Arch Appl Mech (2011) 81: 1195–1202
DOI 10.1007/s00419-010-0478-8

ORIGINAL

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Seismic passive pressures of earth structures by nonlinear optimization

Received: 29 March 2009 / Accepted: 27 September 2010 / Published online: 21 October 2010

Abstract Conventional seismic passive pressures of earth structures are based on a linear failure criterion in earthquake zones. However, experimental evidence shows that the strength envelopes are nonlinear over a wide range of normal stresses. In this paper, the analytical expressions of seismic passive pressures acting on inclined rigid walls are derived with nonlinear failure criterion. Quasi-static representation of earthquake effects using a seismic coefficient is adopted for seismic estimations. Instead of using the nonlinear criterion, a linear failure criterion, which is tangential to the nonlinear criterion, is used to formulate the seismic passive pressure problems as nonlinear programming problems. Analytical results are presented and compared with the previously published solutions using numerical technique. The influences of the parameters in the nonlinear failure criterion on seismic pressures and failure mechanisms are discussed.

Keywords Analytical solutions · Seismic passive pressure · Nonlinear criterion · Earthquakes

1 Introduction

In seismically active zones, earthquakes have the unfavorable effects of decreasing passive earth pressure. Therefore, it is of practical significance to assess the magnitude of seismic passive earth pressure acting on the retaining walls for the seismic deigns of earth structures. The problem of seismic passive pressures of earth structures has been extensively investigated. The exiting methods for evaluating earth pressures are mainly based on the limit equilibrium method, the slip line method, the limit analysis method or the numerical techniques involving the finite element and finite difference. The limit equilibrium method is the most commonly used method for estimating the passive pressures of earth structures. The limit equilibrium principle and its formulation are very simple, and the method is widely accepted by engineers. The seismic earth pressure is conventionally estimated according to the Mononobe–Okabe approach. Earthquake is replaced by a quasi-static inertia force. This calculation of seismic pressure of earth structure is based on the limit equilibrium method applied to a soil wedge. The analysis is a direct modification of the Coulomb wedge, where the earthquake effects are replaced by a quasi-static inertia force whose magnitude is computed on the basis of the seismic coefficient.

An alternative approach that provides more accurate results than the limit equilibrium method is the limit analysis method, which provides a rigorous upper or lower bound to the exact collapse load. The upper bound theorem of limit analysis method has been extended to fulfill a variety of engineering needs and has been applied to evaluate the passive earth pressure [1–3]. Using the linear Mohr-Coulomb (MC) failure criterion, Chen [1] and Soubra and Macuh [3] focused on improving the analytical static earth pressures. Yang and

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Yin [2] investigated the numerical solutions of seismic earth pressures using the limit analysis finite element technique.

This paper presents the derivation of analytical expressions for seismic earth pressures acting on inclined earth structures with the nonlinear failure criterion using a rotational failure mechanism. Quasi-static representation of earthquake effects using a seismic coefficient is adopted for seismic estimations. The seismic coefficient value is considered to be the same throughout the backfill. Based on upper bound theorem of plasticity, the seismic passive earth pressure problem is formulated as a nonlinear programming problem. Nonlinear sequential quadratic programming algorithm is employed to search for the lowest upper bound solution. To assess the validity of the analytical solutions, values of seismic pressure earth pressures for different seismic coefficients and nonlinear parameters in the failure criterion are calculated and compared with previous numerical solutions. The influence of the nonlinear failure criterion on the seismic passive earth pressures is also analyzed.

2 Nonlinear failure criterion

The solution for seismic passive pressures acting against the earth structures is based on the following assumptions: (1) Strong earthquakes may reduce resistance of soils, but mechanical properties of most soils do not change considerably for most moderate earthquakes where the seismic acceleration is less than 0.3 g (see Okamoto [4]). Therefore, there is no strength reduction for the entire soil masses involved. In fact, many researchers have adopted this hypothesis in seismic analysis of structure stability problems using quasi-static approach [2,5–7]. (2) The rigid retaining wall is long enough, and the problem can be regarded as plane strain problem. (3) The homogenous and isotropic backfill is idealized as a perfectly plastic material and follows the associated flow rule. The friction angle δ at the soil-wall interface is assumed to be constant, and the relative velocity at the interface is tangential to the wall. The earthquake forces are computed on the basis of the seismic coefficient concept. A constant seismic coefficient is assumed for the entire backfill.

The above assumptions agree with those used in literature [3], but the difference is that a nonlinear failure criterion and earthquake loading are considered in the present analysis. In many practical problems, a substantial amount of experimental evidence suggests that the failure envelopes of geomaterials are not linear in the (σ_n, τ) stress space [8–16]. This departure from linearity is significant for stability calculations. In general, the nonlinear failure criterion for the nonlinear failure envelope can be expressed as [2,17,21]

$$\tau = c_0(1 + \sigma_n/\sigma_t)^{1/m} \tag{1}$$

where σ_n and τ are the normal and shear stresses on the failure envelope, respectively, c_0 is the initial cohesion ($c_0 \geq 0$), σ_t is the axial tensile stress ($\sigma_t \geq 0$) at failure, and m is a parameter controlling the curvature of the nonlinear envelope ($m \geq 1$). When $m = 1$, Eq. (1) reduces to the well-known linear MC failure criterion. If a stress state reaches the curve of nonlinear failure criterion in the (σ_n, τ) stress space, plastic flow or failure will occur. Mobilized internal friction angle φ_t is defined as $\tan\varphi_t = d\tau/d\sigma_n$. The normal stress on failure surface can then be expressed as

$$\sigma_n = \sigma_t \left(m\sigma_t \tan\varphi_t/c_0\right)^{\frac{m}{1-m}} - \sigma_t \tag{2}$$

In the (σ_n, τ) stress space, the nonlinear failure criterion can be represented by a nonlinear curve. The tangential line to the nonlinear failure criterion is

$$\tau = c_t + \sigma_n \tan\varphi_t \tag{3}$$

where c_t is the intercept of the straight line on the $\tau-$axis, and it is given by

$$c_t = \frac{m-1}{m} c_0 \left(\frac{m\sigma_t \tan\varphi_t}{c_0}\right)^{\frac{1}{1-m}} + \sigma_t \tan\varphi_t \tag{4}$$

In the following analysis, instead of the actual nonlinear failure criterion, Eq. (1), the tangential line in Eq. (3) and the linear MC failure criterion are employed to calculate energy dissipation and external work rate of seismic passive earth pressures.

3 Upper bound theorem

The upper bound theorem of limit analysis has been recently used to calculate the limit load of earth structures [18–20]. This theorem, which uses a rigid perfectly plastic soil model, states that the internal energy dissipated by any kinematically admissible velocity field can be equated to the work done by external loads, and so provides a strict upper bound on the actual solution. The lowest possible upper bound solution is sought using an optimization scheme by trying various possible kinematically admissible failure mechanisms. A kinematically admissible velocity field satisfies compatibility, the plastic flow rule, and the velocity boundary conditions.

A limit load computed from a convex failure surface, which always circumscribes the actual nonlinear failure surface, will be considered as an upper bound on the actual limit load [1]. This is so because the strength of the convex failure surface is equal to or is larger than that of the actual failure surface. The strength of the tangential line equals or exceeds that of the nonlinear failure criterion at the same normal stress. Thus, the linear failure criterion represented by the tangential line will provide an upper bound on the actual load for the material, whose failure is governed by the nonlinear failure criterion. To ensure that the tangential line always lies outside of the curve, and that the strength corresponding to the tangential line is always more than or is equal to the strength corresponding to the nonlinear curve, the condition $m \geq 1$ is required.

In the following analysis, the tangential to the curve of the nonlinear failure criterion is used to calculate the rate of external work and the internal energy dissipation. The φ_t and c_t for the tangential line are hence regarded as known constants in calculating the rate of work and the energy dissipation. Since the location of tangential point is unspecified, the expressions for work rate and energy involve the variable φ_t. The difference is that the value of φ_t is unknown for the soil following the nonlinear criterion rather than the known value of internal friction angle for the soil following a linear Mohr-Coulomb criterion. Based on upper bound theorem, the kinematical admissibility condition requires that the rotational failure mechanisms must be rotational log-spiral surface or rotational circular surface. Therefore, the rotational failure mechanisms referred to log-spiral mechanism is considered for the seismic calculation schemes. Energy dissipation using the failure mechanism is treated as follows.

4 Seismic passive pressures

The constant seismic coefficient is adopted for seismic estimations of earth structures, and the horizontal seismic coefficient k_h is only considered. The vertical seismic coefficient is disregarded in the present study, but it can easily be incorporated into the objective function.

The rotational log-spiral discontinuity mechanism for the present analysis is shown in Fig. 1. The region *ABC* rotates as a rigid body about the center of rotation *O* with the material below the logarithmic spiral surface remaining at rest. Due to the usage of the tangential line (a linear MC failure criterion), the tangential angle φ_t along the entire slip surface is unchangeable (its value being unknown), so that c_t is also unchangeable along the entire slip surface according to Eq. (4). This failure mechanism is controlled by three variables which are θ_0, θ_h and φ_t. The log-spiral failure mechanism satisfies the kinematical admissibility conditions. Since tangential line which is a linear Mohr-Coulomb failure criterion is employed to calculate energy dissipation and external work of seismic passive earth pressures, the log-spiral curve is suitable for the present analysis.

Since the homogeneous soil masses are rigid, the internal energy is only dissipated along the sliding surface, while the external rate of work is done by the surcharge q on the top surface, the soil mass weight W_{soil} bounded by the boundary line *ABC* and sliding surface, the adhesive force $P_f\,[P_f = c_t H \tan\delta/(\sin\beta\tan\varphi_t)]$, and the earthquake forces $k_h W_{\text{soil}}$ and $k_h q$ where k_h is the horizontal seismic coefficient. Note that these earthquake forces represent the inertia masses of soil mass and overburden mass, simplified as a surcharge loading. The computation of the rate of work due to external forces and internal energy dissipation along velocity discontinuity lines is essentially the same as that for an inclined slope considered by Chen [1]. The total rate of work $\dot{W}_{\text{soil}}$ due to the soil mass weight W_{soil} and due to the horizontal inertia force $k_h W_{\text{soil}}$ can be expressed as

$$\dot{W}_{\text{soil}} = -\gamma r_0^3 \Omega [f_1 - f_2 - f_3] + \gamma r_0^3 \Omega k_h \,[f_4 - f_5 - f_6] \tag{5}$$

where γ is the total unit weight of the soil mass, r_0 is the initial radius of the log-spiral and Ω is the angular velocity. The f_1, f_2, f_3, f_4, f_5 and f_6 are non-dimensional functions, which depend on the angles θ_h and θ_0 and the tangential line angle φ_t. The expressions for f_1 through f_6 are given in Appendix. The rate of work

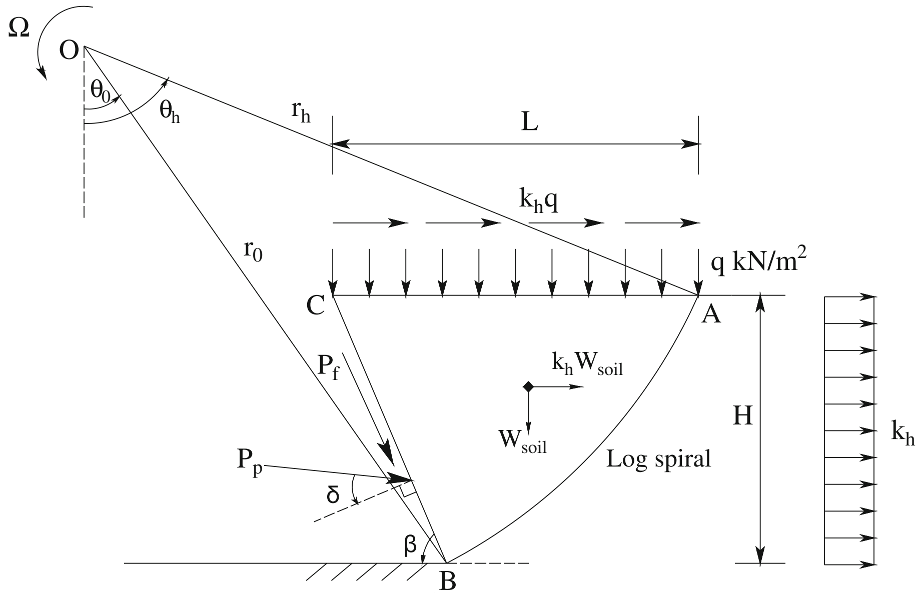

Fig. 1 Rotational failure mechanism for seismic passive earth pressure

$\dot{Q}_q$ due to the surcharge q and the horizontal inertia force $k_h q$ is

$$\dot{Q}_q = -qr_0^2\Omega\,(f_7 - k_h f_8) \tag{6}$$

where q is applied vertical surcharge. The f_7 and f_8 are non-dimensional functions which are given in Appendix. The rate of work $\dot{Q}_{P_p+P_f}$ due to the passive earth pressure P_a and the adhesive force P_f is

$$\dot{Q}_{P_p+P_f} = -P_p r_0 \Omega f_9 - c_t r_0^2 \Omega f_{10} \tag{7}$$

where f_9 and f_{10} are non-dimensional functions which are given in Appendix. For a rigid material, the internal energy is only dissipated along the sliding surface. The rate of energy dissipation $\dot{W}_{\text{int}}$ can thus be written as

$$\dot{W}_{\text{int}} = c_t r_0^2 \Omega f_{11} \tag{8}$$

The work rates of external forces in Eqs. (5), (6) and (7) are equal the internal energy dissipation rate given in Eq. (8), that is, $\dot{W}_{\text{soil}} + \dot{Q}_q + \dot{Q}_{P_p+P_f} = \dot{W}_{\text{int}}$. Substituting the expressions for $\dot{W}_{\text{soil}}$, $\dot{Q}_q$, $\dot{Q}_{P_p+P_f}$ and $\dot{W}_{\text{int}}$ into this equation, we obtain

$$P_p = [-qr_0(f_7 - k_h f_8) - \gamma r_0^2(f_1 - f_2 - f_3) + \gamma r_0^2 k_h(f_4 - f_5 - f_6) - c_t r_0(f_{10} + f_{11})]/f_9 \tag{9}$$

Eq. (9) is obtained assuming that the failure surface follows the shape of a log-spiral. This failure mechanism is controlled by three parameters θ_h, θ_0 and φ_t, that are regarded as three independent variables. The constrained conditions are $\theta_h > \theta_0$, $\theta_0 > 0$ and $\varphi_t > 0$. We employ the sequential quadratic programming to optimize the objective function in Eq. (9) with respect to θ_h, θ_0 and φ_t, to obtain the minimum value of the seismic passive earth pressures.

5 Results and discussions

With the nonlinear failure criterion, the seismic earth pressure can be obtained by optimization method. Nonlinear sequential quadratic programming algorithm is employed to minimize the objective Eq. (9) with respects to θ_h, θ_0 and φ_t for the failure mechanism. Comparisons are summarized in Tables 1 and 2. The effects of the nonlinear coefficient m and the seismic coefficient k_h on the seismic passive earth pressure and the failure mechanisms are studied.

Table 1 Comparisons between the present solutions and numerical results [2] (Example 1)

Solutions (kN/m)	Coefficient m							
	1.1	1.2	1.3	1.4	1.5	1.6	1.8	2.0
Numerical results [2]	255.28	241.04	230.72	222.85	216.70	211.77	204.36	199.07
Present solutions	254.96	240.61	230.18	222.28	216.09	211.13	203.66	198.33
Difference	0.32	0.43	0.54	0.57	0.61	0.64	0.70	0.74

($k_h = 0.05$, $\gamma = 18.0\,\mathrm{kN/m^3}$, H $= 4.0$ m, $c_0 = 4.0$ kPa, $\sigma_t = 15.0$ kPa, $\delta = 0°$, q = 0 and $\beta = 90\circ$)

Table 2 Comparisons between the present solutions and numerical results [2] (Example 2)

Solutions (kN/m)	Coefficient m						
	1.2	1.4	1.6	1.8	2.0	3.0	4.0
Numerical results [2]	303.16	278.85	263.75	253.48	246.05	227.19	219.33
Present solutions	301.74	277.25	262.01	251.65	244.16	225.16	217.25
Difference	1.42	1.60	1.74	1.83	1.89	2.03	2.08

($k_h = 0.05$, $\gamma = 18.0\,\mathrm{kN/m^3}$, H $= 4.0$ m, $c_0 = 8.0$ kPa, $\sigma_t = 25.0$ kPa, $\delta = 0°$, q = 0 and $\beta = 90°$)

5.1 Comparisons

Yang and Yin [2] presented numerical calculations of seismic passive earth pressures based on the upper bound theorem of limit analysis. To evaluate the validity of analytical solutions, an inclined earth structure with horizontal backfill, shown in Fig. 1, is analyzed. The present analytical solutions for seismic passive pressures are compared with seismic passive pressures obtained numerically. Two examples are selected for comparisons.

Table 1 shows values of seismic passive earth pressures obtained by the two methods for $\beta = 70°$, $\delta = 5°$, $\gamma = 17\,\mathrm{kN/m^3}$, H $= 4.0$ m, $c_0 = 9.0$ kPa and $\sigma_t = 15$ kPa, with the nonlinear coefficient m varying from 1.1 to 2.0 (Example 1). Table 2 presents the values of seismic passive earth pressures for $k_h = 0.05$, $\gamma = 18.0\,\mathrm{kN/m^3}$, H $= 4.0$ m, $c_0 = 8.0$ kPa, $\sigma_t = 25.0$ kPa, $\delta = 0°$, q $= 0$ and $\beta = 90°$, with the nonlinear coefficient m varying from 1.2 to 4.0 (Example 2).

It is found from Tables 1 and 2 that the seismic passive pressures using rotational failure mechanism are almost identical with previously published solutions, with the maximum difference of approximately 0.96%. Therefore, this comparison shows that the analytical expressions are applicable to the evaluation of seismic passive earth pressures with the nonlinear failure criterion.

5.2 Effects of nonlinear coefficient m

To investigate the effect of the nonlinear coefficient m on passive earth pressures, an inclined and rough wall is considered. Fig. 2 shows the values of seismic passive earth pressures corresponding to $\beta = 70°$, $\delta = 5°$, $\gamma = 17.0\,\mathrm{kN/m^3}$, H $= 4.0$ m, $c_0 = 9.0$ kPa and $\sigma_t = 15$ kPa with the nonlinear coefficient m varying from 1.5 to 2.0 for $k_h = 0$, 0.1, and 0.2. It can be seen from Fig. 2 that the nonlinear coefficient m has a significant influence on the seismic passive earth pressures. Figure 3 shows the failure surface shapes corresponding to the same parameters given above, with the seismic coefficient being 0.2.

5.3 Effects of seismic coefficient k_h

To investigate the effect of the earthquake loading on passive earth pressures, Fig. 4 shows the values of seismic passive earth pressures corresponding to $\beta = 70°$, $\delta = 5°$, $q = 0$ kPa, $\gamma = 17.0\,\mathrm{kN/m^3}$, H $= 4.0$ m, $c_0 = 20$ kPa and $\sigma_t = 25$ kPa with the seismic coefficient k_h varying from 0.0 to 0.25 for $m = 1.8$, 1.9 and 2.0. It is seen from Fig. 4 that earthquakes have unfavorable effects of decreasing the seismic passive earth pressure. The failure surface shapes are shown in Fig. 5 corresponding to the same parameters given above, with the nonlinear coefficient being 1.8.

In the preceding analysis of seismic earth pressures, an inclined wall with a horizontal backfill is considered. The inclined backfill is not considered. However, there is no difficulty in principle in extending the present method to deal with seismic earth pressures with an inclined backfill.

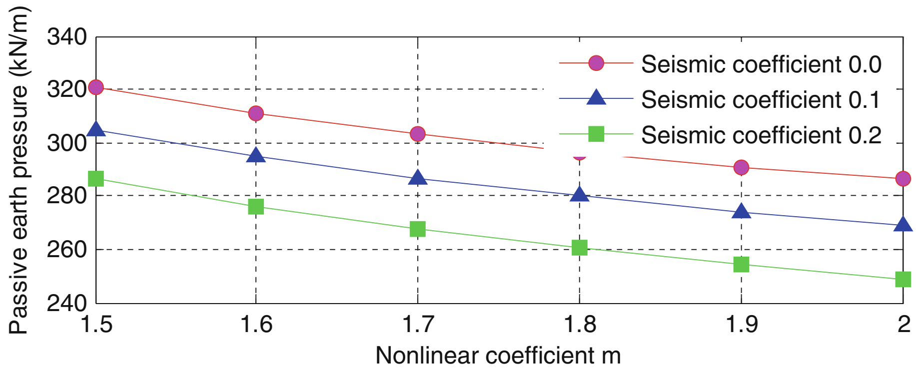

Fig. 2 The effect of nonlinear coefficient m on seismic passive earth pressure ($\beta = 70°$, $\delta = 5°$, $\gamma = 17$ kN/m^3, H = 4.0 m, $c_0 = 9.0$ kPa and $\sigma_t = 15$ kPa)

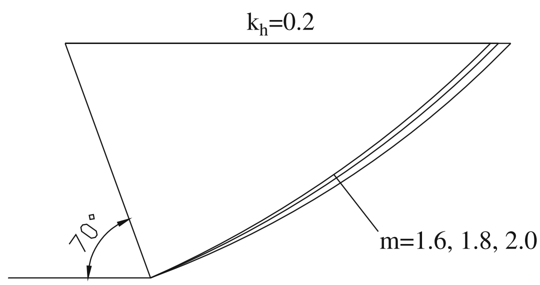

Fig. 3 The effect of nonlinear coefficient m on failure surface shapes ($\beta = 70°$, $\delta = 5°$, $\gamma = 17$ kN/m^3, H = 4.0 m, $c_0 = 9.0$ kPa and $\sigma_t = 15$ kPa)

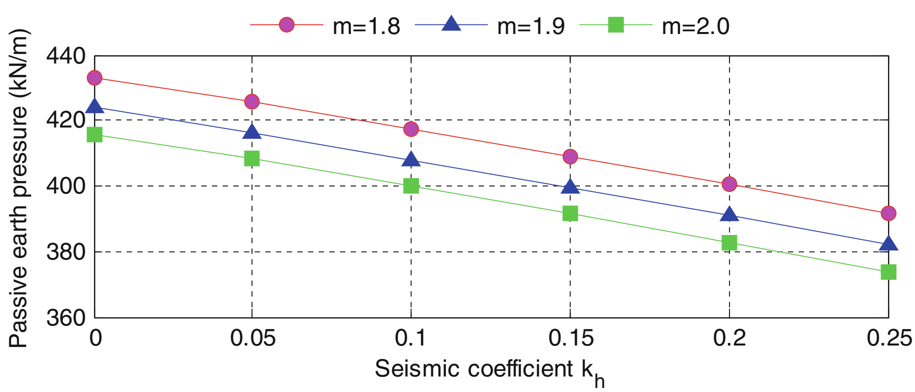

Fig. 4 The effect of seismic coefficient k_h on seismic passive earth pressure ($\beta = 70°$, $\delta = 5°$, $q = 0$, $\gamma = 17$ kN/m^3, H = 4.0 m, $c_0 = 20$ kPa and $\sigma_t = 25$ kPa)

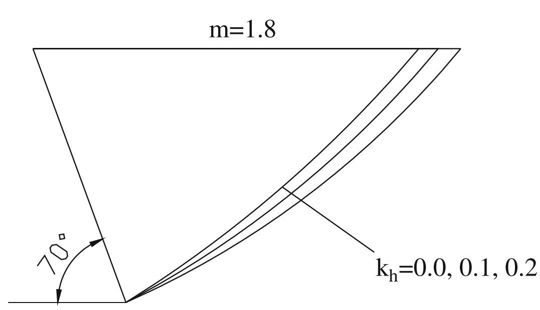

Fig. 5 The effect of seismic coefficient k_h on failure surface shapes ($\beta = 70°$, $\delta = 5°$, $q = 0$, $\gamma = 17$ kN/m^3, H = 4.0 m, $c_0 = 20$ kPa and $\sigma_t = 25$ kPa)

6 Conclusions

This paper presents analytical solutions for seismic passive pressures under earthquake loads with the nonlinear failure criterion. Based on the upper bound theorem of limit analysis, the rate of external work and the internal energy dissipation are calculated. Equating the work rate of external forces to the internal energy dissipation, the objective function is formulated. Seismic pressures are obtained by minimizing the objective function with respects to φ_t, θ_h and θ_0. To verify the validity of the present solutions, the analytical solutions are compared with the numerical results. It is found that seismic passive earth pressures calculated using the analytical solution method are less than those using numerical technique, with the maximum difference approximately of 0.96%. Based on the upper bound theorem of limit analysis, the smallest solution is the best one. Therefore, the present analytical solutions are better than the previous numerical solutions. We conclude with the following observations:

The present analytical solutions are effective for evaluating seismic passive pressures with the nonlinear criterion. The nonlinear criterion has a significant influence on seismic passive pressures and seismic failure mechanisms. Present paper extends static pressure calculations of Soubra and Macuh [3] using the linear MC failure criterion to seismic pressure calculations using the nonlinear failure criterion.

Acknowledgments This research is supported by Hunan Provincial Natural Science Foundation of China (No.09JJ1008) and Foundation for National Excellent Doctoral Dissertation of China (No.200550). These financial supports are gratefully acknowledged.

Appendix

$$\frac{H}{r_0} = \sin\theta_h \cdot \exp\left[(\theta_h - \theta_0)\cdot \tan\varphi_t\right] - \sin\theta_0$$

$$\frac{L}{r_0} = \sin\theta_h \cdot \exp\left[(\theta_h - \theta_0)\cdot \tan\varphi_t\right] - \sin\theta_0 + \frac{H}{r_0 \tan\beta}$$

$$f_1 = \frac{\exp[3(\theta_h - \theta_0)\tan\varphi_t](3\tan\varphi_t \sin\theta_h - \cos\theta_h) + \cos\theta_0 - 3\sin\theta_0\tan\varphi_t}{3(1+9\tan^2\varphi_t)}$$

$$f_2 = \frac{1}{6}\frac{L}{r_0}\cos\theta_h \exp[(\theta_h - \theta_0)\tan\varphi_t]\left\{2\sin\theta_h \exp[(\theta_h - \theta_0)\tan\varphi_t] - \frac{L}{r_0}\right\}$$

$$f_3 = \frac{1}{6}\frac{H}{r_0}\frac{\cos(\beta+\theta_0)}{\sin\beta}\left(\frac{H}{r_0}ctg\beta - 2\sin\theta_0\right)$$

$$f_4 = \frac{\exp[3(\theta_h - \theta_0)\tan\varphi_t](3\tan\varphi_t \cos\theta_h + \sin\theta_h) - \sin\theta_0 - 3\cos\theta_0\tan\varphi_t}{3(1+9\tan^2\varphi_t)}$$

$$f_5 = \frac{1}{3}\frac{L}{r_0}\cos^2\theta_h \exp[2(\theta_h - \theta_0)\tan\varphi_t]$$

$$f_6 = \frac{1}{6}\frac{H}{r_0}\frac{\cos(\beta+\theta_0)}{\sin\beta}\left(\frac{H}{r_0} - 2\cos\theta_0\right)$$

$$f_7 = \frac{1}{2}\frac{L}{r_0}\left\{2\sin\theta_h \exp[(\theta_h - \theta_0)\tan\varphi_t] - \frac{L}{r_0}\right\}$$

$$f_8 = \frac{L}{r_0}\cos\theta_h \exp[(\theta_h - \theta_0)\tan\varphi_t]$$

$$f_9 = -\sin(\theta_0 + \delta + \beta) + \frac{1}{3}\frac{H}{r_0}[ctg\beta\cos(\delta+\beta) + \sin(\delta+\beta)]$$

$$f_{10} = \frac{H}{r_0}\frac{\tan\delta}{\tan\varphi_t}(\sin\theta_0 - ctg\beta\cos\theta_0)$$

$$f_{11} = \frac{\exp[2(\theta_h - \theta_0)\tan\varphi_t] - 1}{2\tan\varphi_t}$$

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