Arch Appl Mech (2016) 86:1449–1464
DOI 10.1007/s00419-016-1128-6

ORIGINAL

Chang-Sheng Lin

Ambient modal identification using non-stationary correlation technique

Received: 6 July 2015 / Accepted: 14 March 2016 / Published online: 28 March 2016

Abstract The topic of this paper is the modal identification from non-stationary ambient response data by applying correlation technique. It is shown theoretically that by assuming the ambient excitation to be non-stationary white noise in the form of a product model, the non-stationary response signals can be converted into free vibration data via the correlation technique. Previous studies have showed that the practical problem of insufficient data samples available for evaluating non-stationary correlation can be approximately resolved by first extracting the amplitude-modulating function from the response and then transforming the non-stationary responses into stationary ones. However, the errors involved in the approximate free-decay response would generally lead to a distortion in the modal identification. In the present paper, we propose that, if the ambient excitation can be represented by a product model with slowly time-varying function, without any additional treatment of transforming the original nonstationary responses, the non-stationary responses of the system can be treated approximately as a stationary random process; then, the nonstationary cross correlation functions of structural response evaluated at an arbitrary, fixed time instants of structural response are of the same mathematical form as that of free vibration of a structure, from which modal parameters of the original system can thus be identified. Numerical simulations, including one example of using the practical earthquake data served as the excitation input acting on a linear two-dimensional model of one-half of a railway vehicle, confirm the validity of the proposed method for identification of modal parameters from non-stationary ambient response data only.

Keywords Non-stationary ambient vibration · Correlation technique · Modal identification

1 Introduction

Modal parameter identification from ambient vibration data has gained considerable attention in recent years [1,2]. A variety of methods have been developed for extracting modal parameters from structures undergoing ambient vibration [3,4], and further applied for structural health monitoring of large-scale structures [5] or structural safety assessment of infrastructure [6]. James et al. [7] developed the so-called Natural Excitation Technique (NExT) using the cross-correlation technique coupled with time-domain parameter extraction. It was shown that the cross-correlation between two response signals of a linear system with classical normal modes and subject to white-noise (stationary) inputs satisfy the free vibration equation of a system. Chiang and Cheng [8] further extended the original correlation technique, which is good for identification of real modes, to identify complex modal parameters of a linear system subjected to stationary ambient excitation. Chiang and Lin extend the correlation technique to perform the modal identification from nonstationary response

C.-S. Lin (✉)
Mechanism and Transmission Development Section, Wind Power Technology Center, Technology Division,
China Steel Corporation, Kaohsiung, Taiwan
E-mail: 189753@mail.csc.com.tw

data only [9] and applied the correlation technique coupled with Eigensystem Realization Algorithm [10] to identify modal parameters of a system subjected to realistic ambient excitation. Spiridonakos and Fassois [11,12] introduce appropriate time varying methods to perform the parametric output-only identification [11] and fault diagnosis [12] of a time-varying structure from nonstationary vibration response data.

Recall that the conventional correlation technique is effective only for a stationary [7,8] or filtered-stationary process [9]. The objective of this research is the modal identification from non-stationary ambient response data via correlation technique only, which is presented for general linear systems excited by non-stationary white noise represented by a product model with slowly time-varying function. It is shown that the non-stationary correlation functions evaluated at an arbitrary, fixed time instants of structural response are of the same form as free vibration decay of the structure with certain initial conditions. Therefore, without any additional treatment of transforming the original nonstationary responses, by treating the sample correlations of measured response corresponding to some fixed time instants as output from free vibration decay, a time-domain modal identification method, such as the ITD method [13], can then be employed to extract modal parameters, including modal frequencies, damping ratios and mode shapes, of the structure with complex modes.

2 Non-stationary correlation technique

2.1 Conventional correlation technique

James et al. [7] developed the so-called Natural Excitation Technique (NExT) using the correlation technique. It was shown that the cross-correlation between two response signals of a linear system with classical normal modes and subjected to white-noise inputs satisfy the system's free vibration equation. In combination with a time-domain parameter extraction scheme, such as the ITD method [11], this concept becomes a powerful tool for the identification analysis of structures under stationary ambient vibration.

When a system is excited by stationary white noise, the cross-correlation function $R_{ij}(\tau)$ between two stationary response signals $x_i(t)$ and $x_j(t)$ can be shown to be [5]:

$$R_{ij}(\tau) = \sum_{r=1}^{n} \frac{\phi_{ir} A_{jr}}{m_r \omega_{dr}} \exp(-\zeta_r \omega_r \tau) \sin(\omega_{dr}\tau + \theta_r) \tag{1}$$

where ϕ_{ir} denotes the i-th component of the r-th mode shape, A_{jr} is a constant, m_r is the r-th modal mass, and ω_{dr} is the rth natural frequency with damping. The result above shows that $R_{ij}(\tau)$ in Eq. (1) is a sum of complex exponential functions (modal responses), which is of the same mathematical form as the free vibration decay or the impulse response of the original system. Thus, the cross-correlation functions evaluated from responses data can be used as free vibration decay or as impulse response in time-domain modal extraction schemes so that measurement of white-noise inputs can be avoided. It is remarkable that the term $\phi_{ir} A_{jr}$ in Eq. (1) will be identified as the mode-shape components. In order to eliminate the A_{jr} term and retain the true mode-shape components ϕ_{ir}, all the measured channels are correlated against a common reference channel, say x_j. The identified components then all possess the common A_{jr} component, which can be normalized out to obtain the desired mode shape ϕ_{ir}.

2.2 Correlation technique for nonstationary ambient responses

In the following, we start by considering a discrete linear system subjected to a non-stationary random excitation $f_k(t)$ [9]. Define the cross-correlation function $R_{ijk}(t,T)$ between two non-stationary response signals $x_{ik}(t)$ and $x_{jk}(t)$ as

$$R_{ijk}(t,T) = E\left[x_{ik}(t+T)\cdot x_{jk}(T)\right]. \tag{2}$$

where $x_{ik}(t)$ is the response at the ith degree of freedom (DOF) due to the input at the kth DOF. Under the assumption of initial-rest system, and through the evaluation of the Duhamel integral, Eq. (2) can be derived as follows

$$R_{ijk}(t,T) = \sum_{r=1}^{n}\sum_{s=1}^{n} \phi_{ir}\phi_{kr}\phi_{js}\phi_{ks} \int_{-\infty}^{t}\int_{-\infty}^{t+T} g_r(t+T-\sigma) g_s(t-\tau) E\left[f_k(\sigma) f_k(\tau)\right] d\sigma d\tau, \tag{3}$$

where $g_r(t) = \frac{1}{m_r\omega_{dr}} \exp(-\zeta_r\omega_{nr}t)\sin(\omega_{dr}t)$. Assume that $f_k(t)$ is non-stationary white noise in the form of a product model, i.e.,

$$f_k(t) = \Gamma_k(t)w_k(t), \tag{4}$$

where $\Gamma_k(t)$ is a deterministic amplitude-modulating function used to describe the change of amplitude with time, and $w_k(t)$ is stationary white noise. The auto-correlation function of excitation force $f_k(t)$ can then be expressed as

$$R_{ffk}(\tau,\sigma) = \Gamma_k(\tau)\Gamma_k(\sigma)E\left[W_k(\tau)W_k(\sigma)\right] = \Gamma_k(\tau)\Gamma_k(\sigma)\alpha_k\delta(\tau-\sigma),$$

where α_k is a constant and $\delta(t)$ is the Dirac delta function. Therefore, Eq. (3) can be evaluated as

$$R_{ijk}(t,T) = \sum_{r=1}^{n}\sum_{s=1}^{n}\alpha_k\phi_{ir}\phi_{kr}\phi_{js}\phi_{ks}\cdot\int_{-\infty}^{t}\Gamma_k^2(\tau)g_r(t+T-\tau)g_s(t-\tau)d\tau. \tag{5}$$

One can, therefore, derive

$$\begin{aligned} R_{ijk}(t,T) &= \sum_{r=1}^{n}\left[G_{ijkr}(t)\exp(-\zeta_r\omega_{nr}T)\cos(\omega_{dr}T) + H_{ijkr}(t)\exp(-\zeta_r\omega_{nr}T)\sin(\omega_{dr}T)\right] \\ &= \sum_{r=1}^{n}\frac{\exp(-\xi_r\omega_{nr}T)}{\sqrt{G_{ijkr}^2(t)+H_{ijkr}^2(t)}}\left[\cos(\omega_{dr}T)\sin\theta(t) + \sin(\omega_{dr}T)\cos\theta(t)\right] \\ &= \sum_{r=1}^{n}\frac{\exp(-\xi_r\omega_{nr}T)}{\sqrt{G_{ijkr}^2(t)+H_{ijkr}^2(t)}}\sin\left[\omega_{dr}T+\theta(t)\right] \end{aligned} \tag{6}$$

where

$$\begin{aligned} \begin{bmatrix} G_{ijkr}(t) \\ H_{ijkr}(t) \end{bmatrix} &= \phi_{ir}\sum_{s=1}^{n}\frac{\alpha_k\phi_{kr}\phi_{js}\phi_{ks}}{m_r\omega_{dr}m_s\omega_{ds}}\cdot \\ &\quad\times\int_0^{\infty}\Gamma_k^2(t+\tau)\exp(-\zeta_r\omega_{nr}-\zeta_s\omega_{ns})\lambda\cdot\sin(\omega_{ds}(t-\tau))\begin{bmatrix}\sin(\omega_{dr}(t-\tau)) \\ \cos(\omega_{dr}(t-\tau))\end{bmatrix}d(t-\tau) \\ &\equiv \frac{\phi_{ir}}{m_r\omega_{dr}}\begin{Bmatrix} G_{jkr}(t) \\ H_{jkr}(t) \end{Bmatrix} \end{aligned} \tag{7}$$

where $G_{ijkr}(t)$ and $H_{ijkr}(t)$ are functions of modal parameters, and independent of T. From Eq. (7), the following equations can then be derived

$$\begin{aligned} G_{ijkr}(t) &= \sum_{s=1}^{n}\frac{\alpha_k\phi_{ir}\phi_{kr}\phi_{js}\phi_{ks}}{m_r m_s\omega_{dr}}\left[\frac{I_{rs}}{J_{rs}{}^2+I_{rs}{}^2}\right] \\ H_{ijkr}(t) &= \sum_{s=1}^{n}\frac{\alpha_k\phi_{ir}\phi_{kr}\phi_{js}\phi_{ks}}{m_r m_s\omega_{dr}}\left[\frac{J_{rs}}{J_{rs}{}^2+I_{rs}{}^2}\right] \end{aligned} \tag{8}$$

where

$$\begin{aligned} I_{rs} &= 2\omega_{dr}(\zeta_r\omega_{nr}+\zeta_s\omega_{ns}) \\ J_{rs} &= (\omega_{ds}{}^2-\omega_{dr}{}^2)+(\zeta_r\omega_{nr}+\zeta_s\omega_{ns}) \end{aligned} \tag{9}$$

To simplify Eq. (9), we define γ_{rs} as

$$\tan(\gamma_{rs}) = \frac{I_{rs}}{J_{rs}} \tag{10}$$

and Eq. (8) can be rewritten as

$$G_{ijkr}(t) = \frac{\phi_{ir}}{m_r\omega_{dr}} \sum_{s=1}^{n} \beta_{jkrs}(J_{rs}^2 + I_{rs}^2)^{-\frac{1}{2}} \sin(\gamma_{rs})$$

$$H_{ijkr}(t) = \frac{\phi_{ir}}{m_r\omega_{dr}} \sum_{s=1}^{n} \beta_{jkrs}(J_{rs}^2 + I_{rs}^2)^{-\frac{1}{2}} \cos(\gamma_{rs}) \tag{11}$$

where $\beta_{jkrs} = \frac{\alpha_k\phi_{kr}\phi_{js}\phi_{ks}}{m_s}$. Substituting Eq. (11) into Eq. (6), and summing over all m input locations, we obtain

$$\begin{aligned} R_{ij}(t,T) &= \sum_{k=1}^{m} R_{ijk}(t,T) \\ &= \sum_{k=1}^{n} \frac{\phi_{ir}}{m_r\omega_{dr}} \exp(-\xi_r\omega_{nr}T) \cdot \sum_{k=1}^{m} [G_{jkr}(t)\cos\omega_{dr}T + H_{jkr}(t)\sin\omega_{dr}T] \\ &= \sum_{r=1}^{n} \frac{\phi_{ir}}{m_r\omega_{dr}} \exp(-\xi_r\omega_{nr}T) \cdot \sum_{k=1}^{m} \left[\frac{1}{\sqrt{G_{jkr}{}^2(t) + H_{jkr}{}^2(t)}} [\sin\omega_{dr}T + \theta_{jkr}(t)] \right] \end{aligned} \tag{12}$$

and Eq. (12) can be further simplified as follows

$$R_{ij}(t,T) = \sum_{r=1}^{n} \frac{\phi_{ir}A_{jr}(t)}{m_r\omega_{dr}} \exp(-\xi_r\omega_{nr}T)\sin(\omega_{dr}T + \Theta_r) \tag{13}$$

The result above shows that for any fixed time instant t, $R_{ij}(t,T)$ in Eq. (13) is a sum of complex exponential functions, which is of the same form as the free vibration decay or the impulse response of the original system [7]. Thus, the cross-correlation functions evaluated at a fixed time instant of responses can be used as free vibration decay or as impulse response in time-domain modal extraction schemes so that measurement of non-stationary white-noise inputs can be avoided. It is remarkable that the term $A_{jr}(t)\,\phi_{\mathrm{ir}}$ with fixed t in the cross-correlation function of Eq. (7) will be identified as the mode-shape component. In order to eliminate the $A_{jr}(t)$ term and retain the true mode-shape component ϕ_{ir}, all the measured channels are correlated against a common reference channel, say x_j. The identified components then all possess the common $A_{jr}(t)$ component, which can be normalized out to obtain the mode shape ϕ_{ir}.

In the following, by considering a discrete linear system subjected to excitation resulted from a single source $w(t)$, which is assumed to be stationary white noise. The equation of motion can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{v}}(t) + \boldsymbol{C}\dot{\boldsymbol{v}}(t) + \boldsymbol{K}\boldsymbol{v}(t) = \boldsymbol{l}w(t), \tag{14}$$

where $\boldsymbol{v}(t)$, $\dot{\boldsymbol{v}}(t)$ and $\ddot{\boldsymbol{v}}(t)$ are the stationary displacement, velocity and acceleration responses, respectively. $\boldsymbol{l}$ is a vector whose elements are the influence factors for each dof and may be thought of a measure of the extent to which the $w(t)$ participates in the total excitation on the structure. Multiplying both sides of Eq. (14) by a slowly time-varying amplitude-modulating function $\Gamma(t)$ we can obtain

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{l}f(t), \tag{15}$$

where $f(t)$ is a non-stationary white noise as represented by the product model:

$$f(t) = \Gamma(t)w(t), \tag{16}$$

and $\boldsymbol{u}(t) = \Gamma(t)\boldsymbol{v}(t)$. Note that in deriving Eq. (15), we assumed that $\Gamma(t)$ is a slowly time-varying function (i.e., $\dot{\Gamma}(t) \approx 0, \ddot{\Gamma}(t) \approx 0$), and so $\Gamma(t)\dot{\boldsymbol{v}}(t) \approx \dot{\boldsymbol{u}}(t)$ and $\Gamma(t)\ddot{\boldsymbol{v}}(t) \approx \ddot{\boldsymbol{u}}(t)$. Denote the time average of $u_i^2(\tau)$ as $\widehat{u}_i^2(t)$, which is defined as [9]:

$$\widehat{u}_i^2(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u_i^2(\tau)d\tau. \tag{17}$$

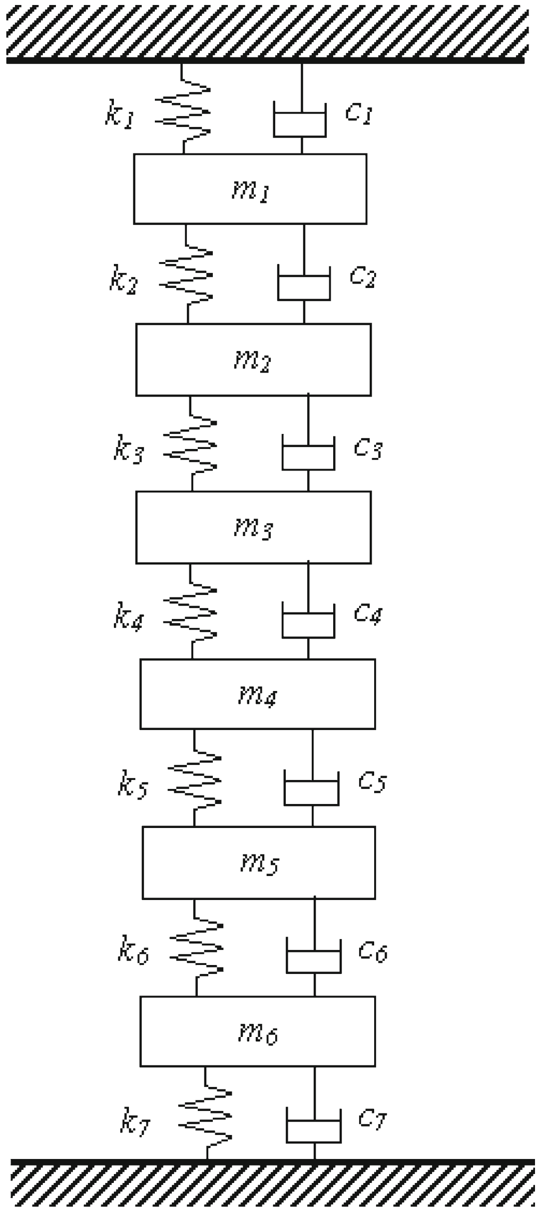

Fig. 1 A Schematic plot of the 6-dof chain system

Recall that we have assumed the $\Gamma(\tau)$ to be a slowly varying function, then from Eq. (17) $\widehat{u}_i^2(t)$ can be approximated as:

$$\widehat{u}_i^2(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \Gamma^2(\tau)\, v_i^2(\tau)d\tau \cong \Gamma^2(t)\,\frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} v_i^2(\tau)d\tau, \tag{18}$$

for T being a short time interval. The temporal mean-square function $\widehat{u}_i^2(t)$ is practically estimated by averaging over short time intervals of the record. If we assume $v_i(\tau)$ is an ergodic process, the integral on the right-hand side of Eq. (18) is just an approximation to $E\left[v_i^2\right]$ and so

$$\widehat{u}_i^2(t) \cong \Gamma^2(t)\, E\left[v_i^2\right], \tag{19}$$

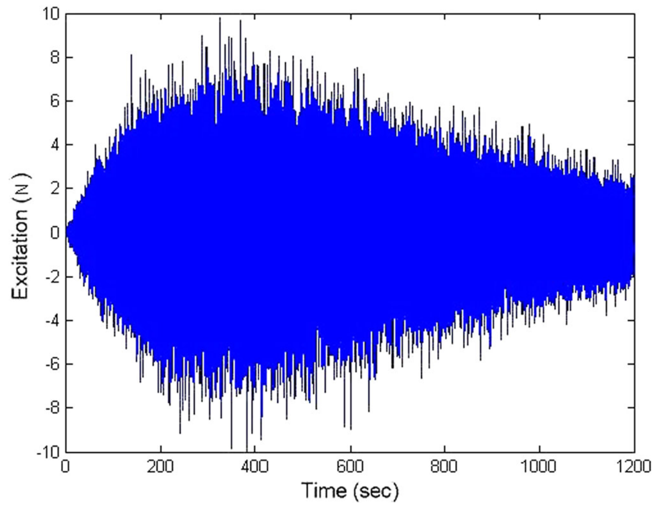

Fig. 2 A sample function of non-stationary white noise with a slowly varying amplitude-modulating function

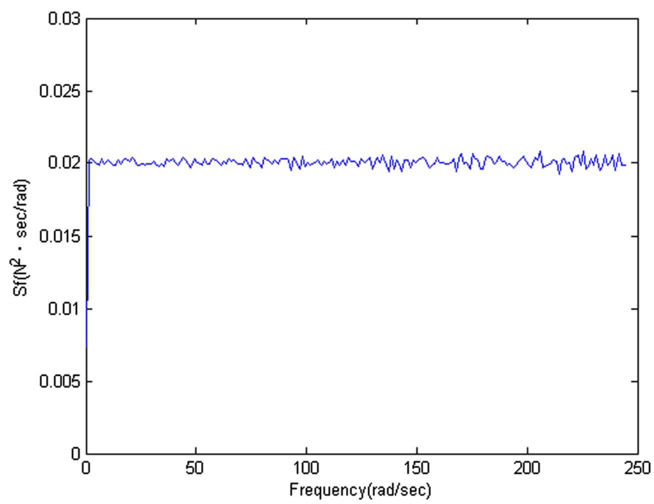

Fig. 3 Power spectrum associated with the stationary part of the simulated non-stationary white noise

Then, the temporal root-mean-square function denoted as $\widehat{\Psi}_i(t)$ can be evaluated by time-averaging over a single sample record as

$$\widehat{\Psi}_i(t) = \left[\widehat{u}_i^2(t)\right]^{\frac{1}{2}} \cong \Gamma(t)\, C_i, \tag{20}$$

where $C_i = \left(E\left[v_i^2\right]\right)^{\frac{1}{2}}$. Note that the temporal root-mean-square function $\widehat{\Psi}_i(t)$ of each dof is proportional to the same envelope function of time, $\Gamma(t)$.

The above result indicates that the temporal root-mean-square functions $\widehat{\Psi}_i(t)$ of the response histories describe the same slowly time-varying variation through interval average as given by the envelope function $\Gamma(t)$. This suggests that if the original non-stationary data could be represented by the product model with a slowly time-varying envelope function, the temporal root-mean-square functions of the data through time average also have the same non-stationary trend as that of the original data. In the relatively short time intervals of the nonstationary sample in the form of the product model with slowly time-varying envelope function $\Gamma(t)$, the variation of amplitude with time of $\Gamma(t)$ is very small, i.e., $\Gamma(t)$ can not significantly describe the time-varying amplitude (variance) in the short time intervals of the aforementioned nonstationary sample process. The short time intervals of the nonstationary sample in the form of the product model with $\Gamma(t)$ can be treated as a quasi stationary sample process, and the nonstationary correlation technique applied to the quasi stationary

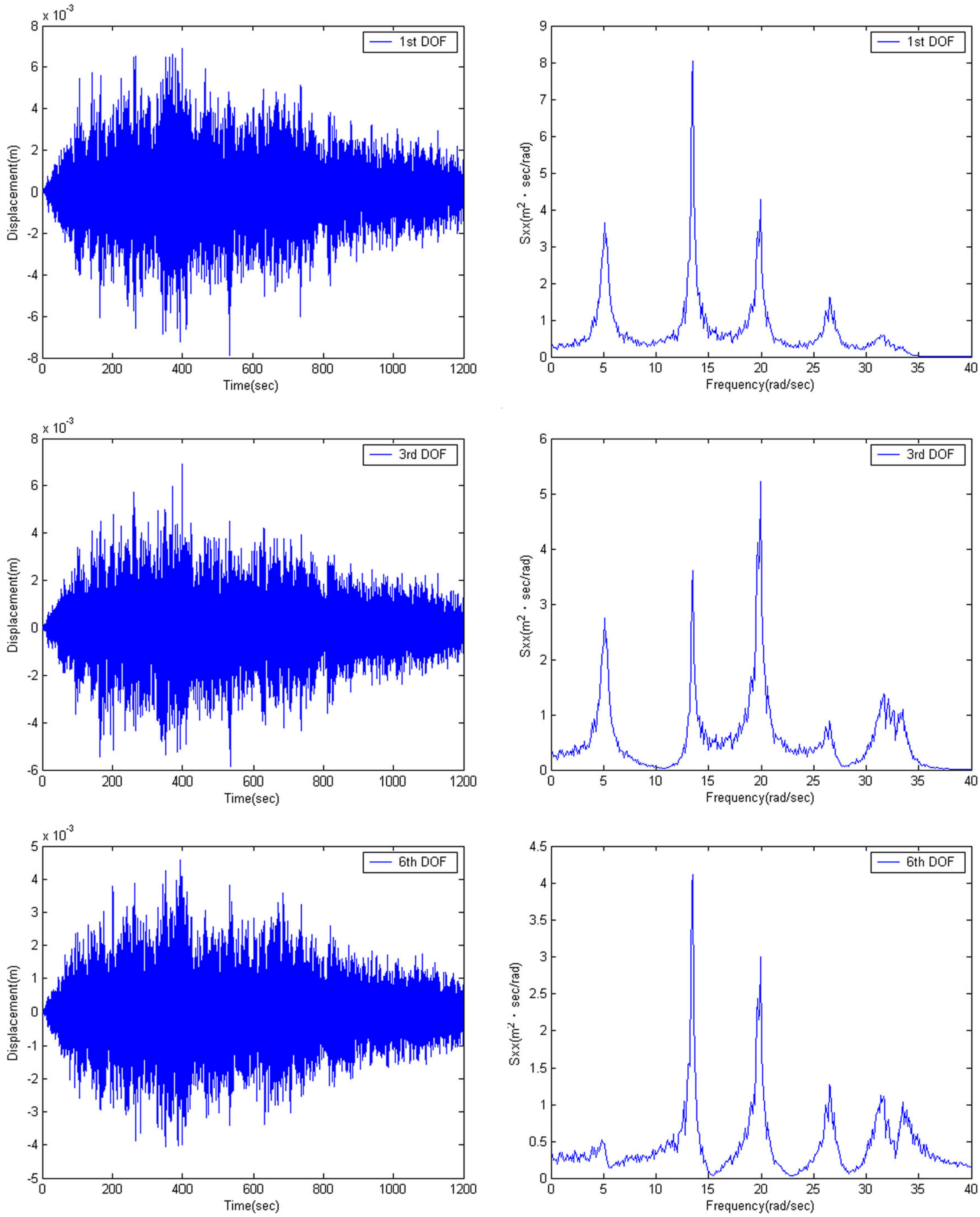

Fig. 4 Typical displacement responses and the corresponding Fourier spectra of the 6-DOF chain system subject to nonstationary white input

sample process can then be approximately transformed into stationary one. The results indicate that if the excitation can be modeled as non-stationary white noise as represented in Eq. (16) with a slowly time-varying envelope function $\Gamma(t)$; then, the non-stationary responses of the system can also be treated approximately as a stationary random process, and the correlation functions therefore can be obtained from a single sample function of time by using the ergodic property of stationary random process.

The preceding results show that the non-stationary problem may reduce to a stationary problem if we evaluate the non-stationary correlation functions at a fixed time instant. Therefore, under appropriate conditions of slowly time-varying envelope function $\Gamma(t)$ in the product model of nonstationary ambient response, we can follow the same procedures as those for stationary problem analysis, the correlation functions thus can be treated as free vibration data and can be obtained from a single sample function of time by using the ergodic property of stationary random process.

3 Numerical simulation

To demonstrate the effectiveness of the proposed method, we consider a linear 6-dof chain model with viscous damping. A schematic representation of this model is shown in Fig. 1. The mass matrix $\boldsymbol{M}$, stiffness matrix $\boldsymbol{K}$, and the damping matrix $\boldsymbol{C}$ of the system are given as follows:

$$\boldsymbol{M} = \begin{bmatrix} 2&0&0&0&0&0\\ 0&2&0&0&0&0\\ 0&0&2&0&0&0\\ 0&0&0&2&0&0\\ 0&0&0&0&3&0\\ 0&0&0&0&0&4 \end{bmatrix} N \times s^2/m, \quad \boldsymbol{K} = 600 \cdot \begin{bmatrix} 1&-1&0&0&0&0\\ -1&2&-1&0&0&0\\ 0&-1&2&-1&0&0\\ 0&0&-1&2&-1&0\\ 0&0&0&-1&3&-2\\ 0&0&0&0&-2&5 \end{bmatrix} N/m,$$

$$\boldsymbol{C} = 0.05\boldsymbol{M} + 0.001\boldsymbol{K} + 0.2 \begin{bmatrix} 1&\dots&1\\ \vdots&\ddots&\vdots\\ 1&\dots&1 \end{bmatrix}_{6\times 6} N \times s/m.$$

Note that the system has non-proportional damping (and so complex modes in general), since the damping matrix $\boldsymbol{C}$ cannot be expressed as a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$. Consider that the ambient vibration input can be modeled as non-stationary white noise as represented by the product model given by Eq. (4). The stationary white noise is generated using the spectrum approximation method [14] as a zero-mean band-pass noise, whose standard deviation is 0.02 $N^2 \times s$/rad with a frequency range from 0 to 50 Hz. The sampling interval is chosen as $\Delta t = 0.01$ s, and the sampling period is $T = N_t \times \Delta t = 1310.72$ s. The stationary white noise simulated is then multiplied by an amplitude-modulating function $\Gamma(t) = 4 \times (e^{-0.002t} - e^{-0.004t})$ to obtain the non-stationary white noise, which serves as the excitation input acting on the 6th mass point of the system. The time signal of a simulated sample of the non-stationary white noise and the power spectrum of the corresponding stationary part are shown in Figs. 2 and 3, respectively.

The simulated displacement responses of the system were obtained using Newmark's method [15]. By examining the Fourier spectra associated with each of the response channel, as shown in Fig. 4, we chose the response of the 6th channel, $X_6(t)$, which contains rich overall frequency information, as the reference channel to compute the correlation functions of the system. According to the theory presented in the previous sections, if the ambient excitation can be represented by a product model with slowly time-varying function, without any additional treatment of transforming the original nonstationary responses, the non-stationary responses of the system can be treated approximately as a stationary random process; then, the nonstationary cross correlation functions of structural response evaluated at an arbitrary, fixed time instants of structural response are of the same mathematical form as that of free vibration of a structure. Therefore, we can follow the same procedures as those for stationary problems, and the correlation functions thus obtained are treated as free vibration data. The Ibrahim time-domain method [13] could then be applied to identify modal parameters of the system.

Table 1 Results of modal parameter identification of the 6-dof chain system subjected to non-stationary white noise input

Mode	Natural frequency (rad/s)			Damping ratio (%)			MAC
	Exact	ITD	Error (%)	Exact	ITD	Error (%)	
1	5.03	5.02	0.23	5.24	5.14	1.91	1.00
2	13.45	13.42	0.20	1.07	0.97	9.35	1.00
3	19.80	19.75	0.24	1.13	1.11	1.77	1.00
4	26.69	26.52	0.62	1.43	1.41	1.40	1.00
5	31.66	31.41	0.79	1.66	1.59	4.22	0.95
6	33.73	33.41	0.94	1.74	1.73	0.57	1.00

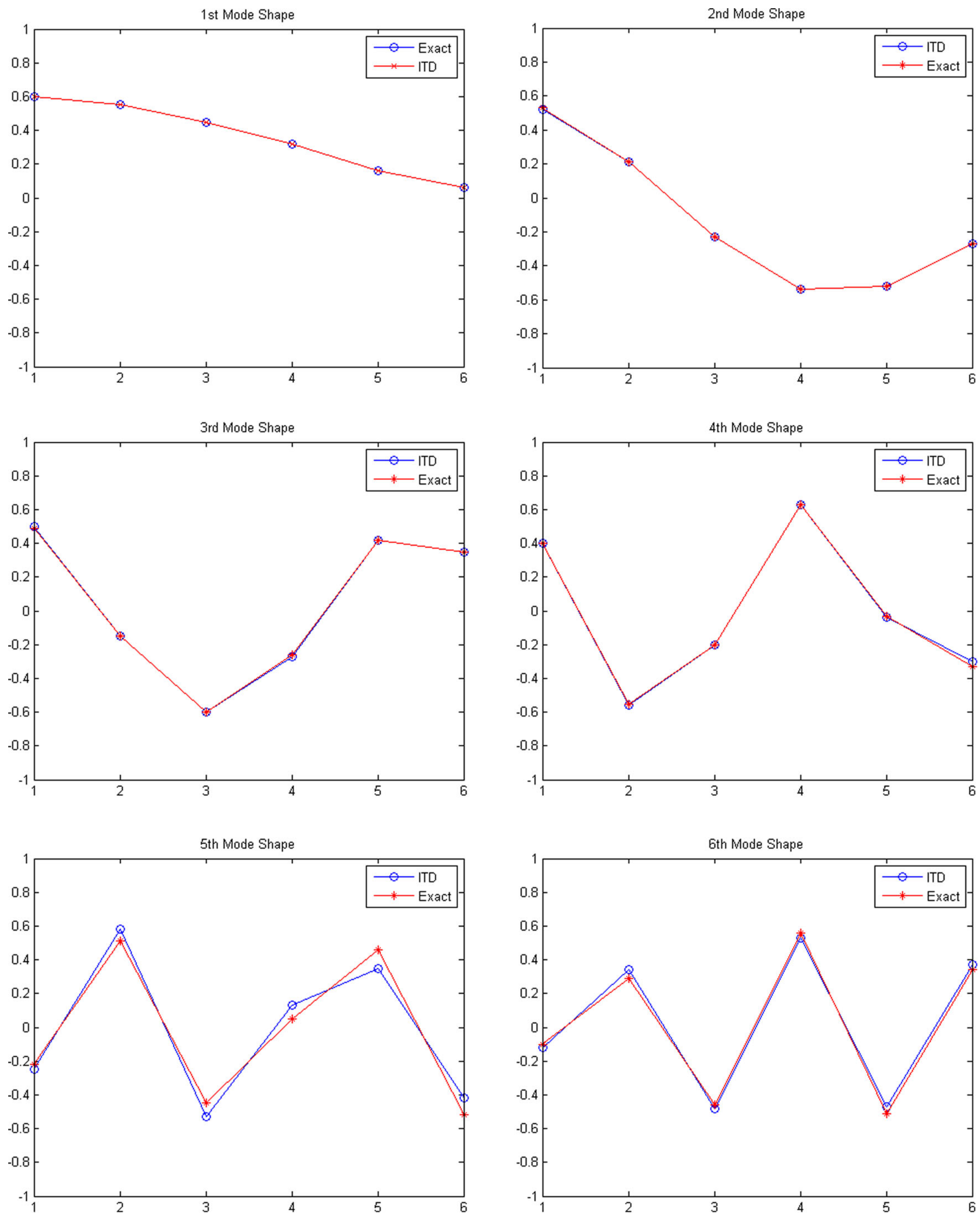

Fig. 5 Comparison between the identified mode shapes and the exact mode shapes of the 6-dof chain system subjected to non-stationary white noise input

The results of modal parameter identification are summarized in Table 1, which shows that the errors in natural frequencies are <1 % and the error in damping ratios is <10%. Note that the "exact" modal damping ratios listed in Table 1 are actually the equivalent modal damping ratios obtained by utilizing ITD from the simulated free vibration data of the non-proportionally damped structure. The identified mode shapes are also compared with the exact values in Fig. 5, where we observe good agreement with the minimum value of MAC (Modal Assurance Criterion) [16] of about 0.95. The errors of identified damping ratios and mode shapes are somewhat higher due to the fact that the system response generally has lower sensitivity to

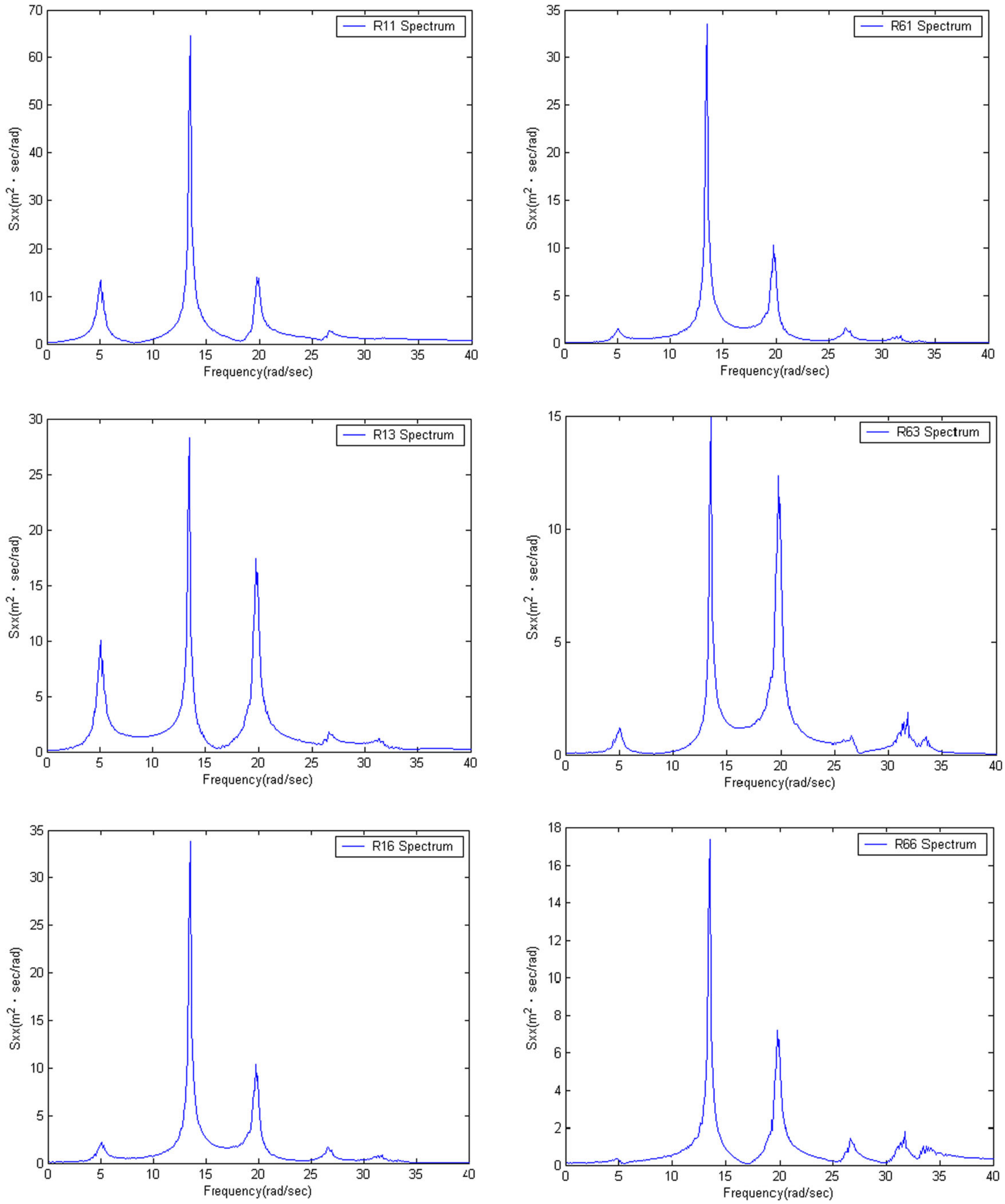

Fig. 6 Comparison between the Fourier spectrum of correlation functions associated with the response of the 1st, 3rd and 6th DOF of the system choosing the response of the 1st and 6th DOF, respectively, as the reference channels

these modal parameters than to the modal frequencies. In addition, by comparing with the Fourier spectra of correlation functions, as shown in Fig. 6, associated with the response of the 1st, 3rd and 6th DOF of the system choosing the response of the 1st and 6th DOF, respectively, as the reference channels, the Fourier spectra of the correlation functions associated with the response of 6th channel (R61, R63, and R66) contains richer over all frequency information, especially for higher modes than that of the 1st channel (R11, R13, and R16). The results indicate that the selection of reference channel for computing correlation functions is also important to the identification results. The richer frequency content the reference channel has, the better results of modal parameters identification can be achieved.

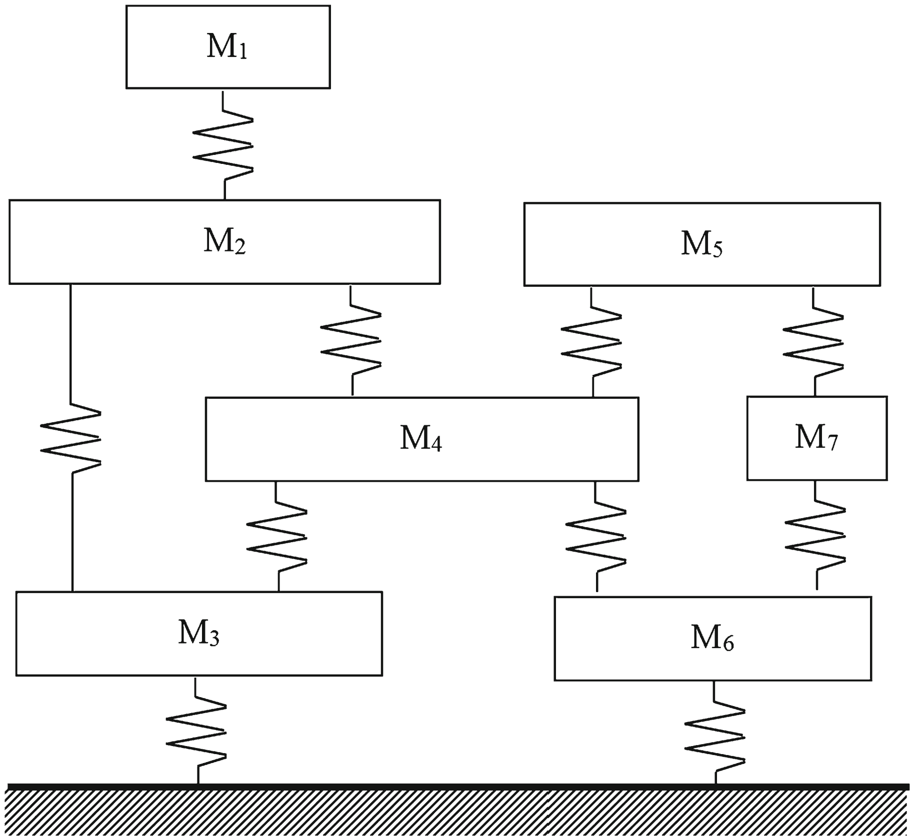

Fig. 7 A Schematic plot of the 7-dof system [17]

Table 2 Results of modal parameter identification of the 7-dof system subjected to non-stationary white noise input

Mode	Natural frequency (rad/s)			Damping ratio (%)			MAC
	Exact	ITD	Error (%)	Exact	ITD	Error (%)	
1	5.08	5.07	0.04	2.22	2.26	1.80	1.00
2	9.75	9.74	0.16	1.51	1.45	3.97	1.00
3	15.74	15.68	0.40	1.42	1.36	4.23	1.00
4	19.48	19.14	1.72	1.49	1.23	17.45	0.86
5	20.65	20.59	0.33	1.52	1.75	15.13	0.95
6	22.31	22.17	0.63	1.56	1.51	3.21	0.94
7	32.44	32.16	0.87	1.93	1.89	2.07	1.00

To further clarify the accuracy and adequacy of the proposed methods, numerical simulations have been performed of another type of a structure, which is not in the form of a chain or truss model used as the test sample. A schematic representation of this model is shown in Fig. 7 [17]. The mass matrix $\boldsymbol{M}$, stiffness matrix $\boldsymbol{K}$, and the proportional damping matrix $\boldsymbol{C}$ of the system are given as follows:

$$\boldsymbol{M} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{N} \cdot \text{s}^2/\text{m}, \quad \boldsymbol{K} = 400 \cdot \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2.7 & -1 & -0.7 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & -0.7 & -1 & 3.7 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 2.85 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix} \text{N/m},$$

$$\boldsymbol{C} = 0.2\boldsymbol{M} + 0.001\boldsymbol{K} \quad \text{N} \cdot \text{s/m}.$$

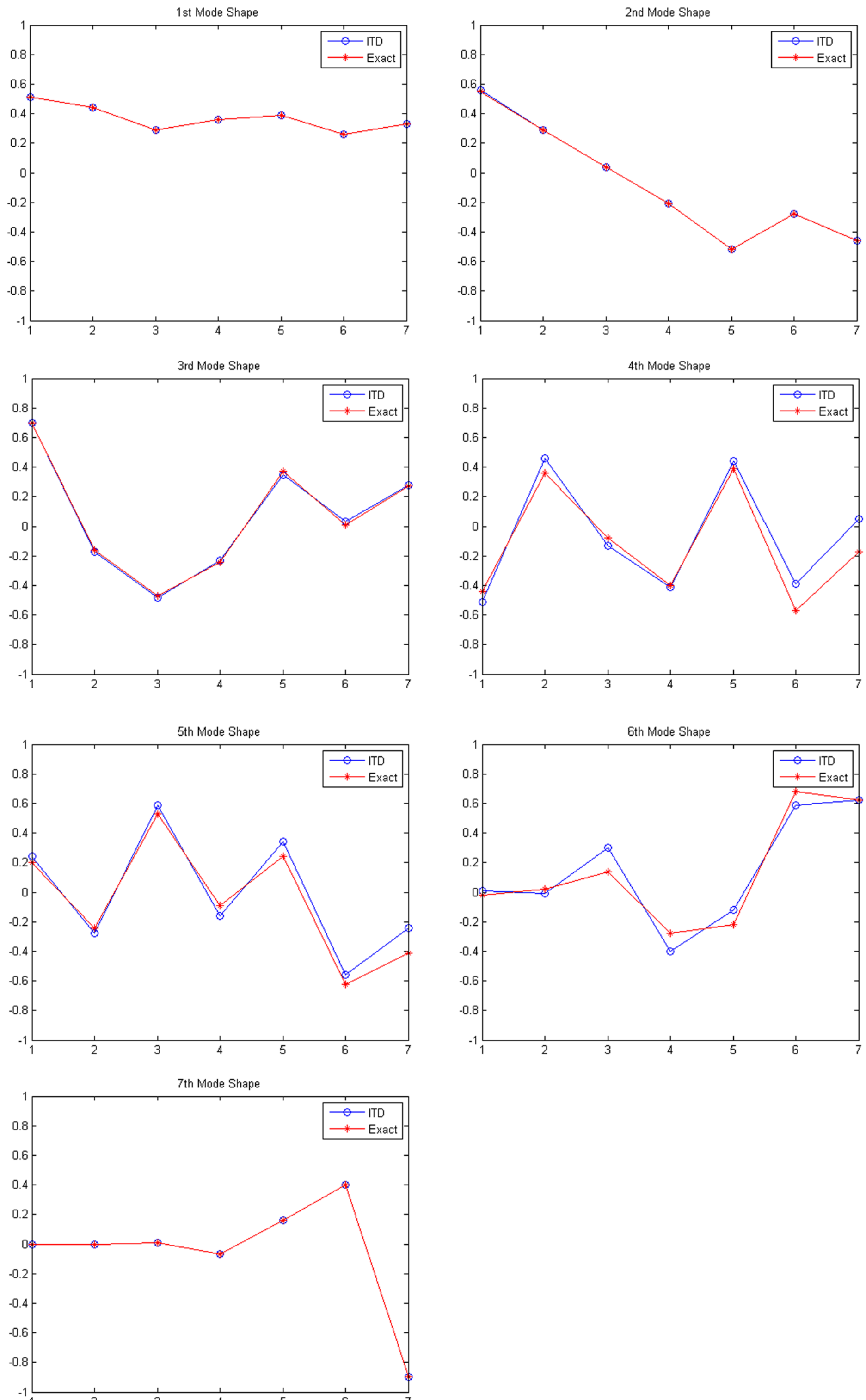

Fig. 8 Comparison between the identified mode shapes and the exact mode shapes of the 7-dof system subjected to non-stationary white noise input

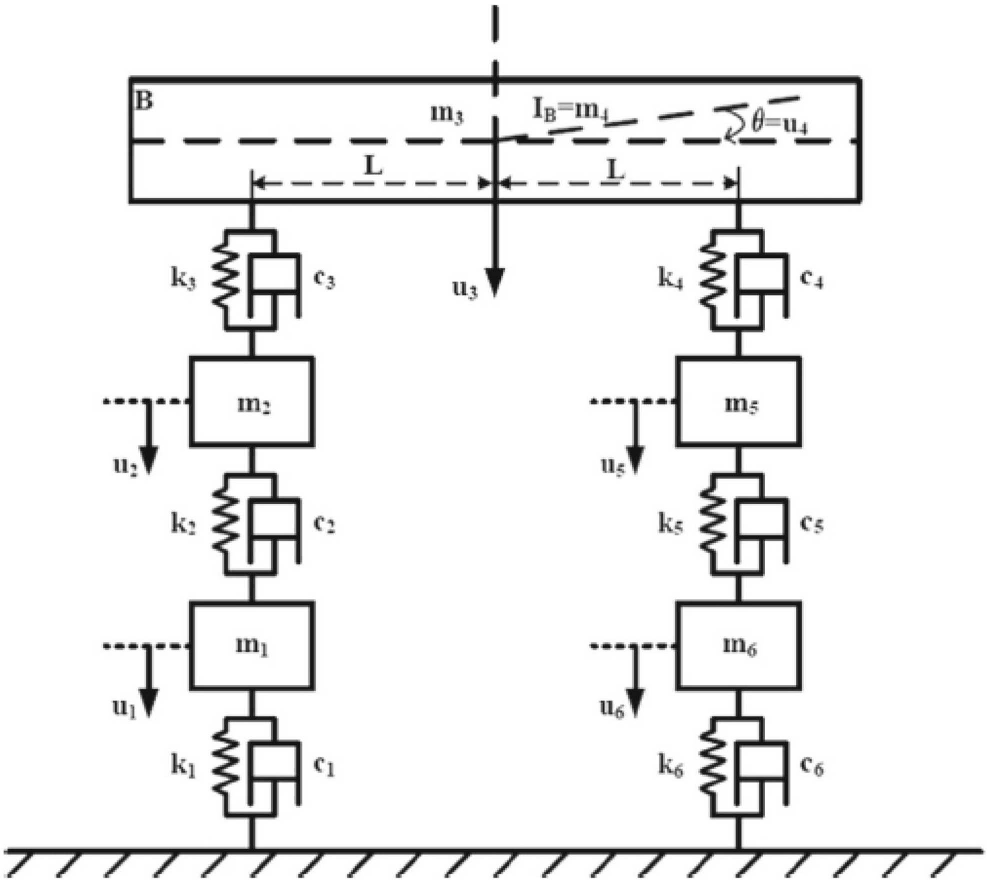

Fig. 9 Schematic plot of the 6-dof system of a linear two-dimensional model of one-half of a railway vehicle [19]

This type of structure is not a chain model system in which the stiffness matrix $\boldsymbol{K}$ is not of a tri-diagonal form. Note that the system has proportional damping, since the damping matrix $\boldsymbol{C}$ can be expressed as a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$. The system considered has two pairs of closely spaced modes, as listed in Table 2. In this example, we still use the previous nonstationary white-noise in the form of a product model as input acting on the 7th mass of the system, and then the corresponding displacement responses were obtained by Newmark's method [15] are used for modal identification. The results of modal identification through the ITD method in conjunction with the proposed nonstationary correlation technique are also summarized in Table 2. From Table 2, we see that the identification results are good. The well-identified mode shapes are also compared with the exact values in Fig. 8. The results indicate that the proposed methods may be applicable to identify the modal parameters of a general structural system subjected to nonstationary white-noise in the form of a product model.

To further examine the effectiveness of the present method for the more complex structural subjected to the realistic ambient excitation, we consider a linear two-dimensional model of one-half of a railway vehicle excited by a practical seismic signal. The dynamic system used in the numerical study (a sketch is shown in Fig. 9) is identical to that in Reference [18, 19]. The system is a 6-DOF system with $\mathbf{u} = [u_1, u_2, u_3, u_4, u_5, u_6]$, where $u_4 = \theta$ is a rotational displacement and others are vertical displacement as shown in Fig. 9. The mass matrix is a diagonal matrix, diag $\mathbf{M} = [m_1, m_2, m_3, m_4, m_5, m_6]$, where $m_4 = \mathrm{I_B}$ is the mass moment of inertia of the rigid body B at the top of the structure. The stiffness matrix can be obtained as

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & -k_3 L & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & k_3 L - k_4 L & -k_4 & 0 \\ 0 & -k_3 L & k_3 L - k_4 L & k_3 L^2 + k_4 L^2 & k_4 L & 0 \\ 0 & 0 & -k_4 & k_4 L & k_4 + k_5 & -k_5 \\ 0 & 0 & 0 & 0 & -k_5 & k_5 + k_6 \end{bmatrix}$$

Table 3 Results of modal parameter identification of the 6-dof system of a linear two-dimensional model of one-half of a railway vehicle subjected to non-stationary white noise input

Mode	Natural frequency (rad/s)			Damping ratio (%)			MAC
	Exact	ITD	Error (%)	Exact	ITD	Error (%)	
1	17.53	17.47	0.36	1.16	1.19	2.32	1.00
2	23.31	23.25	0.27	1.38	1.38	0.28	1.00
3	103.99	102.16	1.75	5.24	6.66	27.16	0.96
4	121.08	117.31	3.11	6.09	7.54	23.95	0.94
5	159.34	154.13	3.27	8.01	9.55	19.24	0.85
6	160.66	163.17	1.56	8.08	9.39	16.28	0.84

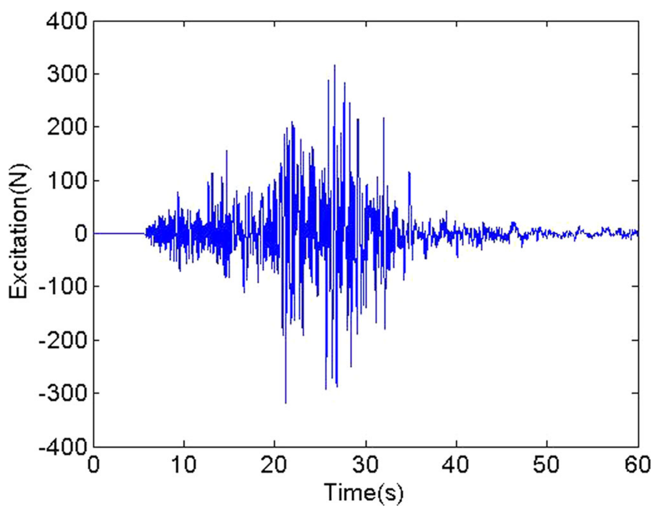

Fig. 10 A recorded sample of the Chi-Chi Earthquake

where L is the horizontal distance between the center of the rigid body B and the springs/dashpots. Throughout this numerical study, $[m_1, m_2, m_3, m_5, m_6] = [1200, 850, 4125, 850, 1220]$ kg, and $m_4 = \mathrm{I_B} = 1.25 \times 10^5$ kg m^2; $k_1 = k_6 = 3.0 \times 10^7$ N/m, $k_2 = k_5 = 1.0 \times 10^6$ N/m and $k_3 = k_4 = 6.0 \times 10^6$ N/m; $L = 8.53$ m; $\boldsymbol{C} = 0.1\boldsymbol{M} + 0.001\boldsymbol{K} N$ s/m. Note that this 6-DOF system of one-half of a railway vehicle has proportional damping, since the damping matrix $\boldsymbol{C}$ can be expressed as a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$. From the numerical values, the analytical modal frequencies and damping ratios are listed in Table 3. This 6-DOF system features the following: 6 modes with considerable frequency ($0 \sim 165$rad/s) range, modal damping levels ranging from low (1.16%) to relatively high (8.08%), and a pair of closely spaced modes (frequency separation smaller than 1.5 rad/s) [18]. Consider that the ambient vibration input is a practical vibration recorded at Sun-Moon Lake on September 21, 1999, when Chi-Chi Earthquake with a moment magnitude of 7.6 occurred in central Taiwan. The sampling interval and period of this seismic record are $\Delta t = 0.005$ s and $T = 59.995$ s, respectively. A sample of the seismic record, which serves as the excitation input acting on the 6th mass of the model, is shown in Fig. 10. The displacement responses of the system were obtained using Newmark's method [15]; then, we perform the modal identification using the simulation responses, and identification results obtained are satisfactory, as summarized in Table 3.

It can be observed from the identification results obtained through the above numerical simulations that, for the finite but sufficient acquisition of available sample time history to perform the correlation technique and ITD method, the modal parameters of a system can be well identified in general. This is because that, in the relatively short time intervals of the nonstationary sample in the form of the product model with slowly time-varying envelope function $\Gamma(t)$, the variation of amplitude with time of $\Gamma(t)$ is very small, i.e., $\Gamma(t)$ can not significantly describe the time-varying amplitude (variance) in the short time intervals of the aforementioned nonstationary sample process. The short time intervals of the nonstationary sample in the form of the product model with $\Gamma(t)$ can be treated as a quasi stationary sample process, and the nonstationary correlation technique applied to the quasi stationary sample process can then be approximately transformed into stationary one. It indicates that when the practical ambient excitation, such as earthquakes, can be approximately modeled as a product model with slowly time-varying function, the proposed method is applicable to identify the modal parameters of a structural system subjected to realistic excitation, which can properly describe the nonstationary process with a slowly time-varying envelope function.

In this paper, we developed modal identification methods under the slowly time-varying nonstationary assumption for ambient excitation. We also demonstrated the validity of these methods through numerical simulations without using the practical response data. From the nonstationary correlation functions of the vibration behavior of realistic ambient excitation, we know that the slowly time-varying nonstationary assumptions are consistent with the time-varying nature of ambient excitation in practice. Thus, the proposed methods are generally applicable in identifying the modal parameters of a structure from the identification results obtained through numerical simulations.

4 Conclusions

To identify dynamic characteristics of structures in nonstationary ambient vibration, modal-identification method of using response data only is studied. If the ambient excitation can be properly represented by a product model with slowly-time-varying envelope function, in the relatively short time intervals, the variation of amplitude with time of temporal root-mean-square functions of the response histories is so small that cannot significantly describe the time-varying amplitude (variance) of the nonstationary sample process. Therefore, without any additional treatment of transforming the original nonstationary responses, the proposed nonstationary cross-correlation functions of structural response evaluated at an arbitrary, fixed time instants of structural response are of the same mathematical form as that of free vibration of a structure, from which modal parameters of the original system can thus be identified. In addition, the choice of the reference channel for computing the correlation functions is important to the identification results. The reference channel is chosen as a response channel whose Fourier spectrum has rich frequency content around the structure modes of interest. The richer frequency content the reference channel has, the better results of modal parameters identification can be achieved.

Acknowledgments The author would like to thank every member in the Wind Power Technology Center of Technology Division, China Steel Corporation (CSC), for their kind assistance and friendship, as well as also thanks Professor Dar-Yun Chiang, Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan, Taiwan, for his valuable support. The author wishes to thank the anonymous reviewers for their valuable comments and suggestions in revising the paper.

References

1. Wang, Z., Chen, G.: Analytical mode decomposition with Hilbert transform for modal parameter identification of buildings under ambient vibration. Eng. Struct. **59**, 173–184 (2014)
2. Au, S.-K., Zhang, F.-L.: Ambient modal identification of a primary-secondary structure by Fast Bayesian FFT method. Mech. Syst. Signal Process. **28**, 280–296 (2012)
3. Osmancikli, G., Bayraktar, A., Türker, T., Uçak, Ş., Mosallam, Ayman: Finite element model calibration of precast structures using ambient vibrations. Constr. Build. Mater. **93**, 10–21 (2015)
4. Amiri, M., Sabbagh-Yazdi, S.R.: Ambient vibration test and finite element modeling of tall liquid storage tanks. Thin-Walled Struct. **49**, 974–983 (2011)
5. Choi, S., Park, S., Hyun, C.-H., Kim, M.-S.: In-operation modal analysis of containments using ambient vibration. Nucl. Eng. Des. **260**, 16–29 (2013)
6. Türker, T., Bayraktar, A.: Structural safety assessment of bowstring type RC arch bridges using ambient vibration testing and finite element model calibration. Measurement **58**, 33–45 (2014)
7. James, G.H., Carne. T.G., Lauffer, J.P.: The Natural Excitation Technique for Modal Parameter Extraction from Operating Wind Turbines. SAND92-1666. UC-261, Sandia National Laboratories (1993)
8. Chiang, D.Y., Cheng, M.S.: Model parameter identification from ambient response. AIAA J. **37**, 513–515 (1999)
9. Chiang, D.Y., Lin, C.S.: Identification of modal parameters from non-stationary ambient vibration data using correlation technique. AIAA J. **46**(11), 2752–2759 (2008)
10. Chiang, D.Y., Lin, C.S.: Identification of modal parameters from ambient vibration data using eigensystem realization algorithm with correlation technique. J. Mech. Sci. Technol. **24**(12), 2377–2382 (2010)
11. Spiridonakos, M.D., Fassois, S.D.: Parametric identification of a time-varying structure based on vector vibration response measurements. Mech. Syst. Signal Process. **23**, 2029–2048 (2009)
12. Spiridonakos, M.D., Fassois, S.D.: An FS-TAR based method for vibration-response-based fault diagnosis in stochastic time-varying structures: experimental application to a pick-and-place mechanism. Mech. Syst. Signal Process. **38**, 206–222 (2013)
13. Ibrahim, S.R., Mikulcik, E.C.: A method for the direct identification of vibration parameters from free response. Shock Vib. Bull. **47**(Pt. 4), 183–198 (1977)

14. Shinozuka, M., Jan, C.-M.: Digital simulation of random processes and its applications. J. Sound Vib. **25**(1), 111–128 (1972)
15. Newmark, N.M.: A method of computation for structural dynamics. J. Eng. Mech. ASCE **85**(EM3), 67–94 (1959)
16. Allemang, R.L., Brown, D.L.: A correlation coefficient for modal vector analysis. In: Proceedings of the 1st international modal analysis conference, Society for Experiment Mechanics, Bethel, CT, pp. 110–116 (1983)
17. Heylen, W., Lammens, S., Sas, P.: Modal analysis theory and testing. Department of Mechanical Engineering, Katholieke Universiteit Leuven, Leuven, Belgium (1998)
18. Petsounis, K.A., Fassios, S.D.: Parametric time-domain methods for the identification of vibrating structures—a critical comparison and assessment. Mech. Syst. Signal Process. **15**, 1031–1060 (2001)
19. Hu, S.L.J., Yang, W.L., Liu, F.S., Li, H.J.: Fundamental comparison of time-domain experimental modal analysis based on high- and first-order matrix models. J. Sound Vib. **333**, 6869–6884 (2014)