} c_{uu} & c_{uv} \\ c_{vu} & c_{vv} \end{bmatrix}$ is the damping matrix of bearings.

(iv) **Modelling of rigid disc** The mass and gyroscopic matrices M_e^d and G_e^d for the disc element corresponding to the degree of freedom $[u \ v \ \theta \ \psi]^T$ are derived using the energy method. The total kinetic energy (T_d) due to the translation and rotation of the disc is given by

$$T_d = \frac{1}{2} m_d (\dot{u}^2 + \dot{v}^2) + \frac{1}{2} I_d (\dot{\theta}^2 + \dot{\psi}^2) + \frac{1}{2} I_p (\Omega^2 - 2\Omega \dot{\psi}) \tag{35}$$

The element matrices are obtained by applying Lagrange's equation to Eq. (35)

$$\begin{pmatrix} \frac{d}{dt} \left(\frac{\partial T_d}{\partial \dot{u}} \right) - \frac{\partial T_d}{\partial u} \\ \cdot \\ \cdot \\ \cdot \\ \frac{d}{dt} \left(\frac{\partial T_d}{\partial \dot{\psi}} \right) - \frac{\partial T_d}{\partial \psi} \end{pmatrix} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix} \begin{pmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} + \Omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \\ 0 & 0 & -I_p & 0 \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \tag{36}$$

where

$$M_e^d = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix} \tag{37}$$

$$G_e^d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \\ 0 & 0 & -I_p & 0 \end{bmatrix} \tag{38}$$

Finally, Q_e representing the gravity force vector and W_e defining the imbalance force for the degree of freedom $[u \ v \ \theta \ \psi]^T$ are given by

$$Q_e = \begin{bmatrix} 0 \\ -mg \\ 0 \\ 0 \end{bmatrix} \tag{39}$$

$$W_e = \begin{bmatrix} m_e d_e \omega^2 \cos(\omega t + \phi) \\ m_e d_e \omega^2 \sin(\omega t + \phi) \\ 0 \\ 0 \end{bmatrix} \tag{40}$$

where m represents the mass for each element, d_e represents the eccentricity, ϕ is the initial angular position of disc with respect to the z axis and m_e represents the mass unbalance.

- (v) **Complete equation of motion** The complete equation of motion for a cracked rotor system is well defined as

$$M\ddot{x} + (C + \omega G)\dot{x} + (K - f(t)K_{\text{crack}})x = Q + W, \quad (41)$$

where M is the global mass matrix including mass matrix of the shaft and rigid discs, G represents the global gyroscopic matrix including gyroscopic matrix of the shaft and rigid discs, C is the external damping matrix, K is the stiffness matrix of the shaft, Q is the gravity force vector, $f(t)$ represents the breathing function describing breathing phenomena and W defines the imbalance force for the complete rotor system.

Wauer [38] has derived the governing dynamic equations of motion for a cracked rotor system. The geometric discontinuity was replaced by a load discontinuity at the crack location to study the torsional vibrations of a circular shaft with a circumferential crack. The vibrational behaviour of rotating shaft having an open surface transverse crack has been analysed by Dimarogonas and Papadopoulos [39,40]. They assumed that the presence of open surface crack in the rotating shaft leads to a system having behaviour similar to that of a rotor with dissimilar M.O.I. along two perpendicular directions. In addition, FE modelling of a two-disc rotor system with a transversely open crack has been proposed by Al-Shudeifat [41]. The formulation of time-periodic finite-element stiffness matrix has been done by using time-varying area M.O.I. at the cracked element cross section of a cracked rotor, which is proposed by Mayes and Davis in 1984 [37]. In another study, the stability analysis of proposed dynamical system has been investigated by Al-Shudeifat [42]. However, Sekhar and Prasad [4] have also developed the flexibility matrix and stiffness matrix for a slant crack present in the rotating shaft which were used later in the finite-element (FE) analysis of the rotor-bearing system. Finally, they concluded that with an increase in crack depth, reduction occurs in all modal frequencies.

Wu et al. [43] have investigated the coupling between lateral and torsional vibrations using finite-element analysis of a rotor with two breathing cracks. They derived the stiffness matrix for the cracked shaft element considering six degrees of freedom per node to study the variation of the system eigenfrequencies and vibrational response. Moreover, the coupling between longitudinal–bending–torsional vibrations of a cracked shaft has been studied by Darpe et al. [44,45] through response-dependent nonlinear breathing crack model by using the FEM. The presence of bending natural frequency and the sum and difference frequencies around it were found in longitudinal vibration spectra which verifies the coupling has been established between bending and longitudinal vibrations as well as between torsion and bending vibrations. In addition, orthogonal natural frequencies (ONFs)-based approach has been presented by Haji and Oyadiji [46,47] for the identification and localization of cracks in rotor systems. The Bernoulli–Euler beam theory along with FEM has been used to simulate the non-rotating cracked rotor. Finally, they investigated the effects of crack depth, crack location and mass of a roving disc on the normalized orthogonal natural frequency curves (NONFCs).

Sinou and Lees [48,49] have analysed the influence of the presence of transversal cracks in the rotating shaft. These research articles address the two main issues, i.e. influence of crack opening and closing and the changes in modal properties on the dynamic behaviour of cracked shaft during operation. The evolution of the orbits at various location of the cracked rotor near $1/2$ and $1/3$ of the first critical speed has been investigated. In another study, the same approach has been also used by Jean-Jacques Sinou [50] to detect the presence of a single transverse breathing crack in a nonlinear disc rotor system through $2\times$ and $3\times$ super-harmonic frequency components. The effects of size of crack and location of crack on the nonlinear dynamic responses and emerging of new resonance—antiresonance peaks at 2^{nd} , 3^{rd} and 4^{th} harmonic frequency components have been analysed numerically. On the other hand, Upadhyay and Satankar [51] have analysed the effect of transverse cracks in a rotating shaft. The influence of (1) presence of crack, (2) changes in position of crack and (3) changes in speed of cracked rotor on the dynamic response were the three main issues addressed in this paper. The dynamic response characteristics of a rotor-disc-bearing system considering one and two cracks have been investigated by Khorrami et al. [52] through modified harmonic balance method. The analytical model was formulated using Timoshenko beam theory, and the effect of crack parameters such as crack location, crack depth and relative angular position on vibrational properties such as the shaft centre orbit, critical speeds and harmonic and super-harmonic components of the unbalance lateral response were investigated. The vibrational response of the cracked rotor-bearing system with Newmark numerical integration method (average acceleration method) was calculated by Mehdi Ghazlane [53]. The change in modal properties by the decrease in natural frequencies, production of $3\times$ harmonic of rotational speed and $2\times$ harmonic of rotating speed modulated with natural frequency were the main parameters to identify the crack.

3.3 Analysis using Hilbert–Huang transform (HHT)

In the recent years, Hilbert–Huang transform (HHT) has been used by the many researchers because it has good computation efficiency. HHT has good potential to become a perfect tool for time–frequency analysis approach for crack detection and identification, especially in the analysis of transient signals. The key component of Hilbert–Huang transform is empirical mode decomposition method (EMD) with the help of which intrinsic mode functions (IMFs), $c_i(t)$, and a remainder, $r_n(t)$, can be found by decomposing any complicated time-series data set.

For an arbitrary time series, $s(t)$ based on IMFs, $c_i(t)$ ($i = 1, 2, 3, \dots, n$), and a remainder, $r_n(t)$, can be defined as follows [58]

$$s(t) = \sum_{i=1}^n c_i(t) + r_n(t). \quad (42)$$

Hilbert transform, $y(t)$, for an arbitrary time series $s(t)$ can be defined as follows:

$$y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{t - \tau} d\tau. \quad (43)$$

Thus, analytic signal $z(t)$ in terms of $s(t)$ and $y(t)$ can be found as follows:

$$z(t) = s(t) + i y(t) = a(t) e^{-i\phi(t)}, \quad (44)$$

where $s(t)$ and $y(t)$ represent complex conjugate pair.

Here,

$$a(t) = [s^2(t) + y^2(t)]^{1/2}, \quad (45)$$

$$\phi(t) = \arctan \frac{y(t)}{s(t)}. \quad (46)$$

The instantaneous frequencies (IFs), $f(t)$, can be found as

$$f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}. \quad (47)$$

Huang et al. [54] have proposed the empirical mode decomposition (EMD) method combined with Hilbert spectral analysis for analysing nonlinear and non-stationary data. There are two stages of analysis: (1) decomposition of any complicated time-series data set into several components with different timescales [i.e. intrinsic mode function (IMF)] by using the empirical mode decomposition (EMD) method and (2) the HHT spectrum of the time-series data constructed by applying Hilbert transformation to every intrinsic mode function (IMF). The effective utilization of the time-series data is one of the main benefits of Hilbert spectral analysis and EMD. An improved HHT based on wavelet packet transform has been presented by Peng and Chu [55] for rolling bearing fault detection. The performance of improved HHT was compared with wavelet-based scalogram and found that HHT has good computation efficiency as well as good resolution in both frequency domain and time domain than scalogram. On the other hand, HHT has been applied by Guo and Peng [56] to investigate nonlinear transient response characteristics of a transversely cracked rotor. The effect of crack propagating ratio (CGR) on the nonlinear dynamic response of transversely cracked rotor near the resonance and subharmonic resonance has been analysed. In addition, Jiao et al. [57] proposed an approach for crack fault detection of the cracked rotor system with a cross and deep crack by using HHT-based time–frequency representation. The experimental results show that phase modulation characteristics due to the torsional vibration of the cross-cracked rotor can be correctly captured by HHT-based method. Finally, they concluded that the HHT-based method has great potential to detect the crack fault effectively. However, a crack detection method for start-up rotor based on HHT has been proposed by Bing Li et al. [58]. In the proposed method, Hilbert transform was used to derive the instantaneous frequency (IF) of each intrinsic mode function (IMF), and hence, the spectrum of instantaneous frequency (IF) is calculated. Finally, an experiment was carried out to study the influence of acceleration and crack depth on the cracked rotor and concluded that Hilbert–Huang transform (HHT) appears superior tool than FFT for crack detection. Moreover, the transient response analysis of a cracked rotor by applying HHT has been presented by Babu et al. [59]. The proposed HHT has good capability to identify very

small crack depths that continuous wavelet transform (CWT) fails to detect. The capability of dealing with large-sized signals without consuming time for computing and analysing these signals is the main advantage of HHT. Hence, HHT seems to be a better signal processing technique compared with CWT and FFT for detection and identification of the presence of crack in the cracked rotor system. In addition, a comprehensive literature study on the industrial application of Hilbert transform (HT) and EMD in the area of mechanical vibration has been published by Michael Feldman [60]. In this work, the author has demonstrated the HT through many applications in order to show the concepts of the HT-based approach on actual vibration signals.

Lin and Chu [61] have applied Hilbert–Huang transform (HHT) to search the acoustic emission (AE) signal time–frequency–energy characteristics of natural fatigue cracks in rotating shafts. The experimental results of HHT, CWT and FFT were compared and concluded that HHT seems to be a superior tool compared to CWT and FFT. Chandra and Sekhar [62] have compared crack detection performance of the three signal processing techniques, namely CWT, HHT and short-time Fourier transform (STFT). The computational time consumed by CWT-based diagnosis is much more than HHT. But, for noisy data HHT is less preferred than CWT. However, Jun Liu et al. [63] have applied Hilbert–Huang marginal spectrum (HHMS) method on the transient response of nonlinear rotor system to extract the crack’s fault features. In addition, an innovative method has been projected to describe the crack’s opening and closing degree and discussed the different rotor parameters responsible for delaying crack propagation. Finally, the effectiveness of the HHMS method for the crack’s fault extraction from the transient response was verified through the experimental and simulation results.

3.4 Approach based on wavelet transform and wavelet finite-element method

The current section presents wavelet transform- and wavelet finite-element method-based approaches to detect the presence of crack in the rotating shaft.

3.4.1 Wavelet transform (WT)-based approach

In the recent years, wavelet transform (WT) is another joint time–frequency analysis technique that has been employed by many researchers to detect the depth and location of a crack by considering the modal damping, natural frequencies and mode shapes. The frequency resolution of wavelet transform is better than the time resolution at low frequencies, while time resolution is better than frequency resolution at high frequencies. There are two types of Wavelet transform based on continuous or discrete signal analysis.

(a) *Continuous wavelet transform (CWT)* The CWT of signal $f(t)$ formalized by the Morlet is defined as

$$\text{CWT}(a, b; \psi) = a^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \psi \left(\frac{t-b}{a} \right) dt, \quad (48)$$

where a represents the scale factor, b is the translating factor and $\psi(t)$ is the wavelet function.

(b) *Discrete wavelet transform (DWT)* Daubechies and Mallat have formulated DWT from discretization of CWT (a, b) as

$$\text{DWT}(i, j; \psi) = a_0^{-i/2} \int_{-\infty}^{\infty} f(t) \psi \left(a_0^{-i} t - j b_0 \right) dt, \quad (49)$$

where i and j are integers and the factor a and b are discretized as $a = a_0^i$ and $b = j a_0^i b_0$, respectively.

Table 3 describes the comparison between the performances of different time–frequency analysis techniques such as continuous wavelet transform (CWT), discrete wavelet transform (DWT), Hilbert–Huang transform (HHT), fast Fourier transform (FFT), short-time Fourier transform (STFT), Wigner–Ville distribution (WVD), Choi–Williams distribution (CWD) and cone-shaped distribution (CSD) used for the analysis of non-stationary signals [64].

Peng and Chu [64] have published an extensive literature survey on the application of wavelet transform in the field of machinery fault detection including the following key facets: (1) the singularity detection for signals, (2) system and parameter identification, (3) the compression of vibration signals, (4) the denoising and separation of the weak signals, (5) the fault feature ablation and (6) the time–frequency investigation of signals. Finally, the demerit of using wavelet transform for the machinery fault detection has been discussed. A novel method for crack diagnosis in beam-like structures has been proposed by Zhong and Oyadiji [65]. The proposed

Table 3 Comparison between the performances of different time–frequency analysis techniques

| Methods | Resolution | Interference term | Response speed |
|---------|--|---------------------------------|------------------|
| CWT | Frequency resolution is better than time resolution at low frequencies and time resolution is better than frequency resolution at high frequencies | Not exist | Fast |
| DWT | High frequency resolution for higher frequencies and high time resolution for lower frequencies | Lesser interference term exists | Slower than CWT |
| HHT | High time resolution and high frequency resolution | Not exist | Faster than CWT |
| FFT | Higher frequency resolution | Not exist | Slower than STFT |
| STFT | Good time resolution or frequency resolution depends on the window function length | Not exist | Slower than CWT |
| WVD | Good time as well as good frequency resolution | Severe interference term exists | Slower than STFT |
| CWD | Good time as well as good frequency resolution | Lesser than WVD | Very slow |
| CSD | Good time as well as good frequency resolution | Lesser than CWD | Very slow |

approach is centred on the stationary wavelet transform (SWT) for crack detection and identification. Finally, they concluded that the SWT is better tool than the conventional discrete wavelet transform (DWT) for the crack diagnosis in simply supported beams. In another study of beam-like structures, Zhong and Oyadiji [66] have proposed another novel approach for detection of smaller cracks as small as 5% crack ratio (crack ratio, $r = Hc/H < 5\%$). The proposed approach is centred on the continuous wavelet transforms (CWTs) that has abundant potential for crack diagnosis in beam-like structures as it is able to detect the damage without baseline modal parameters of an uncracked beam. In addition, a hybrid technique for detection and identification of crack locations through wavelet transform (WT) and fractal dimension (FD) for cantilever beam with two cracks has been proposed by Xiang et al. [67]. In order to decompose the mode shape of the cracked beam, wavelet transform is applied, and thereafter, to compute fractal dimension parameters of detailed signals, FD estimation method is adopted. The crack locations have been identified precisely through FD singularity of the detailed signals. To analyse vibration data for faults identification, Mogal and Lalwani [68] have published an extensive literature review of vibration-based signal processing techniques such as HHT, WT, Wigner–Ville distribution (WVD), fast Fourier transform (FFT) and short-time Fourier transform (STFT). However, a comprehensive literature review on the application of wavelet transform method has been presented by Gomez et al. [69] to diagnose cracked rotors considering the following main aspects: (1) usage and progress in the application WT, (2) significant achievement in using WT for crack diagnosis and (3) the selection of the parameters used. In another study, Gomez et al. [70] applied wavelet transform method for vibration data to identify the presence of cracks in a rotary shaft. Huge variations in energy at $1\times$, $2\times$ and $3\times$ components of rotational speed were observed as soon as crack appears in the rotating shaft. Nine different fault conditions were considered by varying the crack depths from 4 to 50% of the shaft diameter and concluded that the presence of a crack in a rotating shaft can only be identified through the $3\times$ component of the rotational speed.

Huo et al. [71] have proposed a novel method called self-adaptive entropy wavelet (SEW) to identify emerging transverse crack defects on a rotary shaft through CWT, support vector machine (SVM) classifier and Shannon entropy. In addition, a signal processing technique based on support vector machine (SVM) and wavelets has been compared with model-based approach by Soffker et al. [72]. To get a comprehensive study of these two presented techniques, strong and feeble points were compared and discussed.

3.4.2 Wavelet finite-element method (WFEM)-based approach

The basic plan of wavelet finite-element method (WFEM)-based approach is analogous to the standard FEM. Figure 12 shows the procedure of crack detection using WFEM.

Ma et al. [73] have derived wavelet finite-element method (WFEM) in order to take the advantages of both the traditional FEM and wavelet spaces. WFEM-based method has great potential to analyse more complex

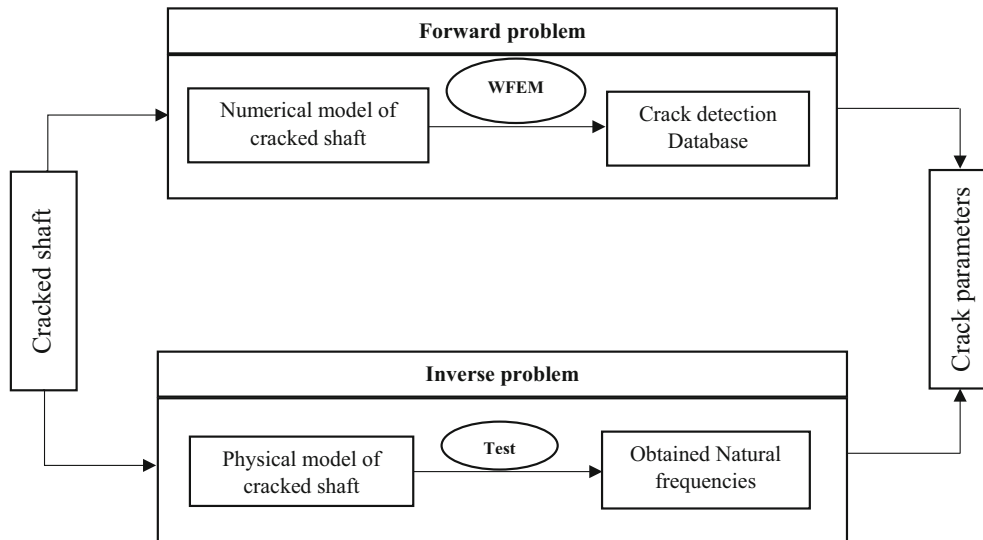


Fig. 12 Procedure of crack detection using WFEM

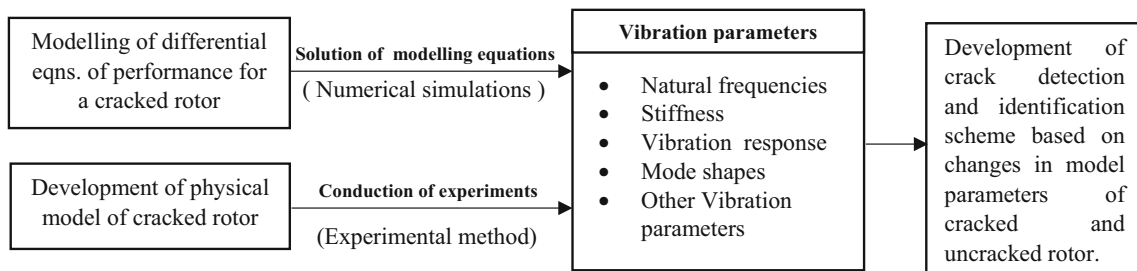


Fig. 13 Procedure of crack detection and identification using numerical simulations and experiments

problems of faults diagnosis in rotating machinery. The benefits of WFEM such as diverse fundamental functions and multi-resolution properties for the analysis of structures have been presented by Chen et al. [74]. They have adopted scaling functions of Daubechies wavelets as trial functions for the construction of two-dimensional (2D) wavelet finite element (WFE). Additionally, bending behaviour of a thin plate has been examined by using WFEM. Numerical results show that WFEM is a highly effective, efficient and accurate technique, especially when dealing with singularity problems.

Xiang et al. [75] proposed a FEM of a B-spline wavelet on the interval (FEM-BSWI) to detect and locate crack in the cantilever beams. The performance of the B-spline wavelet on the interval (BSWI) Euler beam element has been validated through the numerical and experimental investigations. In another study, Xiang et al. [76] have applied FEM-BSWI Rayleigh beam element-based methodology for estimating crack size and crack location in the cracked rotor system. In the proposed model, crack is modelled through change in local stiffness and then surface-fitting technique and contour plotting method have been used to solve inverse as well as forward problems of crack identification. In addition, multiple crack identification approach based on wavelet finite-element methods (WFEM) has been applied by Li and Dong [77] to specify the crack location and depth of each crack. Several crack locations and depths are accurately determined through the changes in natural frequency of a cracked structure by using wavelet finite-element methods (WFEM). Finally, the number and region of cracks have been determined by using damage coefficient method.

3.5 Analysis using numerical simulations and experiments

The current section introduces various numerical simulations and experimental approaches applied to detect the presence of crack in the structural system. The flow diagram shown in Fig. 13 demonstrates the basic procedure of crack detection and identification using numerical simulations and experiment

Through numerical simulations and experiments, Mancilla et al. [78] have presented the analysis of Jeffcott rotor on journal bearings to facilitate cracked shaft detection including the following main aspects: (1) horizontal and vertical vibration responses of the disc and bearing at local resonances, (2) evolution of orbit nearby $1/2$, $1/3$ and $1/4$ of the critical speed for various imbalance-crack orientations. The effect of crack positions and crack ratios on the natural frequencies and buckling loads of a single-edge-cracked cantilever Euler beam has been analysed by Karaagac et al. [79] through the concept of local flexibility. In addition, a comprehensive experimental, numerical and theoretical approach for crack detection and identification in power plant rotating machinery has been presented by Stoisser and Audebert [80]. In the present study, a 3D theoretical cracked beam model has been developed to perform numerical simulations and the results of numerical simulations have been validated through experimental results. On the other hand, Machorro et al. [81] have identified defective shafts by using experimentation and active sensing simulation. Various kinds of defects were considered for the analysis such as misalignment, bent shafts, imbalance, transverse cracks and a combination of them. Through the numerical and experimental results, they demonstrated that there is no significant variation found in mode shapes and natural frequencies of shaft of rotordynamic system when the presence of transverse crack appears in the shaft. However, a non-destructive testing (NDT) method based on eddy current for crack detection has been proposed by Bennoud et al. [82]. In this study, electromagnetic energy of interaction between piece and probe (without defect/with defect) has been calculated through simulation by FEM.

Jain et al. [83] have analysed the vibration response of a cracked rotor system through Umbra Lagrangian formalism considering multiple crack on the rotor. The effects of crack depth on natural frequencies and shaft's stiffness have been investigated by conducting the experiments. It has been also found that the presence of the second crack increases the stress concentration on the first crack. In addition, a concept of operational deflection shape (ODS) and kurtosis of vibration response has been proposed by Saravanan and Sekhar [84] to detect very small cracks. In this study, they utilized amplitude deviation curves and kurtosis deviation curve for crack shaft detection and found significant changes in kurtosis when the crack is nearer to the bearing locations. Moreover, the effects of parametric uncertainties on the vibration response of a transversely cracked shaft have been examined by Chao Fu et al. [85]. The modelling of cracked rotating shaft has been done by using FEM, and the deterministic vibration problem of cracked rotor system was solved by applying harmonic balance method (HBM). Finally, variations in parameters such as crack depth and Young's modulus were analysed by considering various parametric uncertainties with several uncertain degrees. On the other hand, a detailed dynamic model of a transversely cracked rotor system has been constructed by Kumar et al. [86] through the bond graph technique in order to depict the precise and exact dynamics of a cracked rotor system. The flexibility matrix for cracked element has been computed to study the effects of crack depth on dynamic response of cracked rotor.

3.6 Investigation using nonlinear dynamics of cracked rotors

The process of opening and closing (i.e. breathing) the crack in a rotating shaft generates nonlinear behaviour on the dynamics of rotating shafts. This behaviour could be monitored by determining the amplitude of vibration as a function of the angular velocity of the rotating shaft. In this context, several researchers [87–101] have adopted nonlinear dynamics of a cracked rotor as one of the investigation tools.

Muller et al. [87] have analysed the nonlinear dynamics of a cracked rotor through changing the stiffness coefficients of the cracked shaft. The crack was assumed as an external excitation force considering that the presence of a crack causes a change in stiffness coefficients; hence, the system becomes parametrically excited and nonlinear. The nonlinear dynamic response diagnosis using harmonic excitation force has been done by Ishida and Inoue [88] to investigate the presence of crack in the 2DOF cracked rotor. In order to model the breathing mechanism (i.e. open–close mechanism) of crack, they adopted piecewise linear function and power series function. Occurrence of several kinds of resonances due to the presence of crack has been examined. On the other hand, the grazing bifurcation exist in the nonlinear dynamic response of cracked rotor has been analysed by Qin et al. [89]. Several phenomena such as jumps between periodic motions, quasi-periodic motions from the periodic ones, chaos and intermittent chaos were observed due to the presence of a grazing bifurcation in the response. In addition, Wang et al. [90,91] have practically investigated the characteristic changes in a transient response during the start-up, the shutdown or the variable running speeds of rotating shaft having a transverse crack. Numerical and experimental investigation has been accomplished to study the non-stationary vibration response during passages through the main resonance and various kinds of resonances. In another study, Wang et al. [92] have employed three-dimensional FEM and the contact approach to investigate the effects of unbalance on the nonlinear dynamics of cracked shaft. The numerical results show that the nonlinear dynamics of cracked

rotors as well as breathing mechanism of crack were significantly affected by the presence of unbalance. On the other hand, nonlinear dynamic response of rotor system with a real fatigue crack has been experimentally analysed by Zhou et al. [93]. The experimental results obtained were validated through theoretical results as well.

Zhu et al. [94] have analysed the dynamics of a cracked rotating shaft with an active magnetic bearing (AMB). From theoretical analysis, they observed that super-harmonic frequency components of $2\times$ and $3\times$ revolution in the supercritical speed region can be used as an index to detect a crack in the rotor with an active magnetic bearing (AMB), but it cannot be combined with traditional method to detect the crack. The nonlinear dynamics of the cracked rotating shaft with fractional-order damping have been analysed by Cao et al. [95] through a response-dependent breathing crack model. The parameters such as rotational speed, crack depth, fractional damping, crack angle and the imbalance eccentricity level have significant influence on nonlinear dynamic response of the cracked rotating shaft. In addition, the nonlinear dynamic response of a cracked cantilever beam has been analysed by Thatoi et al. [96] through FE method and experimental investigation. The presence of crack in the cantilever beam results in: (1) the deviations in natural frequencies and mode shapes and (2) change in the nonlinear dynamic behaviour which is due to variation in local flexibility. The crack location and its intensity can be predicted through variations in natural frequencies and mode shapes. On the other hand, the super-harmonic dynamic behaviour of the symmetrical rotor with a transversely cracked shaft under inertial excitation has been investigated by Lei and YuShu [97] using the harmonic balance method. By increasing the inertial excitation, the super-harmonic response signals can be amplified in order to detect the early crack faults in transversely cracked rotor systems. Moreover, a slant cracked rotor system attached with two discs has been analysed by Xuelian Chen [98] through the finite-element model. The time-varying stiffness caused by shaft crack has been simulated by adopting a slant crack model. To simulate the bearing, two types of bearing force (linear and nonlinear) were considered. Finally, they studied the effects of eccentric phase differences of two discs on the dynamic responses of the cracked rotor-bearing system under steady-state process and run-up process. In addition, Bovsunovskii [99] has conducted a numerical study on a perfectly elastic cantilever beam with a breathing crack. The effects of the crack location and crack depth on the modes of vibration of the cantilever beam have been investigated through the local changes in the cross section. In the present study, he has derived the expressions of natural frequency for longitudinal and transverse vibrations of a cracked cantilever beam with a close and open crack. In a recent study, Arem and Zid [100, 101] have proposed a systematic approach for modelling and analysis of the cracked rotating shafts behaviour. The EDF-LMS approach was adopted to define the breathing mechanism of the crack, and the breathing mechanism parameters were identified by using three-dimensional (3D) computations. Finally, they concluded that the super-harmonic resonance phenomenon can be a good indicator of the presence of propagating crack.

3.7 Analysis of cracked rotor using artificial intelligence (AI) techniques

Artificial intelligence (AI)-based techniques have great potential for identifying crack location and crack depth accurately in rotating shaft. In the recent years, several kinds of algorithms based on artificial intelligence (AI) techniques have been used by researchers and scientists for crack detection in engineering systems. The main advantage of using AI-based techniques is that the forecasting time for crack detection and identification improves significantly in comparison with other approaches. The flow chart shown in Fig. 14 is useful to understand the procedure of crack detection and identification using AI-based techniques.

He et al. [102] have proposed an approach based on genetic algorithms (GAs) for shaft crack detection. The FEM has been adopted to formulate the shaft crack detection as an optimization problem and applied GAs to search the solution. Finally, they concluded that GAs-based approach has good potential to predict the shaft crack locations and configuration effectively. A crack detection technique for a cracked beam-like structure based on genetic algorithms (GAs) has been presented by Baghmisheh et al. [103]. In this study, the crack location and crack depth have been identified by formulating the cantilever beam as an optimization problem and the binary genetic algorithms (BGAs) and continuous genetic algorithms (CGAs) were applied to find the optimum crack location and depth via minimizing the cost function. The average values of prediction errors were 1.02% and 1.98% for BGA, while they became 0.73% and 1.11% for CGA. In addition, a novel method for crack identification in a shaft by combining genetic algorithm with wavelet-based elements has been offered by Xiang et al. [104]. In this method, wavelet-based elements were adopted to model cracked shaft as well as to obtain the first three natural frequencies. Finally, genetic algorithm (GA)-based approach has been applied to detect the normalized crack depth and location. Moreover, Saridakis et al. [105] have applied fuzzy logic (FL), genetic algorithms (GAs) and artificial neural networks (ANNs)-based approaches to identify crack's

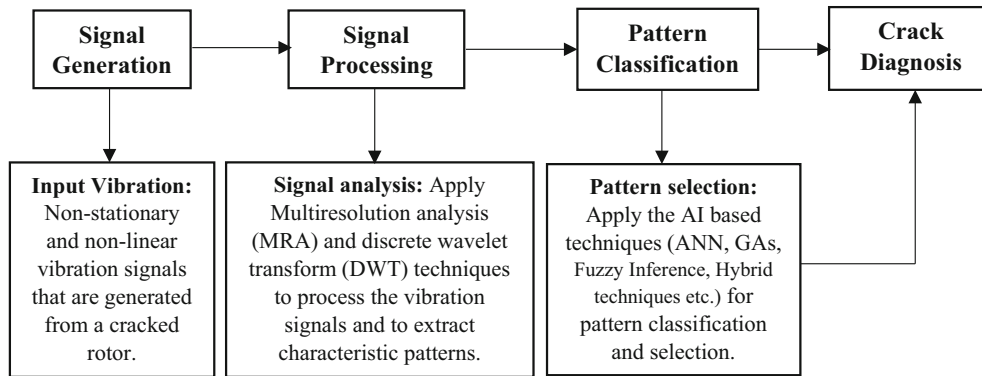


Fig. 14 Procedure of crack detection and identification using AI-based techniques

attributes such as crack relative angle, crack depth and crack position and concluded that significant reduction in computational time was found without any loss of accuracy by using neural networks and the genetic algorithm. On the other hand, an extensive literature review on the various techniques applied in condition monitoring and fault diagnosis has been presented by Thatoi et al. [106]. Forecasting time for crack detection improves significantly by using artificial intelligence techniques in comparison with classical methods.

Baviskar and Tungikar [107] have proposed an approach based on artificial neural networks (ANNs) for crack detection and prediction of multiple crack properties for a simply supported stepped shaft. The results of FEM and experimental analysis have been used to verify predictions of crack location by ANN. In addition, an approach based on the artificial neural networks (ANNs) and CWT has been presented by Babu and Sekhar [108] for detection and identification of crack parameters ,i.e. crack depth and crack position. On the other hand, Mohammed et al. [109] have analysed vibration characteristics of a full-scale cracked rotor considering three dissimilar crack depths by using non-destructive methods. Variations in the spectral content of the transverse vibration of the system have been analysed by applying an artificial neural network (ANN) and power spectral density (PSD) to detect the presence of cracks in rotating shaft. The shifts in the frequency peaks of the PSD characterization demonstrate the presence of three dissimilar crack depths in the rotating shaft. In a recent study, Gomez et al. [110] have analysed vibration signals of rotating shaft by applying the combination of wavelet packets transform energy (WPT) features and a trained artificial neural network (ANN) to detect cracks in a rotating shaft. Ten dissimilar crack conditions were included in the rotating shaft (one with uncracked shaft (healthy) and nine with crack depths varying 4% of the shaft diameter to 50%). The calculated POD curves show that the prediction of crack detection was very much close to 100%. On the other hand, the multiple adaptive neurofuzzy inference system (MANFIS) methodology has been applied by Nanda and Parhi [111] for detecting and identifying the cracks of various depth and location. An analytical method was applied to get the first three mode shapes as well as first three natural frequencies, and the results of MANFIS methodology have been validated through experimental results. In addition, Castejon et al. [112] have applied multi-resolution analysis (MRA) and discrete wavelet transform (DWT) theory to vibration signals for extracting and classifying characteristic patterns of a transversely cracked shaft. The artificial neural networks (ANNs)-based approach was used to analyse characteristic patterns of shafts. The results of proposed method that combines discrete wavelet transform (DWT) and an artificial neural network (ANN) could be an effective tool for the detection and identification of cracks in rotor.

3.8 Analysis of cracked shaft through other miscellaneous techniques

In addition to the techniques mentioned previously, researchers have adopted other tools and methodologies to define and analyse dynamic behaviour of a crack in rotating shafts.

Bachschmid et al. [113] have presented a model-based approach for detection and identification of crack depth and position in a transversely cracked rotor. To evaluate the location of crack and the equivalent bending moments due to the presence of transverse crack, a least square identification method in frequency domain has been adopted. The same approach has been also used by Sekhar [114]. The proposed model-based diagnostic approach combined with least square fitting method has been applied for the identification of various crack depths and crack locations at various angular speeds in a rotor system. In this work, FEM has been applied to

Table 4 Critical observations made from proposed modelling approaches

| Name of approach | Modelling complexity level | Computational efficiency | Result accuracy level | Specific advantages |
|---|-------------------------------|---------------------------------|-------------------------------|---|
| Breathing mechanism (BM) | High | Good | High | Can be used for accurate modelling of time-varying stiffness [32] |
| Finite-element method (FEM) | Medium | High | High | FEM has great potential in the identification and localization of cracks in a cracked rotor [42,46] |
| Hilbert–Huang transform (HHT) | Medium | Better than WT and FFT | High | Good computation efficiency in analysis of transient signals [59,62] |
| Wavelet transform (WT) | Medium | Lesser than HHT better than FFT | Medium | High frequency resolution and high time resolution [64] |
| Wavelet finite-element transform (WFEM) | High | Better than WT and FEM | High | Combined the advantages of WT and FEM [73,74] |
| Numerical simulations and experiments | Medium | Good | Medium | Can easily compares modal parameters of cracked and uncracked shaft [81] |
| Nonlinear dynamics of cracked rotors | Medium | Good | Medium | Crack detection based on changes in the nonlinear dynamic behaviour of cracked rotor [92] |
| Artificial intelligence (AI) techniques | High | Good | High | Can be used for accurate measurement of crack depth and crack location [108] Forecasting time for crack detection improves significantly [105,106] |
| Other miscellaneous techniques | Depends on the method adopted | Depends on the method adopted | Depends on the method adopted | Depends on the method adopted |

model a complex shaft-bearing system. Finally, the crack depth and crack location have been detected through changes in the dynamic behaviour of the shaft. In addition, the vibration response of the cracked rotor system has been analysed by Green and Casey [115] which may be directly ascribed to the existence of a transverse crack in a rotor by using both the local and global asymmetry crack models. The behaviour of $2\times$ harmonic component of cracked rotor system response resulting from introduction of a crack has been monitored by increasing the magnitude of crack depth as well as decreasing the angular speed of shaft. It has been found that magnitude of the $2\times$ harmonic component of cracked rotor system response could be used as a target observation for crack detection and identification. In another study, Green and Varney [116] have developed a test rig to compare the experimental and analytical results of two gaping crack models. Significant differences have been observed in the $2\times$ harmonic response component of two gaping crack models for various crack depths. On the other hand, Sekhar et al. [117] have presented the detection and monitoring of slant crack in a rotating shaft through variations in mechanical impedance. They found that changes in the mechanical impedance affected significantly due to the presence of transverse crack as compared to slant crack; therefore, detection and monitoring of slant crack can also be done effectively by measuring the mechanical impedance.

Jyoti Sinha [118] have applied the higher-order spectra (HOS) to study the existence of higher harmonics in the vibration signal which is a typical case of crack breathing during shaft rotation. A small rotating test rig has been developed to study the nonlinear dynamic response of misaligned and cracked shaft. The experimental results was found encouraging to distinguish these two faults. In addition, a method based on measurement of deflection of long pipes of uniform cross section under bending loads has been proposed by Naik [119] for identification of crack location. The fracture mechanics-based approach has been used to determine stiffness of rotational spring, while crack is modelled as rotational spring. The effectiveness of proposed approach has been verified through experimental investigation. On the other hand, the transfer matrix method has been applied by Jun et al. [120,121] to obtain the vibration response due to the unbalance and crack and the quasi-static response due to gravity. The nonlinear dynamic behaviour of cracked shaft was explained by the changes in additional slope with crack depth and angular speed of shaft. In a recent study, the nonlinear dynamic behaviour

Table 5 Summary of recently published research papers dealing with crack detection in a rotating shaft

| Name of approach used for the analysis | Types of fault analysed | Parameters utilized for crack detection | Techniques used for analysis | References |
|--|--|--|---|--|
| Breathing mechanism (BM)-based approach | Transverse crack | Vibration response, whirl orbits, critical and subcritical speeds | Finite-element method (FEM) + Harmonic balance method (HBM) | Shudeifat et al. [32] |
| | Transverse crack | Stability of the system, eigenvalues of the cracked rotor | Floquet theory | Guo et al. [33] |
| | Breathing crack | 1/2, 1/3, 1/4 and 1/5 subcritical speeds, whirl orbit, frequency spectra | Fourier series + Experimental investigation | Guo et al. [26] |
| | Two transverse cracks Transverse crack | Dynamic response of cracked rotors Vibration response characteristics, double resonance peak characteristics | Three-dimensional finite-element model Phase waterfall plot method | Hossain and Wu [35] Xie et al. [30] |
| Finite-element method (FEM)-based approach | Transverse open crack | Whirl orbits, 1/2 and 1/3 of the backward critical whirling speed | Finite-element method (FEM) + Harmonic balance method (HBM) | AL-Shudeifat [41] |
| | Transverse crack | Orthogonal natural frequencies (ONFs), ONF ratio, normalized orthogonal natural frequency curves (NONFCs) | Finite-element method (FEM) + Bernoulli–Euler beam theory | Haji et al. [46,47] |
| | Transverse crack | Time-varying area moments of inertia, Time-periodic stiffness | Finite-element method (FEM) + Harmonic balance method (HBM) | Al-Shudeifat [42] |
| | Transverse crack | Second harmonic, natural frequency, third harmonic of the rotation speed | Finite-element method (FEM) + NEWMARK | Ghozlane [53] |
| | One and two cracks | Critical speeds, harmonic and super-harmonic components of the unbalance lateral response and the shaft centre orbit | Finite-element method (FEM) + Modified Harmonic balance method (MHBM) | Khorrami et al. [52] |
| | Transverse crack | Peak amplitude of the shaft, harmonic analysis | Finite-element method (FEM) + ANSYS | Upadhyay et al. [51] |
| | Hilbert–Huang transform (HHT)-based approach | Transverse crack | Instantaneous frequency (IF) of each intrinsic mode function | Hilbert–Huang transform (HHT) |
| Transverse crack | | Frequency and energy features of the rotating shaft | Hilbert–Huang transform (HHT) | Lin and Chu [61] |
| Misalignment, rotor cracks and rotor rub | | Signal to Noise Ratio, angular acceleration | Finite-element method (FEM) + Hilbert–Huang transform (HHT) | Chandra et al. [62] |
| Lateral crack | | Vibration signals, Harmonic vibration components, super-harmonic vibration components | Hilbert–Huang marginal spectrum (HHMS) method | Jun Liu et al. [63] |

Table 5 continued

| Name of approach used for the analysis | Types of fault analysed | Parameters utilized for crack detection | Techniques used for analysis | References |
|---|----------------------------|--|--|-----------------------|
| Wavelet transform (WT) and wavelet finite-element transform (WFEM)-based approach | Transverse crack | Mode shape, natural frequencies | Finite-element method (FEM) + Continuous wavelet transforms (CWTs) | Zhong et al. [66] |
| | Breathing cracks | First four Eigen modes | Support vector machine (SVM) and wavelets | Soffker et al. [72] |
| | Transverse crack | 3 × component of the rotational speed, energy level at 3 × component | Wavelet transforms (WTs) | Gomez et al. [70] |
| | Multiple cracks | Natural frequency | Wavelet finite-element method + Damage coefficient method | Li and Dong [77] |
| | Transverse crack | Mode shapes, signal of the mode shape, operating deflection shape (ODS) | Wavelet transform (WT) + Fractal dimension method (FD) | Xiang et al. [67] |
| | Crack faults | Time–frequency spectrums, Mean energy values of coefficients | Continuous Wavelet Transform + Shannon entropy and support vector machine | Huo et al. [71] |
| Numerical simulations and experiments-based approach | Transverse crack | Vibration system response, amplitude of the rotor | Bond graph technique | Kumar et al. [86] |
| | Transverse breathing crack | Operational deflection shape, kurtosis of vibration response | Finite-element method (FEM) | Saravanan et al. [84] |
| | Crack/defects | Electromagnetic energy of interaction with/without defect | Finite-element method (FEM) + Non-destructive testing (NDT) method | Bennoud et al. [82] |
| | Multi-cracked rotor system | Shaft's stiffness and natural frequencies | Umbral Lagrangian formalism | Jain et al. [83] |
| Nonlinear dynamics of cracked rotors-based approach | Transverse crack | Bounds of the nonlinear dynamic responses | Finite-element method (FEM) + Harmonic balance method (HBM) | Chao Fu et al. [85] |
| | Breathing crack | 2 × and 3 × harmonic components and 1/2 × and 3/2 × fractional harmonic components, orbit formation | Runge–Kutta method + CFE-Euler (continued fraction expansion-Euler) method | Cao et al. [95] |
| | Transverse crack | Deviations in mode shapes and natural frequencies | Finite-element method (FEM) + Experimental analysis | Thatoi et al. [96] |
| | Crack fault | Super-harmonic resonances peaks | Harmonic balance method | Lei and YuShu [97] |
| | Slant crack | Time-varying stiffness, super-harmonic resonance, torsional amplitude of the second harmonic frequency | Finite-element method (FEM) + ANSYS | Xuelian Chen [98] |

Table 5 continued

| Name of approach used for the analysis | Types of fault analysed | Parameters utilized for crack detection | Techniques used for analysis | References |
|--|---|--|---|------------------------|
| Artificial intelligence (AI) techniques-based approach | Transverse crack | Non-stationary response of a rotor system, resonance and various kinds of resonances | Theoretical method + Experimental method | Wang et al. [90,91] |
| | Transverse crack | breathing behaviours of crack, stiffness characteristics, whirling orbits | 3D finite-element models + Component mode synthesis method (CMS) | Wang et al. [92] |
| | Transverse crack | Super-harmonic resonance phenomenon | Three-dimensional FE model | Arem et al. [100, 101] |
| | Two transverse cracks | Eigenfrequencies, response of the cracked shaft, computational time | Neural networks (ANN), genetic algorithms (GAs) and fuzzy logic (FL) | Saridakis et al. [105] |
| | Multiple transverse cracks | Modal analysis, natural frequency | FEM + Artificial neural network (ANN) | Baviskar et al. [107] |
| | Crack fault | frequency spectrum, power spectral density (PSD) | Artificial neural network (ANN) | Mohammed et al. [109] |
| Approach based on other miscellaneous techniques | Two transverse cracks | First three natural frequencies, the first three mode shapes | Multiple adaptive neurofuzzy inference system (MANFIS) | Nanda and Parhi [111] |
| | Transversal crack | Characteristic patterns of vibration signals | Discrete wavelet transform (DWT) + Artificial neural network (ANN) | Castejon et al. [112] |
| | Nine different crack levels | Vibration signals, probability of Detection curves | Wavelet Packets transform energy + Artificial Neural Networks | Gomez et al. [110] |
| | Crack is considered parallel to its diameter and perpendicular to the plane of deflection | Measurement of deflection, stiffness of rotational spring | fracture mechanics approach | Naik [119] |
| | Transverse breathing crack | Frequency responses at $1\times$, $2\times$, $3\times$, weighted AWCD curves, ODS curve | Combined operating deflection shape (ODS) and approximate waveform capacity dimension (AWCD) method | ChunLin et al. [123] |
| | Four crack levels | Vibration signals, pattern extraction | Wavelet Packet Transform (WPT) | Castejon et al. [125] |
| Open and breathing crack | Amplitude, frequency vibration modes | Linear fracture mechanics theory (LFM) + Mayes model | Cavalini et al. [124] | |
| Multi-fault rotor | Whirl orbits, orbit morphological characteristics | Fourth-order Runge–Kutta method | Xiang et al. [122] | |
| Singular transverse crack | $1\times$, $2\times$ and $3\times$ harmonic components of response | Finite-element rotor model + MATLAB | Spagnol et al. [126] | |

of a multi-fault rotor system has been analysed by Xiang et al. [122] for crack identification based on whirl orbits using Runge–Kutta method and experimental study. The experimental whirl orbits of a cracked rotor around the $1/2$ subcritical speeds were agreed with the theoretical simulation; therefore, an approach based on the orbit morphological characteristics can be proposed for crack detection in multi-fault rotor system.

Chun Lin et al. [123] have proposed an approach based on operating deflection shape (ODS) combined with approximate waveform capacity dimension (AWCD) method for identification of crack location in a rotating shaft. The effectiveness of proposed approach has been verified by experimental investigation and numerical simulation. In addition, a crack detection and identification approach based on a nonlinear technique has been proposed by Cavalini et al. [124]. In this study, the dynamic behaviour of the breathing and open crack models was compared and the efficiency of the proposed identification technique was discussed. Finally, the numerical results of the developed inverse problem methodology show that the proposed approach is an effective tool for identification of crack location and depth. On the other hand, a methodology based on the relative wavelet energy has been suggested by Castejon et al. [125] in order to automatically select the parameter defined by the WPT (wavelet packet transform). To optimize the process of analysis and selection of patterns as well as making comparisons with high sensitivity levels, an intelligent classification system has been used. Finally, a Jeffcott rotor model with four crack levels has been developed to show the efficiency of the proposed technique and found the best patterns extracted from vibration signals have accuracy level more than 97%. In a recent study, Spagnol et al. [126, 127] have proposed a method for the modelling of breathing mechanism of a crack which includes the influence of unbalance loading and rotor speed. The $1\times$, $2\times$ and $3\times$ components of vertical vibration of rotor with deep crack (80% of radius) were found significantly different for proposed model and an existing weight-dominant model, which show that the weight-dominant breathing model was not appropriate for modelling of the vibration of cracked shaft with large permissible residual unbalance. In addition, the forced vibration of a cracked rotating shaft with an open edge crack has been analysed by Ebrahimi et al. [128] through a new continuous model for flexural vibration. Changes in the pattern of time response, frequency spectrum and orbit diagrams due to the presence of an open edge crack have been investigated for crack identification.

4 Critical observations

The objective of this section is to emphasize some critical observations of the present review on modelling and analysis of cracked rotor. The modelling approaches proposed in this article are critically analysed on the basis of level of modelling complexity, computational efficiency, results accuracy as well as specific advantages of each proposed techniques. Table 4 represents critical observations made from proposed modelling approaches based on the reviewed literature.

The literature reviewed in this article is categorized on the basis of modelling approaches adopted by the various investigators. The summary of recently published research articles has been prepared based on the name approach used for the analysis, type of fault analysed, parameters used for crack detection and techniques used for the analysis. Finally, Table 5 presents the summary of recently published research papers dealing with crack detection in a rotating shaft.

5 Conclusion

An exhaustive literature survey has been carried out in the field of cracked shaft detection. The vibration-based methods for crack detection are broadly categorized to study the features of structural systems with cracks. These methods play a crucial role in the investigations of cracks in rotating machinery. In addition, the modelling approaches adopted by the investigators to detect the cracks in non-rotating or rotating shaft systems have been presented in detail. It has also been observed that the researchers have considered various parameters such as crack depth, crack location, relative angular position of crack and number of cracks as well as opening/closing phenomena of crack in the presented dynamic models. The effects of crack characteristics on vibrational properties, namely, natural frequencies, stiffness, transient responses, damping ratios, whirl orbits, critical speeds and shaft centre orbit evolutions in subcritical speeds have been investigated. The presence of crack on rotor has been identified through time-domain and frequency-domain analysis of captured/simulated vibration signals.

Moreover, the following specific observations have been noticed after analysing various methodologies adopted by researchers.

1. The breathing mechanism-based approach has been implemented by various researchers to detect the crack in the rotating shaft. However, accurate modelling of breathing phenomena is a great challenging task as it introduces time-varying stiffness at crack location during shaft rotation.
2. Wavelet transform-based approach has been adopted by many researchers to derive the vibration indices such as mode shapes and natural frequencies, which are subsequently used for crack detection in the rotating shaft.
3. Finite-element-method-based approach has been employed by various researchers due to its capability of analysing different kinds of crack's geometry such as slant crack or helicoidal cracks, transverse crack, transverse breathing crack and presence of more than one crack. FEM-based approach can be a better choice for modelling and analysis of the presence of crack in the rotating shaft due to its medium level of modelling complexity, good computation efficiency and high level of results accuracy. Moreover, combination of wavelet with FEM is a superior choice to take the advantages of both techniques.
4. Several researchers have employed numerical simulations- and experimental investigations-based approach to study the dynamic behaviour of cracked rotor for crack detection. But this approach requires effective validation of numeral results with experimental results.
5. Fault detection has been enhanced by artificial intelligence (AI) techniques such as artificial neural networks (ANNs), genetic algorithms (GAs), fuzzy logic (FL) and hybrid techniques. Therefore, AI technique-based approach is useful to find location of crack and its depth accurately.

The authors of the present paper believe that this exhaustive review of rotor crack identification techniques and dynamic model analysis will be helpful for researchers in their beginning stage of research and also provide guideline to develop complex and accurate dynamic model for fault detection.

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