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Instability of two streaming conducting and dielectric bounded fluids in porous medium under time-varying electric field

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Abstract In this paper, we consider the instability of the interface between two superposed streaming conducting and dielectric fluids of finite depths through porous medium in a vertical electric field varying periodically with time. A damped Mathieu equation with complex coefficients is obtained. The method of multiple scales is used to obtain an approximate solution of this equation, and then to analyze the stability criteria of the system. We distinguish between the non-resonance case, and the resonance case, respectively. It is found, in the first case, that both the porosity of porous medium, and the kinematic viscosities have stabilizing effects, and the medium permeability has a destabilizing effect on the system. While in the second case, it is found that each of the frequency of the electric field, and the fluid velocities, as well as the medium permeability, has a stabilizing effect, and decreases the value of the resonance point, while each of the porosity of the porous medium, and the kinematic viscosities has a destabilizing effect, and increases the value of the resonance point. In the absence of both streaming velocities and porous medium, we obtain the canonical form of the Mathieu equation. It is found that the fluid depth and the surface tension have a destabilizing effect on the system. This instability sets in for any value of the fluid depth, and by increasing the depth, the instability holds for higher values of the electric potential; while the surface tension has no effect on the instability region for small wavenumber values. Finally, the case of a steady electric field in the presence of a porous medium is also investigated, and the stability conditions show that each of the fluid depths and the porosity of the porous medium ε has a destabilizing effect, while the fluid velocities have stabilizing effect. The stability conditions for two limiting cases of interest, the case of purely fluids), and the case of absence of streaming, are also obtained and discussed in detail.

Keywords Hydrodynamic stability · Conducting and dielectric fluids · Electrohydrodynamics · Flows through porous media · Multiple scales method

1 Introduction

Electrohydrodynamics, which can be explained by the classical theory of electricity and magnetism, has recently received a great deal of attention from many researchers [1–4]. The theory suggests that the Maxwell stresses develop body forces in a fluid, loading to localized fluid motions. Electrohydrodynamic effects and flows are of central importance in many problems of colloidal hydrodynamics especially for the separation of charged particles, as occurs during electrophoresis of colloids, proteins, DNA, cells and many other particles of biological interest. Often in studying the polarization electrohydrodynamics of poorly conducting dielectric liquids, alternating high voltage is employed to prevent the buildup of free charge on the surface of the liquid [5,6]. The liquids used typically have charge relaxation times on the orders of seconds, and if alternating voltage

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of a sufficiently high frequency f is not used, charge accumulation at the surface can result in either of two basic interfacial instabilities [7,8]. The criterion for avoiding these instabilities is simply related to the charge relaxation time τ_0 [5], $f \gg 1/\tau_0 = \sigma_t/\varepsilon_t$, where σ_t and ε_t are the liquid properties of electrical conductivity and dielectric constant. Even if the above inequality is satisfied, however, the surface may be still subject to parametric interfacial instability. Only one of the few papers on parametric surface electrohydrodynamics considers the case of a tangential time-varying electric field at dielectric boundaries [9], but this paper does not consider the situation of a strong electric field gradient. Other papers by Reynolds [10], and Yih [11] are restricted to the case of a perpendicular electric field at fair to good conducting liquid surfaces. On the other hand the parametric interfacial dynamics of oscillating or vibrating liquid systems have been under study for some time. Hasegawa [12] has studied the stability of the interface between two-liquid layers of finite depth under the action of a vertical oscillation, and showed that parametric resonance is possible only when the amplitude of the vertical oscillation exceeds a threshold value. It is possible that a time-varying electric field could be employed in an analogous fashion to suppress this instability [13].

The stability of a horizontal fluid interface between a conducting and a non-conducting fluids in the presence of transverse electric field has been studied experimentally by Taylor and McEwan [14]. It has been shown that the interface becomes unstable under the action of a sufficiently great electric field. Yih [11] extended the investigation to ac electric fields and showed that the interface can be unstable even if the electric field is at all times weaker than that needed for stability in the case of a steady field [15–19], and that when instability occurs the waves may either be synchronous with the electric field or have twice its frequency. On the other hand, the natural frequency of gravitational capillary waves on the interface of a fluid in an electric field depends on the field strength. In an alternating field the natural frequency is a periodically varying parameter. Hence, parametric excitation of instability of the surface and the initiation of parametric waves are possible in an alternating electric field. Due to energy dissipation there must be a threshold of parametric instability. The threshold values of the field intensity can be found within the framework of linear stability theory [20,21]. For excellent reviews about the subject of electrohydrodynamics, see the monographs of Landau and Lifshitz [22], Melcher [23], and Castellanos [24].

The phenomena of parametric resonance arises in many branches of physics and engineering. The treatment of parametric excitation systems having distinct natural frequencies is usually operated by using the multiple time scales method [25]. The behavior of such systems is described by an equation of the Hill or Mathieu type [26]. It is well known that the instability of the solutions of such equations may be described by means of the characteristic curves of Mathieu functions which admit regions of resonance instability. Grigor'ev et al. [27] studied the instability of capillary-gravity waves at the charged flat interface between two media when the upper medium moves parallel to the interface with a velocity that has constant and time-dependent components. They showed that the temporal evolution of capillary wave amplitudes in such a system is described by the Mathieu–Hill equation. González et al. [28] presented a temporal, linear, modal stability analysis for conducting liquid jets in air subjected to time periodic electric fields. The field is originated by a mixed ac-dc potential difference between the jet and a long coaxial cylindrical electrode.

Flows through porous media have been a subject of great interest for the last several decades. This interest was motivated by numerous engineering applications in various disciplines, such as geophysical thermal and insulation engineering, the modelling of packed sphere beds, the cooling of electronic systems, groundwater hydrology, chemical catalytic reactors, ceramic processes, grain storage devices, fiber and granular insulation, petroleum reservoirs, coal combustors, ground water pollution and filtration processes, to name just a few of these applications [29,30]. Much of the recent work on this topic is reviewed by Nield and Bejan [31], Vafai [32], and Pop and Ingham [33]. In most previous studies on porous media, treatments based on Darcy's law have been considered. However, it is well known that Darcy's law is an empirical formula relating the pressure gradient, the bulk viscous resistance and the gravitational force in a porous medium. In this case, the usual viscous term in the equation of motion is replaced by the resistive term $-(\mu/k_1)\mathbf{v}$, where μ is the fluid viscosity, k_1 is the medium permeability, and $\mathbf{v}$ is the Darcian (filter) velocity of the fluid. For an excellent work about electrohydrodynamic flow in porous medium, see ref. [34]

In this paper, the instability of the interface between two superposed streaming conducting and dielectric fluids through porous medium in a vertical electric field varying periodically with time is considered. This problem, which to the best of my knowledge has not been investigated yet, finds its usefulness in chemical engineering and several geophysical situations, since in many geophysical fluid dynamical problems encountered, the fluids are dielectric or conducting and the periodic electric field of the Earth pervades the system. Because of the time dependence of the electric field, the simple equation of force balance can no longer be utilized to obtain the stability criterion, and the hydrodynamics of the fluids as well as of the porous medium

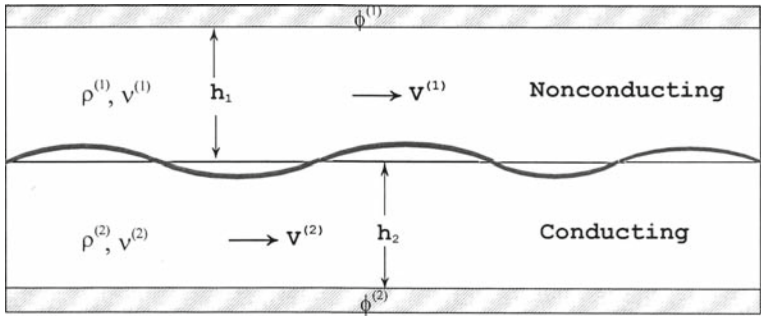

Fig. 1 Defination sketch of the problen

must be taken into account. When this is done, and both the viscosity and medium permeability are included due to Darcy's law, the stability of the interface can be shown to be governed by a damped Mathieu equation with complex coefficients which depend on the gravitational acceleration, surface tension, magnitude and frequency of the periodic electric field, depths, fluid velocities, kinematic viscosities, porosity of the porous medium, and the medium permeability. Some limiting cases are also considered, and the obtained stability results are outlined in a conclusions section at the end of the paper.

2 Basic equations and equilibrium state

We consider here two superposed incompressible inviscid fluids streaming through porous medium in the presence of an unsteady electric field. The upper fluid, which can be a gas or a liquid, with constant density $\rho^{(1)}$ and velocity $(V^{(1)}, 0, 0)$ is taken to be nonconducting, and the lower fluid, which is invariably a liquid, with density $\rho^{(2)}(> \rho^{(1)})$ and velocity $(V^{(2)}, 0, 0)$ is conductive of electricity. The xoy plane is taken to be coincide with the unperturbed middle level separating the two fluids, and the positive z-axis in the upward direction normal to the unperturbed fluid surfaces. The upper dielectric fluid is bounded above by an electrode with potential $\phi^{(1)}$ and below by the interface, has depth h_1, and the lower conducting fluid is bounded below by an electrode with potential [35,36]

$$\phi^{(2)} = \phi_0 \cos(\varpi t) \tag{1}$$

has depth h_2, see Fig. 1. In Eq. (1), t is the time and ϖ is the circular frequency of $\phi^{(2)}$. Both the fluids are assumed to be irrotational, then the fluid velocity $\mathbf{v}$ can be derived from a scalar velocity potential ψ, such that $\mathbf{v} = \nabla\psi$.

The basic equations of motion and continuity of the problem for the nonconducting and conducting fluids are as follows:

1. For nonconducting fluids [37]

$$\frac{\rho^{(1)}}{\varepsilon}\left[\frac{\partial \mathbf{v}^{(1)}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{v}^{(1)}\cdot\nabla)\mathbf{v}^{(1)}\right] = -\nabla P^{(1)} + \frac{K}{8\pi}\nabla E^{(1)2} + \rho^{(1)}\mathbf{g} - \frac{\rho^{(1)}\nu^{(1)}}{k_1}\mathbf{v}^{(1)} \tag{2}$$

$$\nabla\cdot\mathbf{v}^{(1)} = 0 \tag{3}$$

$$\nabla^2\phi^{(1)} = 0 \tag{4}$$

2. For conducting fluids [37]

$$\frac{\rho^{(2)}}{\varepsilon}\left[\frac{\partial \mathbf{v}^{(2)}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{v}^{(2)}\cdot\nabla)\mathbf{v}^{(2)}\right] = -\nabla P^{(2)} + \rho^{(2)}\mathbf{g} - \frac{\rho^{(2)}\nu^{(2)}}{k_1}\mathbf{v}^{(2)} \tag{5}$$

$$\nabla\cdot\mathbf{v}^{(2)} = 0 \tag{6}$$

where $\mathbf{E}^{(1)} = -\nabla\phi^{(1)}$, is the electric field, $\mathbf{g} = (0, 0, -g)$, and K is the dielectric constant.ε is the porosity of the porous medium, $\nu^{(1)}$ and $\nu^{(2)}$ are the kinematic viscosities of the upper and lower fluids, respectively; and k_1 is the medium permeability.

In the equilibrium state, the velocity and pressure distributions in the two fluids are given by [37]

$$\mathbf{v}^{(1)} = (V^{(1)}, 0, 0) \quad \text{and} \quad \mathbf{v}^{(2)} = (V^{(2)}, 0, 0) \tag{7}$$

$$P_0^{(1)} = P_0 - \rho^{(1)} g z + \frac{\rho^{(1)} \nu^{(1)}}{k_1} \psi_0^{(1)} \tag{8}$$

$$P_0^{(2)} = P_0 - \frac{K E_0^2}{8\pi} - \rho^{(2)} g z + \frac{\rho^{(2)} \nu^{(2)}}{k_1} \psi_0^{(2)} \tag{9}$$

where $\psi_0^{(1),(2)} = V^{(1),(2)} x +$ constant, is the zero order of the velocity potential, and P_0 is the hydrostatic pressure at the interface.

The electric potential ϕ is simply $\phi^{(2)}$ in the conducting fluid, so that the electric field in that fluid is zero. In the nonconducting fluid [11]

$$\phi = \phi^{(2)} + \left(\frac{\phi^{(1)} - \phi^{(2)}}{h_1} \right) z \tag{10}$$

so that the vertical electric field in that fluid is [11]

$$\mathbf{E}^{(1)} = \left(\frac{\phi^{(2)} - \phi^{(1)}}{h_1} \right) \mathbf{k} \tag{11}$$

where $\mathbf{k}$ is the unit vector in the z-direction.

3 Formulation of the stability problem

Suppose that the interface of the two fluids is slightly disturbed, as that at any time, it is described by the equation [11]

$$\delta z = \zeta = a(t) \exp(ikx + ily) \tag{12}$$

The electric field and potential in the lower fluid are still given by [11]

$$E^{(2)} = 0 \quad \text{and} \quad \phi^{(2)} = \phi_0 \cos \varpi t \tag{13}$$

But the electric potential of the upper fluid, which must satisfy the Laplace equation and the boundary conditions (if we now specify $\phi^{(1)} = -\phi^{(2)}$) [37]

$$\phi = \phi^{(1)} = -\phi^{(2)} \quad \text{at } z = h_1 \tag{14}$$

$$\phi = \phi^{(2)} \quad \text{at } z = \delta z \tag{15}$$

is given by

$$\phi = \phi^{(2)} \left[1 - \frac{2z}{h_1} - \frac{2 \sinh k(z - h_1)}{h_1 \sinh k h_1} a(t) \exp(ikx + ily) \right] \tag{16}$$

The normal component of the electric stress (tensile) at the interface is, with K indicating the dielectric constant [11]

$$\frac{K}{8\pi} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right] \tag{17}$$

since $z = \exp(ik_x x + ik_y y)$ is a surface of constant potential $\phi^{(2)}$ (although it varies with time). This stress component is [11]

$$\sigma_{en} = \frac{K}{2\pi} \frac{\phi^{(2)2}}{h_1^2} \left[1 + 2\alpha \coth(\alpha h_1) a(t) \exp(ikx + ily) \right] \tag{18}$$

if terms quadratic in $a(t)$ are neglected, where $\alpha = \sqrt{k^2 + l^2}$. The subscript e indicates that σ_e is an electric stress, and the subscript n indicates the normal component.

Now turning to the hydrodynamics of the fluids, we note that since the lower fluid has no electric field or magnetic field, its motion is governed by the ordinary hydrodynamic equations. The potential $\psi_1^{(2)}$ for the motion of the lower fluid is [11]

$$\psi_1^{(2)} = \widetilde{A}_2(t)\cosh\alpha(z+h_2)\exp(ikx+ily)+G_2(t) \tag{19}$$

which satisfies the boundary condition [37]

$$\frac{\partial\psi_1^{(2)}}{\partial z}=0 \quad \text{at } z=-h_2 \tag{20}$$

Similarly, the upper fluid, which is free of electric charges, has a constant K, and therefore possesses a velocity potential [11]

$$\psi_1^{(1)} = \widetilde{A}_1(t)\cosh\alpha(z-h_1)\exp(ikx+ily)+G_1(t) \tag{21}$$

which satisfied [37]

$$\frac{\partial\psi_1^{(1)}}{\partial z}=0 \quad \text{at } z=h_1 \tag{22}$$

We are left to deal with the interfacial conditions. There are two such conditions, one kinematic, and the other dynamic. The kinematic condition is [11]

$$\frac{\partial\psi_1^{(1),(2)}}{\partial z}=\left(\varepsilon\frac{\partial}{\partial t}+V^{(1),(2)}\frac{\partial}{\partial x}\right)\delta z \quad \text{at } z=0 \tag{23}$$

and the dynamic condition is [16]

$$p^{(2)}-p^{(1)}+\frac{K}{8\pi}|\mathbf{E}|^2-\frac{1}{k_1}(\rho^{(2)}\nu^{(2)}\psi_1^{(2)}-\rho^{(1)}\nu^{(1)}\psi_1^{(1)})=\alpha^2T\delta z \quad \text{at } z=0 \tag{24}$$

T being the surface tension and p the pressure.

Conditions (20), (22), and (23), on using Eqs. (19) and (21) give

$$\widetilde{A}_1=-\frac{1}{\alpha\sinh\alpha h_1}\left(\varepsilon\frac{\partial a}{\partial t}+ikV^{(1)}a\right) \tag{25}$$

$$\widetilde{A}_2=\frac{1}{\alpha\sinh\alpha h_2}\left(\varepsilon\frac{\partial a}{\partial t}+ikV^{(2)}a\right) \tag{26}$$

Hence, Eqs. (19) and (21) can be written, respectively, in the form

$$\psi_1^{(1)}=-\frac{1}{\alpha\sinh\alpha h_1}\left(\varepsilon\frac{\partial a}{\partial t}+ikV^{(1)}a\right)\cosh\alpha(z-h_1)\exp(ikx+ily)+G_1(t) \tag{27}$$

$$\psi_1^{(2)}=\frac{1}{\alpha\sinh\alpha h_2}\left(\varepsilon\frac{\partial a}{\partial t}+ikV^{(2)}a\right)\cosh\alpha(z+h_2)\exp(ikx+ily)+G_2(t) \tag{28}$$

In order to utilize Eq. (24), it is necessary to use the Bernoulli equations [11]

$$\frac{\rho^{(1)}}{\varepsilon^2}\left[\varepsilon\frac{\partial\psi_1^{(1)}}{\partial t}+V^{(1)}\frac{\partial\psi_1^{(1)}}{\partial x}+\frac{1}{2}q_1^2\right]+p^{(1)}+\rho^{(1)}g\delta z=F^{(1)}(t) \tag{29}$$

$$\frac{\rho^{(2)}}{\varepsilon^2}\left[\varepsilon\frac{\partial\psi_1^{(2)}}{\partial t}+V^{(2)}\frac{\partial\psi_1^{(2)}}{\partial x}+\frac{1}{2}q_2^2\right]+p^{(2)}+\rho^{(2)}g\delta z=F^{(2)}(t) \tag{30}$$

in which q is the speed. Since an arbitrary functions of time has been added to $\psi_1^{(1)}$ and to $\psi_1^{(2)}$, we can take $F^{(1)}(t)$ and $F^{(2)}(t)$ to be zero. Neglecting q^2, we have [11]

$$p^{(2)}-p^{(1)}=g(\rho^{(1)}-\rho^{(2)})\delta z+\frac{\rho^{(1)}}{\varepsilon^2}\left[\varepsilon\frac{\partial\psi_1^{(1)}}{\partial t}+V^{(1)}\frac{\partial\psi_1^{(1)}}{\partial x}\right]-\frac{\rho^{(2)}}{\varepsilon^2}\left[\varepsilon\frac{\partial\psi_1^{(2)}}{\partial t}+V^{(2)}\frac{\partial\psi_1^{(2)}}{\partial x}\right] \tag{31}$$

Substituting from Eqs. (12), (27), and (28) into Eq. (31), we obtain

$$
\begin{aligned}
p^{(2)} - p^{(1)} &= g(\rho^{(1)} - \rho^{(2)}) a \exp(ikx + ily) \\
&\quad - \frac{\rho^{(1)}}{\alpha} \coth \alpha h_1 \left[\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{2ikV^{(1)}}{\varepsilon} \frac{\mathrm{d}a}{\mathrm{d}x} - \frac{k^2 V^{(1)2}}{\varepsilon^2} a \right] \exp(ikx + ily) - \frac{\rho^{(2)}}{\alpha} \coth \alpha h_2 \\
&\quad \times \left[\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{2ikV^{(2)}}{\varepsilon} \frac{\mathrm{d}a}{\mathrm{d}x} - \frac{k^2 V^{(2)2}}{\varepsilon^2} \right] \exp(ikx + ily) + \frac{\rho^{(1)}}{\varepsilon} \frac{\mathrm{d}G_1}{\mathrm{d}t} - \frac{\rho^{(2)}}{\varepsilon} \frac{\mathrm{d}G_2}{\mathrm{d}t}
\end{aligned} \tag{32}
$$

Putting Eqs. (12), (27), (28), and (32) into Eq. (24), and using the σ_{en} in Eq. (18) for the electric term in Eq. (24), we obtain [11]

$$
\frac{K}{2\pi} \left(\frac{\phi^{(2)}}{h_1} \right)^2 = \frac{\rho^{(2)}}{\varepsilon} \left(\frac{\mathrm{d}}{\mathrm{d}t} + \frac{\varepsilon \nu^{(2)}}{k_1} \right) G_2(t) - \frac{\rho^{(1)}}{\varepsilon} \left(\frac{\mathrm{d}}{\mathrm{d}t} + \frac{\varepsilon \nu^{(1)}}{k_1} \right) G_1(t) \tag{33}
$$

and

$$
F_1 \frac{\mathrm{d}^2 a}{\mathrm{d}t^2} + i\,(F_2 - i F_3) \frac{\mathrm{d}a}{\mathrm{d}t} - \left[F_4 - i F_5 + \left\{ \frac{K\alpha^2}{\pi} \left(\frac{\phi_0}{h_1} \right)^2 \cos^2(\varpi t) \coth \alpha h_1 - F_6 \right\} \right] a = 0 \tag{34}
$$

where

$$
F_1 = \rho^{(1)} \coth \alpha h_1 + \rho^{(2)} \coth \alpha h_2 \tag{35}
$$
$$
F_2 = \frac{2k}{\varepsilon} \left[\rho^{(1)} V^{(1)} \coth \alpha h_1 + \rho^{(2)} V^{(2)} \coth \alpha h_2 \right] \tag{36}
$$
$$
F_3 = \frac{\varepsilon}{k_1} \left[\rho^{(1)} \nu^{(1)} \coth \alpha h_1 + \rho^{(2)} \nu^{(2)} \coth \alpha h_2 \right] \tag{37}
$$
$$
F_4 = \frac{k^2}{\varepsilon^2} \left[\rho^{(1)} V^{(1)2} \coth \alpha h_1 + \rho^{(2)} V^{(2)2} \coth \alpha h_2 \right] \tag{38}
$$
$$
F_5 = \frac{k}{k_1} \left[\rho^{(1)} \nu^{(1)} V^{(1)} \coth \alpha h_1 + \rho^{(2)} \nu^{(2)} V^{(2)} \coth \alpha h_2 \right] \tag{39}
$$
$$
F_6 = (\rho^{(2)} - \rho^{(1)}) \alpha g + \alpha^3 T \tag{40}
$$

Equation (34) is the well known damped Mathieu equation with complex coefficients. We now need to determine the structure of the stability conditions of Eq. (34).

4 Multiple time scales method

We use the method of multiple scales as described by Nayfeh and Mook [38] to obtain an approximate solution of the damped Mathieu equation (34), and then to analyze the stability criteria of the considered system. We introduce a fast time scale $T_0 = t$, and a slow time scale $T_1 = \sigma t$, where σ is a small parameter. The differential operators can now be expressed as the derivative expansions

$$
\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial T_0} + \sigma \frac{\partial}{\partial T_1} + \sigma^2 \frac{\partial}{\partial T_2} + \cdots \tag{41}
$$
$$
\frac{\mathrm{d}^2}{\mathrm{d}t^2} = \frac{\partial^2}{\partial T_0^2} + 2\sigma \frac{\partial^2}{\partial T_0 \partial T_1} + \sigma^2 \left(\frac{\partial^2}{\partial T_1^2} + 2 \frac{\partial^2}{\partial T_0 \partial T_2} \right) + \cdots \tag{42}
$$

We assume also that the solution of Eq. (34) can be written as

$$
a(t;\sigma) = a_0(T_0, T_1) + \sigma a_1(T_0, T_1) + \sigma^2 a_2(T_0, T_1) + \cdots \tag{43}
$$

Inserting Eqs. (41)–(43) into the damped Mathieu equation (34), collecting terms of like powers of σ, and then equating these coefficients to zero, because powers of σ are linearly independent, we obtain

$$F_1 \frac{\mathrm{d}^2 a_0}{\mathrm{d}T_0^2} + i\,(F_2 - i\,F_3)\,\frac{\mathrm{d}a_0}{\mathrm{d}T_0} - [(F_4 - F_6) - i\,F_5]\,a_0 = 0 \tag{44}$$

and

$$F_1 \frac{\partial^2 a_1}{\partial T_0^2} + i\,(F_2 - i\,F_3)\,\frac{\partial a_1}{\partial T_0} - [(F_4 - F_6) - i\,F_5]\,a_1 = -2F_1 \frac{\partial^2 a_0}{\partial T_0 \partial T_1}$$
$$-i\,(F_2 - i\,F_3)\,\frac{\partial a_0}{\partial T_1} - \left[\frac{K\alpha^2}{\pi}\left(\frac{\phi_0}{h_1}\right)^2 \cos^2(\varpi T_0)\coth\alpha h_1\right] a_0 \tag{45}$$

The solution of Eq. (44) can be written in the form

$$a_0(T_0, T_1) = F(T_1)\exp[(\Lambda + i\Omega)T_0] + C.C. \tag{46}$$

where $F(T_1)$ is an unknown complex function of T_1, $C.C.$ represents the complex conjugate of the preceding term, Λ and Ω are real. Substituting from Eq. (46) into Eq. (44), and separating the real and imaginary parts, we obtain

$$F_1(\Lambda^2 - \Omega^2) + F_3\Lambda - F_2\Omega - (F_4 - F_6) = 0 \tag{47}$$

and

$$2\Lambda\left[\Omega F_1 + \frac{F_2}{2}\right] = -(F_3\Omega + F_5) \tag{48}$$

Eliminate Λ between Eqs. (47) and (48), we obtain

$$4F_1^3\Omega^4 + 8F_1^2F_2\Omega^3 + \left[F_1F_3^2 + 5F_1F_2^2 + 4F_1^2(F_4 - F_6)\right]\Omega^2 + \left[F_2F_3^2 + F_2^3 + 4F_1F_2(F_4 - F_6)\right]\Omega$$
$$+\left[-F_1F_5^2 + F_2F_3F_5 + F_2^2(F_4 - F_6)\right] = 0 \tag{49}$$

The dispersion relation (49) is a quartic equation in Ω with real coefficients. To study the properties of the roots of Eq. (49) can judge the stability of the system. The given system whose characteristic polynomial

$$L(\omega) = \Omega^4 + b_1\Omega^3 + b_2\Omega^2 + b_3\Omega + b_4 \tag{50}$$

has no zero repeated root is stable if and only if [39]

$$b_1 > 0, b_2 > 0, b_3 \geq 0, b_4 \geq 0, \quad \text{and} \quad b_1b_2b_3 - b_1^2b_4 - b_3^2 \geq 0 \tag{51}$$

Comparing Eqs. (49) and (50), and applying the stability conditions (51) to Eq. (49), we find that the first condition in Eq. (51) is trivially satisfied, and the second and third conditions of Eqs. (51) are satisfied only if the following condition holds:

$$F_6 \leq F_4 + \left(\frac{F_3^2 + F_2^2}{4F_1}\right) \tag{52}$$

while the fourth and fifth boundary conditions in Eq. (51) are satisfied only if the following conditions hold:

$$F_1F_5^2 + F_6F_2^2 \leq F_2[F_3F_5 + F_2F_4] \tag{53}$$

and

$$\left[F_3^2 + 5F_2^2 + 4F_1(F_4 - F_6)\right]\left[F_3^2 + F_2^2 + 4F_1(F_4 - F_6)\right] - 8F_1\left[-F_1F_5^2 + F_2F_3F_5 + F_2^2(F_4 - F_6)\right]$$
$$-\frac{F_2}{8F_1^3}\left[F_3^2 + F_2^2 + 4F_1(F_4 - F_6)\right]^3 \geq 0 \tag{54}$$

respectively, then the system under consideration is stable only if the conditions in (52)–(54) are simultaneously satisfied.

Also, substitute from Eq. (46) into Eq. (45), we obtain

$$F_1 \frac{\partial^2 a_1}{\partial T_0^2} + i(F_2 - iF_3)\frac{\partial a_1}{\partial T_0} - [(F_4 - F_6) - iF_5]a_1 =$$
$$- \left[\{(2F_1\Lambda + F_3) + i(2F_1\Omega + F_2)\} \frac{\mathrm{d}F}{\mathrm{d}T_1} + \left\{ \frac{K\alpha^2}{2\pi} \left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1 \right\} F \right]$$
$$\times \exp[(\Lambda + i\Omega)T_0] - \left\{ \frac{K\alpha^2}{4\pi} \left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1 [\exp\{i(\Omega + 2\varpi)T_0\} \right.$$
$$\left. + \exp\{i(\Omega - 2\varpi)T_0\}] \right\} F \exp[\Lambda T_0] + C.C. \tag{55}$$

Equation (55) contains non-homogeneous terms.

5 Resonance and non-resonance cases

Now, a uniform solution for Eq. (55) is required to eliminate the secular terms. This elimination introduces the solvability condition corresponding to the terms containing the factor $\exp[(\Lambda + i\Omega)T_0]$. Thus, in order to analyze the solution of Eq. (55), we need to distinguish between two cases. The first one is the non-resonance case, when the frequency ϖ of the oscillating electric field is not near the frequency Ω, and the second one is the resonance case which arises when the frequency ϖ is near Ω.

5.1 The non-resonance case

In order to obtain a uniformly valid expansion, the coefficient of the factor $\exp[(\Lambda + i\Omega)T_0]$ in Eq. (55) must vanish. Thus, we have

$$\frac{\mathrm{d}F}{\mathrm{d}T_1} + \frac{\frac{K\alpha^2}{2\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1}{\{(2F_1\Lambda + F_3) + i(2F_1\Omega + F_2)\}} F = 0 \tag{56}$$

This equation can be simplified, using Eqs. (47) and (48), to the form

$$\frac{\mathrm{d}F}{\mathrm{d}T_1} - (P_1 + iP_2)F = 0 \tag{57}$$

where

$$P_1 = \frac{(F_2\Lambda + F_5)\left\{\frac{K\alpha^2}{2\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1\right\}}{\Omega\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda + \Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}} \tag{58}$$

$$P_2 = \frac{(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{2\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda + \Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}} \tag{59}$$

The solution of Eq. (57) can be written as

$$F(T_1) = \widehat{A} \exp[(P_1 + iP_2)T_1] \tag{60}$$

and this solution shows that stability occurs in the non-resonance case when $P_1 < 0$. Using Eq. (52), we find that the denominator in Eq. (58) is positive. Therefore the stability condition $P_1 < 0$ holds if $F_2\Lambda + F_5 < 0$, i.e., when $\Lambda < -(F_5/F_2)$, i.e., when the parameter Λ is less than a negative value. Substituting from Eqs. (26) and (29), respectively, in this condition, we can conclude that both the porosity of porous medium ε, and the kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$ have stabilizing effects on the considered system in the non-resonance case, while the medium permeability k_1 has a destabilizing effect.

5.2 The resonance case

In order to obtain a solution in the neighborhood of the resonance case, we express the nearness of ϖ to Ω by introducing the detuning parameter ζ according to

$$\varpi = \Omega + \sigma\zeta \tag{61}$$

and hence

$$-i(\Omega - 2\varpi)T_0 = i\Omega T_0 + 2i\zeta T_1 \tag{62}$$

Thus, the secular terms can be eliminated when

$$\frac{dF}{dT_1} + \frac{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1}{\{(2F_1\Lambda + F_3) + i(2F_1\Omega + F_2)\}}\left\{2F + \overline{F}\exp(2i\zeta T_1)\right\} = 0 \tag{63}$$

Equation (63) admits a non-trivial solution of the form [40]

$$F(T_1) = \left[\beta(T_1) + i\gamma(T_1)\right]\exp(i\zeta T_1) \tag{64}$$

with real functions $\beta(T_1)$ and $\gamma(T_1)$.

Substituting from Eq. (64) into Eq. (63), and separating the solvability condition into real and imaginary parts, we obtain the equations governing β and γ in the form

$$\left[\frac{d}{dT_1} - \frac{3(F_2\Lambda + F_5)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\right\}}{\Omega\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right]\beta$$
$$-\left[\zeta - \frac{(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right]\gamma = 0 \tag{65}$$

and

$$\left[\zeta - \frac{3(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right]\beta$$
$$+\left[\frac{d}{dT_1} - \frac{(F_2\Lambda + F_5)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\right\}}{\Omega\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right]\gamma = 0 \tag{66}$$

These coupled linear equations have the solutions

$$\beta(T_1) = \left[\zeta - \frac{(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right]\exp(QT_1) \tag{67}$$

and

$$\gamma(T_1) = \left[Q - \frac{3(F_2\Lambda + F_5)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\right\}}{\Omega\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right]\exp(QT_1) \tag{68}$$

where the constant Q satisfies the equation

$$Q^2 - \frac{4(F_2\Lambda + F_5)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}}{\Omega\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}} Q + \frac{3(F_2\Lambda + F_5)^2\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}^2}{\left[\Omega\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}\right]^2}$$
$$+ \left[\zeta - \frac{3(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right] \times \left[\zeta - \frac{(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\right] = 0 \quad (69)$$

The dispersion relation (69) is a quadratic equation in the growth rate Q. Necessary and sufficient conditions for stability are therefore governed by the inequalities $\Lambda < -(F_5/F_2)$, and

$$\zeta^2 - \frac{4(2F_1\Omega + F_2)\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}}\zeta + \frac{3\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}^2\left\{(2F_1\Omega + F_2)^2 + (2F_1\Lambda + F_3)^2\right\}}{\left[\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}\right]^2} > 0 \quad (70)$$

This inequality can be satisfied when

$$(\zeta - \zeta_1^*)(\zeta - \zeta_2^*) > 0 \quad (71)$$

i.e., when

$$\zeta > \zeta_1^* \quad \text{and} \quad \zeta < \zeta_2^* \quad (\zeta_1^* > \zeta_2^*) \quad (72)$$

where

$$\zeta_{1,2}^* = \frac{\left\{\frac{K\alpha^2}{4\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right\}}{\left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\}} \times \left[2(2F_1\Omega + F_2) \pm \sqrt{(2F_1\Omega + F_2)^2 - 3(2F_1\Lambda + F_3)^2}\right] \quad (73)$$

In view of Eq. (72), the stability conditions in the resonance case can be sought in the form

$$\phi_1 < \phi_0^2 < \phi_2 \quad (74)$$

where

$$\Phi_{1,2} = \frac{4\pi h_1^2(\varpi - \Omega)}{3K\alpha^2\sigma \coth\alpha h_1\left[(2F_1\Omega + F_2)^2 + (2F_1\Lambda - F_3)^2\right]} \times \left[2(2F_1\Omega + F_2) \pm \sqrt{(2F_1\Omega + F_2)^2 - 3(2F_1\Lambda + F_3)^2}\right] \times \left\{8F_1^2\Omega^2 + 4F_1F_2(\Lambda+\Omega) + 4F_1(F_4 - F_6) + F_2^2 + F_3^2\right\} \quad (75)$$

The values of ϕ_0^2 as described by Eqs. (74) and (75) are the critical values of the disturbances. These critical values, which are known as the transition curves, separate the stable from the unstable regions. We shall

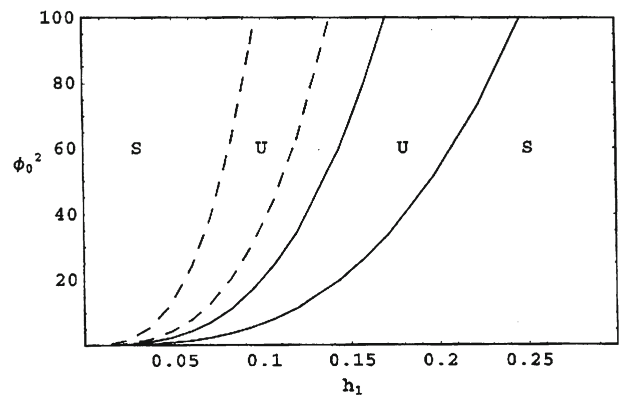

Fig. 2 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $h_1 \times 10^{-2}$ (m) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $V^{(1)} = 0.003\,\text{m/sec}$, $V^{(2)} = 0.005\,\text{m/sec}$, $k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\nu^{(1)} = 0.1 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\nu^{(2)} = 0.6 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\varepsilon = 3.3\,\text{sec/m}$, $h_2 = 0.008\,\text{m}$, $k = \alpha = 50\,\text{m}^{-1}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, when $\varpi = 20\,\text{sec}^{-1}$ (*solid curves*), and $\varpi = 50\,\text{sec}^{-1}$ (*dashed*)

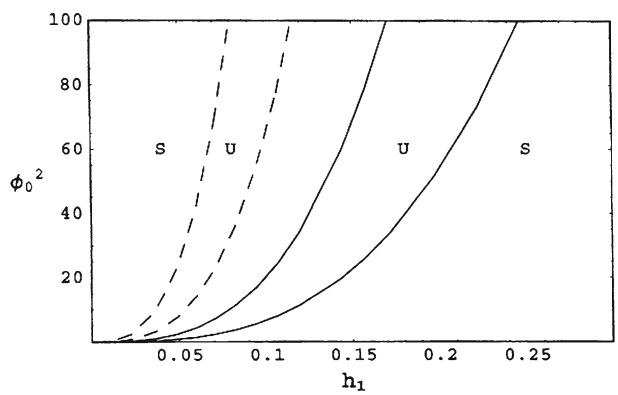

Fig. 3 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $h_1 \times 10^{-2}$ (m) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\nu^{(1)} = 0.1 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\nu^{(2)} = 0.6 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\varepsilon = 3.3\,\text{sec/m}$, $h_2 = 0.008\,\text{m}$, $k = \alpha = 50\,\text{m}^{-1}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, $\varpi = 20\,\text{sec}^{-1}$, when $V^{(1)} = 0\,\text{m/sec}$, $V^{(2)} = 0\,\text{m/sec}$ (*solid curves*), and $V^{(1)} = 0.15\,\text{m/sec}$, $V^{(2)} = 0.2\,\text{m/sec}$ (*dashed*)

give numerical discussions for the effects of various physical parameters on the stability of the system under consideration by drawing the transition curves $\phi_0^2 = \phi_1$ and $\phi_0^2 = \phi_2$, respectively, in the (ϕ_0^2, h_1) plane. In the following figures, Figs. 2, 3, 4, 5 and 6, the letter S denotes stable region, while the letter U denotes unstable region, respectively. According to the Floquet's theory [25], the region bounded by the two branches of the transition curves is unstable, while the area outside them is stable. Note that the value of the parameter Ω appears in Eq. (75) is determined by solving Eq. (49) for Ω and we choose the real root of the solutions.

In Figs. 2, 3, 4, we plot ϕ_0^2 versus h_1 for different values of the frequency of the periodic electric field ϖ, the fluid velocities $V^{(1)}$, $V^{(2)}$, and the medium permeability k_1, respectively. It is found that, when the frequency ϖ is changed from $\varpi = 20\,\text{sec}^{-1}$ to the value $\varpi = 50\,\text{sec}^{-1}$ in Fig. 2, the fluid velocities $V^{(1)}$, $V^{(2)}$ are changed from $V^{(1)} = 0\,\text{m/sec}$, $V^{(2)} = 0\,\text{m/sec}$ to $V^{(1)} = 0.15\,\text{m/sec}$, $V^{(2)} = 0.2\,\text{m/sec}$ in Fig. 3, and the medium permeability k_1 is changed from $k_1 = 0.015 \times 10^{-12}\,\text{m}^2$ to $k_1 = 0.098 \times 10^{-12}\,\text{m}^2$ in Fig. 4, that the instability region U has decreased, and the resonance point moves towards the left. Then the increase of each of the frequency of the electric field ϖ, and the fluid velocities $V^{(1)}$, $V^{(2)}$, as well as the medium permeability k_1, has a stabilizing effect on the system, and decreases the value of the resonance point.

In Figs. 5 and 6, we plot ϕ_0^2 versus h_1 for different values of the porosity of the porous medium ε, and the kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$, respectively. It is found, when the porosity ε is changed from $\varepsilon = 0.07\,\text{sec/m}$ to $\varepsilon = 0.66$ sec/m in Fig. 5, and the kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$ are changed from $\nu^{(1)} = 0\,\text{m}^2/\text{sec}$, $\nu^{(2)} = 0\,\text{m}^2/\text{sec}$ to $\nu^{(1)} = 0.002\,\text{cm/sec}$, $\nu^{(2)} = 0.0029\,\text{cm/sec}$ in Fig. 6, respectively, that the instability region

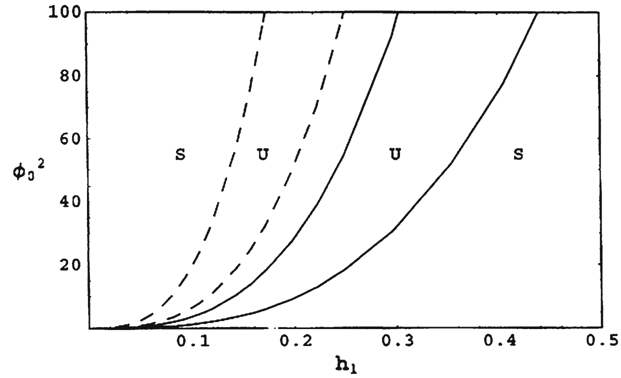

Fig. 4 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $h_1 \times 10^{-2}$ (m) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $V^{(1)} = 0.003\,\text{m/sec}$, $V^{(2)} = 0.005\,\text{m/sec}$, $\nu^{(1)} = 0.1 \times 10^{-4}\,\text{m}^2\text{/sec}$, $\nu^{(2)} = 0.6 \times 10^{-4}\,\text{m}^2\text{/sec}$, $\varepsilon = 3.3\,\text{sec/m}$, $h_2 = 0.008\,\text{m}$, $k = \alpha = 50\,\text{m}^{-1}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2\text{/kg m}^3$, $\varpi = 20\,\text{sec}^{-1}$, when $k_1 = 0.15 \times 10^{-12}\,\text{m}^2$ (*solid curves*), and $k_1 = 0.098 \times 10^{-12}\,\text{m}^2$ darcy (*dashed*)

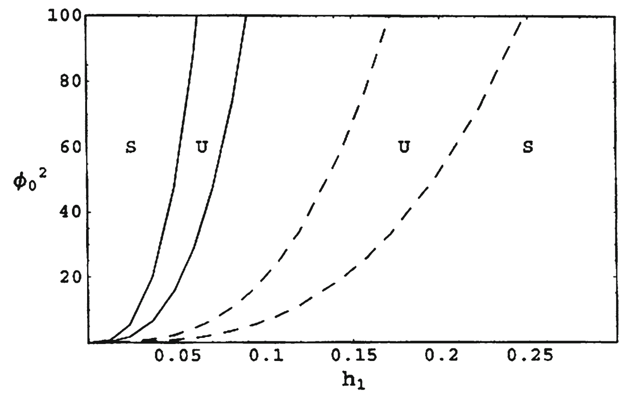

Fig. 5 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $h_1 \times 10^{-2}$ (m) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879$ kg/m^3, $V^{(1)} = 0.003\,\text{m/sec}$, $V^{(2)} = 0.005\,\text{m/sec}$, $k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\nu^{(1)} = 0.1 \times 10^{-4}\,\text{m}^2\text{/sec}$, $\nu^{(2)} = 0.6 \times 10^{-4}\,\text{m}^2\text{/sec}$, $h_2 = 0.008$ m, $k = \alpha = 50\,\text{m}^{-1}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2\text{/kg m}^3$, $\varpi = 20\,\text{sec}^{-1}$, when $\varepsilon = 0.07\,\text{sec/m}$ (*solid curves*), and $\varepsilon = 0.66\,\text{sec/m}$ (*dashed*)

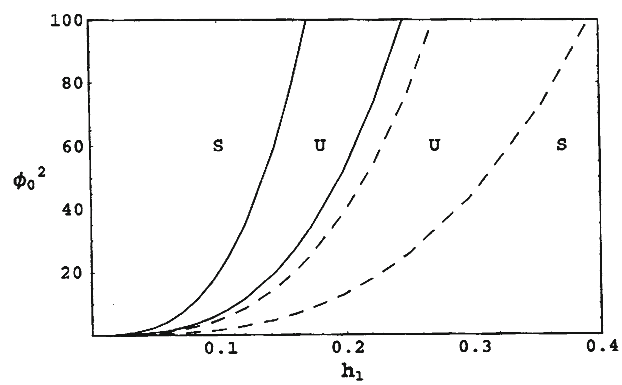

Fig. 6 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $h_1 \times 10^{-2}$ (m) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $V^{(1)} = 0.003\,\text{m/sec}$, $V^{(2)} = 0.005\,\text{m/sec}$, $k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\varepsilon = 3.3\,\text{sec/m}$, $h_2 = 0.008\,\text{m}$, $k = \alpha = 50\,\text{m}^{-1}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2\text{/kg m}^3$, $\varpi = 20\,\text{sec}^{-1}$, when $\nu^{(1)} = 0\,\text{m}^2\text{/sec}$, $\nu^{(2)} = 0$ m^2/sec (*solid curves*), and $\nu^{(1)} = 0.002$ m^2/sec, $\nu^{(2)} = 0.0029\,\text{m}^2\text{/sec}$ (*dashed*)

U has increased, and the resonance point moves towards the right. Then the increase of each of the porosity of the porous medium ε, and the kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$ has a destabilizing effect on the system, and increases the value of the resonance point.

6 Absence of streaming and porous medium

In the absence of both streaming velocities and porous medium, then Eqs. (33) and (34) reduce to the following forms:

$$\frac{K}{2\pi}\left(\frac{\phi^{(2)}}{h_1}\right)^2 = \rho^{(2)}G_2'(t) - \rho^{(1)}G_1'(t) \tag{76}$$

and

$$\left[\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2\right]\frac{\mathrm{d}^2a}{\mathrm{d}t^2} + \left\{(\rho^{(2)} - \rho^{(1)})\alpha g + \alpha^3 T - \frac{K\alpha^2}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\cos^2(\varpi t)\coth\alpha h_1\right\}a = 0 \tag{77}$$

or

$$\frac{\mathrm{d}^2a}{\mathrm{d}t^2} + \left[\omega_0^2 - b - b\cos(2\varpi t)\right]a = 0 \tag{78}$$

in which

$$\omega_0^2 = \frac{(\rho^{(2)} - \rho^{(1)})\alpha g + \alpha^3 T}{\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2}, \tag{79}$$

$$b = \frac{\frac{K\alpha^2}{2\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1}{\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2} \tag{80}$$

Equation (78) can be put in the canonical form of the Mathieu equation

$$\frac{\mathrm{d}^2a}{\mathrm{d}\tau^2} + [\delta_1 - 2\delta_2\cos(2\tau)]\,a = 0 \tag{81}$$

if we put

$$\tau = \varpi t,\ \ \delta_1 = \frac{\omega_0^2 - b}{\varpi^2},\ \ \text{and}\ \ \delta_2 = \frac{b}{2\varpi^2} \tag{82}$$

Equations (81) and (82) are the same equations obtained earlier by Yih [11] i.e., they can be obtained from our results in the limiting case of absence of both streaming and porous medium. The stability of the solutions of Eq. (81) is determined entirely by the coefficients δ_1 and δ_2. This solution can be written in the form $a = \widehat{a}(\tau)\exp(\pm\chi\tau)$, where $\chi = \chi(\delta_1, \delta_2)$ is the exponential growth rate. The function $\widehat{a}(\tau)$ is periodic with fundamental frequency equal to $m\varpi$ ($m = 1, 2, 3, \ldots$). Jones [13] have shown that the $m = 1$ solution has its fundamental harmonic at the frequency of the voltage, and over-all stability of the continuum is determined by considering all possible wavenumbers. The stability diagram for Eq. (81) is standard [41], and for completeness is given in Fig. 7 (and also in ref. [11]), which shows that even if b is very small, there may be regions of instability. Since ϕ_0^2 has a frequency of 2ϖ, not ϖ, the various regions correspond to double frequency and synchronism of the hydrodynamic oscillations in relation to ϕ_0, instead of synchronism and half-frequency, respectively. It should be remarked that the instability found here is akin to the one found by Benjamin and Ursell [42] for the free surface of a liquid in vertical periodic motion.

The stability condition for Mathieu equation (81) reduced to the problem of the bounded regions of the Mathieu functions for which McLachlan [26] gives the condition for stability as

$$\delta_2^2 - 4\delta_1\delta_2 + 2\delta_1(1 - \delta_1) > 0 \tag{83}$$

Substituting from Eq. (82) into the stability condition (83), we get

$$8\varpi^2(\omega_0^2 - b) + \left[b^2 - 8\omega_0^2(\omega_0^2 - b)\right] > 0 \tag{84}$$

For any arbitrary frequency ϖ, the condition (84) can be satisfied when the following two conditions hold:

$$\omega_0^2 > b \tag{85}$$

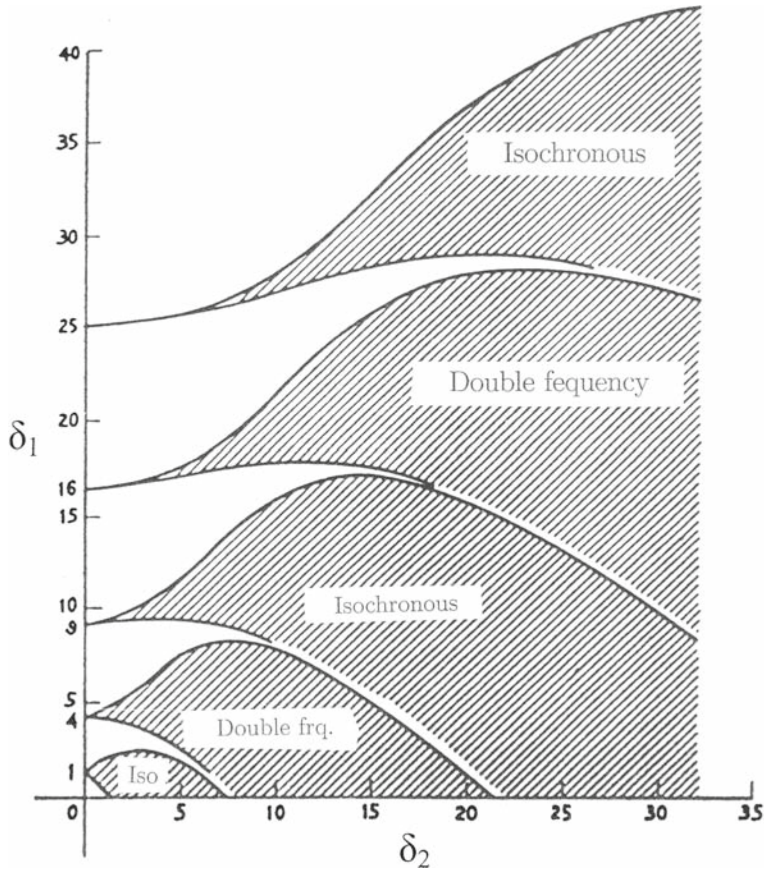

Fig. 7 The stability diagram for the Mathieu equation (81)

and

$$b^2 + 8\omega_0^2 b - 8\omega_0^4 > 0 \tag{86}$$

Substitute from Eqs. (79) and (80) into the conditions (85) and (86), we obtain

$$\phi_0^2 < H_c, \quad H_c > 0 \tag{87}$$

and

$$\phi_0^4 + 8H_c\phi_0^2 - 8H_c^2 > 0 \tag{88}$$

where

$$H_c = \frac{2\pi h_1^2 F_6}{K\alpha^2 \coth \alpha h_1} \tag{89}$$

The inequality (88) can be written in the form

$$\left[\phi_0^2 - 2(\sqrt{6}-2)H_c\right]\left[\phi_0^2 + 2(\sqrt{6}+2)H_c\right] > 0 \tag{90}$$

For $H_c > 0$ (since $\rho^{(2)} > \rho^{(1)}$)

$$\left[\phi_0^2 - 2(\sqrt{6}-2)H_c\right] > 0 \tag{91}$$

The stability occurs only when

$$3.6H_c < \phi_0^2 < H_c \tag{92}$$

Otherwise the system will be unstable, when

$$H_c < \phi_0^2 < 3.6H_c \tag{93}$$

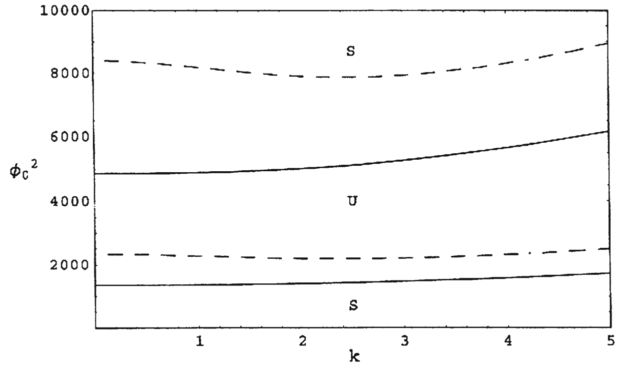

Fig. 8 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $k \times 10^{-2}$ (m^{-1}) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $h_2 = 0.8\,\text{cm}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, when $h_1 = 0.005\,\text{m}$ (*solid curves*), and $h_1 = 0.006\,\text{m}$ (*dashed*)

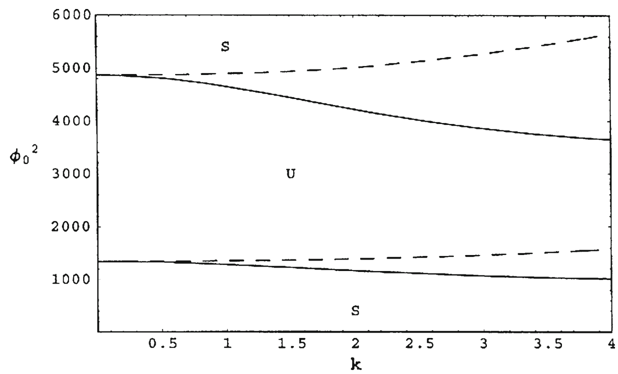

Fig. 9 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $k \times 10^{-2}$ (m^{-1}) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $h_1 = 0.005\,\text{m}$, $h_2 = 0.008\,\text{m}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, when $T = 0.03\,\text{kg/sec}$ (*solid curves*), and $T = 0.076$ kg/sec (*dashed*)

The values of ϕ_0^2 as described by Eqs. (92) or (93) are the critical values of the disturbances. These critical values, which are known as the transition curves, separate the stable from the unstable regions. We shall give numerical discussions for the stability of the system under consideration by drawing the transition curves $\phi_0^2 < H_c$ and $\phi_0^2 = 3.6H_c$, respectively, in the (ϕ_0^2, k) plane. In the following Figs. 8 and 9, the letter S denotes stable region, while the letter U denotes unstable region, respectively. According to the Floquet's theory [25], the region bounded by the two branches of the transition curves is unstable, while the area outside them is stable.

In Figs. 8 and 9, we plot ϕ_0^2 versus k for different values of the porosity of the fluid depth h_1, and the surface tension T, respectively. It is found, when the depth h_1 is changed from $h_1 = 0.005\,\text{m}$ to $h_1 = 0.006\,\text{m}$ in Fig. 8, and the surface tension T is changed from $T = 0.03\,\text{kg/sec}$ to $T = 0.076\,\text{kg/sec}$ in Fig. 9, respectively, that the instability region U has increased. Then the increase of each of the fluid depth h_1, and the surface tension T has a destabilizing effect on the system. Figure 8 shows also that the instability sets in for any value of the fluid depth h_1, and by increasing the depth, the instability holds for higher values of ϕ_0^2; while Fig. 9 shows that the surface tension T has no effect on the instability region for small wavenumber values k till a critical wavenumber value k_c after which the instability region increases by increasing the surface tension parameter T.

7 The case of steady electric field

If the applied electric field is steady, then $\varpi = 0$; in this case put $a(t) = \exp(i\omega t)$ in Eq. (34), we thus obtain the following dispersion relation

$$F_1\omega^2 + (F_2 - iF_3)\,\omega + \left[F_4 - iF_5 + \frac{K\alpha^2}{\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1 - F_6\right] a = 0 \tag{94}$$

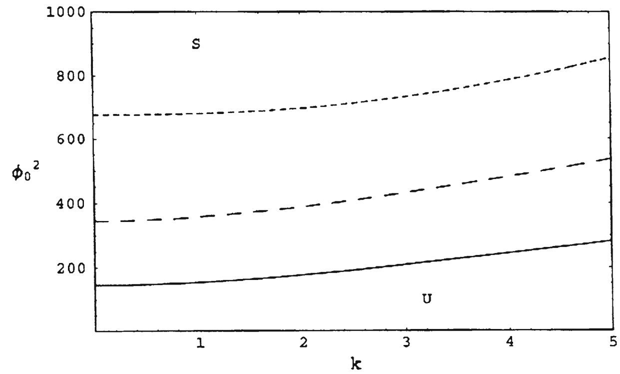

Fig. 10 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $k \times 10^{-2}$ (m^{-1}) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $V^{(1)} = 0.003\,\text{m/sec}$, $V^{(2)} = 0.005\,\text{m/sec}$, $k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\nu^{(1)} = 0.6 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\nu^{(2)} = 0.8 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\varepsilon = 3.3\,\text{sec/m}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, when $h_1 = 0.003\,\text{m}$, $h_2 = 0.005\,\text{m}$ (*solid curve*), $h_1 = 0.004\,\text{m}$, $h_2 = 0.006\,\text{m}$ (*dashed*), and $h_1 = 0.005\,\text{m}$, $h_2 = 0.007\,\text{m}$ (*dotted*)

The dispersion relation (94) is a quadratic equation in ω with complex coefficients. Solving this equation, we have

$$\omega = \frac{1}{2}\left[-\frac{F_2 - iF_3}{F_1} \pm \sqrt{a_1 + ib_1}\right] \tag{95}$$

where

$$a_1 = \frac{F_2^2 - F_3^2 - 4F_1\left[F_4 - F_6 + \frac{K\alpha^2}{\pi}\left(\frac{\phi_0}{h_1}\right)^2 \coth\alpha h_1\right]}{F_1^2} \tag{96}$$

$$b_1 = \frac{4F_1F_5 - 2F_2F_3}{F_1^2} \tag{97}$$

From Eq. (95), we have

$$\text{Re}(\omega) = \frac{1}{2}\left[-\frac{F_2}{F_1} \pm \sqrt{\frac{1}{2}\left(\sqrt{a_1 + ib_1} + a_1\right)}\right] \tag{98}$$

$$\text{Im}(\omega) = \frac{1}{2}\left[\frac{F_3}{F_1} \pm \sqrt{\frac{1}{2}\left(\sqrt{a_1 + ib_1} - a_1\right)}\right] \tag{99}$$

It can be shown that $\text{Im}(\omega) < 0$ if

$$b_1^2 < 4\left(\frac{F_3}{F_1}\right)^4 + 4a_1\left(\frac{F_2}{F_1}\right)^2 \tag{100}$$

Thus the flow will be unstable if

$$\phi_0^2 < \phi_3, \quad \phi_3 = \frac{\pi h_1^2}{K\alpha^2}\left[\frac{F_5(F_2F_3 - F_1F_5) + F_3^2(F_6 - F_4)}{F_3^2 \coth\alpha h_1}\right] \tag{101}$$

The values of ϕ_0^2 as described by Eq. (101) are the critical values (or the transition curves) of the disturbances which separate the stable from the unstable regions. We shall give numerical discussions for the effects of various physical parameters on the stability of the system under consideration by drawing the transition curves $\phi_0^2 = \phi_3$, in the (ϕ_0^2, k) plane. In the following Figs. 10, 11, 12, the letter S denotes stable region, while the letter U denotes unstable region, respectively.

In Figs. 10 and 11, we plot ϕ_0^2 versus k for different values of the fluid depths h_1, h_2 and the porosity of the porous medium ε, respectively. It is found, when fluid depths h_1, h_2 are changed from $h_1 = 0.003\,\text{m}$, $h_2 = 0.005\,\text{m}$ to $h_1 = 0.004\,\text{m}$, $h_2 = 0.006\,\text{m}$, and then to $h_1 = 0.005\,\text{m}$, $h_2 = 0.007\,\text{m}$ in Fig. 10, and the porosity of the porous medium ε is changed from $\varepsilon = 0.0033\,\text{sec/m}$ to $\varepsilon = 0.0066\,\text{sec/m}$, and then to

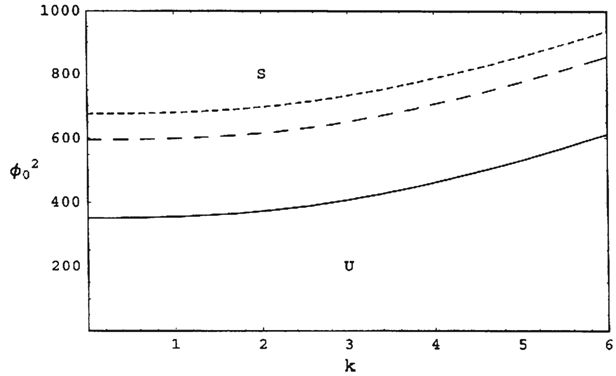

Fig. 11 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $k \times 10^{-2}$ (m^{-1}) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$, $V^{(1)} = 0.003\,\text{m/sec}$, $V^{(2)} = 0.005\,\text{m/sec}$, $k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\nu^{(1)} = 0.6 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\nu^{(2)} = 0.8 \times 10^{-4}\,\text{m}^2/\text{sec}$, $h_1 = 0.005\,\text{m}$, $h_2 = 0.008\,\text{m}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, when $\varepsilon = 0.0033\,\text{sec/m}$ (*solid curve*); $\varepsilon = 0.0066\,\text{sec/m}$ (*dashed*); and $\varepsilon = 6.54\,\text{sec/m}$ (*dotted*)

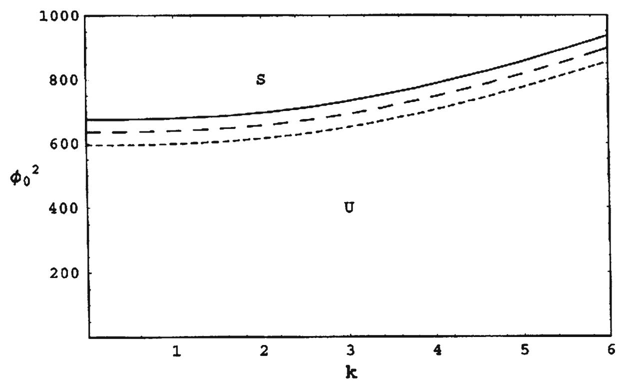

Fig. 12 Variation of $\phi_0^2 \times 10^{-5}$ (kg m^2 s^{-3}/A) with $k \times 10^{-2}$ (m^{-1}) for a system having $\rho^{(1)} = 1.293\,\text{kg/m}^3$, $\rho^{(2)} = 879\,\text{kg/m}^3$,$k_1 = 0.79 \times 10^{-12}\,\text{m}^2$, $\nu^{(1)} = 0.1 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\nu^{(2)} = 0.6 \times 10^{-4}\,\text{m}^2/\text{sec}$, $\varepsilon = 0.033\,\text{sec/m}$, $h_1 = 0.005\,\text{m}$, $h_2 = 0.008\,\text{m}$, $T = 0.076\,\text{kg/sec}$, $g = 9.8\,\text{m/sec}^2$, $K = 0.56 \times 10^{-10}\,\text{sec}^2\text{c}^2/\text{kg m}^3$, when $V^{(1)} = 0\,\text{m/sec}$, $V^{(2)} = 0\,\text{m/sec}$ (*solid curves*); $V^{(1)} = 0.4\,\text{m/sec}$, $V^{(2)} = 1.1\,\text{m/sec}$ (*dashed*), and $V^{(1)} = 1\,\text{m/sec}$, $V^{(2)} = 2\,\text{m/sec}$ (*dotted*)

$\varepsilon = 6.54\,\text{sec/m}$ in Fig. 11, that the instability region U has increased, Then the increase of each of the fluid depths h_1, h_2 and the porosity of the porous medium ε has a destabilizing effect on the system.

In Fig. 12, we plot ϕ_0^2 versus k for different values of the fluid velocities $V^{(1)}$, $V^{(2)}$. It is found, when the fluid velocities $V^{(1)}$, $V^{(2)}$ are changed from $V^{(1)} = 0\,\text{m/sec}$, $V^{(2)} = 0\,\text{m/sec}$ to $V^{(1)} = 0.4\,\text{m/sec}$, $V^{(2)} = 1.1\,\text{m/sec}$, and then to $V^{(1)} = 1\,\text{m/sec}$, $V^{(2)} = 2\,\text{m/sec}$, that the instability region U has decreased. Then the increase of the fluid velocities $V^{(1)}$, $V^{(2)}$ has a stabilizing effect on the system.

In the next, we shall discuss the stability conditions for two limiting cases of interest, the case of pure fluids (non-porous media), and the case of absence of streaming (the Rayleigh–Taylor instability), as follows.

7.1 The case of pure fluids

For non-porous medium, i.e., when $\varepsilon \to 1$ and $k_1 \to \infty$, then Eq. (94), on using Eqs. (35)–(40), reduces to

$$\begin{aligned}
&\omega^2 \left[\rho^{(1)} \coth \alpha h_1 + \rho^{(2)} \coth \alpha h_2\right] + 2k\omega \left[\rho^{(1)} V^{(1)} \coth \alpha h_1 + \rho^{(2)} V^{(2)} \coth \alpha h_2\right] \\
&\quad + k^2 \left[\rho^{(1)} V^{(1)^2} \coth \alpha h_1 + \rho^{(2)} V^{(2)^2} \coth \alpha h_2\right] \\
&\quad + \alpha \left[\frac{K\alpha}{\pi} \left(\frac{\phi_0}{h_1}\right)^2 \coth \alpha h_1 - (\rho^{(2)} - \rho^{(1)}) g - \alpha^2 T\right] = 0,
\end{aligned} \tag{102}$$

which gives

$$\begin{aligned}
&\omega[\rho^{(1)}\coth\alpha h_1+\rho^{(2)}\coth\alpha h_2]+k[\rho^{(1)}V^{(1)}\coth\alpha h_1\\
&\quad+\rho^{(2)}V^{(2)}\coth\alpha h_2]=\pm i\sqrt{\alpha[\rho^{(1)}\coth\alpha h_1+\rho^{(2)}\coth\alpha h_2]}\\
&\quad\times\left\{\frac{k^2\rho^{(1)}\rho^{(2)}(V^{(1)}-V^{(2)})^2\coth\alpha h_1\coth\alpha h_2}{\alpha[\rho^{(1)}\coth\alpha h_1+\rho^{(2)}\coth\alpha h_2]}\right.\\
&\quad\left.+\left[\frac{K\alpha}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1-(\rho^{(2)}-\rho^{(1)})g-\alpha^2T\right]\right\}^{1/2}
\end{aligned}\tag{103}$$

As $\rho^{(2)}>\rho^{(1)}$, we note from Eq. (103) that the interface will be unstable if the function

$$\begin{aligned}
F=&\frac{k^2\rho^{(1)}\rho^{(2)}(V^{(1)}-V^{(2)})^2\coth\alpha h_1\coth\alpha h_2}{\alpha[\rho^{(1)}\coth\alpha h_1+\rho^{(2)}\coth\alpha h_2]}\\
&+\left[\frac{K\alpha}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1-(\rho^{(2)}-\rho^{(1)})g-\alpha^2T\right]
\end{aligned}\tag{104}$$

is positive, and stable when it is negative [37]. Therefore, the system will be stable or unstable, if the following conditions are satisfied, respectively:

$$\begin{aligned}
&\frac{k^2\rho^{(1)}\rho^{(2)}(V^{(1)}-V^{(2)})^2\coth\alpha h_1\coth\alpha h_2}{\alpha[\rho^{(1)}\coth\alpha h_1+\rho^{(2)}\coth\alpha h_2]}+\frac{K\alpha}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1\\
&\lesseqgtr[(\rho^{(2)}-\rho^{(1)})g+\alpha^2T]
\end{aligned}\tag{105}$$

Now, considering the wave motion in x-direction only, i.e., when $l=0$ (or $\alpha=k$), and $h_1=h_2$ then from Eq. 104, F can be written in the following form:

$$F(x)=(a+b)x\coth x-(1+cx^2)\tag{106}$$

where

$$a=\frac{\rho^{(1)}(V^{(1)}-V^{(2)})^2}{gh_1\left(1+\rho^{(1)}/\rho^{(2)}\right)(\rho^{(2)}-\rho^{(1)})}\tag{107}$$

$$b=\frac{K\phi_0^2}{\pi gh_1^3(\rho^{(2)}-\rho^{(1)})}\tag{108}$$

$$c=\frac{T}{gh_1^2(\rho^{(2)}-\rho^{(1)})}\tag{109}$$

$$x=kh_1\tag{110}$$

Equations (106)–(110) are the same equations obtained earlier by Murhty [37]; i.e., they can be obtained from our results in the limiting case of steady electric field and non-porous medium. From the analytic behavior of the function $F(x)$, we make the following conclusions:

(1) Absence of surface tension, i.e., when $c=0$. In this case, Eq. (106) becomes

$$F(x)=(a+b)x\coth x-1\tag{111}$$

We find from Eq. (111), and Fig. 13 (see also ref. [37]) that, if $(a+b)>1$, curve (12), then the system is unstable for all wavenumbers, and if $(a+b)=1$, curve (11), then the system is unstable only for wavenumber values greater than 0.4; while if $(a+b)<1$, curves (5)–(10), then the system is stable for wavenumbers less than the critical wavenumber x_0, beyond which the system is unstable. Note that the critical value x_0 increases by increasing the value of $(a+b)$. It is known that, the relative streaming between two fluids destabilizes short wavelength perturbations [43]. The above results show that this is the case even when the electric part of the Maxwellian stress is present and further that this short wavelength instability is enhanced by the presence of Maxwellian stresses.

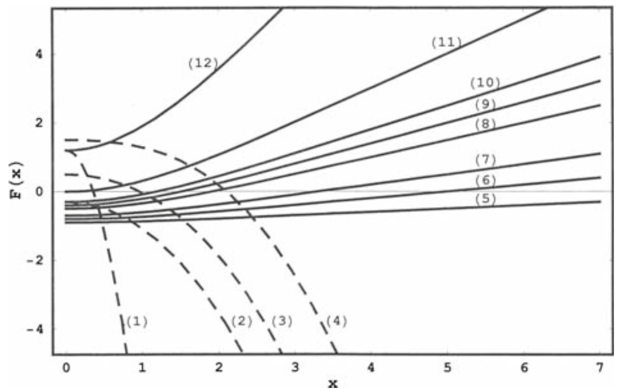

Fig. 13 The stability diagram for the function $F(x)$ defined by Eq. (106) versus x for the following cases: (1) $a + b = 2.2$, $c = 10$, (2) $a + b = 0.7$, $c = 1$, (3) $a + b = 1.5$, $c = 1$, (4) $a + b = 2.5$, $c = 1$, (5) $a + b = 0.1$, $c = 0$, (6) $a + b = 0.2$, $c = 0$, (7) $a + b = 0.3$, $c = 0$, (8) $a + b = 0.5$, $c = 0$, (9) $a + b = 0.6$, $c = 0$, (10) $a + b = 0.7$, $c = 0$, (11) $a + b = 1$, $c = 0$, and (12) $a + b = 2.2$, $c = 0$

(2) Presence of surface tension, i.e., when $c \neq 0$. In this case, we find from Eq. (106), and Fig. 13 that for $(a + b) > 1$, curves (3) and (4), and for small values of c, the system is unstable for $x < x_0'$, and then it is stable for $x > x_0'$. As we decrease the surface tension parameter c it appears that the unstable wavenumbers band is decreased. Note also that by increasing the values of c and $(a + b)$, curve (1), then the unstable wavenumbers band is decreased in comparison with curves (3) and (4). When $(a + b) < 1$, curve (2), and whatever may be the value of c, the surface tension parameter, the system is always stable. Thus we conclude that for $(a + b) > 1$, and for any value of c, the system has a stabilizing effect due to the presence of surface tension [44]. In this case, we may say that the surface tension inhibits the Kelvin–Helmholtz instability.

7.2 Rayleigh-Taylor instability

If $V^{(1)} = V^{(2)} = 0$, then Eq. (94), on using Eqs. (35)–(40), reduces to

$$(i\omega)^2\left[\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2\right] + (i\omega)\left(\frac{\varepsilon}{k_1}\right)\left[\rho^{(1)}\nu^{(1)}\coth\alpha h_1 + \rho^{(2)}\nu^{(2)}\coth\alpha h_2\right]$$
$$-\alpha\left[\frac{K\alpha}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1 - (\rho^{(2)} - \rho^{(1)})g - \alpha^2 T\right] = 0 \tag{112}$$

The roots of this equation are given by

$$i\omega = -\left(\frac{\varepsilon}{2k_1}\right)\left(\frac{\rho^{(1)}\nu^{(1)}\coth\alpha h_1 + \rho^{(2)}\nu^{(2)}\coth\alpha h_2}{\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2}\right)$$
$$\pm\left\{\left(\frac{\varepsilon}{2k_1}\right)^2\left(\frac{\rho^{(1)}\nu^{(1)}\coth\alpha h_1 + \rho^{(2)}\nu^{(2)}\coth\alpha h_2}{\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2}\right)^2\right.$$
$$\left.+\alpha\left[\frac{\frac{K\alpha}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1 - (\rho^{(2)} - \rho^{(1)})g - \alpha^2 T}{\rho^{(1)}\coth\alpha h_1 + \rho^{(2)}\coth\alpha h_2}\right]\right\}^{1/2} \tag{113}$$

If

$$\frac{K\alpha}{\pi}\left(\frac{\phi_0}{h_1}\right)^2\coth\alpha h_1 < [(\rho^{(2)} - \rho^{(1)})g + \alpha^2 T] \tag{114}$$

then both the values of $(i\omega)$ are either real negative or complex conjugates with negative real parts, and the system is thus stable when the condition (114) is satisfied. Otherwise the system will be unstable.

8 Concluding remarks

In this paper, the instability of the interface between two superposed streaming conducting and dielectric fluids through porous medium in a vertical electric field varying periodically with time is considered. Because of the time dependence of the electric field, the simple equation of force balance can no longer be utilized to obtain the stability criterion, and the hydrodynamics of the fluids as well as of the porous medium must be taken into account. When this is done, and both the viscosity and medium permeability are included due to Darcy's law, the stability of the interface can be shown to be governed by a damped Mathieu equation with complex coefficients which depend on the gravitational acceleration, surface tension, magnitude and frequency of the periodic electric field, depths, fluid velocities, kinematic viscosities, porosity of the porous medium, and the medium permeability. The method of multiple scales is used to obtain an approximate solution of the damped Mathieu equation, and then to analyze the stability criteria of the considered system. In order to analyze the solution of this equation, we need to distinguish between two cases. The first one is the non-resonance case, when the frequency ϖ of the oscillating electric field is not near the frequency Ω, and the second one is the resonance case which arises when the frequency ϖ is near Ω. The stability conditions for the damped Mathieu equation are obtained in both cases.

A simple condition for the stability in the non-resonance case show that stability occurs when Λ is less than a negative critical value from which we conclude that

(1) Both the porosity of porous medium ε, and the kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$ have stabilizing effects on the considered system.
(2) The medium permeability k_1 has a destabilizing effect on the system.

While the numerical calculations of the stability conditions for the resonance case indicate that

(1) The frequency of the electric field ϖ, and the fluid velocities $V^{(1)}$, $V^{(2)}$, as well as the medium permeability k_1, has a stabilizing effect on the system, and decreases the value of the resonance point.
(2) The porosity of the porous medium ε, and the kinematic viscosities $\nu^{(1)}$, $\nu^{(2)}$ has a destabilizing effect on the system, and increases the value of the resonance point.

In the absence of both streaming velocities and porous medium, then the obtained equation can be put in the canonical form of the Mathieu equation. The stability condition for Mathieu equation is obtained, and the numerical calculations indicate that the fluid depth h_1, and the surface tension T has a destabilizing effect on the system. The instability sets in for any value of the fluid depth h_1, and by increasing the depth, the instability holds for higher values of ϕ_0^2; while the surface tension T has no effect on the instability region for small wavenumber values k till a critical wavenumber value k_c after which the instability region increases by increasing the surface tension parameter T.

Finally, the case of steady electric field in the presence of porous medium is also investigated, and the stability conditions are obtained analytically and confirmed numerically, which show that

(1) Each of the fluid depths h_1, h_2 and the porosity of the porous medium ε has a destabilizing effect on the system.
(2) The fluid velocities $V^{(1)}$, $V^{(2)}$ have stabilizing effects on the system.

In the latter case, the stability conditions for the two limiting cases of interest, the case of purely fluids (non-porous media) and the case of absence of streaming (the Rayleigh-Taylor instability), are also obtained and discussed in detail

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