Arch Appl Mech (2015) 85:713–724
DOI 10.1007/s00419-015-0983-x

ORIGINAL

Zihao Liu · Wanyou Li · Huajiang Ouyang · Donghua Wang

Eigenstructure assignment in vibrating systems based on receptances

Received: 11 June 2014 / Accepted: 13 January 2015 / Published online: 24 January 2015

Abstract This paper presents a method for structural modifications for achieving desired eigenstructures based on receptances, by adding multiple mass–spring systems to some locations of the primary system. This method has the benefit that not only neither analytical nor modal models are needed, but also the original mass and stiffness of the primary system are maintained. Moreover, when a complex structure or machine is designed for some special functions so that its inner structure is not allowed to be modified, it is an effective way in practice to achieve desired dynamical performance resulted from adding several external simple substructures. The theory is presented in this paper, which is suitable for linear undamped systems. Numerical experiment is set up, and the results of the modifications are compared with the method proposed by Braun and Ram. Both theoretical derivation and numerical results demonstrate the effectiveness of this method.

Keywords Eigenstructure assignment · Structural modification · Receptance · Mass–spring system · Undamped systems

1 Introduction

In many engineering cases, in order to improve the dynamic characteristics of a structure, structural modifications are needed. Generally speaking, they mean modifications of the system mass, stiffness and damping parameters to meet certain dynamic performance [1], such as the need to avoid resonance or creation of a node on the system at a certain frequency.

Structural modification problems can be divided into forward and inverse problems. The forward problem aims to predict the dynamic behaviour of the modified structure. The inverse problem aims to determine the modifications required so that the modified structure would have the desired dynamic behaviour specified a priori [2–5]. This paper deals with the inverse modification problem. Inverse problems are difficult to solve

Z. Liu · W. Li (✉) · D. Wang
College of Power and Energy Engineering, Harbin Engineering University,
Harbin 150001, People's Republic of China
E-mail: liwanyou@hrbeu.edu.cn

Z. Liu
E-mail: fageiliuzihao@126.com

H. Ouyang
School of Engineering, University of Liverpool, Liverpool, UK

H. Ouyang
State Key Laboratory of Structural Analysis of Industrial Equipment, Dalian University of Technology,
Dalian 16023, People's Republic of China

because issues such as the existence of solutions of the problem is unknown, and the solution may not be unique. Thus, inverse structural modifications remain an area of active research, as can be seen from [6–8].

Structural modification problems can be solved by firstly constructing the system model and then casting the problem of achieving certain dynamic performance as an optimisation problem. Optimisation generally aims to find a solution that minimises or maximises an objective function from a given set of feasible solutions that satisfy some constraints. An optimisation problem can be solved by a number of methods [9], such as projected Landweber iterative method, genetic algorithms, simulated annealing and so on.

In principle, when solving structural modification problems, information of the original structure can be obtained from the physical model, which includes the mass, stiffness and damping matrices [10]. The original structure may also be characterised by modal data [11–14]. The accuracy of the original system parameters directly affects the accuracy and feasibility of structural modifications. However, these methods are difficult to apply to some complex and uncertain systems in engineering, because inverse problems are usually ill-posed and their solutions may not exist or are sensitive to errors caused by model simplification and numerical methods used. To overcome these problems, the authors of [15–18] used the receptances (frequency response functions, or FRFs for short) of the original system that could be measured quite accurately. This receptance-based method is accurate and fairly easy to use.

One popular way of structural modifications is to change the values of existing masses and stiffnesses of a structure [19–21]. This works well for many structures. However, there are two shortcomings in this approach. The first one is that for some complex structures, it is difficult to modify existing masses and stiffnesses. The second one is that the effectiveness in achieving a certain goal of structural modifications can be limited if the modifications are only allowed at existing masses and stiffnesses. In some cases, the original structure was designed for certain specific functions and requirements, which should not be modified. Therefore, an alternative way of making structural modifications should be established and used. This involves adding a subsystem to the original structure instead of modifying the existing degrees of freedom (dof) of the original structure. In [22], one spring and one mass were added to a mass–spring system to assign the system with one desired natural frequency. However, it is difficult to assign more than one frequency using this method. Moreover, modes were not considered in the method proposed in [22]. It would be very useful to assign both frequencies and modes (that is, eigenstructure).

This paper presents a structural modification strategy that adding multiple mass–spring systems to the original structure instead of directly modifying the mass and stiffness values at the existing degrees of freedom, based on receptances. Hence, this method has two main advantages. Firstly, it makes direct use of FRFs (measured FRF data when real structures are concerned), so that the method proposed in this paper is neither affected by the ill-conditioning of the eigenstructure extraction, nor it needs knowledge of the original system mass matrix [8]. Secondly, this strategy is to add several external simple substructures, rather than to change the original design, which overcomes both difficulties in accurately modifying stiffness values of the system in practice and meanwhile avoids influencing the function of the machine or structure resulted from modification of original purposeful design. It combines the idea of structural modifications in [22] and the approach in [8] and casts eigenstructure assignment as an optimal problem. By means of numerical simulations, feasibility and effectiveness of this method are verified.

2 Theoretical development

Although damping is always present, it is very small in most structures. Even if a damping ratio is 10 %, the difference between a damped natural frequency and an undamped natural frequency is different by only 0.5 %. Therefore, when free vibration or natural frequencies or modes are studied (as done in this paper), neglecting damping is acceptable and the eigenstructure assignment made in this paper will not be affected. Then, a general linear discrete conservative dynamic system is described by

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\ddot{\mathbf{x}}$ is the vector of acceleration, $\mathbf{x}$ is the vector of displacement and $\mathbf{f}$ is the force vector. In this paper, only mass–spring type of structures is studied.

Structural modifications cause the mass and stiffness matrices to change by $\delta\mathbf{M}$ and $\delta\mathbf{K}$, respectively, which are treated as forcing terms on the unmodified structure. Therefore, Eq. (1) can then be re-written as

$$(\mathbf{M} + \delta\mathbf{M})\,\ddot{\mathbf{x}} + (\mathbf{K} + \delta\mathbf{K})\,\mathbf{x} = \mathbf{f} \tag{2}$$

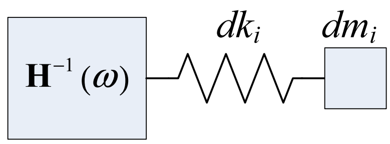

Fig. 1 Conservative n-degree-of-freedom spring–mass system modified by a simple oscillator attached at the ith degree of freedom

Assuming harmonic response $\mathbf{x} = \mathbf{u}e^{i\omega t}$ and substituting it into Eq. (2) yields

$$\left(-\omega^2\mathbf{M} + \mathbf{K}\right)\mathbf{u} = \left(\omega^2\delta\mathbf{M} - \delta\mathbf{K}\right)\mathbf{u} + \mathbf{f} \tag{3}$$

The frequency response function (FRF) matrix of the original system is defined as $\mathbf{H}(\omega) = \left(-\omega^2\mathbf{M} + \mathbf{K}\right)^{-1}$. Then, Eq. (3) becomes

$$\mathbf{H}^{-1}\mathbf{u} = \left(\omega^2\delta\mathbf{M} - \delta\mathbf{K}\right)\mathbf{u} + \mathbf{f} \tag{4}$$

It is assumed that a mass–spring subsystem is added at the ith freedom of original system, and the mass and stiffness are dm and dk, as shown in Fig. 1, and the relative amplitude of vibration is du. Thus, an extra degree of freedom is introduced. Consequently, the matrices in Eq. (4) are enlarged by one row and column. Then, the equation of motion of the modified system is described by

$$\begin{pmatrix} \mathbf{H}_{n\times n}{}^{-1} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \\ du \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -dk & \cdots & 0 & dk \\ \vdots & \vdots & & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & dk & \cdots & 0 & -dk + \omega^2 dm \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \\ du \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \\ 0 \end{pmatrix} \tag{5}$$

The last row of the above equation is

$$(-\omega^2 dm + dk)du + dku_i = 0 \tag{6}$$

Solving for du in terms of u_i yields

$$du = \left(\frac{dk}{dk - \omega^2 dm}\right)u_i \tag{7}$$

By substituting Eq. (7) into Eq. (5), the following equation is derived:

$$\begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -dk & \cdots & 0 & dk \\ \vdots & \vdots & & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & dk & \cdots & 0 & -dk + \omega^2 dm \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \\ \left(\frac{dk}{dk-\omega^2 dm}\right)u_i \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \\ 0 \end{pmatrix} \tag{8}$$

The ith and $(n+1)-$th rows of the above equation can be written as

$$-dku_i + \left(\frac{(dk)^2}{dk - \omega^2 dm}\right)u_i + f_i = \left(\frac{\omega^2 dmdk}{dk - \omega^2 dm}\right)u_i + f_i \tag{9}$$

$$dku_i + \left(\frac{\omega^2 dm - dk}{dk - \omega^2 dm}\right)dku_i = 0 \tag{10}$$

Then, Eq. (5) becomes

$$\mathbf{H}_{n\times n}^{-1}\begin{pmatrix}u_1\\u_2\\\vdots\\u_i\\\vdots\\u_n\end{pmatrix}=\begin{pmatrix}0&0&\cdots&0&\cdots&0\\0&0&\cdots&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots&&\vdots\\0&0&\cdots&\frac{\omega^2 dmdk}{dk-\omega^2 dm}&\cdots&0\\\vdots&\vdots&&\vdots&\ddots&\vdots\\0&0&\cdots&0&\cdots&0\end{pmatrix}\begin{pmatrix}u_1\\u_2\\\vdots\\u_i\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}f_1\\f_2\\\vdots\\f_i\\\vdots\\f_n\end{pmatrix} \tag{11}$$

The derivation process above is about adding a single mass–spring subsystem. The general situation of adding multiple mass–spring subsystems is investigated below.

It is assumed that the desired natural frequency and mode are, respectively, ω_h and $\mathbf{u}_h$, and the FRFs for the desired natural frequency are $\mathbf{H}_h$. Moreover, denote that the number of mass–spring subsystems used for modifications is j. The vector of modifications $\mathbf{y}$ is written as

$$\mathbf{y}^{\mathrm{T}}=\left[\left[dm_1\quad dk_1\right]\left[dm_2\quad dk_2\right]\cdots\left[dm_j\quad dk_j\right]\right] \tag{12}$$

Each couple of parameters $\left[dm_i\quad dk_i\right]$, where i=1,…,j in vector $\mathbf{y}$ stands for an added mass–spring subsystem. They can be collected into the first matrix on the right-hand side of Eq. (11), through the derivation process above. The rearranged matrix in Eq. (11) contains both the desired natural frequency ω_h and parameters from $\mathbf{y}$, and it is named $\mathbf{G}\left(\mathbf{y},\omega_h\right)$. By collecting all the parameters from vector $\mathbf{y}$ into matrix $\mathbf{G}$, it is possible to rearrange the single eigenpair assignment problem in Eq. (3) as

$$\mathbf{u}_h=\mathbf{H}\left(\omega_h\right)\mathbf{G}\left(\mathbf{y},\omega_h\right)\mathbf{u}_h \tag{13}$$

The eigenstructure assignment problem can therefore be cast as

$$\min_{\mathbf{x}}\left\{\sum_{h=1}^{n}\alpha_h\left\|\mathbf{H}(\omega_h)\mathbf{G}(\mathbf{y},\omega_h)\mathbf{u}_h-\mathbf{u}_h\right\|_2^2\right\} \tag{14}$$

where weighting coefficient α_h is a positive scalar.

Equation (14) can be solved by optimisation algorithms. There are many of them, which have been applied in various fields [23,24]. But the focus of this paper is to study a novel strategy for structural modifications and its feasibility. Hence, the algorithms for solving this optimisation problem will not be studied in this paper.

This receptance-based formulation does not require information of the original system mass and stiffness matrices. This method is particularly suitable for those systems in which mass and stiffness matrices are unknown or difficult to be measured. Moreover, it is particularly suitable when the external subsystems can be added to the original system. The accuracy of this method depends on the accuracy of the FRF. Currently, since techniques for measuring FRFs are quite mature, it is easy to get enough accurate data. It should be pointed out that the current method cannot avoid spillover, that is, the unassigned frequencies and modes would get changed unintentionally, like other passive vibration control methods. This thorny issue was overcome by a different method of structural modifications for some simple structures in a very recent paper [25].

3 Numerical analysis

In this section, three simulated examples are analysed by the current method and another well-known method [14] to demonstrate the effectiveness of the current method.

3.1 Numerical experiment set-up

In Fig. 2, mass ($m_i, i=1,\ldots,4$) and mass ($m_j, i=2,\ldots,5$) are connected through spring $k_{ij}(j=i+1)$, and each of the five masses is also connected to the rigid ground through ground spring $k_{gi}(i=1,\ldots,5)$.

For a comparison with the well-established technique proposed by Braun and Ram in [14], all the system parameters used here are identical to those of the real example in [8] which was used to compare the method in [8] with the method proposed by Braun and Ram, as listed in Table 1.

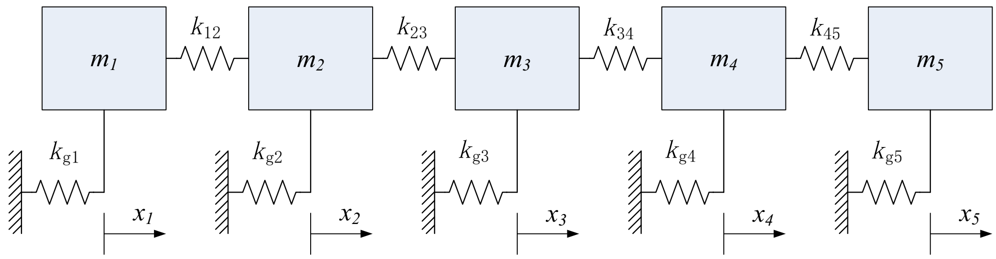

Fig. 2 Simulated 5-dof system

Table 1 System parameters

Parameter	Value
Mass (kg)	
m_1	1.73
m_2	5.12
m_3	8.21
m_4	2.61
m_5	1.34
Stiffness (N/m)	
k_{12}	7.36e4
k_{23}	6.82e4
k_{34}	7.35e4
k_{45}	8.21e4
k_g	9.89e4

Table 2 Desired eigenstructure

Mode number h	1	2
f_h (Hz)	39.00	55.00
$u_h(1)$	1.00	0
$u_h(2)$	−0.55	0.01
$u_h(3)$	0.20	−0.10
$u_h(4)$	0	0.80
$u_h(5)$	0.05	1.00

The desired eigenstructure is also the same as that in [8], namely, it contains two frequencies 39 and 55 Hz and the corresponding modes, summarised in Table 2.

The structural modifications method proposed by Braun and Ram [14] is to change the values of existing masses and stiffnesses of a structure, which requires the knowledge of the system's left eigenvectors. Moreover, regularization techniques, which in general are based on the knowledge of the system mass matrix, need to be used in order to calculate eigenvectors reliably. However, for some complex systems, the system mass matrix is not easy to obtain accurately or even unknown in practice.

On the other hand, the proposed method is to add multiple mass–spring systems to the original structure based only on the frequency responses of the original system. In the numerical experiment of this paper, the system FRFs are obtained from $[\omega_h^2\mathbf{M} - \mathbf{K}]^{-1}$ at the natural frequencies of the desired modes, while they are measured by experiment in practical applications.

3.2 Modification for one frequency and mode

3.2.1 Adding two mass–spring subsystems

Aimed at assigning the one mode listed in Table 2 at f_1 (39 Hz), two mass–spring subsystems are employed firstly to modify the system, which are mounted on masses m_1 and m_2, as shown in Fig. 3. Two masses (m_6, m_7) are, respectively, added to masses (m_1, m_2) through springs (k_{16}, k_{27}). The range of computed parameters is assumed and listed in Table 3.

In the above figure, two masses (m_6 and m_7) are, respectively, added to masses (m_1, m_2) through springs (k_{16}, k_{27}) as a modification.

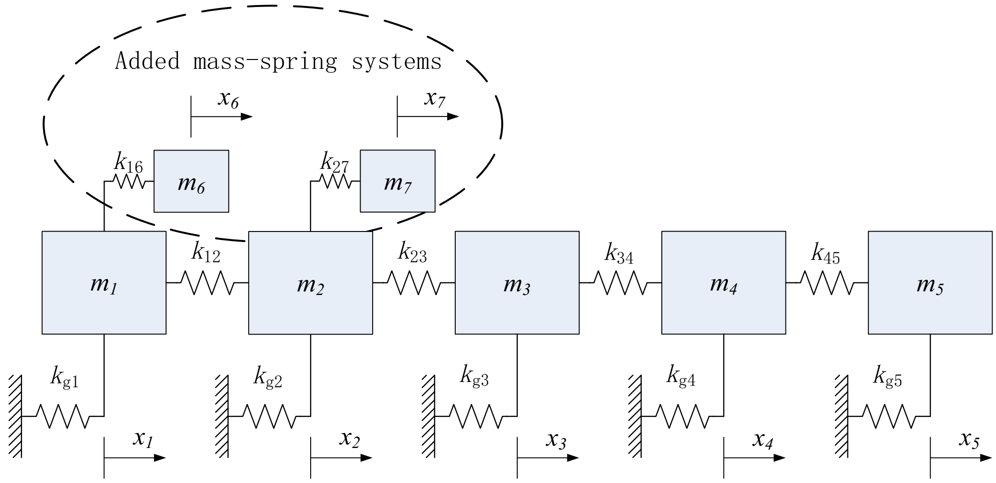

Fig. 3 The modified system by adding two mass–spring subsystems

Table 3 Modification bounds

Parameter	Lower bound	Upper bound
Mass (kg)		
m_6	0	2
m_7	0	2
Stiffness (kN/m)		
k_{16}	0	300
k_{27}	0	300

Table 4 Parameters of added mass–spring systems

Mass (kg)	Value (kg)	Stiffness (kN/m)	Value
m_6	0.9153	k_{16}	116.69
m_7	0.6895	k_{27}	70.98

Table 5 Modified mode shapes and eigenstructure comparison

	Goal	Method BR	Proposed method
f_i (Hz)	39.00	38.99	39.00
$u_i(1)$	1.000	1.000	1.000
$u_i(2)$	−0.550	−0.547	−0.474
$u_i(3)$	0.200	0.192	0.076
$u_i(4)$	0.000	0.055	0.179
$u_i(5)$	0.050	0.031	0.146
Desired mode number, h	–	1	1
$\|f_h - f_i\|$ (Hz)	–	0.014	0.003
$\cos(u_i,\mathrm{u}_h)$	–	0.9987	0.9765

The parameters of mass–spring subsystems obtained by applying the modifications are listed in Tables 4 and 5. As a further proof, Fig. 4 shows the absolute values of FRFs ($\mathbf{H}_{i,5}(\omega), i = 1, \ldots, 5$) of the system modified by the proposed method (solid line) and the original system (dotted line). The frequencies and modes after modification are collected in Table 5. The cosines between the desired eigenvectors and the attained ones are also given.

It is easily seen that the attained frequencies are very accurate, but the attained modes are not very close to the desired one, and particularly, the desired node is not realised. It is believed that this is because the number of desired modal data (1 frequency and 5 modal elements) is much greater than the number of modifying quantities (2 added masses and springs). Hence, in order to increase the chance of obtaining better solutions, more mass–spring subsystems are considered in the next section.

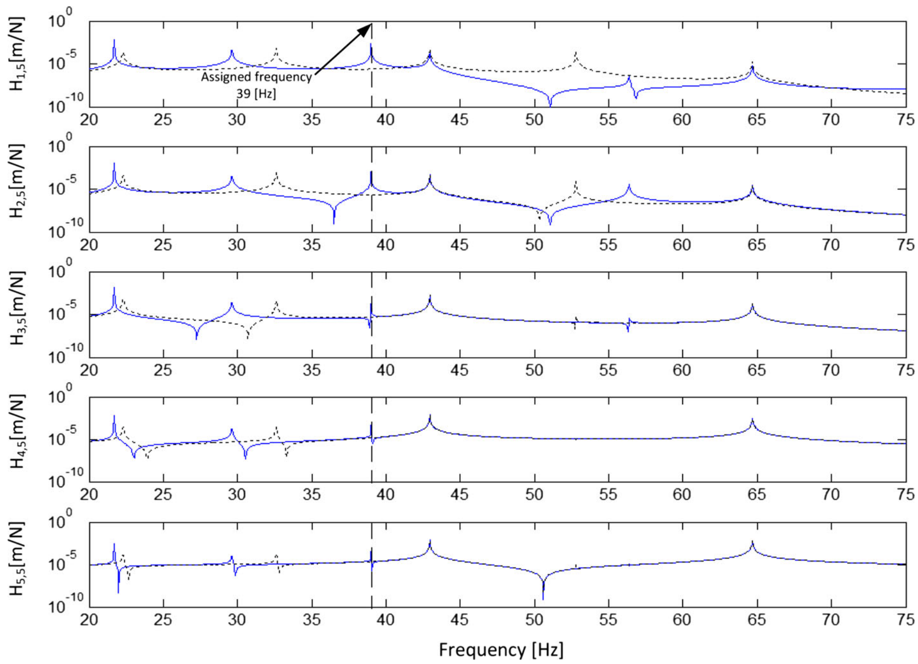

Fig. 4 System FRFs and assigned natural frequency

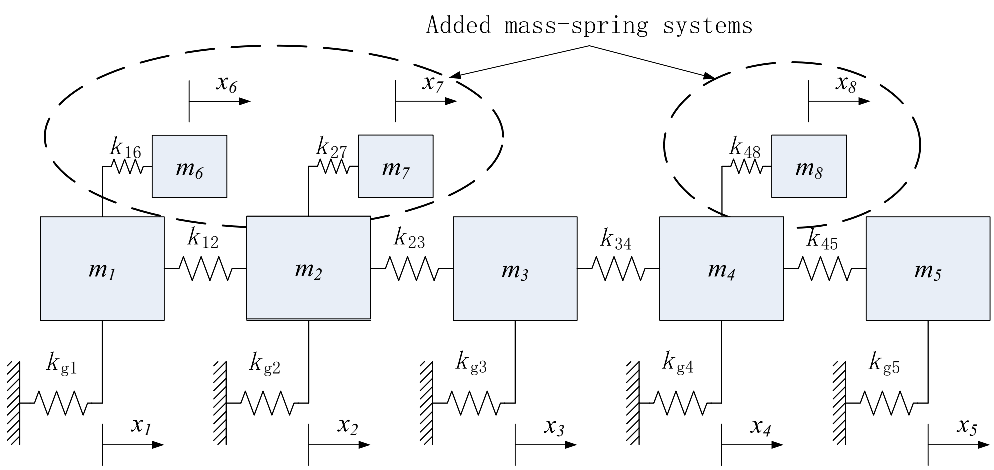

Fig. 5 The modified system by adding three mass–spring subsystems

3.2.2 Adding three mass–spring subsystems

Three mass–spring subsystems (m_6, m_7 and m_8) are employed to modify the vibration system in this section, which are, respectively, connected to masses (m_1, m_2 and m_4) through springs (k_{16}, k_{27} and k_{48}), as shown in Fig. 5. The range of computed parameters is assumed and listed in Table 6.

In Fig. 5, the added three mass–spring subsystems (m_6, m_7 and m_8) are, respectively, added to masses (m_1, m_2 and m_4) through springs (k_{16}, k_{27} and k_{48}).

The parameters of the mass–spring subsystems obtained by applying the modifications are listed in Tables 7 and 8. Figure 6 shows the absolute values of FRFs ($\mathbf{H}_{i,5}(\omega), i = 1, \ldots, 5$) of the systems modified by the

Table 6 Modification bounds

Parameter	Lower bound	Upper bound
Mass (kg)		
m_6	0	2
m_7	0	2
m_8	0	2
Stiffness (kN/m)		
k_{16}	0	300
k_{27}	0	300
k_{48}	0	300

Table 7 Parameters of added mass–spring systems

Mass (kg)	Value (kg)	Stiffness (kN/m)	Value
m_6	0.8194	k_{16}	93.46
m_7	0.6690	k_{27}	64.34
m_8	1.9318	k_{48}	262.85

Table 8 Modified mode shapes and eigenstructure comparison

	Goal	Method BR	Proposed method
f_i (Hz)	39.00	38.99	39.01
$u_i(1)$	1.000	1.000	1.000
$u_i(2)$	−0.550	−0.547	−0.475
$u_i(3)$	0.200	0.192	0.134
$u_i(4)$	0.000	0.055	−0.020
$u_i(5)$	0.050	0.031	−0.016
Desired mode number, h	–	1	1
$\|f_h - f_i\|$ (Hz)	–	0.014	0.010
$\cos(\mathrm{u}_i,\mathrm{u}_h)$	–	0.9987	0.9951

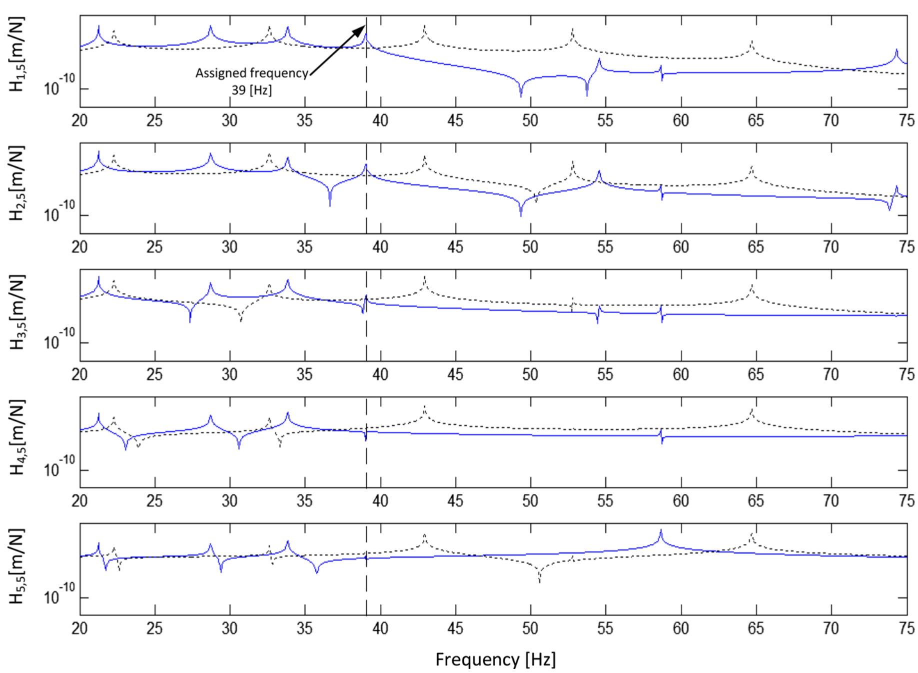

Fig. 6 System FRFs and assigned natural frequency

proposed method (solid line) and the original system (dotted line). The frequencies and modes after modification are collected in Table 8. The cosines between the desired eigenvector and the attained one are shown in the last row of Table 8.

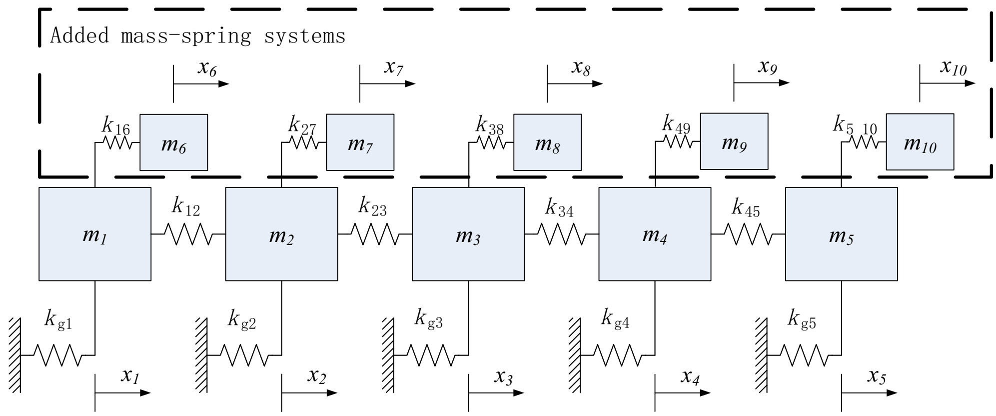

Fig. 7 The modified system by adding five mass–spring subsystems

Table 9 Modification bounds

Parameter	Lower bound	Upper bound
Mass (kg)		
m_6	0	2
m_7	0	2
m_8	0	2
m_9	0	2
m_{10}	0	2
Stiffness (kN/m)		
k_{16}	0	200
k_{27}	0	200
k_{38}	0	200
k_{49}	0	200
$k_{5,10}$	0	200

It is obvious that the modification in this section has better performance in mode assignment when using more mass–spring subsystems in modification. Therefore, it is easier to find better solution through increasing the number of added mass–spring subsystems.

However, it is important to note that the solution of this problem relies on the solution strategy chosen. In another word, it does not mean the more mass–spring subsystems, and the better performance would be achieved for modification. It is difficult to get a group of commendable solutions by some solution methods, especially when dealing with many unknown parameters. Thus, a compromise must be considered in practice.

3.3 Modification for two frequencies and modes

In order to further assess the effectiveness of the method proposed, five mass–spring subsystems ($m_i, i = 6, \ldots, 10$) are added to modify the original 5-dof system in this section, for assigning the same desired frequencies and corresponding modes as [14], as shown in Fig. 7. The added masses are connected through springs ($k_{mn}, m = 1, \ldots, 5; n = m + 5$). The upper and lower bounds of the mass–spring subsystems are listed in Table 9. Factors such as the economy and feasibility of the modifications needed are considered.

It is important to note that the solutions of inverse problems may not exist or not be unique. Therefore, a different original system may need a different number of added mass–spring subsystems.

Five mass–spring subsystems ($m_i, i = 6, \ldots, 10$) are now added to modify the original 5-dof system, connected to masses ($m_i, i = 1, \ldots, 5$), respectively, through springs ($k_{mn}, m = 1, \ldots, 5; n = m + 5$).

In the application of the numerical method described herein, the system FRFs come from inverting matrix $\left[\omega_h^2\mathbf{M} - \mathbf{K}\right]$ at the two targeted natural frequencies (ω_1 and ω_2). In order to treat them equally in the minimisation of Eq. (14), of the two weighting parameters α_1 and α_2 in Eq. (14) are set to 1, which are the same as those used by method BR.

Table 10 Parameters of added mass–spring systems

Mass (kg)	Value (kg)	Stiffness (kN/m)	Value
m_6	0.9044	k_{16}	108.62
m_7	0.7336	k_{27}	93.94
m_8	0.7187	k_{38}	73.46
m_9	0.9962	k_{49}	61.78
m_{10}	1.5344	k_{510}	38.81

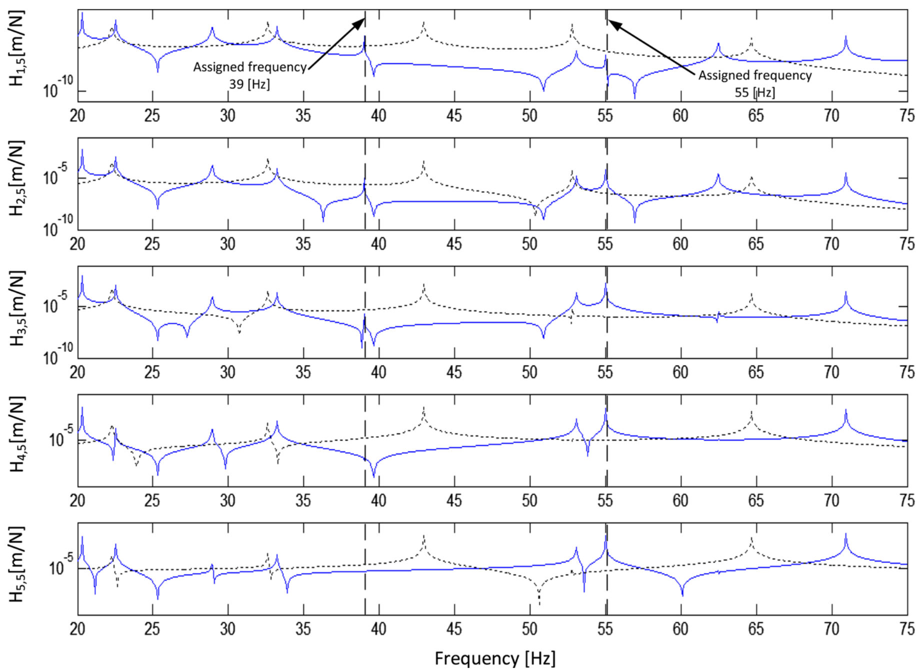

Fig. 8 System FRFs and assigned natural frequencies

Through solving the optimisation problem, the values of the added masses and stiffnesses are shown in Table 10. Using the modification parameters, the attained FRFs (solid line) and the original FRFs (dashed line) are given in Fig. 8. The desired frequencies are marked in the figure.

Except the resonance peaks at desired frequencies (39 and 55 Hz), the modified system has gained five new degrees of freedom, and thus, several other resonance peaks appear within the scope of Fig. 8. Thus, it is important to make sure that the new natural frequencies as a result of the modifications would not cause degradation of the modified system. Often, it may not matter that the vibration amplitude of added mass–spring subsystem is large, as long as the original system has a good performance in engineering practice.

For comparison, the attained natural frequencies and corresponding modes are listed in Table 11. The cosines between the desired modes and the attained ones are shown in the last row of Table 11. The attained modes and desired modes of 39 and 55 Hz are, respectively, shown in Fig. 9a, b.

Through comparing with the results of method BR, apparently the performance of the proposed method is good. The proposed method also possesses the remarkable advantage that there is no need to have the theoretical model of the original structure. In other words, the correctness of the proposed modifications computed depends on the quality of measured FRFs (simulated FRFs in this theoretical paper though), avoiding the need of exact knowledge of mass and stiffness matrices of the original structure. In case that the accuracy of the measured FRFs is not adequate, before performing the inverse structural modification proposed, one must reduce the noise in measurement. Another remarkable benefit of this method is addition of some subsystems on certain points of the original structure, which do not changes the design of the original structure.

Table 11 Modified mode shapes and eigenstructure comparison

	Goal		Method BR		Proposed method	
	1	2	1	2	1	2
f_i (Hz)	39.00	55.00	38.99	54.93	39.00	55.01
$u_i(1)$	1.000	0.000	1.000	−0.003	1.000	0.000
$u_i(2)$	−0.550	0.010	−0.547	0.011	−0.538	0.011
$u_i(3)$	0.200	−0.100	0.192	−0.090	0.104	−0.278
$u_i(4)$	0.000	0.800	0.055	0.799	−0.004	0.864
$u_i(5)$	0.050	1.000	0.031	1.000	−0.002	1.000
Desired mode number, h	–	–	1	2	1	2
$\lvert f_h - f_i \rvert$ (Hz)	–	–	0.014	0.066	0.004	0.005
$\cos(u_i, u_h)$	–	–	0.9987	1.0000	0.9955	0.9910

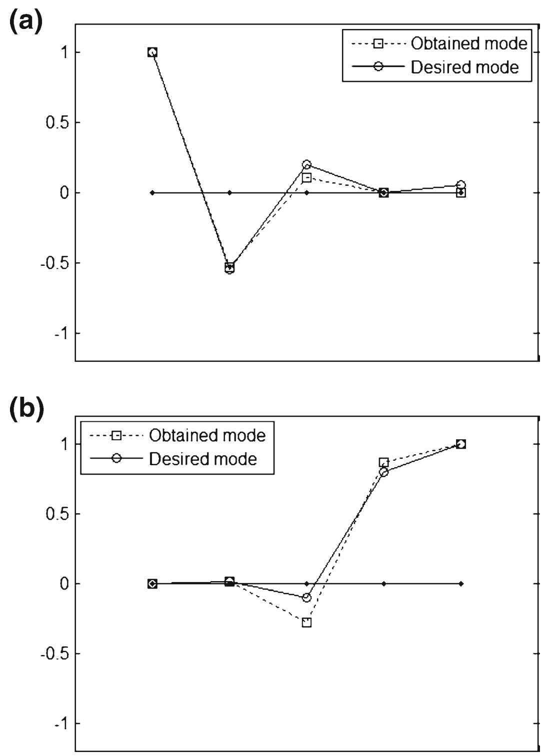

Fig. 9 The desired and the attained modes: **a** 39 Hz, **b** 55 Hz

4 Conclusions

This paper presents a method for structural modifications based on receptances. It achieves the modification by adding multiple mass–spring subsystems to the original structure instead of directly modifying the mass and stiffness values at the existing degrees of freedom of the original structure. This method not only has the benefit that does not need knowledge of original system mass and stiffness matrices, but also maintains the original structure designed for certain specific functions and requirements which should not be modified. The information needed is the measured FRFs of the original structure, which can be fairly easily obtained in practice with sufficient accuracy.

Firstly, the theory of the proposed method is deduced in this paper. Then, numerical simulation for assignment of two frequencies and two modes of a five-dof system is carried out to validate the effectiveness of the method, using various numbers of added masses and springs. A comparison with a certified well-known method by Ram and Braun is also made. It is obvious that the proposed method has very good performance.

Acknowledgments This work is partly funded by the National Natural Science Foundation of China (No. 51375104).

References

1. Li, T., He, J.: Local structural modification using mass and stiffness changes. Eng. Struct. **21**(11), 1028–1037 (1999)
2. He, J., Li, Y.: Relocation of anti-resonances of a vibratory system by local structural changes. Int. J. Anal. Exp. Modal Anal. **10**(4), 224–235 (1995)
3. Tsuei, Y., Yee, E.K.: A method for modifying dynamic properties of undamped mechanical systems. J. Dyn. Syst. Meas. Control **111**(3), 403–408 (1989)
4. Ram, Y., Braun, S.: An inverse problem associated with modification of incomplete dynamic systems. J. Appl. Mech. **58**(1), 233–237 (1991)
5. Joseph, K.: Inverse eigenvalue problem in structural design. AIAA J. **30**(12), 2890–2896 (1992)
6. Olsson, P., Lidström, P.: Inverse structural modification using constraints. J. Sound Vib. **303**(3), 767–779 (2007)
7. Çakar, O.: Mass and stiffness modifications without changing any specified natural frequency of a structure. J. Vib. Control **17**(5), 769–776 (2011)
8. Ouyang, H., Richiedei, D., Trevisani, A., Zanardo, G.: Eigenstructure assignment in undamped vibrating systems: a convex-constrained modification method based on receptances. Mech. Syst. Signal Process. **27**, 397–409 (2012)
9. Datta, B.N.: Numerical Methods for Linear Control Systems: Design and Analysis. Academic Press, London (2004)
10. Xu, S., Qian, J.: Orthogonal basis selection method for robust partial eigenvalue assignment problem in second-order control systems. J. Sound Vib. **317**(1), 1–19 (2008)
11. Farahani, K., Bahai, H.: An inverse strategy for relocation of eigenfrequencies in structural design. Part I: first order approximate solutions. J. Sound Vib. **274**(3), 481–505 (2004)
12. Liangsheng, W.: Direct method of inverse eigenvalue problems for structure redesign. J. Mech. Des. **125**(4), 845–847 (2003)
13. Bucher, I., Braun, S.: The structural modification inverse problem: an exact solution. Mech. Syst. Signal Process. **7**(3), 217–238 (1993)
14. Braun, S., Ram, Y.: Modal modification of vibrating systems: some problems and their solutions. Mech. Syst. Signal Process. **15**(1), 101–119 (2001)
15. Mottershead, J.: Structural modification for the assignment of zeros using measured receptances. J. Appl. Mech. **68**(5), 791–798 (2001)
16. Kyprianou, A., Mottershead, J.E., Ouyang, H.: Structural modification. Part 2: assignment of natural frequencies and antiresonances by an added beam. J. Sound Vib. **284**(1), 267–281 (2005)
17. Ouyang, H.: Prediction and assignment of latent roots of damped asymmetric systems by structural modifications. Mech. Syst. Signal Process. **23**(6), 1920–1930 (2009)
18. Mottershead, J.E., Ram, Y.M.: Receptance method in active vibration control. AIAA J. **45**(3), 562–567 (2007)
19. Park, Y.-H., Park, Y.-S.: Structural modification based on measured frequency response functions: an exact eigenproperties reallocation. J. Sound Vib. **237**(3), 411–426 (2000)
20. Chu, E.: Pole assignment for second-order systems. Mech. Syst. Signal Process. **16**(1), 39–59 (2002)
21. Mermertaş, V., Gürgöze, M.: Preservation of the fundamental natural frequencies of rectangular plates with mass and spring modifications. J. Sound Vib. **276**(1), 440–448 (2004)
22. Kyprianou, A., Mottershead, J.E., Ouyang, H.: Assignment of natural frequencies by an added mass and one or more springs. Mech. Syst. Signal Process. **18**(2), 263–289 (2004)
23. Bagheri, M., Jafari, A., Sadeghifar, M.: A genetic algorithm optimization of ring-stiffened cylindrical shells for axial and radial buckling loads. Arch. Appl. Mech. **81**(11), 1639–1649 (2011)
24. Nicknam, A., Hosseini, M.: Structural damage localization and evaluation based on modal data via a new evolutionary algorithm. Arch. Appl. Mech. **82**(2), 191–203 (2012)
25. Ouyang, H., Zhang, J.: Passive modifications for partial assignment of natural frequencies of mass–spring systems. Mech. Syst. Signal Process. **50**, 214–226 (2015)