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ORIGINAL

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The mechanical behaviour of rubber under hydrostatic compression and the effect on the results of finite element analyses

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Abstract Using finite element tools for the dimensioning of rubber components is state of the art. When conducting finite element simulations, a split of the strain energy function, which results in stresses, into a deviatoric and a volumetric part is made. The mechanical behaviour of reinforced natural rubber under hydrostatic pressure is determined to prove the validity of this assumption. Furthermore, a nearly incompressible material behaviour is assumed in simulations of rubber, which may cause an insufficient outcome quality especially for rubber components that are exposed to hydrostatic pressure like highly confined bushings. In this paper, a method is presented to determine the compressibility, or its reciprocal the bulk modulus of rubber. The effect of the bulk modulus of a natural rubber on the simulation results of a bearing is pointed out. The obtained results are compared to test data to show the significance of the value of the bulk modulus for achieving a satisfactory outcome quality. Therefore, reliable information about the in-use behaviour of rubber components is obtained to reduce the costs and the effort in the dimensioning process.

Keywords Rubber · Incompressibility · Hydrostatic pressure · Bulk modulus · Finite element simulation

1 Introduction

The simulation and the dimensioning of rubber components by using finite element (FE) tools is state of the art in industry and science. In commonly used simulation tools, a various number of implemented material models are provided to simulate the hyperelastic material behaviour of these components. These models are based on either a strain energy density function, for example, Mooney-Rivlin [1], Ogden [2], or molecular-statistical approaches, for example, Arruda-Boyce [3], Kilian [4]. Stress–strain relations obtained by mechanical tests are normally used to calibrate these models by assuming a perfectly incompressible behaviour even for carbon black–filled technical rubber compounds. This assumption is equal to an approximately infinite or at least a very large bulk modulus, compared to the shear modulus, or its reciprocal, nearly zero compressibility (see Ogden [8]). It will be worked out in this contribution that the bulk modulus is an important material parameter to optimise the quality of the simulation results achieved by hyperelastic material models in finite element analyses. Figure 1 shows a typical dependence of pressure on volume ratio for a rubber component as it is reported, for example, by Adams and Gibson [9]. The bulk modulus corresponds to the slope of the graph. In practice, the dependence of the bulk modulus on the value of the hydrostatic pressure is normally not available. Therefore, simulations are usually carried out by the assumption of incompressible material behaviour or by

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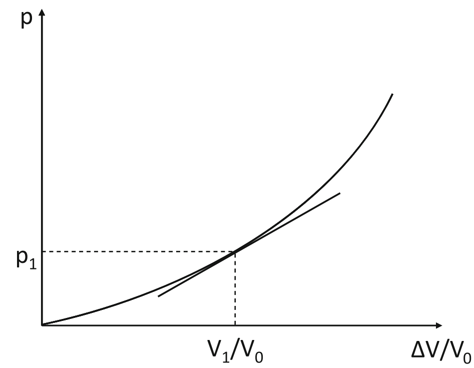

Fig. 1 Compression behaviour of a rubber component

Fig. 2 Example of a bushing used in automotive vehicles (Vorwerk Automotive GmbH & Co KG)

setting the bulk modulus K to a constant value which is several orders of magnitude higher than the applied shear modulus. This method offers a satisfactory outcome quality for most of the technical applications rubber is used in, since they are not exposed to higher hydrostatic pressure values up to several 100 bar.

This simplified assumption is no longer valid in case of rubber components under hydrostatic pressure. This hydrostatic pressure state sets up, for example, in highly confined rubber components such as embedded mounts or bushings (see Fig. 2). The large fraction of rubber surface bonded to metal restrains the lateral movements of rubber under load and leads to the build-up and increase in hydrostatic pressure. Similar problems occur for rubber components under pressure loads like sealing. This led to the development of several volumetric strain energy functions that are summarised and extended by a new proposal by Hartmann and Neff [5]. Furthermore, Le Cam [7] points out the significance of the change in volume during deformation for characterising rubber components. Therefore, a large deviation between simulation results and the components' real life behaviour of characteristics like material stiffness dependent on the approximated bulk modulus is obtained for the components mentioned above.

Besides the bulk modulus, the mechanical behaviour of rubber components under hydrostatic pressure plays an important role in many applications like sealing or bearings. In Fig. 3, the simulation results of a unidirectional compression and tensile test for different values of the bulk modulus are presented and compared to the results obtained by assuming incompressibility. There is no significant deviation in this case between the resulting stresses and hence no dependence on the bulk modulus. But when conducting the same simulation on specimens under hydrostatic pressure, the results deviate considerably despite the fact that every chosen value of the bulk modulus fulfils the condition of being orders of magnitude higher than the shear modulus. Figure 4 shows exemplarily the dependence of the nominal stress on the bulk modulus for a natural rubber (NR) in a tensile and compression test on a rubber component under a constant hydrostatic pressure of 300 bar. These examples show the importance of the bulk modulus in simulating rubber components that are partially

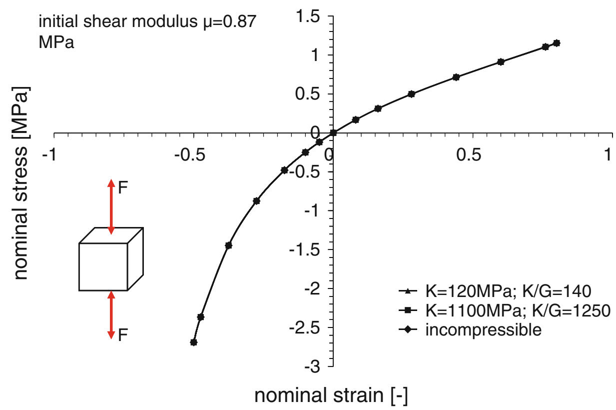

Fig. 3 Dependence of simulation result for tensile stress on bulk modulus

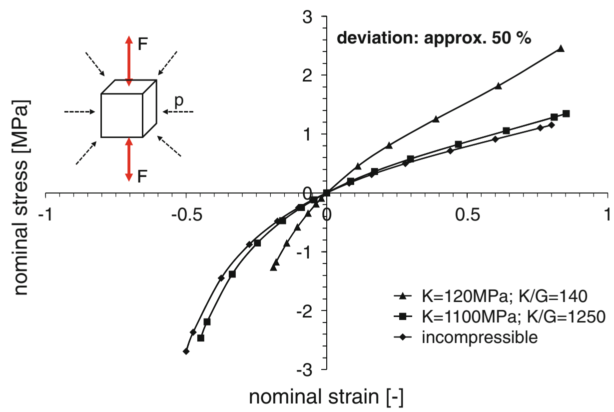

Fig. 4 Dependence of simulation result for tensile stress on bulk modulus under hydrostatic compression

or completely exposed to hydrostatic pressure. The exact knowledge of the material's bulk modulus at different hydrostatic pressure values is essential to achieve a satisfactory outcome quality of the FE simulations to avoid or at least reduce the effort of elaborate tests under real circumstances.

Furthermore, when conducting FE simulations of rubber components, a second assumption is usually made. Material models based on a strain energy density function assume a split of the respective function into two parts reflecting the deviatoric, incompressible deformation that covers the isochoric change in shape due to the applied load, and the volume changing deformation dependent on the hydrostatic pressure. This results in the split of the stress as the derivation of the strain energy function.

$$W = W^{dev}\left(\bar{I}_1, \bar{I}_2\right) + W^{vol}(J) \tag{1}$$

$$\boldsymbol{\sigma} = S - pI \tag{2}$$

with

$$\bar{I}_1 = \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2 \tag{3}$$

$$\bar{I}_2 = \bar{\lambda}_1^2\bar{\lambda}_2^2 + \bar{\lambda}_1^2\bar{\lambda}_3^2 + \bar{\lambda}_2^2\bar{\lambda}_3^2 \tag{4}$$

$$\bar{\lambda}_i = J^{-1/3}\lambda_i \tag{5}$$

$$J = \det F = \sqrt{\lambda_1^2\lambda_2^2\lambda_3^2} \tag{6}$$

$\bar{I}_i$: principal invariants of the deviatoric Cauchy–Green tensor F: deformation gradient; λ: principal stretch; S: deviatoric stress; p: hydrostatic pressure; I: Identity tensor.

In the present paper, the validity of these assumptions is examined. The test bench introduced in [12] enables the determination of the compression behaviour of rubber components for a hydrostatic pressure range from 1 to 300 bar. Furthermore, tensile tests under hydrostatic pressure can be conducted.

2 Test bench

A direct measurement of the change in pressure due to a reduction in volume of a rubber specimen is used to determine the rubber's bulk modulus. Therefore, the specimen is placed in a metal cylinder filled with degassed water. The increase in pressure is realised by a reduction in volume due to screwing in a spindle of a choke valve. The bulk modulus can be determined for a hydrostatic pressure range from 1 to 300 bar by this test set-up. The test specimen is cylindrically shaped with a volume fraction of rubber compared to the total chamber of 0.2.

In Fig. 5, the test bench for conducting tensile tests under hydrostatic pressure is shown. The specimen's elongation is applied by another spindle in the head and a nut, which implicitly gives a measure of the axial stretch due to the known thread pitch. The resulting force can be measured indirectly via a compression spring combined with a path sensor. The known spring stiffness enables the determination of the resulting tensile stress. The specimen is held by one clamp on each of its ends, so that homogenous deformation states are achieved. The strain rate and the maximum elongation and therefore the applied strain can be varied.

3 Results

In the following, the obtained results of the tensile tests under hydrostatic pressure and the hydrostatic compression tests are presented. The tests were conducted for a technical NR with a hardness of 56 Shore A. The achieved results of the tensile tests are compared to the simulation results for different values of the bulk modulus. Furthermore, simulations of a bushing were conducted and compared to test data, so that the dependence of these results on the choice of the bulk modulus is carried out. The behaviour of the test bench (Fig. 5) without a specimen shows no relaxation over a period of time one order of magnitude higher than the test time. This is due to the applied materials. There are only two small rubber gaskets, whereas the rest of the design consists of metal. In combination with the degassed water, this leads to the observed negligible relaxation behaviour in this case.

3.1 Tensile tests

The tensile behaviour of rubber under hydrostatic pressure is analysed. The test specimens used are of 2 mm thickness with a cross-section area of 16 mm^2 (see Fig. 5). For this type of specimen, tests were conducted at hydrostatic pressure values of 1, 100, 200 and 300 bar.

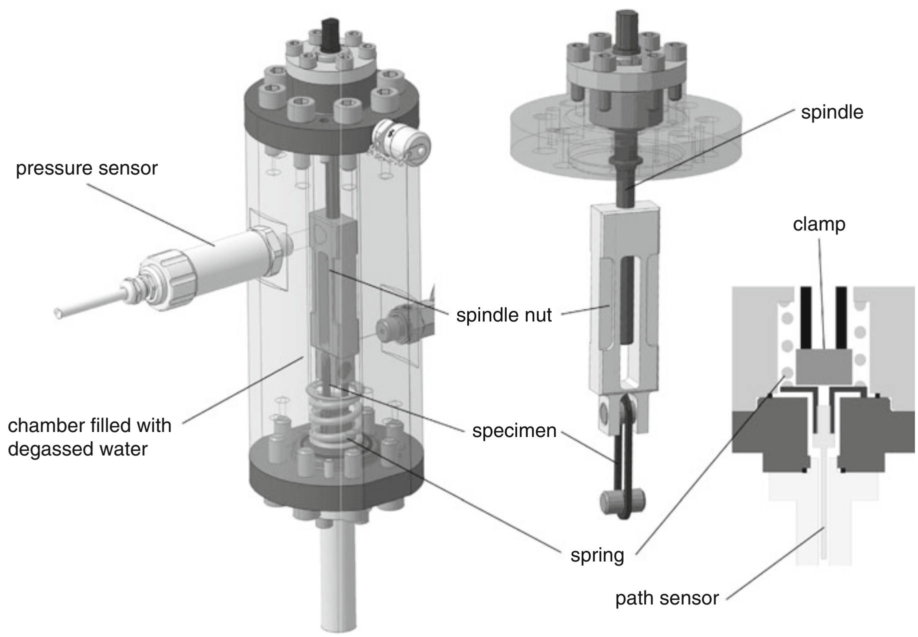

Fig. 5 Test bench for conducting tensile tests under hydrostatic pressure

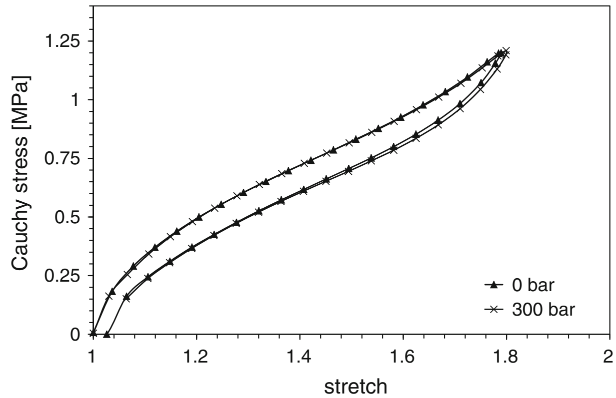

Fig. 6 Comparison of nominal stress for two different states of hydrostatic pressure (Δp)

The presented test set-up does not yet enable the determination of the viscoelastic material behaviour which is shown for rubber, for example, by Haupt and Sedlan [14]. Therefore, the tests were conducted with a low strain rate to achieve results, which allow comparative conclusions for the influence of the hydrostatic pressure on the rubber behaviour. Comparative conclusions can be drawn, when the stress is corrected by the volumetric strain due to the hydrostatic pressure of 300 bar (J determined by Eq. (5) and the results presented in chapter 3.2).

$$S = S_{300\text{bar}} \cdot J^{2/3} \tag{7}$$

The influence of the elongation of a rubber component on the change in volume is summarised by LeCam [7] or Horgan and Murphy [6]. There, it is stated that for lower stretch values, these effects are not that intense. Since within the measurement accuracy of this test bench there is no deviation in pressure detected during the elongation, which matches the statements in [7,6], the tensile behaviour up to the regarded strain of 80 % is assumed to be incompressible. This enables the determination of the Cauchy stress as follows:

$$\boldsymbol{\sigma} = \boldsymbol{\lambda} \cdot S \tag{8}$$

with

$$\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}, \boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_3 = \frac{1}{\sqrt{\boldsymbol{\lambda}}}, J = \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2 \cdot \boldsymbol{\lambda}_3 \tag{9}$$

Figure 6 shows the resulting Cauchy stress component in loading direction of the fourth load cycle. The results for hydrostatic pressure values of 1 and 300 bar are exemplarily represented. All tests conducted show approximately the same material behaviour. The maximum deviation of the curves is about 1 % in the relaxing path, which is even lower than the standard deviation of 1.4 %. The results confirm the assumption that there is no apparent influence of hydrostatic pressure up to 300 bar on the deviatoric behaviour of an NR for stretches up to 80 %. Since the influence of hydrostatic pressure on the resulting stress has proved negligible, the split of the strain energy function is valid for the investigated NR.

3.2 Hydrostatic compression tests

Along with the tensile tests, hydrostatic compression tests were performed, where $\lambda_1 = \lambda_2 = \lambda_3 < 1$ [11]. These tests were conducted with the same NR compound as used in the tensile tests. The specimen is cylindrically shaped with a length of 100 and 18 mm in diameter. In Fig. 7, the result for the tested rubber specimen corrected by the test bench compliance is shown. The standard deviation for the tested NR is 1 %. The bulk modulus of the tested rubber specimen corresponds to the slope of the displayed graph. In the beginning of the hydrostatic compression, for volume ratios up to 0.0005, the test data show a slight increase in curvature. Due to the low pressure values, this results in a approximated bulk modulus, which corresponds to the initial value of the gradient, of

$$K = \frac{\partial p}{\partial\left(\boldsymbol{\Delta} V/V_0\right)} \approx 120 \text{ MPa} \tag{10}$$

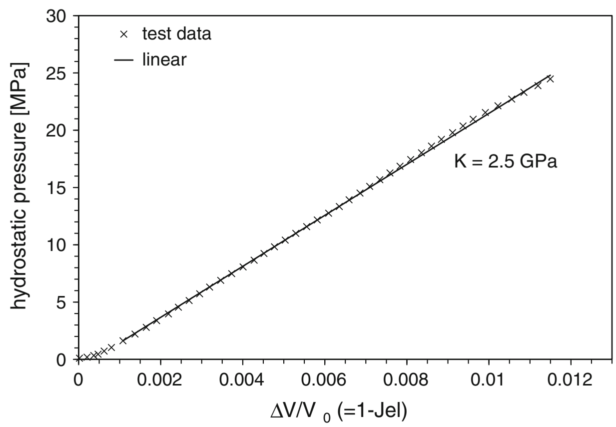

Fig. 7 Results of hydrostatic compression testing

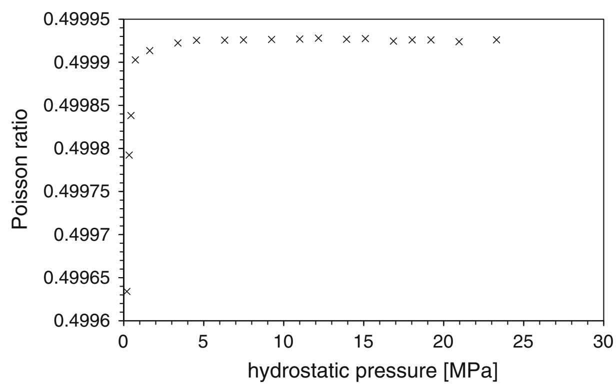

Fig. 8 Poisson ratio determined by the hydrostatic compression test

where

$$\mathbf{\Delta} V/V_0 = 1 - J \tag{11}$$

For volume ratios greater than 0.001, there is an approximately linear relationship with pressure. The resulting bulk modulus is $K = 2.5$ GPa, which can be found in the literature as well (see for example [11]). A linear relationship between the volume change and the pressure is reported, for example, in [10], which is found to be incorrect for lower compression values.

Several further tests by using other types of rubber and shore hardness show the same effect, a slight increase in curvature followed by a nearly linear relationship between volume change and hydrostatic pressure for higher volume ratios. There is an increase in bulk modulus of about one order of magnitude. Thus, it is evident that the bulk modulus is an important feature to influence the outcome quality of FE simulations especially for rubber components that are partially or completely exposed to hydrostatic pressure. In many commonly implemented material models in FE tools, the bulk modulus is a fixed number. The nonlinear behaviour of the pressure dependent on J is determined by the volumetric part of the strain energy function (see chapter 3.3). To achieve a satisfactory result quality by assuming a constant bulk modulus, a least-square method for the first part of the curve up to a hydrostatic pressure value of 15 MPa leads to a first approximation of a bulk modulus of 600 MPa. This is the technical relevant range of pressure for most applications, because most of these rubber components are just partially exposed to high hydrostatic pressure values.

Since in the hydrostatic compression test only very small uniform strains occur in contrast to mechanical tests (e.g. uniaxial tensile testing), the material behaviour under hydrostatic compression enables the approximation of the Poisson ratio (see Fig. 8) according to Krevelen [13] as follows

$$\nu = \frac{3K - 2G}{6K - 2G}. \tag{12}$$

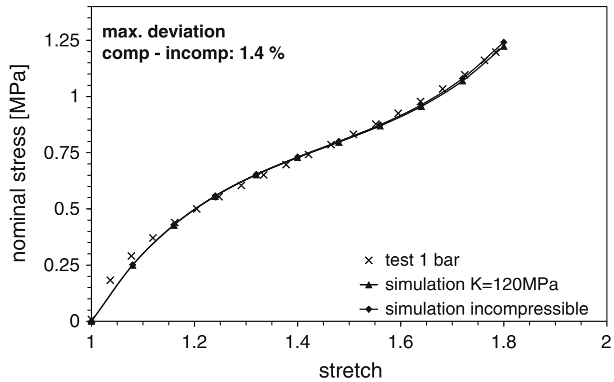

Fig. 9 Comparison of simulation results (incompressible and compressible) with tensile test results

The shear modulus G (here: 1.08 MPa) is obtained by unidirectional tensile and compression tests. For hydrostatic pressure values greater than 4 MPa, the Poisson ratio is nearly constant at 0.49992, which again clearly indicates the compressibility of the rubber component.

3.3 Comparison of test results with FE simulations

In the following, the effects of these results on the quality of FE simulations with Abaqus FEA 6.10 are determined. The FE model used is based on Kilian [4] with the strain energy function formulated as follows. The material parameters for the deviatoric part of the strain energy function were determined by fitting them to the results of tensile and compression tests.

$$W = \underbrace{\boldsymbol{\mu} \left\{ -\left(\boldsymbol{\lambda}_m^2 - 3\right) [\ln(1-\boldsymbol{\eta}) + \boldsymbol{\eta}] - 2/3\, \boldsymbol{\alpha} \left(\frac{\tilde{I}-3}{2}\right)^{\frac{3}{2}} \right\}}_{W^{dev}} + \underbrace{\frac{K}{2}\left(\frac{J_{el}^2 - 1}{2} - \ln J_{el}\right)}_{W^{vol}} \tag{13}$$

with

$$\boldsymbol{\eta} = \sqrt{\frac{\tilde{I}-3}{\boldsymbol{\lambda}_m^2 - 3}} \tag{14}$$

$$\tilde{I} = (1-\boldsymbol{\beta})\,\bar{I}_1 + \boldsymbol{\beta}\bar{I}_2 \tag{15}$$

$\mu = 1.08$ (initial shear modulus); $\lambda_m = 5.92$ (locking stretch); $\alpha = 0.18$ (global interaction parameter); $\beta = 0$ (linear mixture parameter); $\bar{I}_i$ = invariants of the Cauchy–Green tensor; J = total volume change

3.4 Comparison of tensile tests

The influence of the bulk modulus on the tensile behaviour of rubber components in FE simulations based on the material model shown in Eq. (13) is determined, and the achieved results are compared to the test data. A simple tensile test under normal pressure is simulated using two different settings. At first, the rubber is assumed to be incompressible, which means the volumetric part of the strain energy function is neglected. The second simulation is conducted with an assumed bulk modulus of 120 MPa, which corresponds to the first range in Fig. 7. As shown in Fig. 9, the deviation between both the simulation results and the test data is negligible. That means, the assumption of incompressibility is valid, provided that the hydrostatic pressure values are sufficiently low. In this case, there is no significant influence of the bulk modulus on the tensile behaviour of these rubber components. The limitation for this assumption depends on the rubber used and its behaviour under hydrostatic compression as shown in Fig. 7.

In a second step, according to the test conducted, simulations on a rubber component under a hydrostatic pressure of 300 bar are carried out. Viscoelastic effects are neglected in this comparison. Instead of assuming an

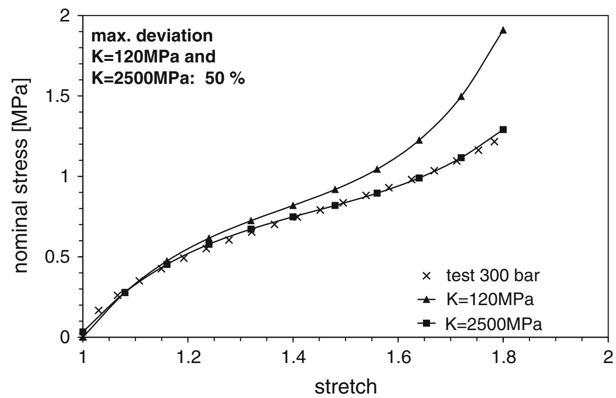

Fig. 10 Comparison of simulation results with different value of K and tensile test results under 300 bar

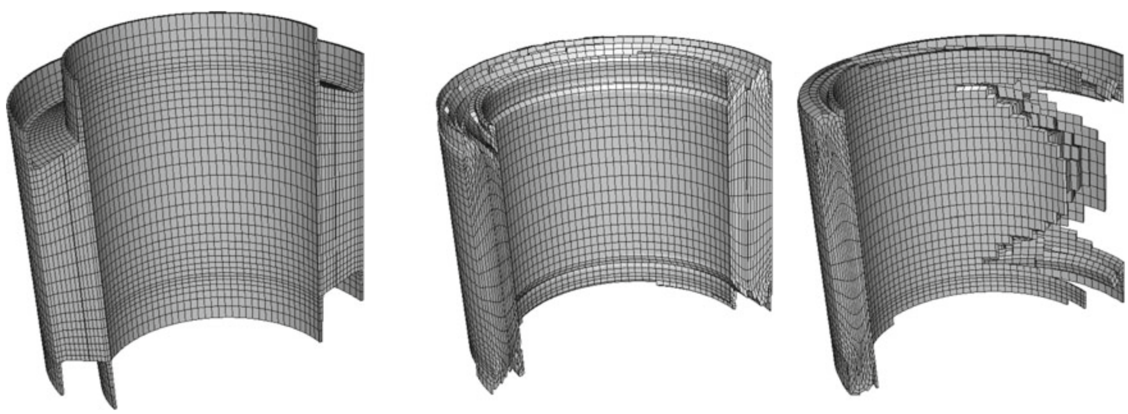

Fig. 11 Cross-sections of a finite element model of a bushing, elements exposed to a hydrostatic pressure exceeding 2 MPa, middle: after calibration (*step one*), right: after radial deflection (*step two*)

incompressible material behaviour, a bulk modulus of 2,500 MPa is chosen. This corresponds to the measured value for this range of pressure (see Fig. 7). The results shown in Fig. 10 emphasise the importance of the knowledge of the exact volumetric behaviour of rubber in simulating rubber components exposed to higher hydrostatic pressure values.

The bulk modulus measured in the hydrostatic compression tests for 300 bar leads to a satisfying result quality for the simulation of a tensile test under hydrostatic pressure. The maximum deviation between the test data and the simulation results is 2 %. In contrast, a factor 20 lower bulk modulus shows a different material response in the simulation of a tensile test. The nominal stress deviates at a nominal stretch of 1.8 about 50 % from the experimental results.

3.5 Influence on the simulation of confined rubber components

As shown in the previous chapter, the determination of the bulk modulus shows significant effects on the simulated material behaviour of rubber components under hydrostatic pressure. In the following, the influence of this effect on rubber components in technical applications is determined. Therefore, a bushing as a practical rubber component that is partially exposed to hydrostatic pressure up to 30 MPa will exemplarily be investigated (see Fig. 2). This kind of bushing is used in automotive vehicles and influences significantly their road handling. A crucial parameter for its in-use performance is the material stiffness, which is determined in mechanical tests on the bushing under different load cases, such as radial or axial deflection, or torsion. The material stiffness corresponds to the required reaction force to realise the radial deflection and serves as an indicator for the simulation outcome quality.

The simulation process contains two steps. In the first one, the outer sleeve is radially calibrated which leads to the development of a hydrostatic pressure in the rubber between the both sleeves. Then, the inner sleeve is

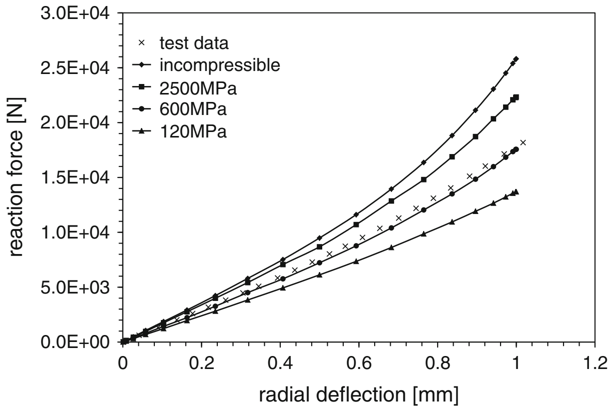

Fig. 12 Simulation results of bushing for different values of the bulk modulus

radially deflected which relieves pressure on one half and leads to an increase on the other one. In Fig. 11, the elements of the FE model (C3D8) for both steps are shown that are exposed to a hydrostatic pressure higher than 2 MPa. This corresponds to the lower limit of the nearly linear area in Fig. **7** and thus to a bulk modulus of 2,500 MPa. The simulations are carried out for three different values for the bulk modulus: 120 MPa that covers the very first part of the obtained curve, 2,500 MPa for the second part and 600 MPa as the least-square value (see chapter 3.2). The obtained results are presented in Fig. 12 and compared to the test data as well as the simulation result assuming incompressible material behaviour.

The assumption of incompressibility leads to a deviation between the simulated and the measured reaction force of about 40 %, the simulation result for a bulk modulus of 2,500 MPa to a deviation of about 20 %. Both simulations show a stiffer material behaviour compared to the component's test data. On the other hand, a bulk modulus of 120 MPa causes a 20 % lower reaction force and therefore a too soft simulation result of the material behaviour. The simulation results for $K = 600$ MPa prove a good conformity with the test data. These results emphasise the significance of the bulk modulus for achieving a satisfactory result quality in FE simulations of rubber components under hydrostatic pressure.

4 Conclusions

This paper shows the influence of the bulk modulus on the outcome quality of FE simulations of rubber components under hydrostatic pressure as well as a method to determine the bulk modulus for a wide range of hydrostatic pressure. Various tests were conducted with the presented test device. First of all, the stress–strain behaviour of rubber under hydrostatic pressure was determined in tensile tests. It is shown that the influence of the state of pressure on the deviatoric stress is negligible. Therefore, the split of the strain energy function in a deviatoric and a volumetric part is valid. For rubber components that are not exposed to hydrostatic pressure, the assumption of nearly incompressibility is either valid.

In contrast to this, for confined rubber components, the exact knowledge of the volumetric behaviour dependent on the hydrostatic pressure is important for the simulation outcome quality. Compression tests were conducted to obtain the dependence of the volume ratio on the hydrostatic pressure to determine the rubber's bulk modulus. The influence of the bulk modulus on the result quality is shown for a bushing used in automotive vehicles. Both too high values and too low values for the bulk modulus lead to a significant deviation between the obtained simulation results and the test data. Therefore, the bulk modulus is a crucial parameter to influence the outcome quality for rubber components under hydrostatic pressure.

5 Outlook

After the significant influence of the bulk modulus on material behaviour under hydrostatic pressure in FE simulations for a NR has been shown, further tests will be conducted. The aim is to determine more precisely the effect of filler and other compound ingredients particularly the portion and type of carbon black on the behaviour under hydrostatic pressure. Besides, the influence of the pressure on the bulk modulus should be considered more precisely in FE material models to achieve a better result quality. Furthermore, the interaction between volume changes due to stretching [7] and hydrostatic pressure will be pointed out.

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