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ORIGINAL

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Buckling analysis of stringer-stiffened laminated cylindrical shells with nonuniform eccentricity

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Abstract In this study, the influence of nonuniformity of eccentricity of stringers on the general axial buckling load of stiffened laminated cylindrical shells with simply supported end conditions is investigated. The critical loads are calculated using Love's First-order Shear Deformation Theory and solved using the Rayleigh-Ritz procedure. The effects of the shell length-to-radius ratio, shell thickness-to-radius ratio, number of stringers, and stringers depth-to-width ratio on the buckling load of nonuniformly eccentric shells, are examined. The research demonstrates that an appropriate nonuniform distribution of eccentricity of stringers leads the buckling load to increase significantly.

Keywords Stringer-Stiffened laminated cylindrical shells · Nonuniform eccentricity · General axial buckling load

1 Introduction

Stiffened circular cylindrical shells are widely used in the aerospace industries, offshore structures, and submarine hulls. In most of these applications, shell is subjected to compressive loads and, therefore, it may buckle. It is essential that the shell stiffness-to-weight ratio is as large as possible in order to resist the applied loadings. There exist a number of ways to increase the shell's buckling capacity, including constructing the shell from composite materials, stiffening of the shell with stiffeners, and using a combination of these ways.

In the field of buckling analysis of stringer-stiffened isotropic cylindrical shells, extensive studies are performed. The magnitude of buckling loads is affected by the eccentricity of stringers (longitudinal stiffeners) and rings (circumferential stiffeners). Van der Neut [1] pointed out the importance of the eccentricity of stiffeners in the buckling of cylindrical shells under axial compression. Singer et al. [2] studied the effect of eccentricity of stiffeners on the buckling load of cylindrical shells under axial compressive load. They concluded that the behavior of eccentricity effect depends very strongly on the geometry of the shell. Rosen and

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Singer [3] investigated buckling of axially loaded stiffened cylindrical shells with elastic restraints theoretically and experimentally. The approach they used was based on averaging of the stiffeners. Ji and Yeh [4] analyzed the nonlinear buckling of ring- and stringer-stiffened cylindrical shells. Jarmai et al. [5] investigated the minimum cost design of a stiffened cylindrical shell under buckling constraint. Simoes et al. [6] presented reliability-based optimum design of a welded stringer-stiffened steel cylindrical shell column fixed at the bottom and free at the top subject to axial compression and horizontal force acting on the top of the column. Jiang et al. [7] presented the buckling analysis of stiffened circular cylindrical panels using differential quadrature element method. A comprehensive computational process of design optimization of stiffened storage tank for spacecraft was provided by chen et al [8]. The main objectives of the research were the optimization of the structure weight and the investigation of the structural performances of CHAN type and BAR type sections of stiffeners.

Some studies are carried out in the context of buckling and vibration analyses of stiffened composite cylindrical shells. In fact, most of the studies are confined to vibration of stiffened shells, including Lee and Kim [9,10], Zhao et al. [11], and Talebitooti et al. [12]. Reddy and Starnes [13] studied the buckling analysis of circumferentially or axially stiffened laminated cylindrical shells subjected to simply supported end conditions. The layerwise theory and the smeared stiffener approach were utilized. Rikards et al. [14] employed a triangular finite element model to study the buckling and vibration of laminated stiffened shells and plates using First-order Shear Deformation Theory (FSDT). Spagnoli [15] studied the different modes of instability in stiffened conical shells under axial compressive load through a linear eigenvalue finite element analysis. Abramovich et al. [16] studied repeated buckling and its influence on the geometrical imperfections of stiffened cylindrical shells under combined loading. Kidane et al. [17] determined the global buckling load for a generally cross and horizontal grid stiffened cylindrical shell with simply supported and clamped end conditions, by developing an analytical model using the smeared method. Prusty [18] carried out the free vibration and buckling response of stiffened panels under general loading. Lene et al. [19] presented an advanced methodology for optimum buckling design of a stiffened cylindrical shell under various loadings. Najafizadeh et al. [20] investigated the buckling analysis of functionally graded cylindrical shells stiffened by rings and stringers under axial compression.

The review of the literature signifies that no effort is exerted to analyze the buckling of stiffened circular cylindrical shells with nonuniform-eccentricity stringers. In this paper, new buckling results are presented for laminated stiffened cylindrical shells with nonuniform stringers. The influences of various parameters including the shell length-to-radius ratio, shell thickness-to-radius ratio, number of stringers, and stringers depth-to-width ratio on the buckling load are investigated, serving for academic researches and relevant industrial applications.

2 Theory and formulation

Consider a longitudinally stiffened laminated circular cylindrical shell with thickness h, length L, and mean radius R, as depicted in Fig. 1. The arbitrary point in middle surface is measured on the axial, circumferential and radial axes (x, θ, z), and the origin of the coordinate system is located at one of the two edges of the shell. The fth stringer is located at $\theta = \theta_{sf}$. The stringers may be made up of an isotropic material with mass density ρ_s, Young's modulus E_s, shear modulus G_s, and Poisson's ratio υ_s. The stringers are of rectangular cross-section with a width b_s and a depth d_s. Eccentricity e_s is the distance between the centroid of cross-section of the stringer and the middle surface of the shell at the position of the stringer.

The displacement field based on First-order Shear Deformation Theory (FSDT) is expressed in the cylindrical coordinate system as follows

$$\begin{aligned} u(x,\theta,z) &= u_0(x,\theta) + z\beta_x(x,\theta) \\ v(x,\theta,z) &= v_0(x,\theta) + z\beta_\theta(x,\theta) \\ w(x,\theta,z) &= w_0(x,\theta) \end{aligned} \tag{1}$$

where (u, v, w) are the orthogonal components of displacement of an arbitrary point (x, θ, z) in the shell along the coordinates (x, θ, z), (u_0, v_0, w_0) are the displacements of the shell midsurface at point $(x, \theta, 0)$, and β_x, β_θ are the rotations of the normals to the midplane about the θ and x axes, respectively.

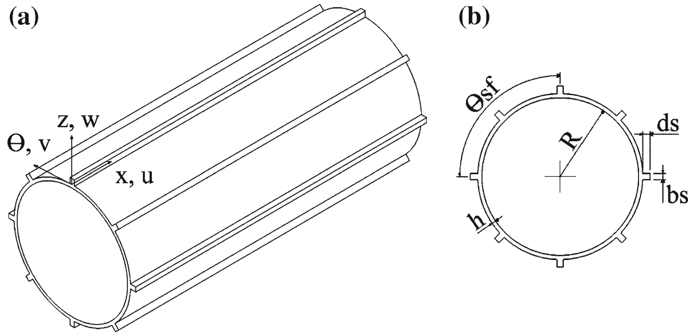

Fig. 1 Configuration, geometrical properties and coordinate system of a longitudinally stiffened cylindrical shell: **a** isometric view, **b** cross-sectional view

The displacements $(u_0, v_0, w_0, \beta_x, \beta_\theta)$ which satisfy the simply supported geometric boundary conditions $w = v = 0$, can be defined as

$$u_0 = c_1 \cos\frac{m\pi x}{L}\cos n\theta, \quad v_0 = c_2 \sin\frac{m\pi x}{L}\sin n\theta, \quad w_0 = c_3 \sin\frac{m\pi x}{L}\cos n\theta$$
$$\beta_x = c_4 \cos\frac{m\pi x}{L}\cos n\theta, \quad \beta_\theta = c_5 \sin\frac{m\pi x}{L}\sin n\theta \tag{2}$$

in which c_1, c_2, c_3, c_4, c_5 are the displacement amplitudes, and m and n are the axial and circumferential mode numbers, respectively.

The strain-displacement relations from Love's FSDT are expressed by

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{(0)} + z\boldsymbol{\kappa} \tag{3}$$
$$\boldsymbol{\gamma} = \boldsymbol{\gamma}^{(0)}$$

$$\boldsymbol{\varepsilon}^{(0)} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{\theta\theta}^{(0)} \\ \gamma_{x\theta}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{1}{R}\frac{\partial v_0}{\partial\theta} + \frac{w_0}{R} \\ \frac{1}{R}\frac{\partial u_0}{\partial\theta} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \boldsymbol{\kappa} = \begin{Bmatrix} \kappa_{xx} \\ \kappa_{\theta\theta} \\ \kappa_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial\beta_x}{\partial x} \\ \frac{1}{R}\frac{\partial\beta_\theta}{\partial\theta} \\ \frac{1}{R}\frac{\partial\beta_x}{\partial\theta} + \frac{\partial\beta_\theta}{\partial x} \end{Bmatrix} \tag{4}$$

$$\boldsymbol{\gamma}^{(0)} = \begin{Bmatrix} \gamma_{\theta z}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{R}\frac{\partial w_0}{\partial\theta} + \beta_\theta - \frac{v_0}{R} \\ \frac{\partial w_0}{\partial x} + \beta_x \end{Bmatrix} \tag{5}$$

in which $\varepsilon_{xx}^{(0)}, \varepsilon_{\theta\theta}^{(0)}$ and $\gamma_{x\theta}^{(0)}, \gamma_{xz}^{(0)}, \gamma_{\theta z}^{(0)}$ are the normal and shear strains at the middle surface $(z = 0)$, $\kappa_{xx}, \kappa_{\theta\theta}$ are the midsurface changes in curvature, and $\kappa_{x\theta}$ is the midsurface twist.

The strain energy of the shell is presented as

$$U_{sh} = \frac{1}{2}\int_0^L\int_0^{2\pi} \boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{S}\boldsymbol{\varepsilon} R\,\mathrm{d}\theta\,\mathrm{d}x \tag{6}$$

where the strain vector $\boldsymbol{\varepsilon}$ and the stiffness matrix $\mathbf{S}$ are defined by

$$\boldsymbol{\varepsilon} = \left\{\boldsymbol{\varepsilon}^{(0)}\boldsymbol{\kappa}\boldsymbol{\gamma}^{(0)}\right\}^{\mathrm{T}} \tag{7}$$

$$S = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & H_{44} & H_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & H_{45} & H_{55} \end{bmatrix} \tag{8}$$

where A_{ij}, B_{ij}, C_{ij} and H_{ij} are in turn the elements of the extensional, coupling, bending, and thickness shear stiffness matrices, expressed as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2)\mathrm{d}z, \quad H_{ij} = K_0 \int_{-h/2}^{h/2} Q_{ij}\mathrm{d}z \tag{9}$$

in which A_{ij}, B_{ij}, C_{ij} are defined for $i, j = 1, 2, 6$, H_{ij} is defined for $i, j = 4, 5$, and K_0 is the shear correction factor introduced by Mindlin and is equal to $\pi^2/12$. Q_{ij} are the engineering constants written as

$$\begin{aligned} &Q_{11} = \frac{E_{11}}{1 - \upsilon_{12}\upsilon_{21}}, \quad Q_{12} = \frac{\upsilon_{12}E_{22}}{1 - \upsilon_{12}\upsilon_{21}}, \quad Q_{22} = \frac{E_{22}}{1 - \upsilon_{12}\upsilon_{21}}, \\ &Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \end{aligned} \tag{10}$$

where E_{11} and are the elastic moduli in the principle material coordinates, G_{12}, G_{13} and G_{23} are the shear moduli, and υ_{12} and υ_{21} are Poisson's ratios.

For a shell composed of different layers of orthotropic material, the A_{ij}, B_{ij}, C_{ij} and H_{ij} are given by

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{\mathrm{NL}} \overline{Q}_{ij}^{k}(h_k - h_{k+1}), \quad B_{ij} = \frac{1}{2}\sum_{k=1}^{\mathrm{NL}} \overline{Q}_{ij}^{k}\left(h_k^2 - h_{k+1}^2\right), \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{\mathrm{NL}} \overline{Q}_{ij}^{k}\left(h_k^3 - h_{k+1}^3\right), \quad H_{ij} = K_s \sum_{k=1}^{\mathrm{NL}} \overline{Q}_{ij}^{k}(h_k - h_{k+1}) \end{aligned} \tag{11}$$

in which h_k and h_{k+1} denote the distances from the shell middle surface to the outer and inner surfaces of the kth layer. NL is the number of layers in the laminated shell, and $\overline{Q}_{ij}$ are the elements of transformed reduced stiffness matrix ($\overline{\mathbf{Q}}$) for the kth layer given by

$$\overline{\mathbf{Q}} = \mathbf{T}^{-1}\mathbf{Q}\mathbf{T}^{-T} \tag{12}$$

where $\mathbf{Q}$ is the reduced stiffness matrix written as

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \tag{13}$$

and $\mathbf{T}$ is the transformation matrix for the principal material coordinates and the shell coordinates, presented by

$$\mathbf{T} = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 2\sin\alpha\cos\alpha & 0 & 0 \\ \sin^2\alpha & \cos^2\alpha & -2\sin\alpha\cos\alpha & 0 & 0 \\ -\sin\alpha\cos\alpha & \sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{14}$$

in which α is the orientation of the fibers.

The displacements of centroid of the cross-section of the fth stringer, i.e. (u_{sf}, v_{sf}, w_{sf}), may be related to the displacements of the shell midsurface at the position of the stringer as follows

$$\left.\begin{aligned} u_{sf} &= u_0 + e_s(x)\beta_x \\ v_{sf} &= v_0\left(1 + \frac{e_s(x)}{R}\right) + e_s(x)\beta_\theta \\ w_{sf} &= w_0 \end{aligned}\right\} at\ \theta = \theta_{sf} \tag{15}$$

where $e_s(x)$ is the eccentricity of the stringer.

It is assumed that the stringers' geometrical and material properties are the same. Besides, the stringers are equally spaced around the circumference of the shell. For rectangular stringers, geometrical properties including the cross-sectional area $A_s(x)$, the second moment of areas about θ and z axes $I_{\theta s}(x)$, $I_{zs}(x)$, the polar second moment of area $J_s(x)$, and the eccentricity $e_s(x)$ are written as

$$\begin{aligned} A_s(x) =& b_s d_s(x), \quad I_{zs}(x) = \frac{b_s^3 d_s(x)}{12}, \quad I_{\theta s}(x) = \frac{b_s d_s(x)^3}{12}, \\ J_s(x) =& \frac{b_s d_s(x)}{12}\left[b_s^2 + d_s(x)^2\right], \quad e_s(x) = \frac{h + d_s(x)}{2} \end{aligned} \tag{16}$$

in which the depth $d_s(x)$ and, as a result, the other geometrical properties of stringers are written in general forms, as variables in terms of x, which can be used for stringers with both uniform $[d_s]$ and nonuniform $[d_s(x)]$ depth.

Using the discrete stiffener theory, the strain energy of the fth stringer at the location θ_{sf} can be expressed by [21]

$$U_{sf} = \frac{1}{2}\int_0^L \left[E_s A_s(x)\left(\frac{\partial u_{sf}}{\partial x}\right)^2 + E_s I_{\theta s}(x)\left(\frac{\partial^2 w_{sf}}{\partial x^2}\right)^2 + E_s I_{zs}(x)\left(\frac{\partial^2 v_{sf}}{\partial x^2}\right)^2 + \frac{G_s J_s(x)}{R^2}\left(\frac{\partial^2 w_{sf}}{\partial x \partial \theta}\right)^2\right]_{\theta=\theta_{sf}} \mathrm{d}x \tag{17}$$

By substituting Eq. (15) into Eq. (17), the energy expression for the fth stringer is arranged in terms of the displacements of the shell midsurface.

The work done by the axial compressive load (N_x) during buckling may be expressed as

$$V_{N_x} = -\int_0^L \int_0^{2\pi} \left\{\frac{N_x R}{2}\left[\left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial v_0}{\partial x}\right)^2 + \left(\frac{\partial w_0}{\partial x}\right)^2\right]\right\} \mathrm{d}\theta \mathrm{d}x \tag{18}$$

where the negative sign shows that the compressive load N_x losses potential energy.

Thus, the total potential energy U_{tot} of the system may be obtained as

$$U_{\mathrm{tot}} = U_{sh} + \sum_{f=1}^{N_s} U_{sf} + V_{N_x} \tag{19}$$

where N_s is the number of stringers.

By applying the Rayleigh-Ritz technique, we have

$$\frac{\partial U_{\mathrm{tot}}}{\partial c_1} = \frac{\partial U_{\mathrm{tot}}}{\partial c_2} = \frac{\partial U_{\mathrm{tot}}}{\partial c_3} = \frac{\partial U_{\mathrm{tot}}}{\partial c_4} = \frac{\partial U_{\mathrm{tot}}}{\partial c_5} = 0 \tag{20}$$

which leads to a set of governing eigenvalue equations

$$[\mathbf{K} - N_x \mathbf{G}]\{c_1 c_2 c_3 c_4 c_5\}^{\mathrm{T}} = 0 \tag{21}$$

where $\mathbf{K}$ is the stiffness matrix and $\mathbf{G}$ is the geometric or stress stiffness matrix. The critical load N_{xcr} is obtained when the determinant of matrix $\mathbf{K} - N_x\mathbf{G}$ vanishes.

3 Shells stiffened by equal mass uniform- and nonuniform-eccentricity stringers

Consider a circular cylindrical shell with length L, radius R, and thickness h, stiffened by Ns equally spaced, external stringers. All of the stringers have width bs and depth ds. It is investigated that for a laminated stiffened cylindrical shell, is it feasible to obtain higher buckling loads by changing eccentricity (depth) of stringers along their lengths without having to increase their mass?

To answer to this, keeping the same number and width of stringers as those of the stringers of the aforementioned stiffened shell, we define two depth distribution functions designated as Stringer Designs I and II

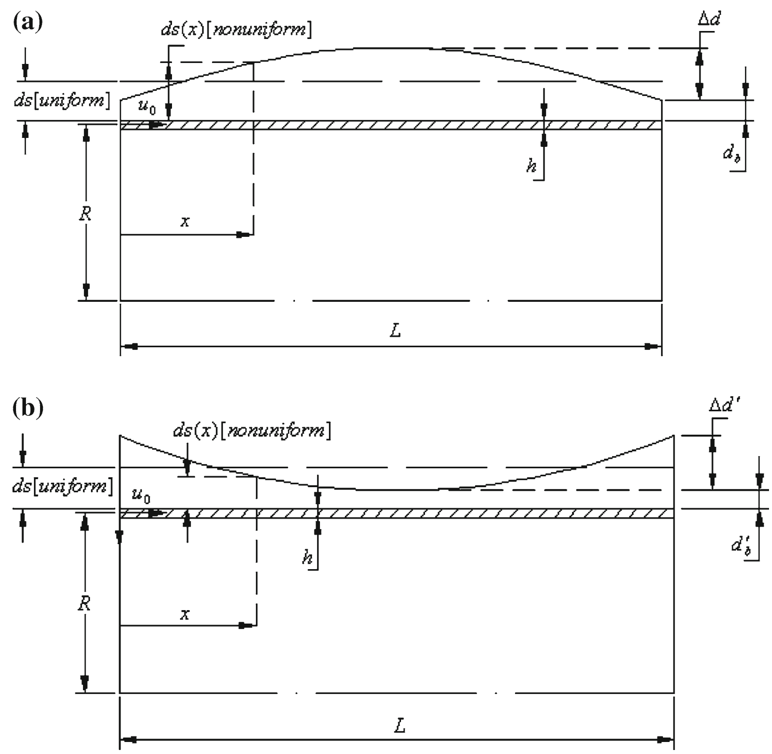

Fig. 2 Status of variation of stringer depth $ds(x)$: **a** Stringer Design I, **b** Stringer Design II

as follows (see Fig. 2)
Stringer Design I:

$$ds(x) = d_b \left\{ 1 + \frac{\Delta d}{d_b} \left[4\left(\frac{x}{L}\right)\left(1 - \frac{x}{L}\right) \right]^q \right\} \tag{22}$$

in which q represents status of variation of depth distribution function along the shell length, d_b is the minimum value of stringer depth located at its two ends, and Δd is the difference between the maximum value of stringer depth located at its middle section ($x = L/2$) and the minimum one of stringer depth d_b. Note that $q = 0$ denotes the case of uniformly eccentric shell. Also note that in principle, it is possible to adopt q up to any desired degree, but here we consider the values of $q = 1$ and 2 for determination $ds(x)$ because of being feasible and appropriate for practical applications.
Stringer Design II:

$$ds(x) = d'_b \left[1 + \frac{\Delta d}{d'_b} \left(\frac{2x}{L} - 1 \right)^2 \right] \tag{23}$$

where d'_b is the minimum value of stringer depth located at its middle section, and $\Delta d'$ is the difference between the maximum value of stringer depth located at its two end sections and the minimum one of stringer depth d'_b.

The volumes of stringers in the cases of uniform and nonuniform eccentricity distributions are written as

$$Vs_{\text{uniform}} = As_{\text{uniform}} L \tag{24}$$

$$Vs_{\text{nonuniform}} = \int_0^L As_{\text{nonuniform}} \mathrm{d}x = \int_0^L As(x) \mathrm{d}x \tag{25}$$

where Vs_{uniform}, As_{uniform} and $Vs_{\text{nonuniform}}$, $As_{\text{nonuniform}} = As(x)$ are the volume and cross-sectional area of stringers with uniform and nonuniform eccentricity, respectively. For equality of the volume (and, therefore, the mass) of stringers in the cases of uniform and nonuniform eccentricities, we have

$$Vs_{\text{uniform}} = Vs_{\text{nonuniform}} \tag{26}$$

For rectangular stringers, the above relation may be simplified to

$$dsL = \int_0^L ds(x)\mathrm{d}x \tag{27}$$

from which, Δd can be obtained for definite values of d_b, q and d_b', for shells reinforced by stringers with the depth function defined by Eqs. (22) and (23), respectively.

4 Results and discussion

4.1 Verification of the formulation

Before starting the analysis, the validity of the derived formulation should be ensured. The results are compared against those of Refs. [13] and [3] respectively for unstiffened laminated and stringer-stiffened isotropic cylindrical shells subjected to the simply supported end conditions. The 'smeared stiffener' method was utilized in both references to study general buckling of shells. The geometrical and material properties of the shell and stringers are given in Table 1. The comparisons are furnished in Table 2. As can be viewed from this table, good agreements between the present analysis results and those of the references are observed.

4.2 Stiffened laminated shells with uniform- and nonuniform-eccentricity stringers

To study the influence of nonuniformity of eccentricity of stringers on the critical buckling load of stiffened shells, we consider a circular cylindrical shell stiffened by four equally spaced, external stringers, which acts as a reference design. The shell is simply supported on its edges. The shell is also laminated of orthotropic, equal thickness layers with [0/90/0] scheme. The geometrical and material properties of the shell with uniform stringers are given in Table 3.

The stiffened shell has the critical buckling load of 87.4 kN/m. The variation of the critical buckling load (N_{xcr}) with respect to the depth ratio (d_b/ds or d_b'/ds), for stringers with Designs I and II, are displayed in

Table 1 The geometrical and material properties of the shell and stringers used in Refs. [13] and [3]

Ref.	Geometrical properties	Material properties
[13]	$h = 3.048$ mm $R = 254$ mm $L = 879.856$ mm Lamination scheme [0/90/0]	$E_1 = 207$ GPa $E_2 = 5.17$ GPa $G_{12} = 2.59$ GPa $\upsilon_{12} = 0.25$
[3]	$h = 0.247$ mm $R = 486h$ $L = 1.5R$ $bs = 0.9$ mm $ds = 0.498$ mm $b = 36.3$ h $Ns = 2\pi R/b = 84$	$E = Es = 73.58$ GPa $\upsilon = \upsilon s = 0.3$

Table 2 Comparisons of the critical buckling loads $N_{xcr}[N/m]$ of unstiffened and stringer-stiffened cylindrical shells

Shell	Ref. [13]	Ref. [3]	Present	Diff (%)[a]
Unstiffened	346401 (3,6)[b]	–	336822 (3,6)	−2.77
Stringer-stiffened	–	25597 (1,9)	25845 (1,9)	0.97

[a]Difference (%)=$(N_{xcr,PresentAnalysis} - N_{xcr,Ref[13or3]}) \times 100/N_{xcr,Ref[13or3]}l$

[b]The numbers in parentheses refer to the numbers of longitudinal and circumferential buckling modes (m, n)

Table 3 The geometrical and material properties of the shell and stringers used in this study

Geometrical properties	Material properties
$L = 2\,\text{m}$, $R = 1\,\text{m}$, $h = 0.0015\,\text{m}$ $Ns = 4$, $bs = 0.010\,\text{m}$, $ds = 0.020\,\text{m}$	$E_1 = 135\,\text{GPa}$, $E_2 = 13\,\text{GPa}$ $G_{13} = G_{12} = 6.4\,\text{GPa}$, $G_{23} = 4.3\,\text{GPa}$ $\upsilon_{12} = 0.38$ $Es = 200\,\text{GPa}$, $\upsilon s = 0.3$

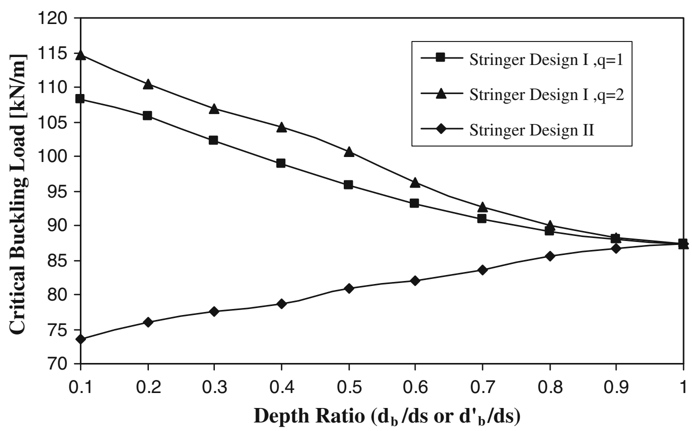

Fig. 3 Variation of the critical buckling load versus the depth ratio for the shells stiffened by Stringer Designs I and II

Fig. 3. As can be seen in this figure, with decreasing the depth ratio, the critical load of the shells reinforced by stringers with Design I (for both $q = 1$ and 2) increases. The difference in the critical load of the shells stiffened by stringers with $q = 1$ and 2, is more severe for depth ratios ranging from 0.1 to 0.5. This difference is maximum at the minimum value of depth ratio, i.e. $d_b/ds = 0.1$. Moreover, for either value of q, the largest critical load of the nonuniformly eccentric stiffened shells takes place at $d_b/ds = 0.1$. The largest buckling load occurs at $q = 2$. This load is equal to 114.6 kN/m, being 31.1% greater than the buckling load of the uniformly eccentric shell. This percentage increase is clearly large enough to ensure using of nonuniform stringers in stiffening shells. From Fig. 3, it is also viewed that the critical load of the shells stiffened by stringers with Design II drops with decreasing the depth ratio, yielding the largest critical load to be the critical load of the uniformly eccentric shell ($d'_b/ds = 1$). This demonstrates that an inappropriate distribution of eccentricity of stringers can result in an undesirable reduction in the buckling capacity.

It needs mentioning that increasing q denotes distancing from the case of uniform eccentricity, while increasing the depth ratio indicates approaching the case of uniform eccentricity, as is observable in Fig. 4. This figure illustrates the variation of the depth distribution of stringers with Designs I and II along the length of a longitudinally stiffened cylindrical shell with $L = 2\,\text{m}$, $ds = 0.020\,\text{m}$, for depth ratios 0.2, 0.4, 0.6 and 0.8. It is observed that for Stringer Design I, the stringers eccentricity at their midsection rises with the decrement of the depth ratio. This is reverse for stringers with Design II. Conversely, with the increment of the depth ratio, the depth distribution for both Stringer Designs becomes closer and closer to the case of uniform depth.

It is worthy to note that for stringers with Design I, either decreasing the depth ratio or increasing q, leads the stringers eccentricity (and as a result, the stringers stiffness) to increase at their midsection and decrease at their two ends, yielding an increase in the buckling load. This phenomenon is because the midpoints of the shell owing to being farther from the supports have much more freedom and less resistance than the end points, leading to the buckling initiation to be much probable from the midpoints. Raising the stringers eccentricity at their midsection causes the shell midpoints to become more constrained and, hence, further force would be required to move them. The reverse is true for shells stiffened by stringers with Design II. It is thus emphasized that stringers of Design I with $q = 2$ and $d_b/ds = 0.1$, which their eccentricity at their midsection is more than the two ends, are much better than the others.

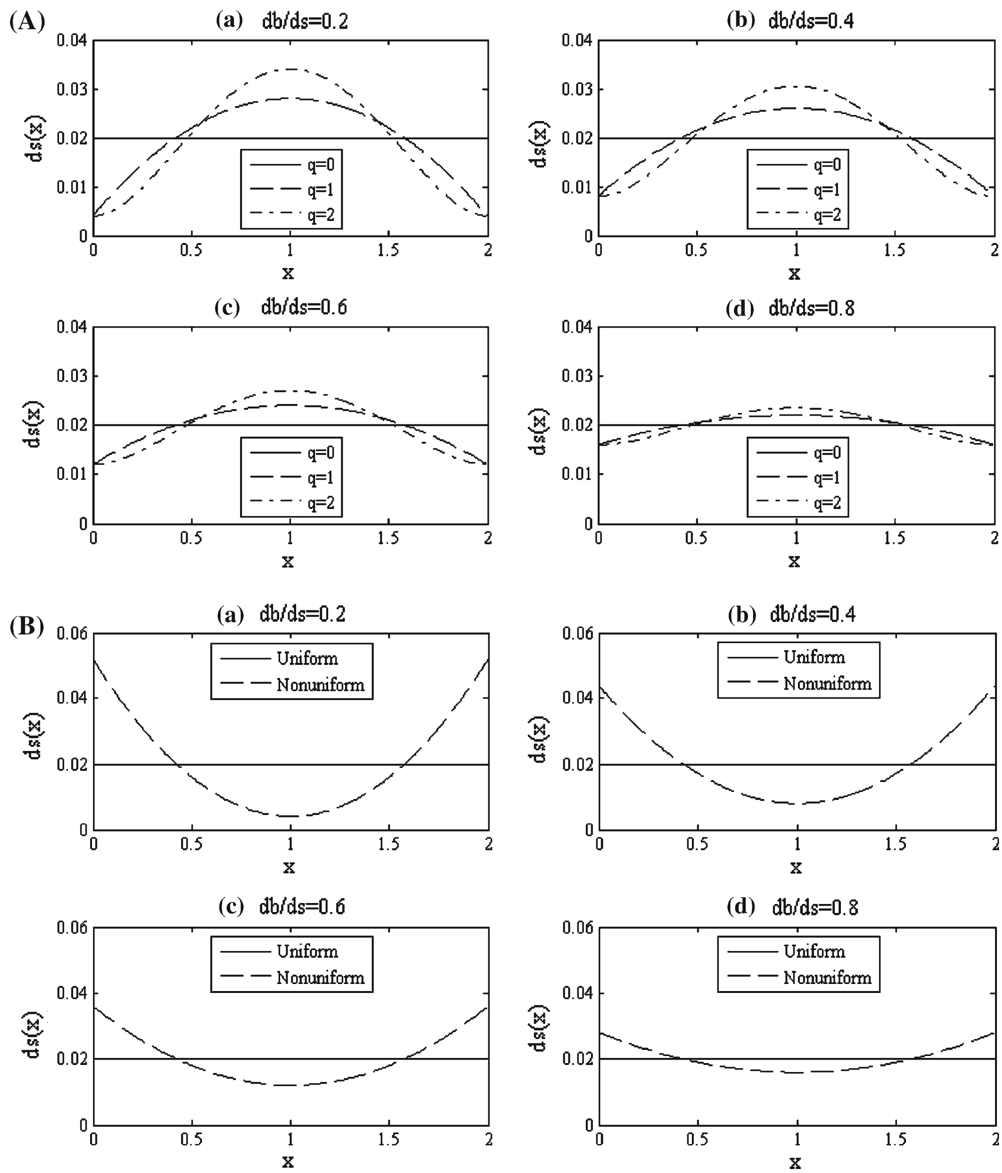

Fig. 4 Variation of depth distribution of stringers along the shell length for the depth ratios (**a**) 0.2, (**b**) 0.4, (**c**) 0.6, and (**d**) 0.8 ($L = 2$ m, $ds = 0.020$ m): (A) Stringer Design I, (B) Stringer Design II

4.3 Influence of geometrical parameters on the buckling load

In this section, the influence of some geometrical parameters of the stiffened shell on the critical buckling load of the nonuniformly and uniformly eccentric reinforced shells is examined. It is noted that the shell is reinforced by stringers with Design I for $q = 2$ and $d_b/bs = 0.1$. The geometrical and material properties of the shell and stringers are those presented in Table 3.

Figure 5 illustrates the variation of the critical buckling load versus the shell length-to-radius ratio (L/R). As viewed in the figure, the buckling load of nonuniformly eccentric shells is more than that of uniformly ones for all the L/R ratios. This rise decreases with increasing the L/R ratio (from 31.1 % at $L/R = 2$ to 9.4 % at $L/R = 16$). It is, therefore, concluded that employing nonuniform stringers (with Design I) in lieu of uniform ones is more useful for stiffening short shells. Moreover, the two curves level off with rising the L/R ratio, denoting that the value of rise in the buckling load becomes constant for the higher L/R ratios.

The critical loads of the nonuniformly and uniformly eccentric shells are plotted against the shell thickness-to-radius ratio (h/R) in Fig. 6. From the figure, it is observed that the augmentation of the buckling

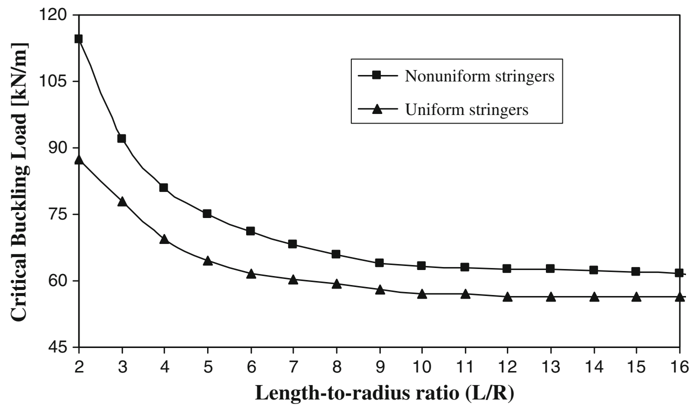

Fig. 5 Variation of the critical buckling load with the shell length-to-radius ratio

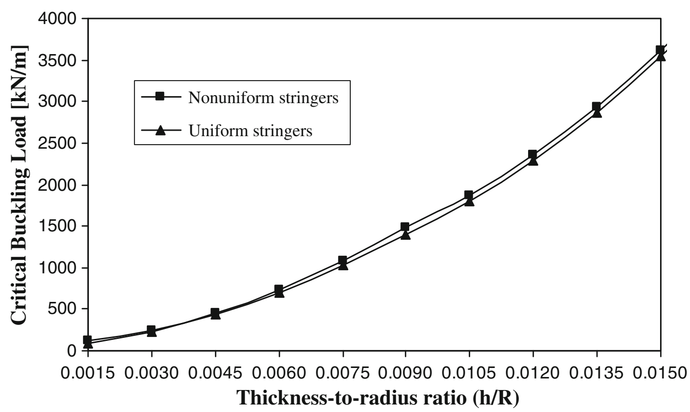

Fig. 6 Variation of the critical buckling load versus the shell thickness-to-radius ratio

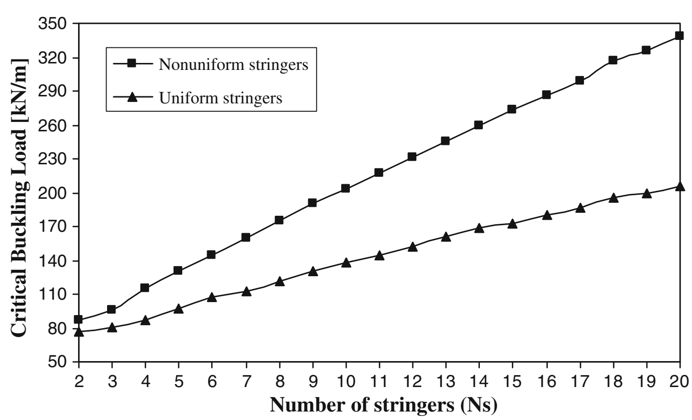

Fig. 7 Variation of the critical buckling load with respect to the number of stringers

load from the uniformly to nonuniformly eccentric shells drops with increasing the h/R ratio (from 31.1 % at $h/R = 0.0015$ to 2.3 % at $h/R = 0.015$). This can be attributed to the fact that the stringers contribution to the critical load recedes with rising the h/R ratio.

Figure 7 shows plots of the critical buckling load with respect to the number of stringers (Ns). It is seen that the percentage increase in the buckling load rises continuously with the increment of the number of stringers (from 14.4 % at $Ns = 2$ to 64.2 % at $Ns = 20$). The slope of the curve of the nonuniformly eccentric shell is more than that of the uniformly eccentric one, indicating that the critical load is more sensitive to the number of stringers when the cylindrical shell is reinforced by nonuniform stringers.

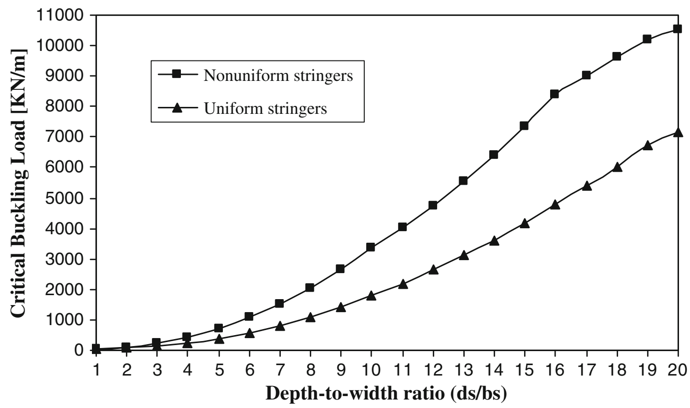

Fig. 8 Variation of the critical buckling load against the stringers depth-to-width ratio

Table 4 A comparison of the buckling loads of nonuniformly and uniformly stringer-stiffened shells, for either of the externally or internally stiffened shells

Stiffening type	Critical buckling load N_{xcr} [kN/m]		Discrepancy (%)
	Uniformly stiffened	Nonuniformly stiffened	
		$(d_b/ds = 0.1, q = 2)$	
Internal	51.8 (1,10)	79.4 (1,10)	53.3
External	87.4 (1,10)	114.6 (1,9)	31.1

A study of the variation of the critical load with respect to the stringers depth-to-width ratio (ds/bs) is presented in Fig. 8. The augmentation in the load due to the nonuniformity of stringers first increases to a maximum (from 9.7 % at $ds/bs = 1$ to 93.9 % at $ds/bs = 6$) and then decreases with the increment of the ds/bs ratio (47.5 % at $ds/bs = 20$). For $ds/bs \geq 2$, the increases are larger than 30%, warranting the use of nonuniform stringers for reinforcing shells.

Table 4 contains a comparison of the buckling loads of the nonuniformly and uniformly eccentric shells, for either of the externally and internally stiffened shells. As seen in the table, the improvement in the buckling load from the internally stiffened shell with uniform stringers to the one with nonuniform stringers is more than that for the externally stiffened shell. However, it is viewed that the buckling load of the internally, nonuniformly eccentric shell is lower than that of the externally, uniformly one. For the shell stiffened by uniform stringers, the increase in the load from the internally to externally stiffened shell is 68.7%. This is much higher than the corresponding value for the shell stiffened by nonuniform stringers, which is 44.3%.

5 Concluding remarks

The effect of nonuniform eccentricity of stringers on the axial buckling load of stiffened laminated circular cylindrical shells is studied. Assuming the identical width for stringers, we have considered some eccentricity distributions such a way that leading to the same mass for stringers. The stringers are handled as discrete elements and the Rayleigh-Ritz energy approach is employed to compute the critical buckling load. Based on the numerical results obtained, the following conclusions are made:

1. It is feasible to achieve a much greater buckling load for a stringer-stiffened laminated shell without requiring augmenting the stringers mass.
2. The considerable increases in the buckling load from the uniformly to nonuniformly eccentric shells reveal the potency of nonuniform stringers in reinforcing shells.
3. The buckling loads of the nonuniformly eccentric shells may either be raised or be lowered markedly, depending on the status in which the depth of stringers is distributed. For shells with simply supported end conditions, raising the stringers eccentricity (and therefore, the stringers stiffness) at the midspan of the shell increases the magnitude of the buckling load much further as compared with uniform stringers. Inversely, raising the stringers eccentricity at the two ends of the shell decreases the buckling load when compared to uniform stringers

4. Reinforcing shells by appropriate nonuniformly eccentric stringers are more efficient for short cylindrical shells.
5. Stiffening shells with appropriate nonuniform stringers is more effective when the cylindrical shell is thin.
6. The larger the number of stringers is, the greater the increase in the buckling load of shells owing to the nonuniformity of stringers is.
7. Using appropriate nonuniformly eccentric stringers yields more efficiency for moderately stiffened shells.
8. The increase in the critical load from the internally stiffened shell with nonuniform stringers to the one with uniform stringers is greater than that for the externally stiffened shell. Nevertheless, the critical load of the internally, nonuniformly eccentric shell is smaller than that of the externally, uniformly one.

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