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Wave propagation in thermally conducting linear fibre-reinforced composite materials

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Abstract The propagation of plane waves in a fibre-reinforced, anisotropic, generalized thermoelastic media is discussed. The governing equations in x–y plane are solved to obtain a cubic equation in phase velocity. Three coupled waves, namely quasi-P, quasi-SV and quasi-thermal waves are shown to exist. The propagation of Rayleigh waves in stress free thermally insulated and transversely isotropic fibre-reinforced thermoelastic solid half-space is also investigated. The frequency equation is obtained for these waves. The velocities of the plane waves are shown graphically with the angle of propagation. The numerical results are also compared to those without thermal disturbances and anisotropy parameters.

Keywords Fibre-reinforced · Plane waves · Anisotropy · Angle of propagation · Thermal waves

1 Introduction

Fibre-reinforced composites are used in a variety of structures due to their low weight and high strength. The analysis of stress and deformation of fiber-reinforced composite materials has been an important subject of solid mechanics for last three decades. Spencer [1], Pipkin [2] and Rogers [3,4] did pioneer works on the subject. Sengupta and Nath [5] discussed the problem of surface waves in fibre-reinforced anisotropic elastic media. Recently, Singh and Singh [6] discussed the reflection of plane waves at the free surface of a fibre-reiforced elastic half-space.

Lord and Shulman [7] introduced a theory of generalized thermoelasticity with one relaxation time for an isotropic body. The theory was extended for anisotropic body by Dhaliwal and Sherief [8]. In this theory, a modified law of heat conduction including both the heat flux and its time derivatives replaces the conventional Fourier's Law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both coupled and uncoupled theories of thermoelasticity. Erdem [9] derived heat conduction equation for a composite rigid material containing an arbitrary distribution of fibers.

The impact of earthquakes on the artificial structures is of great concern to engineers and architects. During an earthquake and similar disturbances, a structure is excited into a more or less violent vibration, with resulting oscillatory stresses, which depend upon both ground vibration and physical properties of the structure. Most concrete constructions need steel reinforcing to some extent. The study of plane and surface wave propagation in thermally conducting fibre-reinforced composites has applications in civil engineering and geophysics.

Chadwick and Seet [10] and Singh and Sharma [11] have discussed the propagation of plane harmonic waves in anisotropic thermoelastic materials. Singh [12] studied a problem on wave propagation in an anisotropic generalized thermoelastic solid and obtained a cubic equation, which gives the dimensional velocities of various plane waves.

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Chadwick [13] and Nayfeh and Nasser [14] have discussed the propagation of surface waves in homogeneous thermoelastic media. Chadwick and Windle [15] studied the effect of heat conduction on the propagation of Rayleigh waves in the semi-infinite media (i) when the surface is maintained at constant temperature and (ii) when the surface is thermally insulated. Sharma and Singh [16] have studied thermoelastic surface waves in a transversely isotropic half-space. In the present paper, the governing equations of thermally conducting, linear fiber-reinforced rigid bodies are generalized by using the theory of Dhaliwal and Sherief [8]. The governing equations in x–y plane are solved analytically to obtain a cubic equation which gives the phase velocities of three coupled waves, namely, quasi-P, quasi-SV and quasi-thermal waves. The dependence of velocities of these plane waves upon the angle of propagation are shown graphically. The propagation of Rayleigh waves in stress free thermally insulated and transversely isotropic fibre-reinforced thermoelastic solid half-space is also investigated.

2 Basic Equations

The linear equations governing thermoelastic interactions in homogeneous anisotropic solid are

(a) Strain–displacement relations

$$e_{ij} = \frac{u_{i,j} + u_{j,i}}{2}, \quad i, j = 1, 2, 3. \tag{1}$$

(b) Stress-strain-temperature relations

$$t_{ij} = c_{ijkl} e_{kl} - \beta_{ij} T, \quad i, j, k, l = 1, 2, 3. \tag{2}$$

(c) Equation of motion

$$t_{ij,j} + \rho F_i = \rho \ddot{u}_i, \quad i, j = 1, 2, 3. \tag{3}$$

(d) Heat conduction equation

$$K_{ij} T_{,ij} = T_0 \beta_{ij} \dot{u}_{i,j} + \rho C_e \dot{T}, \quad i, j = 1, 2, 3. \tag{4}$$

where ρ is the mass density, u_i the mechanical displacement, e_{ij} the strain tensor, t_{ij} the stress tensor, T the temperature change of a material particle, T_0 the reference uniform temperature of the body, K_{ij} the heat conduction tensor, c_{ijkl} the isothermal elastic parameters tensor, β_{ij} the thermal elastic coupling tensor, C_e the specific heat at constant strain. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

3 Formulation of the Problem

Consider a homogeneous thermally conducting transversely isotropic fibre-reinforced medium in the undeformed state at uniform temperature T_0. The medium is assumed transversely isotropic in such a way that planes of isotropy are perpendicular to y-axis.

The origin is taken on the thermally insulated and stress-free plane surface and y-axis pointing vertically downward into the half-space which is represented by $y \geq 0$. The fibre direction is assumed parallel to x-axis. The geometry of the problem is given in Fig. 1. The body forces and heat sources are assumed absent. Following Dhaliwal and Sherief [8], the linear governing equations for generalized thermoelasticity of fibre-reinforced elastic material in x-y plane are expressed as

$$c_{11} u_{1,11} + (c_{13} + c_0) u_{2,12} + c_0 u_{1,22} - \beta_1 T_{,1} = \rho \ddot{u}_1, \tag{5}$$

$$c_{33} u_{2,22} + (c_{13} + c_0) u_{1,12} + c_0 u_{2,11} - \beta_2 T_{,2} = \rho \ddot{u}_2, \tag{6}$$

$$K_1 T_{,11} + K_2 T_{,22} - \rho C_e (\dot{T} + \tau_0 \ddot{T}) = T_0 [\beta_1 (\dot{u}_{1,1} + \tau_0 \ddot{u}_{1,1}) + \beta_2 (\dot{u}_{2,2} + \tau_0 \ddot{u}_{2,2})], \tag{7}$$

where $\beta_1 = (c_{11} + c_{13})\alpha_1 + c_{13}\alpha_2$, $\beta_2 = (c_{13} + c_{33} - c_{55})\alpha_1 + c_{33}\alpha_2$, $c_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta$, $c_{13} = c_{12} = \lambda + \alpha$, $c_{33} = c_{22} = \lambda + 2\mu_T$, $c_0 = c_{44}/2$, $c_{44} = c_{66} = 2\mu_L$, $c_{55} = 2\mu_T$, $c_{23} = c_{33} - c_{55}$, and λ, α, β, μ_L, μ_T are material constants, K_1, K_2 are coefficients of thermal conductivity, α_1, α_2 are coefficients of linear thermal expansion, τ_0 is thermal relaxation time, u_1, u_2 are the components of displacement vector.

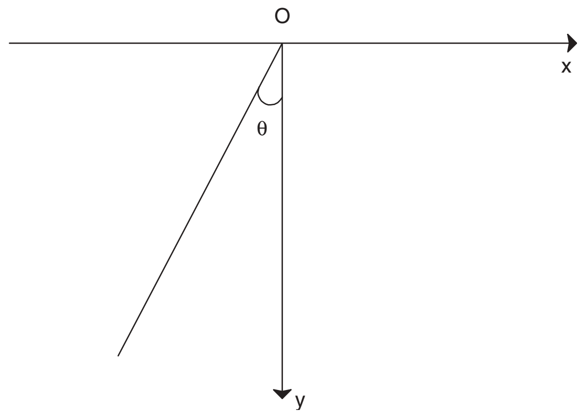

Fig. 1 Geometry of the problem

4 Analytical solution

Solutions of the Eqs. (5)–(7) are now sought in the form of the harmonic traveling wave

$$\{u_1, u_2, T\} = \{\mathrm{Ad}_1, \mathrm{Ad}_2, kB\} \exp\{ik(xp_1 + yp_2 - vt)\}, \tag{8}$$

where k is the wave number, A, B are arbitrary constants, d_1, d_2 are components of unit displacement vector and p_1, p_2 are components of unit propagation vector.

Making use of Eq. (8), Eqs. (5)–(7) take the form

$$(D_1 - \zeta)\mathrm{Ad}_1 + L_1 p_1 p_2 \mathrm{Ad}_2 + i\beta_1 p_1 B = 0, \tag{9}$$

$$L_1 p_1 p_2 \mathrm{Ad}_1 + (D_2 - \zeta)\mathrm{Ad}_2 + i\beta_2 p_2 B = 0, \tag{10}$$

$$\beta_1 p_1 \mathrm{Ad}_1 + \beta_2 p_2 \mathrm{Ad}_2 + i\left(\rho \frac{C_e}{T_0}\right)\left[\left(\frac{D_0}{C_e \tau \zeta}\right) - 1\right] B = 0, \tag{11}$$

where $\zeta = \rho v^2$,

$$D_1 = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)p_1^2 + \mu_L p_2^2,$$

$$D_2 = \mu_L p_1^2 + (\lambda + 2\mu_T)p_2^2, D_0 = (K_1 p_1^2 + K_2 p_2^2),$$

$$L_1 = \lambda + \alpha + \mu_L, \tau = \tau_0 + \left(\frac{i}{\omega}\right).$$

The homogeneous system of Eqs. (9)–(11) admits nontrivial solution and enables to conclude that ζ satisfies the cubic equation

$$(D_3 - L_2\zeta)[(D_1 - \zeta)(D_2 - \zeta) - L_1^2 p_1^2 p_2^2] - \zeta[(D_1 - \zeta)\gamma^2 p_2^2 + (D_2 - \zeta)p_1^2 - 2L_1\gamma p_1^2 p_2^2] = 0, \tag{12}$$

where $D_3 = D_0/\epsilon C_e \tau v_0^2$, $L_2 = 1/\epsilon v_0^2$, $\gamma = \beta_2/\beta_1$, $v_0^2 = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)/\rho$, $\epsilon = T_0\beta_1^2/\rho C_e v_0^2$.

The Eq. (12) may be expressed as

$$\zeta^3 + L\zeta^2 + M\zeta + N = 0, \tag{13}$$

where $L = -[D_1 + D_2 + \epsilon v_0^2(D_3 + p_1^2 + \gamma^2 p_2^2)]$, $M = D_1D_2 - L_1^2 p_1^2 p_2^2 + \epsilon v_0^2(D_1D_3 + D_2D_3 + D_1\gamma^2 p_2^2 + D_2 p_1^2 - 2L_1\gamma p_1^2 p_2^2)$, $N = -\epsilon v_0^2 D_3[D_1D_2 - L_1^2 p_1^2 p_2^2]$.

The Eq. (13) gives three values of ζ. Each value of ζ corresponds to a wave if v^2 is real and positive. Hence, three positive values of v will be the velocities of propagation of three possible waves. The expressions for velocities of these waves may be obtained from Eq. (13) by using Cardan's method. Equation (13) takes form

$$\Lambda^3 + 3H\Lambda + G = 0, \tag{14}$$

where $\Lambda = \zeta + \frac{L}{3}$, $H = \frac{3M-L^2}{9}$, $G = \frac{27N-9LM+2L^3}{27}$.

For all the three roots of Eq. (14) to be real, $\Delta_0(= G^2 + 4H^3)$ should be negative. Assuming the Δ_0 to be negative, we obtain the three roots of Eq. (14) as

$$\Lambda_n = 2(-H)^{1/2} cos\left[\frac{\phi + 2\pi(n-1)}{3}\right], \quad n = 1, 2, 3. \tag{15}$$

where

$$\phi = \tan^{-1}\{(|\Delta_0|)^{1/2}/(-G)\}. \tag{16}$$

Hence,

$$v_n = \left\{\frac{(\Lambda_n - L/3)}{\rho}\right\}^{1/2}, \quad n = 1, 2, 3 \tag{17}$$

are velocities of propagation of the three possible quasi waves. The waves with velocities v_1, $v_2$and v_3 may be called as quasi-P (qP) wave, quasi-Shear Vertical (qSV) wave and quasi-Thermal (qT) wave, respectively. This fact may be verified, when we solve the Eq. (13) by using a computer program of Cardan's method. The velocities of these plane waves are found to depend upon angle of propagation and various other thermal parameters.

The Eq. (12) may be rewritten as a cubic equation in p_2

$$p_2^6 - Up_2^4 + Vp_2^2 - W = 0, \tag{18}$$

where $p_1 = \sin\theta$ and p_2 is unknown parameter, and $U = [L_1^2 p_1^2 K_0 - K_0(c_0+c_{33}) - a_0c_0c_{33} + \gamma^2\zeta c_0]/K_0c_0c_{33}$, $V = [b_0d_0K_0 + a_0(c_0+c_{33}) - a_0L_1^2p_1^2 - b_0\gamma^2\zeta - c_{33}\zeta p_1^2 + 2L_1\gamma\zeta p_1^2]/K_0c_0c_{33}$, $W = [d_0\zeta p_1^2 - a_0b_0d_0]/K_0c_0c_{33}$, and $a_0 = (K_1/\epsilon C_e\tau v_0^2)p_1^2 - L_2\zeta$, $b_0 = c_{11}p_1^2 - \zeta$, $d_0 = c_0p_1^2 - \zeta$, $K_0 = K_1/\epsilon C_e\tau v_0^2$.

The characteristic Eq. (18) is cubic in p_2^2 and hence possesses three roots m_i^2, $i = 1, 2, 3$. For surface waves, it is essential that motion is confined to free surface $y = 0$ of the half-space so that the characteristic roots m_i^2 must satisfy the radiation condition Im $(m_i) \geq 0$. Then the formal solution for displacements and temperature change is written as

$$u_1 = (A_1 e^{-m_1y} + A_2 e^{-m_2y} + A_3 e^{-m_3y})e^{i(kx-\omega t)}, \tag{19}$$

$$u_2 = (a_1A_1 e^{-m_1y} + a_2A_2 e^{-m_2y} + a_3A_3 e^{-m_3y})e^{i(kx-\omega t)}, \tag{20}$$

$$T = (b_1A_1 e^{-m_1y} + b_2A_2 e^{-m_2y} + b_3A_3 e^{-m_3y})e^{i(kx-\omega t)}, \tag{21}$$

where a_j and b_j $(j = 1, 2, 3)$ are given as

$$a_j = \frac{\{m_j(L_1\beta_1p_1^2 - \beta_2M_j)\}}{\{p_1(L_1\beta_2m_j^2 - \beta_1N_j)\}}, \tag{22}$$

$$b_j = \frac{ik\{L_1p_1^2m_j^2 - M_jN_j)\}}{\{p_1(L_1\beta_2m_j^2 - \beta_1N_j)\}}, \tag{23}$$

where $M_j = c_{11}p_1^2 + c_0m_j^2 - \zeta$, $N_j = c_0p_1^2 + c_{33}m_j^2 - \zeta$.

5 Frequency equation

The boundary conditions at the thermally insulated surface $y = 0$ are given by

$$t_{yy} = 0, \quad t_{xy} = 0, \quad \frac{\partial T}{\partial y} = 0, \tag{24}$$

where $t_{yy} = c_{13}u_{1,1} + c_{33}u_{2,2} - \beta_2 T,\ t_{xy} = c_0(u_{1,2} + u_{2,1})$.

Substituting u_1, u_2 and T from Eqs. (19)–(21) in boundary conditions (24), we get

$$(ikc_{13} - a_1m_1c_{33} - \beta_2b_1)A_1 + (ikc_{13} - a_2m_2c_{33} - \beta_2b_2)A_2 + (ikc_{13} - a_3m_3c_{33} - \beta_2b_3)A_3 = 0, \tag{25}$$

$$(ika_1 - m_1)A_1 + (ika_2 - m_2)A_2 + (ika_3 - m_3)A_3 = 0, \tag{26}$$

$$m_1b_1A_1 + m_2b_2A_2 + m_3b_3A_3 = 0, \tag{27}$$

The linear homogeneous system of Eqs. (25)–(27) will have a nontrivial solution for $A_j,\ j = 1, 2, 3$, only if the determinant of their coefficients vanishes, that is

$$\begin{aligned}(m_1a_1c_{33} - ikc_{13} + \beta_2b_1)\{m_3b_3(m_2 - ika_2) - m_2b_2(m_3 - ika_3)\}\\ +(m_2a_2c_{33} - ikc_{13} + \beta_2b_2)\{m_1b_1(m_3 - ika_3) - m_3b_3(m_1 - ika_1)\}\\ +(m_3a_3c_{33} - ikc_{13} + \beta_2b_3)\{m_2b_2(m_1 - ika_1) - m_1b_1(m_2 - ika_2)\} = 0,\end{aligned} \tag{28}$$

The Eq. (28) is the frequency equation which depends on wave number k and phase velocity v.

6 Discussion of roots of Eq. (12)

(a) Fibre-reinforced elastic case
In absence of thermal disturbances, the Eq. (12) reduces to

$$(D_1 - \zeta)(D_2 - \zeta) - L_1^2p_1^2p_2^2 = 0, \tag{29}$$

which gives the expressions for velocities of qP and qSV waves in fibre-reinforced media as

$$2\rho v^2 = [(D_1 + D_2) \pm \{(D_1 - D_2)^2 + 4L_1^2p_1^2p_2^2\}^{1/2}]. \tag{30}$$

(b) Isotropic elastic case
For $\mu_L = \mu_T = \mu$ and $\alpha = \beta = 0$, the Eq. (30) gives the velocities of P and SV waves in an isotropic elastic solid as $\{(\lambda + 2\mu)/\rho\}^{1/2}$ and $\{\mu/\rho\}^{1/2}$, respectively.

7 Discussion of the frequency Eq. (28)

Equation (28) contains the complete information about the phase velocity, wavenumber and attenuation coefficient of the surface waves in a fibre-reinforced thermoelastic half-space. In general, wavenumber and hence the phase velocities of the waves are complex quantities, therefore the waves are attenuated in space. If we write

$$v^{-1} = V^{-1} + i\omega^{-1}Q \tag{31}$$

so that $k = R + iQ$, where $R = \omega/V$ and V, Q are real numbers. Also the roots of characteristic Eq. (18) are, in general complex and hence we assume that $m_j = \alpha_j + i\beta_j,\ (j = 1, 2, 3)$ so that the exponent in solutions (19) to (21) become $-R[(Q/R)xp_1 + m_j^I y] - iR\{xp_1 - m_j^R y - Vt\}$, where $m_j^R = \alpha_j - \beta_j Q/R$, $m_j^I = \beta_j + \alpha_j Q/R$. This shows that V is the propagation velocity and Q is the attenuation coefficient of the wave. Using (31) in frequency Eq. (28), the values of propagation speed V and attenuation coefficient Q may be obtained.

(a) Fibre-reinforced elastic case
In absence of thermal disturbances, the Eq. (28) reduces to

$$(a_1{}' - a_2{}')(n_1n_2c_{33} + k^2c_{13}) + ikc_{13}(n_1 - n_2)(1 - a_1{}'a_2{}') = 0, \tag{32}$$

where n_1^2, n_2^2 are roots of equation

$$c_0c_{33}x^2 + (c_{11}c_{33}p_1^2 + c_0^2p_1^2 - L_1^2p_1^2 - c_0\zeta - c_{33}\zeta)x + (\zeta^2 - \zeta c_{11}p_1^2 - \zeta c_0p_1^2 + c_0c_{11}p_1^4) = 0, \tag{33}$$

and $a_1{}', a_2{}'$ are given by

$$a_j{}' = -\frac{M_j}{L_1p_1n_j}, \quad j = 1, 2. \tag{34}$$

The Eq. (32) represents the Rayleigh type of waves in the fibre-reinforced elastic medium. Sengupta and Nath [5] also obtain a similar equation by assuming the displacements in terms of potentials. Singh [17] showed that, for wave propagation in fibre-reinforced anisotropic media, the decoupling can not be achieved by the introduction of displacement potentials. Equation (32) is obtained by the method suggested by Singh [17].

(b) Isotropic elastic case
For $\mu_L = \mu_T = \mu$ and $\alpha = \beta = 0$, the Eq. (32) reduce to

$$(a_1{}^* - a_2{}^*)[n_1{}^*n_2{}^*(\lambda + 2\mu) + k^2\lambda] + ik\lambda(n_1{}^* - n_2{}^*)(1 - a_1{}^*a_2{}^*) = 0, \tag{35}$$

$$\text{where } a_1{}^* = n_1{}^*/p_1, a_2{}^* = -p_1/n_2{}^*,\ n_1{}^* = [(\zeta/(\lambda + 2\mu)) - p_1^2]^{1/2}, n_2{}^* = [(\zeta/\mu) - p_1^2]^{1/2}. \tag{36}$$

8 Numerical example

For the purpose of numerical computations, the unit propagation vector $(p_1, p_2, 0)$ is taken as $(\sin\theta,\ \cos\theta, 0)$, where θ is the angle of propagation with the vertical axis. Using FORTRAN PROGRAM of Cardan's method, Eq. (6) is solved. The numerical values of the velocities of the plane waves at each angle of propagation are computed with the following physical constants for generalized fibre-reinforced thermoelastic materials.

$\rho = 2,660\,\text{kg/m}^3$, $\lambda = 5.65 \times 10^{10}\,\text{N/m}^2$, $\mu_T = 2.46 \times 10^{10}\,\text{N/m}^2$,
$\mu_L = 5.66 \times 10^{10}\,\text{N/m}^2$, $\alpha = -1.28 \times 10^{10}\,\text{N/m}^2$, $\beta = 220.90 \times 10^{10}\,\text{N/m}^2$,
$K_1 = 0.0921 \times 10^3\,\text{J m}^{-1}\,\text{deg}^{-1}\,\text{s}^{-1}$, $K_2 = 0.0963 \times 10^3\,\text{J m}^{-1}\text{deg}^{-1}\,\text{s}^{-1}$, $\alpha_1 = 0.017 \times 10^4\,\text{deg}^{-1}$,
$\alpha_1 = 0.015 \times 10^4\,\text{deg}^{-1}$, $C_e = 0.787 \times 10^3\,\text{J kg}^{-1}\,\text{deg}^{-1}$, $T_0 = 293\text{K}$, $t_0 = 0.05\,\text{s}$, $\omega = 2\,\text{s}^{-1}$.

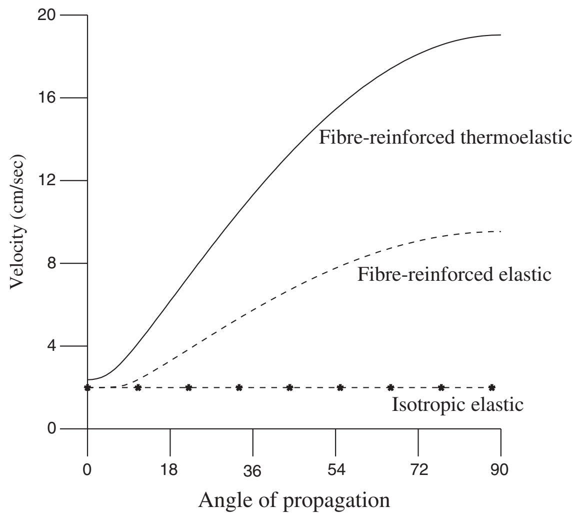

Fig. 2 Velocity of quasi-P wave as a function of angle between direction of propagation and vertical axis

The numerical results are shown by plotting the graphs of velocities of plane waves with the angle of propagation. Figure 2 shows the comparison of velocity of quasi-P wave. The solid curve shows the velocity of quasi-P wave in a generalized fibre-reinforced thermoelastic media. The velocity of this wave increases sharply with the angle of propagation. After neglecting thermal effects, the solid curve reduces to dotted curve in Figure 1. The dotted curve with center symbols represents the velocity of P wave in an isotropic media. Similarly, the curves in Fig. 3 shows the velocity of quasi-SV wave. The anisotropy effect on quasi-SV wave is found considerable, whereas the thermal effect is negligible. The velocity of quasi-thermal wave with angle of propagation is also shown in Fig. 4. It first decreases sharply and then very slowly.

From theoretical as well as numerical analysis, the following points are observed

1. Three types of plane waves, quasi-P, quasi-SV and quasi-thermal waves are shown to exist in a two-dimensional model of a generalized fibre-reinforced thermoelastic media.
2. The velocities of these plane waves are function of angle between direction of propagation and vertical axis.

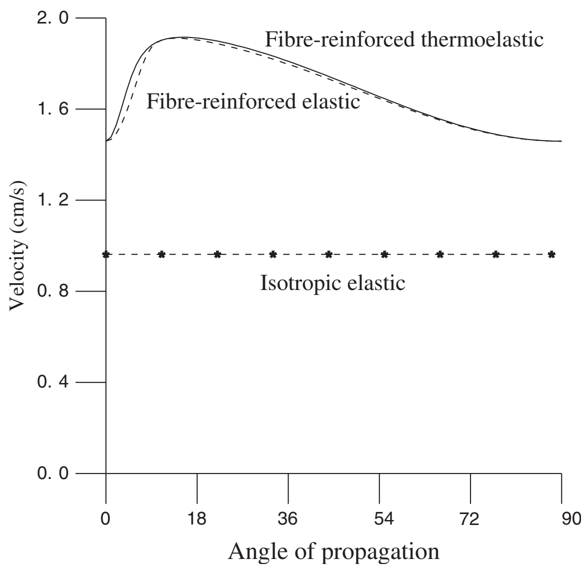

Fig. 3 Velocity of quasi-SV wave as a function of angle between direction of propagation and vertical axis

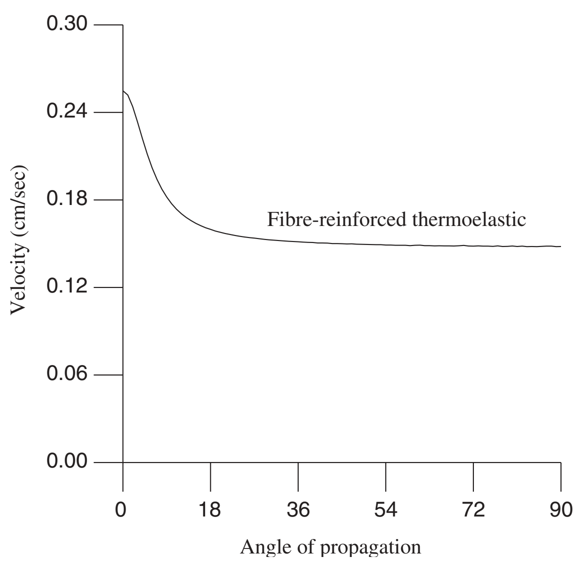

Fig. 4 Velocity of thermal wave as a function of angle between direction of propagation and vertical axis

3. The velocities of these plane waves are affected by presence of anisotropy and thermal disturbances.
4. The frequency equation is obtained which contains all information regarding phase velocity and wave number of the Rayleigh waves.

The research work is supposed to be useful in further studies, both theoretical and observational, of wave propagation in the more realistic models of the thermoelastic solids present in the earth's crust.

References

1. Spencer, A.J.M.: Deformation of fibre-reinforced materials. Clarendon, Oxford (1941)
2. Pipkin, A.C.: Finite deformations of ideal fiber-reinforced composites. In: Sendeckyj, G.P. (ed) Composites materials, 2. Academic, New York, pp 251–308 (1973)
3. Rogers, T.G.: Finite deformations of strongly anisotropic materials. In: Hutton, J.F., Pearson, J.R.A. Walters, K. (eds) Theoretical rheology. Applied Science Publication, London, pp 141–168 (1975)
4. Rogers, T.G.: Anisotropic elastic and plastic materials. In: Thoft-Christensen, P. (ed) Continuum mechanics aspects of geodynamics and rock fracture, Mechanics. Reidel, pp 177–200 (1975)
5. Sengupta, P.R., Nath, S.: Surface waves in fibre-reinforced anisotropic elastic media, Sadhana. Proc Indian Acad Sci **26**, 363–370 (2001)
6. Singh, B., Singh, S.J.: Reflection of plane waves at the free surface of a fibre-reinforced elastic half-space, Sadhana. Proc Indian Acad Sci **29**, 249–257 (2004)
7. Lord, H.W., Shulman, Y.: A generalized dynamical theory of thermoelasticity. J Mech Phys Solids **15**, 299–309 (1967)
8. Dhaliwal, R.S., Sherief, H.H.: Generalized thermoelasticity for anisotropic media. Q Appl Math **33**, 1–8 (1980)
9. Erdem, A.U.: Heat Conduction in fiber-reinforced rigid bodies,"10 Ulusal Ist Bilimi ve Tekmgi Kongrest (ULIBTK), 6–8 Eylul, Ankara (1995)
10. Chadwick, P., Seet, L.T.C.: Wave propagation in transversely isotropic heat conducting elastic materials. Mathematica **17**, 255–274 (1970)
11. Singh, H., Sharma, J.N.: Generalized thermoelastic waves in transversely isotropic media. J Acoust Soc Am **77**, 1046–1053 (1985)
12. Singh, B.: Wave propagation in anisotropic generalized thermoelastic solid. Indian J Pure Appl Math **34**, 1479–1485 (2003)
13. Chadwick, P.: In: Hill R., Sneddon I.N. (eds) Progress in solid mechanics, vol 1. North-Holland, Amsterdam (1960)
14. Nayfeh, A., Nasser, S.N.: Thermoelastic waves in solids with thermal relaxation. Acta Mech **12**, 53–69 (1971)
15. Chadwick, P., Windle, D.W.: Propagation of Rayleigh isothermal and insulated boundaries. Proceedings of the Royal Society of America **280**, 47–71 (1964)
16. Sharma, J.N., Singh, H.: Thermoelastic surface waves in a transversely isotropic half-space with thermal relaxation. Indian J Pure Appl Math **16**, 1202–1219 (1985)
17. Singh, S.J.: Comments on "Surface waves in fibre–reinforced anisotropic elastic media, Sadhana. Proc Indian Acad Sci **27**, 1–3 (2002)