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SH-waves in viscoelastic heterogeneous layer over half-space with self-weight

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Abstract The effect of gravity, heterogeneity and internal friction on propagation of SH-waves (horizontally polarised shear waves) in viscoelastic layer over a half-space has been studied. Using the method of separation of variables, dispersion equation has been obtained and used to recover the damped velocity of SH-waves. Both the real and imaginary parts of dispersion equation are in well agreement with the classical Love wave equation. It has been observed that heterogeneity of the medium affects the velocity profile of SH-wave significantly. Some other peculiarities have been observed and discussed in our study.

Keywords SH-wave · Viscoelastic · Gravity · Internal friction · Heterogeneity

1 Introduction

The earth is considered to be a layered elastic medium with a variation in density and rigidity in constituent's layers. The study of body waves in a half-space is important to seismologists due to its possible applications in geophysical prospecting and in understanding the cause and estimation of damage due to earthquakes.

In seismological studies, the phenomenon 'liquefaction' denotes a state in which solid deposit of sands inside the ground is transformed into a state of suspension, so that they behave as a viscous liquid. Studies of wave propagation in the earth stratum under loads have been done with assumption that the earth behaves to a first approximation as an ideal elastic or viscoelastic material. The theory of viscoelasticity is of great importance in the broad field of solid mechanics and particularly in seismology, exploration geophysics, etc. Geophysical studies reveal the fact that the interior of earth, similar to the outer, is layered. To study the effect of viscoelasticity in wave propagation, some attempts have been done earlier. Several papers have been published on the propagation of seismic waves in elastic medium with different types of inhomogeneity. Bhattacharya [1] pointed out some possible exact solution of SH-wave equation for inhomogeneous media. Cooper [2], Shaw and Bugl [3], Schoenberg [4], Borcherdt [5], Kaushik and Chopra [6], Gogna and Chander[7] and Romeo [8] have studied the propagation of SH-waves in viscoelastic media. The propagation of waves in a homogeneous viscoelastic layer overlying a viscoelastic medium was studied by Kanai [9].

Lockett [10] discussed the reflection and refraction of waves in viscoelastic materials. Cerveny [11] studied the propagation of SH-waves in viscoelastic media with and without heterogeneity. Chattopadhyay et al. [12]

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studied the propagation of torsional surface waves in heterogeneous anisotropic half-space under initial stress. Wang et al. [13] studied the wave propagation in an inhomogeneous transversely isotropic material obeying the generalized power law model. Roy [14] studied the propagation of SH-wave in laterally heterogeneous medium. The asthenosphere that forms the transition zone between the low dense crust and higher density mantle is viscous in nature, and most of the dynamic earth processes responsible for the earthquake take place in this zone. These together with the basic characteristic of the earth (anisotropy, heterogeneity and gravity) motivate us towards the present study.

This paper studies the effect of gravity, internal friction and heterogeneity on the propagation of SH-waves in a viscoelastic layer. The dispersion equation has been derived and found to be in agreement with the Love wave equation.

2 Formulation and solution of the problem

We consider a medium consisting of a viscoelastic layer of thickness H lying over a half- space with self-weight, which generates the initial hydrostatic stress $S'_{11} = S'_{33} = -\rho_2 gz$. Rectangular coordinates with the origin at the interface and the z-axis towards the interior of the elastic half-space have been used. The interface between the layer and half-space is given by $z = 0$, and the upper boundary may be described as $z = -H$. Let ρ, μ and η be the density, elastic constant and viscosity of the layer, respectively.

2.1 Solution for layer

For the heterogeneity of the layer, we have considered that the properties of the medium change only in z-direction.

For SH-wave propagating in the x-direction and causing displacement in the y-direction only, we shall assume that

$$u_1 = w_1 = 0, \quad v_1 = v_1 (x, z, t) \quad \text{and} \quad \frac{\partial}{\partial y} \equiv 0. \tag{1}$$

The only non-vanishing equation of motion in the absence of body force due to above assumption [15] is given as

$$\frac{\partial}{\partial x} p_{xy} + \frac{\partial}{\partial z} p_{yz} = \rho \frac{\partial^2 v_1}{\partial t^2} \tag{2}$$

where

$$p_{xy} = \left(\mu + \eta \frac{\partial}{\partial t}\right) \frac{\partial v_1}{\partial x} \quad \text{and} \quad p_{yz} = \left(\mu + \eta \frac{\partial}{\partial t}\right) \frac{\partial v_1}{\partial z}. \tag{3}$$

In the upper layer, ρ, μ and η are assumed to be function of depth only and are given by (Bhattacharya [1])

$$\begin{aligned} &\mu = \mu_0 (1 - \sin \alpha z) \\ &\eta = \eta_0 (1 - \sin \alpha z) \\ &\text{and} \\ &\rho = \rho_0 (1 - \sin \alpha z) \end{aligned} \tag{4}$$

where ρ_0, μ_0 and η_0 are the constant values of ρ, μ and η at the interface, and α is an arbitrary constant having dimension inverse of the length.

For heterogeneous viscoelastic layer, Eq. (2) becomes

$$\left(\mu + \eta \frac{\partial}{\partial t}\right) \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial}{\partial z} \left[\left(\mu + \eta \frac{\partial}{\partial t}\right) \frac{\partial v_1}{\partial z}\right] = \rho \frac{\partial^2 v_1}{\partial t^2}. \tag{5}$$

Assuming that

$$v_1 = V(z) e^{i(\omega t - kx)} \tag{6}$$

corresponding to a wave propagating in the positive x-direction and with amplitude which depends only on depth z, we find from Eq. (5) the following equation for V

$$\frac{d^2V}{dz^2} + \frac{1}{\overline{\mu_1}}\frac{d\overline{\mu_1}}{dz}\frac{dV}{dz} + \left\{\frac{\omega^2\rho}{\overline{\mu_1}} - k^2\right\} V = 0 \tag{7}$$

where

$$\overline{\mu_1} = \mu + i\omega\eta.$$

Now, we take the following substitution

$$V(z) = \frac{Y_1(z)}{\sqrt{\overline{\mu_1}}}$$

the above substitution transforms the Eq. (7) into the form

$$\frac{d^2Y_1}{dz^2} + \left[\frac{1}{4\overline{\mu_1^2}}\left(\frac{d\overline{\mu_1}}{dz}\right)^2 - \frac{1}{2\overline{\mu_1}}\frac{d^2\overline{\mu_1}}{dz^2} + \left(\frac{\omega^2\rho}{\overline{\mu_1}}\right) - k^2\right] Y_1 = 0. \tag{8}$$

Now, with the help of Eq. (4), Eq. (8) becomes

$$\frac{d^2Y_1}{dz^2} + m^2Y_1 = 0 \tag{9}$$

where

$$m^2 = \frac{\alpha^2}{4} + \frac{\omega^2\rho_0}{\overline{\mu_0}} - k^2 \quad \text{and} \quad \overline{\mu_0} = \mu_0 + i\omega\eta_0. \tag{10}$$

The solution of Eq. (9) is

$$Y_1 = A\cos(mz) + B\sin(mz)$$

where A and B are arbitrary constants.

Therefore, we get solution for viscoelastic layer as

$$v_1 = \frac{1}{\sqrt{\overline{\mu_0}}}(1 - \sin\alpha z)^{\frac{-1}{2}}\,[A\cos mz + B\sin mz]\,e^{i(\omega t - kx)}. \tag{11}$$

2.2 Solution for half-space

The initial stresses in the half-space with self-weight are hydrostatic and are given by

$$S'_{11} = S'_{33} = -\rho_2 gz, \quad S'_{22} = S'_{12} = S'_{23} = S'_{31} = 0 \tag{12}$$

where ρ_2 is the density of the half-space, g is the acceleration due to gravity and components of the body forces are $X = 0$, $Y = 0$ and $Z = g$.

The dynamical non-vanishing equations of motion of initially stressed half-space due to gravity is given by Biot [16]

$$\frac{\partial S'_{12}}{\partial x} + \frac{\partial S'_{22}}{\partial y} + \frac{\partial S'_{23}}{\partial z} - \rho_2 g\tilde{\omega}'_{23} - \rho_2 gz\frac{\partial\tilde{\omega}'_{23}}{\partial z} + \rho_2 gz\frac{\partial\tilde{\omega}'_{12}}{\partial x} = \rho_2\frac{\partial^2 v_2}{\partial t^2} \tag{13}$$

where S'_{ij} are the incremental stresses, $\tilde{\omega}'_{ij}$ are rotational components in the half-space and (u_2, v_2, w_2) are the displacement components.

The stress-strain relations under the hydrostatic initial stress are given by

$$S'_{ij} = \lambda_2\overline{e}\delta_{ij} + 2\mu_2 e_{i,j} \tag{14}$$

where $\delta_{ij}, \overline{e}, \lambda_2$ and μ_2 are Kronecker delta, the cubical dilatation and elastic constants of the half-space, respectively, given by

$$2e_{i,j} = u_{i,j} + u_{j,i}, \overline{e} = \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_2}{\partial z} \quad \text{and} \quad \tilde{\omega}'_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}). \tag{15}$$

Now, again with the conditions of SH-wave propagation, we take $v_2 = f_2(z)e^{-i(\omega t - kx)}$ and using Eqs. (14), (15) and (16), Eq. (13) becomes

$$f_2''(z) + \frac{b}{a+bz} f_2'(z) + k^2 \left(\frac{c^2}{a+bz} - 1 \right) f_2(z) = 0 \tag{16}$$

where

$$a = \frac{\mu_2}{\rho_2} = \beta_2^2, \quad b = -\frac{g}{2}. \tag{17}$$

Substituting $f_2(z) = \frac{\psi(z)}{(a+bz)^{1/2}}$ in Eq. (16), we obtain

$$\psi''(z) + \left\{ \frac{b^2}{4(a+bz)^2} + k^2 \left(\frac{c^2}{a+bz} - 1 \right) \right\} \psi(z) = 0 \tag{18}$$

taking $\sigma = -\frac{2k}{b}(a+bz)$ and $s = -\frac{c^2 k}{2b}$ Eq. (18) changes to

$$\psi''(\sigma) + \left\{ -\frac{1}{4} + \frac{s}{\sigma} + \frac{1}{4\sigma^2} \right\} \psi(\sigma) = 0 \tag{19}$$

which is Whittaker's equation.

The solution of the Eq. (19) can be written as

$$\psi(\sigma) = DW_{s,0}(\sigma) + EW_{-s,0}(-\sigma) \tag{20}$$

where $W_{s,0}(\sigma)$ and $W_{-s,0}(-\sigma)$ are Whittaker's functions.

We consider the appropriate solution in view of the condition $\psi \to 0$ as $z \to \infty$

$$\psi(\sigma) = EW_{-s,0}(-\sigma). \tag{21}$$

Hence, we get the solution of Eq. (19) as

$$v_2 = E \left(\frac{2a - gz}{2} \right)^{-\frac{1}{2}} W_{-s,0} \left(-\frac{2k(2a - gz)}{g} \right) e^{-i(\omega t - kx)}. \tag{22}$$

Now, introducing Biot's gravity parameter $G = \frac{\rho_2 g}{\mu_2 k}$, the Eq. (22) may be written as

$$v_2 = E \left(\frac{2a - gz}{2} \right)^{-\frac{1}{2}} W_{-s,0} \left\{ -\left(\frac{4}{G} - 2kz \right) \right\} e^{-i(\omega t - kx)}. \tag{23}$$

Now, using the asymptotic expansion (Whittaker and Watson, [17]) of Whittaker's function for large argument and retaining up to the second term, $W_{-s,0}\left(-\frac{2k(2a-gz)}{g}\right)$ may be approximated as

$$W_{-s,0} \left\{ -\left(\frac{4}{G} - 2kz \right) \right\} \sim e^{-\left(kz - \frac{2}{G}\right)} \left(2kz - \frac{4}{G} \right)^{-s} \left[1 - \frac{(s+0.5)^2}{\left(2kz - \frac{4}{G}\right)} \right]. \tag{24}$$

3 Boundary conditions

For the SH-wave propagation, the following boundary conditions are to be satisfied

(i) The upper layer is stress free, i.e.

$$\left(\mu+\eta\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}=0 \quad \text{at } z=-H. \tag{25}$$

(ii) Displacements are continuous at the interface, i.e.

$$v_1=v_2 \quad \text{at } z=0. \tag{26}$$

(iii) Stresses are continuous at the interface, i.e.

$$\left(\mu+\eta\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}=\mu_2\frac{\partial v_2}{\partial z} \quad \text{at } z=0. \tag{27}$$

Using Eqs. (11) and (24) with above boundary conditions, we get

$$\frac{A}{B}=\frac{\alpha\cos\alpha H\sin mH-2m\left(1+\sin\alpha H\right)\cos mH}{\alpha\cos\alpha H\cos mH+2m\left(1+\sin\alpha H\right)\sin mH} \tag{28}$$

$$\frac{A}{\sqrt{\mu_0}}=ED^* \tag{29}$$

$$\sqrt{\mu_0}\left(\alpha\frac{A}{2}+mB\right)=\mu_2 N^*. \tag{30}$$

Now, eliminating the constants A, B and E from Eqs. (28), (29) and (30), we get the dispersion equation as

$$\begin{aligned}&\alpha^2\cos\alpha H\sin mH-2m\alpha\cos mH\left(1+\sin\alpha H\right)+2m\alpha\cos mH\cos\alpha H+4m^2(1+\sin\alpha H)\sin mH\\&=2\left(\eta_1+i\eta_2\right)\left[\alpha\cos\alpha H\sin mH-2m\left(1+\sin\alpha H\right)\cos mH\right]\frac{N^*}{D^*}.\end{aligned} \tag{31}$$

By separating the real and imaginary parts from Eq. (31), we have finally the dispersion equation as

$$\begin{aligned}&\tan\left(\sqrt{r}H\cos\frac{\theta}{2}\right)\\&=\frac{(1+\sin\alpha H)\left[2\alpha x+8xy\tanh yH-4\eta_1 x\frac{N^*}{D^*}+4\eta_2\frac{N^*}{D^*}\right]-2\alpha x\cos\alpha H-2\alpha\eta_2\tanh yH\cos\alpha H\frac{N^*}{D^*}}{(1+\sin\alpha H)\left[4\left(x^2-y^2\right)-4y\eta_1\tanh yH\frac{N^*}{D^*}-2\alpha y\tanh yH+4x\eta_2\tanh yH\frac{N^*}{D^*}\right]-2\alpha\eta_1\cos\alpha H\frac{N^*}{D^*}+\alpha\cos\alpha H\left[\alpha+2y\tanh yH\right]}\end{aligned} \tag{32}$$

and

$$\begin{aligned}&\tan\left(\sqrt{r}H\cos\frac{\theta}{2}\right)\\&=\frac{(1+\sin\alpha H)\left[2\alpha y-4(x^2-y^2)\tanh yH-4\eta_1 y\frac{N^*}{D^*}-4\eta_2 x\frac{N^*}{D^*}\right]-2\alpha y\cos\alpha H+2\alpha\eta_1\tanh yH\cos\alpha H\frac{N^*}{D^*}-\alpha^2\tanh yH\cos\alpha H}{(1+\sin\alpha H)\left[8xy-4x\eta_1\tanh yH\frac{N^*}{D^*}+2\alpha x\tanh yH-4y\eta_2\tanh yH\frac{N^*}{D^*}\right]-2\alpha\eta_2\cos\alpha H\frac{N^*}{D^*}-2\alpha\cos\alpha H\tanh yH}\end{aligned} \tag{33}$$

where

$$m=\left(re^{i\theta}\right)^{\frac{1}{2}},$$

$$x=\sqrt{r}\cos\frac{\theta}{2},\quad y=\sqrt{r}\sin\frac{\theta}{2},$$

$$r\cos\theta=\frac{\alpha^2}{4}-k^2+\rho_0\frac{\mu_0\omega^2}{\left(\mu_0^2+\eta_0^2\omega^2\right)},$$

$$r\sin\theta = -\rho_0 \frac{\eta_0\omega^3}{(\mu_0^2+\eta_0^2\omega^2)},$$

$$\frac{N^*}{D^*} = \frac{\frac{d}{\mathrm{d}z}\left(\left(\frac{2a-gz}{2}\right)^{-\frac{1}{2}} W_{-s,0}\left(-\frac{2k(2a-gz)}{g}\right) e^{-i(\omega t-kx)}\right)\Big\}_{z=0}}{\left(\left(\frac{2a-gz}{2}\right)^{-\frac{1}{2}} W_{-s,0}\left(-\frac{2k(2a-gz)}{g}\right) e^{-i(\omega t-kx)}\right)\Big\}_{z=0}},$$

$$\eta_1 = \frac{\mu_0}{(\mu_0^2+\eta_0^2\omega^2)}$$

and

$$\eta_2 = -\frac{\omega\eta_0}{(\mu_0^2+\eta_0^2\omega^2)}.$$

4 Particular cases

Considering $\alpha = 0$, $\eta_0 = 0$ and neglecting the higher-order terms in the expansion of Whittaker's function, Eqs. (32) and (33) reduce to standard Love wave equation

$$\tan\left[kH\left\{\frac{c^2}{\beta_1^2}-1\right\}^{1/2}\right] = \frac{\mu_2\left\{1-\frac{c^2}{\beta_2^2}\right\}^{1/2}}{\mu_0\left\{\frac{c^2}{\beta_1^2}-1\right\}^{1/2}}$$

where

$$\beta_1^2 = \frac{\mu_0}{\rho_0}.$$

5 Numerical results and discussion

We have obtained the phase velocity and damping coefficient of SH-wave from Eq. (31). The following data have been used for viscoelastic layer and gravitational half-space (Fig. 1).

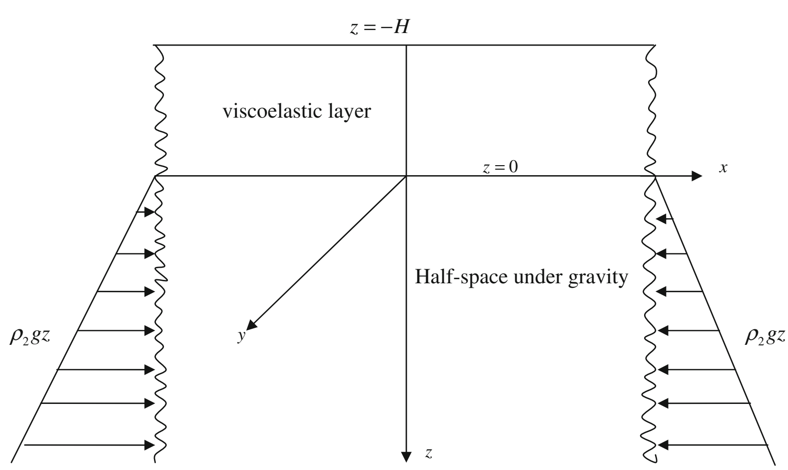

Fig. 1 Geometry of the problem

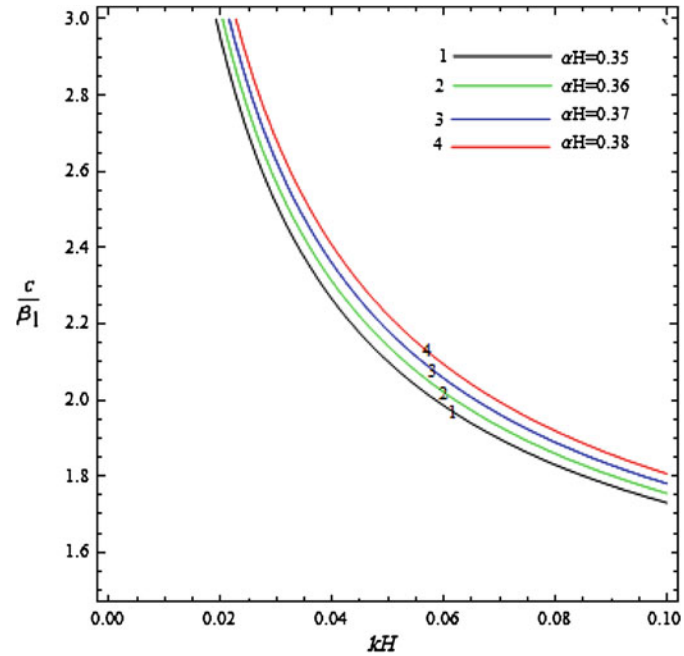

Fig. 2 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH) for different values of inhomogeneity parameter (αH)

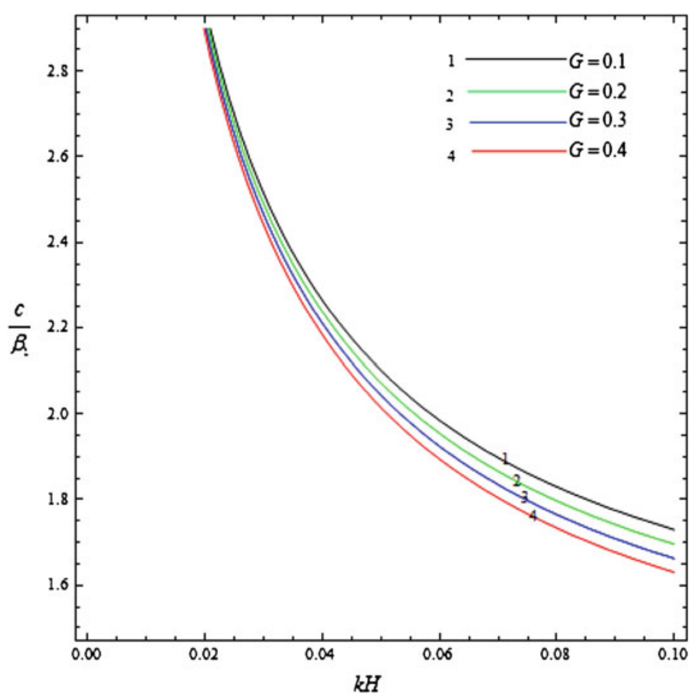

Fig. 3 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH) for different values of Biot gravity parameter G

(i) Viscoelastic layer [18]

$$\rho_0 = 3323 \frac{\text{kg}}{\text{m}^3}, \quad \mu_0 = 6.77 \times 10^{10} \frac{\text{N}}{\text{m}^2}, \quad \frac{\mu_0}{\eta_0} = 50\,\text{s}^{-1}$$

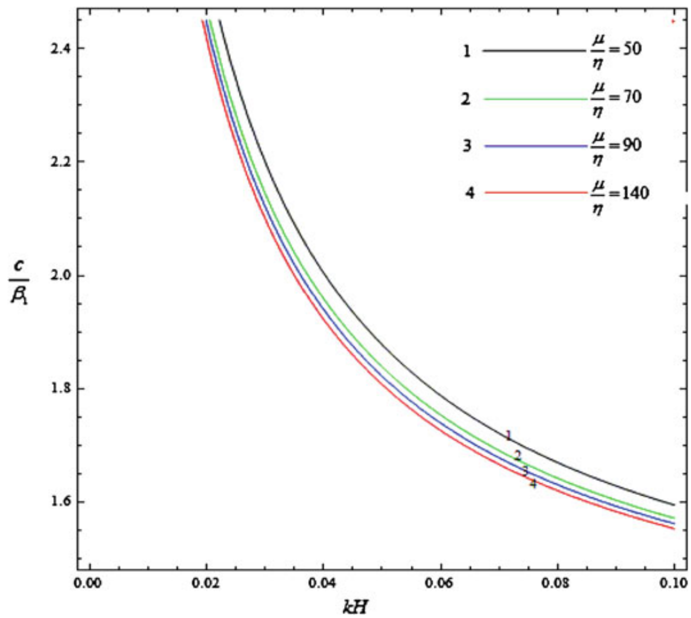

Fig. 4 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH) for different values of internal friction (μ/η)

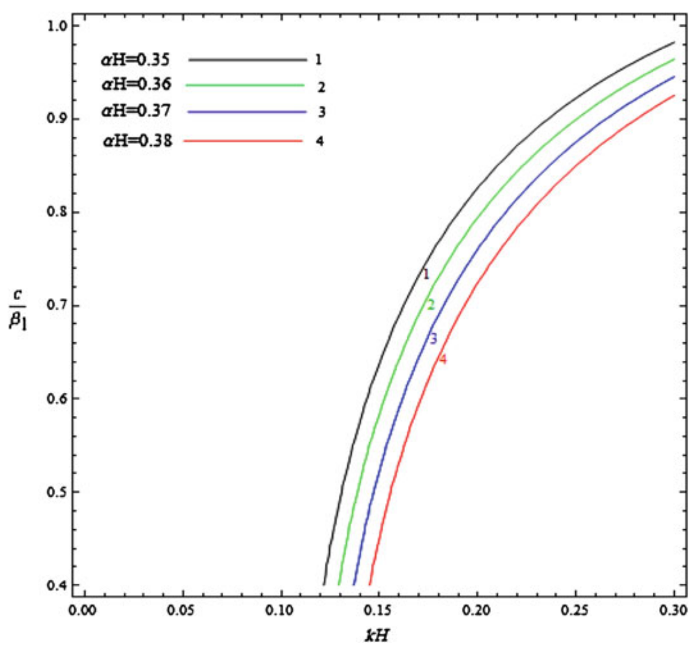

Fig. 5 Variation in dimensionless damping velocity (c/β_1) against dimensionless wave number (kH) for different values of inhomogeneity parameter (αH)

(ii) Half-space under gravity [19]

$$\rho_2 = 2.72 \times 10^3 \frac{\text{kg}}{\text{m}^3}, \quad \mu_2 = 4.53 \times 10^{10} \frac{\text{N}}{\text{m}^2}$$

Using the above numerical data and Eqs. (32) and (33), we have the following graphs.

From Fig. 2, we conclude that increment in inhomogeneity parameter increases the phase velocity. By Figs. 3 and 4, we observe that increment in gravity and internal friction decreases the phase velocity of SH-wave. In Figs. 5 and 6, it can be observed that damping velocity of the wave increases with the increase in wave

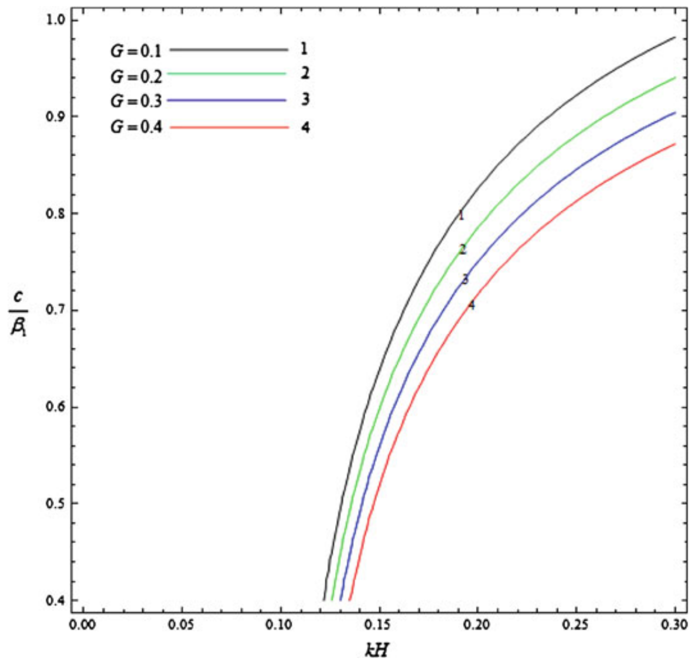

Fig. 6 Variation in dimensionless damping velocity (c/β_1) against dimensionless wave number (kH) for different values of Biot gravity parameter

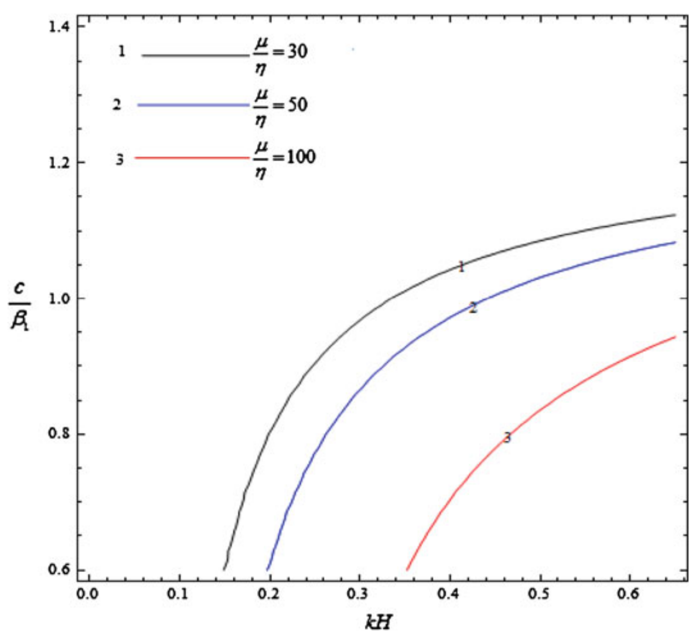

Fig. 7 Variation in dimensionless damping velocity (c/β_1) against dimensionless wave number (kH) for different values of internal friction (μ/η)

number. It is also observed that the inhomogeneity and gravity decrease the damping velocity of SH-wave. From Fig. 7, it is clear that damping velocity of the wave increases with the increase in wave number; then, there is a decrease in phase velocity to a greater extent for the value of internal friction $\mu/\eta = 50S^{-1}$. Keeping in mind the dependence of phase velocity (c/β_1) on wave number(kH), surface plot of phase velocity against varying wave number, heterogeneity parameter and gravity has been shown in Figs. 8 and 9, respectively.

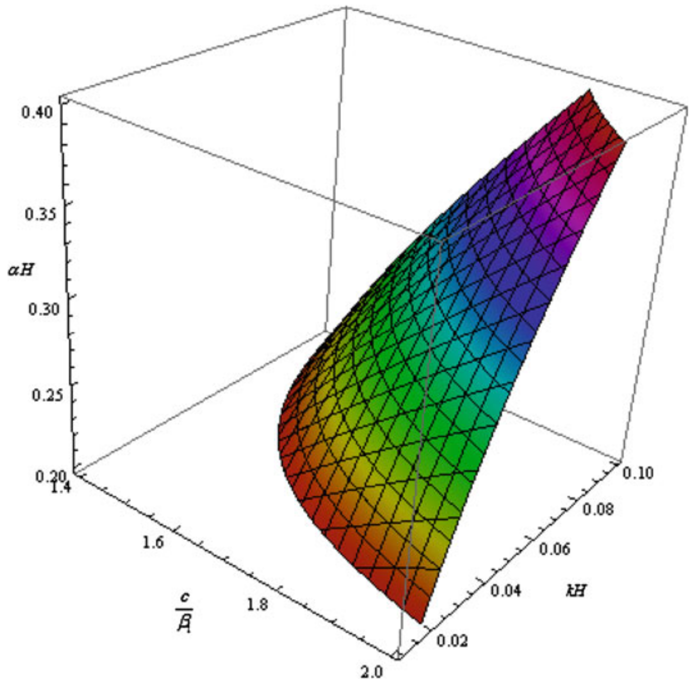

Fig. 8 Variation in dimensionless phase velocity (c/β_1) with respect to dimensionless wave number (kH) and inhomogeneity parameter (αH)

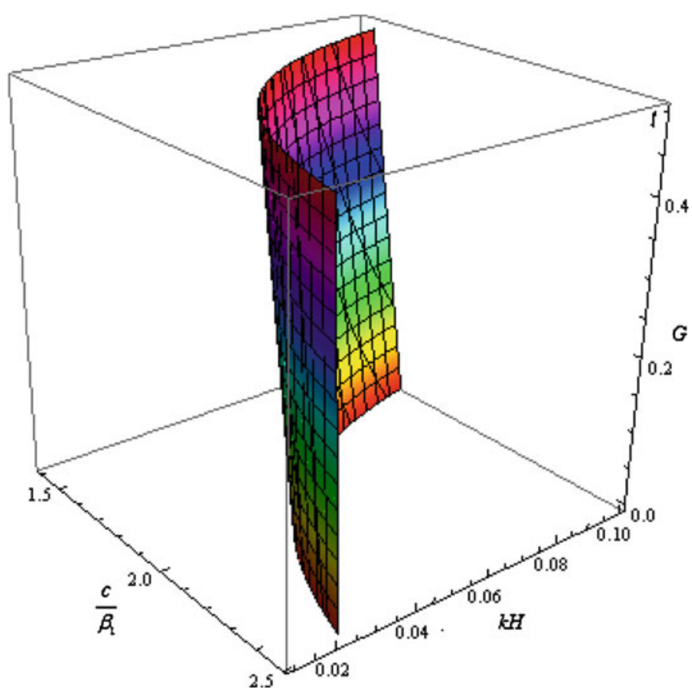

Fig. 9 Variation in dimensionless phase velocity (c/β_1) with respect to dimensionless wave number (kH) and garvity parameter G

6 Conclusions

It is found that the heterogeneity, gravity and internal friction have significant effect on the propagation of SH-waves in viscoelastic layer over half-space with self-weight. The dispersion equation has been obtained, which coincides with the classical result of Love wave when the initial stresses and inhomogeneity parameters are neglected. Graphical representations reveal that inhomogeneity of the medium increases the phase velocity, whereas increment of gravity and internal friction decreases the phase velocity of SH-waves. We observed that increment in inhomogeneity and gravity decreases the damping velocity of SH-waves, whereas internal friction decreases the damping velocity up to greater extent. This study may have possible applications in the field of seismology and geophysics.

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