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Strips of localization of plastic deformation

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Abstract Model of slip band propagation in materials with yielding plateau is introduced. Development of a slip band is modeled in a form of loss of stability during transition of the material from an elastic state to hardening. This transition is the generalization of crack mode *I* development model by Novozhilov (J Appl Math Mech 3:201–210, 1969) in elastic solids. Possibility for slip bands of limited length is shown in the model in contrast to ideal plasticity model that only leads to infinite slip bands. Problems of localization in a form of slip bands in a state for the pure shear and for intermaterial layer are considered. For different external loads and various mechanical properties of the interlayer, the lengths of the localization zone of plastic deformations, the graphs of the tangential displacement jump in this zone, and the shear stress on their continuation are found.

Keywords Localization strip of plastic deformation · Piecewise analytic function · Shear displacement jumps

1 Introduction

It is known that in materials having a clearly defined yield platform the isolated yield strips may appear in the presence of an inhomogeneous stress field. These strips occupy an insignificant volume of the body as compared to the elastic part. The corresponding discontinuous problems of the linear elasticity theory were considered in [8]. An original concept, that considers cracks-cuts in elastic bodies (the surfaces of the normal displacements jumps) as nontrivial equilibrium states of a physically nonlinear elastic medium, was proposed in [10,11]. Such a concept was applied in [2] to study the localization strips of plastic deformation for a homogeneous stress field in a homogeneous material, provided the deformation diagram of the material has a peak tooth under the conditions of hard loading (Fig. 1).

It is obvious that a similar deformation pattern can also occur in case of piecewise homogeneous materials. However, the localization strips of plastic deformation in such cases will initially appear in interlayers providing adhesion of the material components. This can be explained by the fact that these interlayers resulting from the welding or gluing of dissimilar materials are, as a rule, the weakest components of composites. This paper is devoted to the study of the strips of localization of plastic deformation in the region of separation of two materials, provided that the diagram of deformation of the interlayer has a peak tooth.

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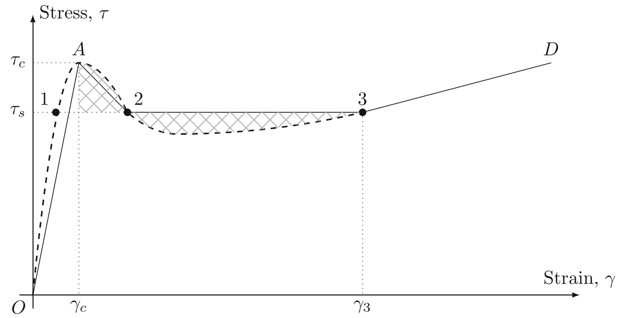

Fig. 1 Stress–strain curve $\tau \sim \gamma$ built according to Eq. (1)

2 Slip model for Lüders band

For modeling of development of Lüders bands in soft steel that consists of ferrite with inclusions of sustainably more solid pearlite, slip η ($\eta = \gamma d$ where η is shear strain, d is characteristic size of domain under consideration) of two neighboring grains of pearlite and ferrite under shear force T ($T = \tau d$, where τ is shear stress) is considered. Considering elastic-plastic model for the ferrite grain and brittle collapse for the pearlite grain, dependency of shear stress τ from shear deformation γ can be expressed in a form of relation that generalize model of crack mode I in an elastic solid by Novozhilov [10,11].

$$\tau = \tau_1 \gamma/\gamma_c \exp(-\gamma/\gamma_c) + \tau_2(1 + a\gamma/\gamma_c)\big[1 - \exp(-\gamma/\gamma_c)\big] \tag{1}$$

where τ_1, τ_2, a are the material constants and γ_c is deformation for ultimate shear stress τ_c.

$$\tau_c = \tau_0 \exp(-1) + \tau_1(1 - a)[1 - \exp(-1)]$$

The terms in (1) correspond to disruption of pearlite grain and elastic-plastic deformation of ferrite, respectively.

It follows from (1) that system of two grains may be held in three equilibrium states denoted as 1, 2, and 3 on the stress–strain curve $\tau \sim \gamma$ (Fig. 1). First state stands for an ascending slope of the stress–strain curve $\tau \sim \gamma$, second state stands for the descending slope, and the third state stands for hardening. Points $2 and 3$ are states of stable equilibrium, while 1 is unstable. A pair of grains interacting according to a descending segment of the stress–strain curve $\tau \sim \gamma$ inevitably transits to hardening state at point 3. If all pair of grains of two contiguous layers crossings a body transformed to such state, then the whole body passed to the state of ideal plasticity. Thus, elastic body being in a state of stable elastic deformation, interaction defined by the by law of descending area of stress–strain curve $\tau \sim \gamma$ may exist only locally. Hereupon, it is possible to describe similar areas as the lines of displacement discontinuity in solid body or slip bands. All grains are in a state of stable interaction described by the law of ascending stress–strain curve $\tau \sim \gamma$ around these lines; thus, there is no displacement discontinuities.

During theoretical research of equilibrium deformations of elastic-plastic bodies, it is always possible to interpret a body as a continuous environment using the methods of the plasticity theory. However, it is possible to take into account not only the forms of equilibrium, when all grains interact according to the law of ascending (stable) stress–strain curve $\tau \sim \gamma$, but also the forms with displacement discontinuities with interaction occurring according to the law of descending stress–strain curve $\tau \sim \gamma$ between its edges (Fig. 1). The form and dimensions of these bands are unknown beforehand. They can be obtained from relations of elasticity theory describing the edge of each displacement band for corresponding boundary conditions, following from (1), at $\gamma > \gamma_c$.

Approximations according to the following assumptions are proposed since strictly formulating and solving of this nonlinear problem impose certain inconveniences:

1. Relation between stresses and strains on ascending (stable) segment ($\gamma < \gamma_c$), i.e., in the volume where solidity remains, is the linear Hooke's law;
2. The problem is treated as geometrically linear;
3. Segment $\gamma_c < \gamma < \gamma_3$ (hardening starts at γ_3) of $\tau \sim \gamma$ relation is approximated in the simplest way:

$$\tau = \tau_c H(\gamma - \gamma_c) \tag{2}$$

$H(x)$ is Heaviside step function:

$$H(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{3}$$

Introduce yield stress τ_s that is determined from the condition

$$\int_{\gamma_c}^{\gamma_3} (\tau - \tau_s)\mathrm{d}\gamma = 0 \tag{4}$$

demanding the area of an approximating curve (Fig. 1) to be zero in an interval $\gamma_c \le \gamma \le \gamma_3$. This condition is equivalent to the requirement for the approximating dependency to give the same value surface energy density. Accepted simplifications lead to linearization of all equations of the problems and enable to get its approximate solution.

The model of slip bands formation described reminds of the model dislocation motion, when the "quantum" of plastic slip is defined as the displacement of the dislocation on distance equal to length of the Burgers vector. In the model, it is accepted for polycrystalline material that plastic deformation develops due to the plastic slips of separate grains (crystallites) and the "quantum" of plastic strain is defined as a slip within the limits of a pair of grains δ.

Within the limits of the localization band, only shear stresses will be considered. Taking into account the simplifications accepted higher, it is possible to assume that on the edges of the localization band in the area $0 \le |x| \le l$ only shear stresses τ_s are active, and on areas $l \le |x| \le b$ shear stresses can change in between τ_s to τ_c. Such model of loading of band edges is similar to Dugdale model [3,7], but with the substantial difference, that stresses within the limits of the band are not equal to zero ($\tau_0 > 0$).

3 Problem solution for localization bands

The method of displacement discontinuities will be used for the problem, which describe a band longitudinal slip as aggregate of regional dislocations with the proper Burgers vector b_0.

It is known [11] that in case there exists displacement discontinuity in the form regional distribution with the Burgers vector b_0 at the origin of co-ordinates, that is parallel to the x-axis, then the stress field in some point (x, y) is determined from known relations:

$$\begin{aligned} \sigma_x x &= -\frac{\mu\delta_0 y(3x^2 + y^2)}{2\pi(1-\nu)(x^2+y^2)^2}, \quad \sigma_y y = -\frac{\mu\delta_0 y(x^2 - y^2)}{2\pi(1-\nu)(x^2+y^2)^2}, \\ \sigma_{xy} &= \frac{\mu\delta_0 x(x^2 - y^2)}{2\pi(1-\nu)(x^2+y^2)^2} \end{aligned} \tag{5}$$

where μ is elastic shear modulus and ν is Poisson's ratio.

Consider dislocation discontinuity line is on the x axis and dislocations distributed along this line have density function $b_0 f(\xi)$. Dislocation discontinuities in an infinitesimal neighborhood of length $d\xi$ of a point $(\xi, 0)$ cause stresses in a point $(x, 0)$ that can be expressed in a form:

$$d\sigma_{xy} = \frac{\mu\delta_0}{2\pi(1-\nu)} \frac{f(\xi)}{\xi - x} \tag{6}$$

Thus, one can obtain the following equation:

$$\int_{-b}^{b} \frac{f(\xi)}{\xi - x}\mathrm{d}\xi = \frac{2\pi(1-\nu)\tau(x)}{\mu\delta_0} \tag{7}$$

where $\tau(x)$ is stress distribution along x axis. Consider piecewise constant distribution $\tau(x)$ in a form of:

$$\tau(x) = \begin{cases} \tau_c - \tau_0, & l < |x| < b \\ \tau_0 - \tau_s, & 0 < |x| < l \end{cases} \tag{8}$$

Equation (7) is singular equation of the first type with Cauchy kernel function with both limits of integration fixed. Its solution with regard to condition of boundness of function $f(\xi)$

$$\int_{-b}^{b} \frac{\tau(\xi)}{\sqrt{b^2-\xi^2}} \mathrm{d}\xi = 0 \tag{9}$$

is known, and dislocation discontinuity distribution function:

$$f(x) = \frac{2\pi(1-\nu)\sqrt{b^2-\xi^2}}{\mu\pi b_0} \int_{-b}^{b} \frac{\tau(\xi)}{(\xi-x)\sqrt{b^2-\xi^2}} \mathrm{d}\xi \tag{10}$$

Substituting stress distribution $\tau(x)$ from conditions (8) into Eq. (9), one obtains transcendent equation for determining relative size $\theta = b/l$:

$$\pi(\beta-\alpha) + 2(\alpha-1)F(\theta) = 0 \tag{11}$$

where $\beta = \tau_0/\tau_c$, $\alpha = \tau_s/\tau_c$ and

$$F(\theta) = \arccos\left(\frac{1}{\theta}\right) \tag{12}$$

Solution leads to a conclusion that length of the localized band still stays unidentified. For formulating an additional condition that helps to determine the required length, consider the fact that the deformation in the band must be limited to a value connected to the length of the yielding plane BC, since the solution obtained is possible only before strains enter hardening phase in the point C. For implementation of this condition, dislocation discontinuity should be determined for localization band.

Consider the discontinuity $\delta(x)$ in the point x of the interval $(-b, b)$ using the function of dislocation discontinuity distribution (10)

$$\delta(x) = b_0 \int_{-b}^{x} f(\xi)\mathrm{d}\xi$$

It is shown in [2] that maximal dislocation discontinuity can be observed in the middle point $x = 0$ of the band

$$\delta = b_0 \int_{-b}^{0} f(\xi)\mathrm{d}\xi \tag{13}$$

Substituting the dislocation distribution function (10) into equation, one can obtain the solution for the maximal dimensionless displacement discontinuity on the band $\bar{\delta} = \delta/b$

$$\bar{\delta} = \frac{4}{\pi}(1-\alpha)(1-\nu)\gamma_s \frac{\ln\left(\theta + \sqrt{\theta^2-1}\right)}{\theta} \tag{14}$$

where γ_s is ultimate yield strain.

Dependency of size of dimensionless displacement discontinuity strip $\bar{\delta}/(1-\nu)/\gamma_s$ from the parameter of loading β for some fixed values of parameter α, that characterize the distinction of top and lower yielding limits, is represented in Fig. 2. It ensues from the results that for each value of α there are two different values of β for the same relation $\bar{\delta}/(1-\nu)/\gamma_s$. In addition, curves, corresponding to the fixed value $\bar{\delta}$, have the obvious maximum. It means that for each α, it is possible to find a maximal relation $\bar{\delta}/(1-\nu)/\gamma_s$ which defines minimum length of a localization band.

Consider an estimation of the minimum length of localized band. For this purpose, δ is written in the next form:

$$\delta = h\gamma_L \tag{15}$$

where h is conventional band width and γ_L is slip strain on the yielding plateau (Lüders strain). In this case for the length of the localized band, we obtain

$$b = \frac{1}{(1-\nu)} \frac{\gamma_L}{\gamma_s} h \tag{16}$$

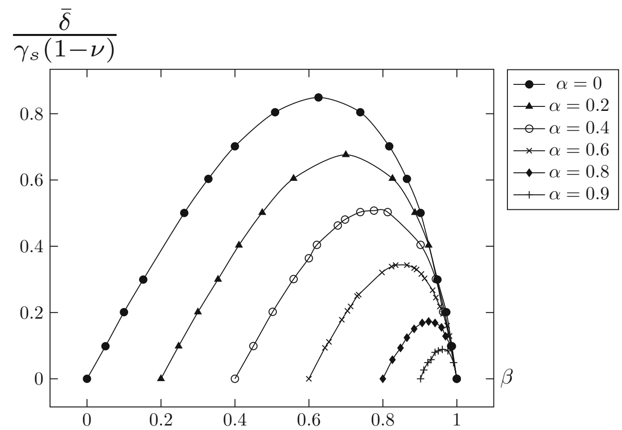

Fig. 2 Dependency of $\bar{\delta}/(1-\nu)/\gamma_s$ from parameter $\beta = \tau_0/\tau_c$ with different values of $\alpha = \tau_s/\tau_c$

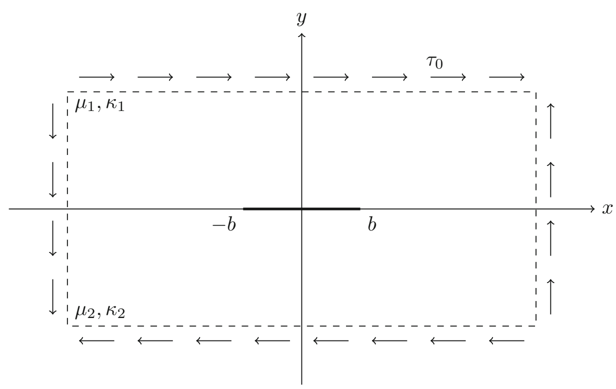

Fig. 3 The strip of localization of plastic deformation between two materials

From data for steel 1045 [13], we obtain $\gamma_c = 1.5\times10^{-2}$, $\gamma_s = 2.5\times10^{-3}$. It is more problematic to obtain h. To determine this value, we assume that the characteristic width of a localized band is determined by the mean size of grains in polycrystalline material, since dimensions of grains influence Lüders deformation and morphology of stripes, especially for low-carbon steels [1,13]. In accordance with data [13], mean diameter of grain in steel 1045 is 10 μm. It follows for $\alpha = 0$ that relation $\bar{\delta}/(1-\nu)/\gamma_s$ will be of order 0.848 and width of the band b is 100 μm.

4 Problem solution for interface slip band

Consider the generalized plane stress state of an infinite plate consisting of two welded half planes $y > 0$ and $y < 0$ (Fig. 3) with mechanical characteristics $\mu_1, \kappa_1 (y > 0)$ and $\mu_2, \kappa_2 (y < 0)$. A uniformly distributed shear load τ_0 is prescribed at infinity.

Considering that the interlayer is usually very thin, we will direct its thickness to zero and prescribe the mechanical properties of the interlayer to the interface. Let the interval $(-b, b)$ of the material interface is the zone of localization of plastic deformations. The presence of such zone is associated with the appearance of a nontrivial solution of the considered problem, which takes place along with a homogeneous solution $\sigma_{xx}(x, y) = 0, \sigma_{yy}(x, y) = 0, \sigma_{xy}(x, y) = \tau_0$.

We assume that shear stresses τ_c occur in the sections $l \le |x| \le b$ of zone of localization of plastic deformations and the shear stresses τ_s take place in the interval $|x| \le l$ of this zone. Then, mentally cutting the strip of localization of plastic deformation along the axis x and replacing the effect of plastic bonds between the faces with shear stresses τ_c and τ_s on the corresponding parts of the shores, we arrive at the problem of linear fracture mechanics with unknown positions of the points b and l (Fig. 4). Taking into account a subcritical homogeneous stress state $\sigma_{xy}(x, y) = \tau_0$, we have the following conditions on the boundary of the localization

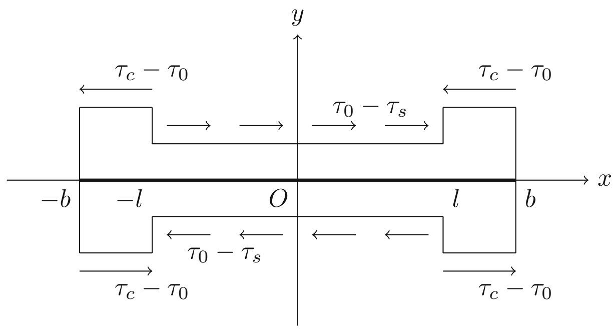

Fig. 4 Piecewise constant shear stresses on the faces of the localization strip of plastic deformation

zone:

$$\sigma_{xy}^{(i)}(x,0) = \tau,\ \left[u_2(x,0)\right] = 0,\ -b < x < b \tag{17}$$

where $\tau(x)$ is piecewise function

$$\tau(x) = \begin{array}{ll} \tau_0 - \tau_s, & 0 < |x| < l \\ -\tau_0 + \tau_c, & l < |x| < b \end{array} \tag{18}$$

Here $\left[u_2(x,0)\right] = u_2^{(1)}(x,0) - u_2^{(2)}(x,0)$.

The following expressions for the displacements jumps and stresses at the interface were given in [5] for two bonded elastic isotropic half spaces provided the stress–strain state does not depend on the coordinate z.

$$\sigma_{yy}^{(1)}(x,0) - i\sigma_{xy}^{(1)}(x,0) = g\left[F_1^+(x) + \gamma F_1^-(x)\right] \tag{19}$$

$$[u_1'(x)] + i[u_2'(x)] = F_1^+(x) - F_1^-(x) \tag{20}$$

where $[u_i'(x)] = \frac{\partial u_i^{(1)}(x,0)}{\partial x} - \frac{\partial u_i^{(2)}(x,0)}{\partial x}$, $g = \frac{2\mu_1\mu_2}{\mu_1+\mu_2\kappa_1}$, $\varepsilon = \frac{\mu_1+\mu_2\kappa_1}{\mu_2+\mu_1\kappa_2}$.

The function $F_1(z)$ is analytic in the entire plane except of the localization zone $(-b, b)$. Satisfying the boundary conditions (17)–(18) with the aid of (19), (20), we obtain

$$\mathrm{Im}\left(F_1^+(x) + \varepsilon F_1^-(x)\right) = -\frac{1}{g}\tau(x),\ \mathrm{Im}\left(F_1^+(x) - F_1^-(x)\right) = 0 \text{ for } -b < x < b$$

The last two relations are equivalent to the following Dirichlet problem for a piecewise linear analytic function $F_1(z)$

$$\mathrm{Re}\left(iF_1^{\pm}(x)\right) = -\frac{\tau(x)}{g(1+\varepsilon)} \quad \text{for } -b < x < b \tag{21}$$

with the condition at infinity

$$F_1(x)|_{x\to\infty} = 0 \tag{22}$$

The function $iF_1(z)$ bounded at both ends has the form [4] (46.25).

$$iF_1(z) = -\frac{\sqrt{(z-b)(z+b)}}{2\pi i g(1+\varepsilon_1)}\int_{-b}^{b}\frac{\tau(t)\mathrm{d}t}{(t-z)\sqrt{(t-b)(t+b)}} \tag{23}$$

under the additional condition

$$-\frac{2}{g(1+\varepsilon)}\int_{-b}^{b}\frac{\tau(t)\mathrm{d}t}{\sqrt{(t-b)(t+b)}} = 0 \tag{24}$$

It follows from (24) that

$$\int_{-b}^{b} \frac{\tau(t)\mathrm{d}t}{\sqrt{(t-b)(t+b)}} = \left(\int_{-b}^{-l} + \int_{l}^{b}\right) \frac{(\tau_0-\tau_c)\mathrm{d}t}{\sqrt{(t-b)(t+b)}} + \int_{-l}^{l} \frac{(\tau_0-\tau_s)\mathrm{d}t}{\sqrt{(t-b)(t+b)}} = 0$$

Evaluating the integrals, one obtains

$$\frac{2}{i}\left[(\tau_0-\tau_c)\arccos\frac{l}{b} + (\tau_0-\tau_s)\arcsin\frac{l}{b}\right] = 0 \tag{25}$$

We introduce the designations $\rho = l/b$. Note that there should be $\alpha < \beta$. Equation (25) can be rewritten in the form

$$\pi(\beta-\alpha) + 2(1-\alpha)\arccos\rho = 0 \tag{26}$$

The solution of this equation is the following

$$\rho = \cos\frac{\pi(\beta-\alpha)}{2(1-\alpha)}$$

The stresses at the crack continuation are found on the basis of formula (19). Taking into account $F_1^+(x) = F_1^-(x) = F_1(x)$ for $|x| > b$, one gets

$$\sigma_{yy}^{(1)}(x,0) - i\sigma_{xy}^{(1)}(x,0) = g(1+\varepsilon)F_1(x) \tag{27}$$

Calculating further the integrals and using (23), we have

$$\begin{aligned}\sigma_{yy}^{(1)}(x,0) - i\sigma_{xy}^{(1)}(x,0) = \tau_c + \frac{\tau_c-\tau_s}{\pi}\Bigg(&\arcsin\left(\frac{b^2-lx}{b(x-l)}\right)\\ &-\arcsin\left(\frac{b^2+lx}{b(x+l)}\right)\Bigg)\end{aligned} \tag{28}$$

The derivative of the tangential crack facing displacement jump due to formula (20) is defined as

$$\left[u_1'(x)\right] = \mathrm{Re}\left(F_1^+(x) - F_1^-(x)\right) \tag{29}$$

Using the Sokhotski-Plemelj formulas [9] gives

$$\left[u_1'(x)\right] + i\left[u_2'(x)\right] = F_1^+(x) - F_1^-(x) = -\frac{\sqrt{x^2-b^2}}{\pi g(1+\varepsilon)}\int_{-b}^{b}\frac{\tau(t)\mathrm{d}t}{(t-x)\sqrt{t^2-b^2}}$$

Calculating further the integral, we obtain

$$\left[u_1'(x)\right] = \frac{(1-\alpha)\tau_c}{4\pi(1+\varepsilon)g}\left(\Gamma(x,l,b) - \Gamma(x,-l,b)\right)$$

where

$$\Gamma(x,l,b) = \ln\frac{b^2-lx-\sqrt{(b^2-l^2)(b^2-x^2)}}{b^2-lx+\sqrt{(b^2-l^2)(b^2-x^2)}}$$

Performing the integration and using [12], the displacement jump $u_1(x,0)$ is determined by the formula

$$\left[u_1(x)\right] = \frac{-(1-\alpha)\tau_c}{2\pi(1+\varepsilon)g}\left((x-l)\Gamma(x,l,b) - (x+l)\Gamma(x,-l,b)\right) \tag{30}$$

For $x = 0$, we have

$$\delta = \left[u_1(x)\right] = \frac{2l(1-\alpha)\tau_c}{\pi(1+\varepsilon)g}\ln\frac{b-\sqrt{b^2-l^2}}{l} \tag{31}$$

Table 1 Value of ρ, b, l and dimensionless discontinuity $\bar{\delta} = \delta/l$

α, β	$\tau_s\ 10^7 \tau_0\ 10^7$	ρ	b, l	$\bar{\delta}$
0.1	1.66016	0.766044	0.000212151	0.00707042
0.5	8.30079		0.000162517	
0.2	3.32031	0.707107	0.000223811	0.00067021
0.6	9.96095		0.000158258	
0.3	4.98047	0.900969	0.000644167	0.000232859
0.5	8.30079		0.000580375	
0.8	13.2813	0.707107	0.000895242	0.000167552
0.9	14.9414		0.000633032	

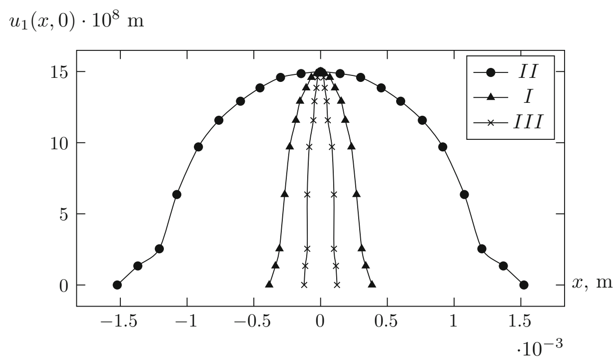

Fig. 5 The jump of the tangential displacement $u_1(x, 0) \times 10^8$ m on the interval $[-b, b]$

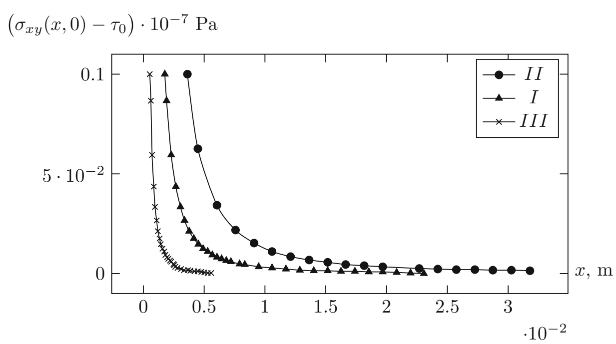

Fig. 6 Shear stresses $\left(\sigma_{xy}(x, 0) - \tau_0\right) \times 10^{-7}$ Pa on the interval $[b, b + d]$

5 Results of numerical analysis

Steel 1045 with $E_1 = 1.7 \times 10^{11}$ Pa, $\nu_1 = 0.28$, $\mu_1 = 6.64063 \times 10^{10}$ Pa, $\gamma_s = 0.0025$, $\gamma_L = 0.015$ (upper material) and iridium with $E_2 = 5.28 \times 10^{11}$ Pa, $\nu_2 = 0.26$, $\mu_2 = 2.09524 \times 10^{11}$ Pa (lower) were considered.

It was assumed that for the intermaterial layer with $\gamma = 1.4456$, $g = 5.43881 \times 10^{10}$ Pa, $\gamma_s = 0.0025$, shear deformation at the flow site (Lüders strain) $\gamma_L = 0.015$ [6,13]. Table 1 shows the solution of the transcendental equation (25) $\rho = l/b$, value b and l, value of dimensionless discontinuity $\bar{\delta} = \delta/b$ for different values of the lower yield strength τ_s and the shear load at infinity τ_0 (parameters α and β) at an upper yield strength $\tau_c = 1.66016 \times 10^8$ Pa.

Figures 5 and 6 show the graphs of the jump of the tangential displacement $u_1(x, 0) \times 10^8$ m on the interval $\left[-b, b\right]$ and shear stresses $\left(\sigma_{xy}(x, 0) - \tau_0\right) \times 10^{-7}$ Pa on the interval $\left[b, b + d\right]$ for $\tau_s/\tau_c = 0.2, 0.8$, $\tau_0/\tau_c = 0.6, 0.9$ (curves I and II, respectively). Curve III is constructed for a homogeneous material (steel 1045) for $\tau_s/\tau_c = 0.2$, $\tau_0/\tau_c = 0.6$. The results were obtained under the assumption $\delta = h\gamma_L$, where h is the conditional thickness of the localization zone. The value h in the numerical calculations was chosen equal to the average grain size of the steel 1045 ($h = 10^{-5}$ m).

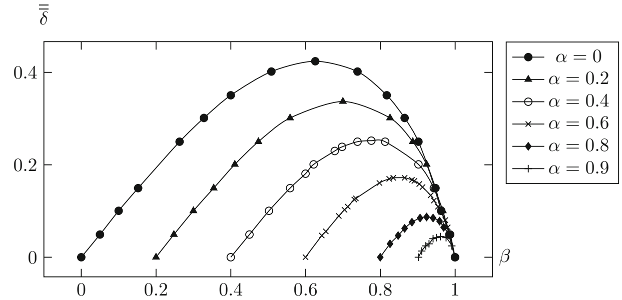

Fig. 7 Dependence of the dimensionless displacements jump $\bar{\bar{\delta}}(\beta)$ on the parameter β

Table 2 The maximum value $\bar{\bar{\delta}}(\beta)$ for each value of α

α	β	$\bar{\bar{\delta}}$
0	0.627398	0.421916
0.1	0.664658	0.379724
0.2	0.701919	0.337533
0.3	0.739179	0.295341
0.4	0.776439	0.253150
0.5	0.813699	0.210958
0.6	0.850959	0.168766
0.7	0.888219	0.126575
0.8	0.92548	0.084383
0.9	0.96274	0.0421916

Figure 7 illustrates the dependence of the dimensionless discontinuity jump $\bar{\bar{\delta}} = \bar{\delta}(1 + \varepsilon)g/\tau_c$ on the parameter $\beta = \tau_0/\tau_c$ at different values $\alpha = \tau_s/\tau_c$.

For each value of α you can uniquely find the maximum value $\bar{\bar{\delta}}(\beta)$ (Table 2).

6 Conclusions

By introducing a model of slip band propagation in materials with yielding plateau, a piecewise linear homogeneous material with an adhesive interlayer whose stress–deformation curve has a peak tooth is considered. We show the possibility of the existence of nontrivial equilibrium states associated with the appearance of a plastic deformation localization zones in such a layer. For different external loads and various mechanical properties of the interlayer, the lengths of the localization zone of plastic deformations, the graphs of the tangential displacement jump in this zone, and the shear stress on their continuation can be found.

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