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Modal parameter estimation based on the wavelet transform of output data

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Summary The paper presents the results of the research on modal parameter estimation based alone on output measurements. A system is excited randomly and random decrement functions are used to separate the random responses from the determination free vibrations. It is shown that on estimation of the natural frequencies and damping ratios of the system is possible using the wavelet transform of the system's free response. A particular form of the son wavelet function improves the results compared to those obtained with the Morlet wavelet function. An optimal son wavelet function is obtained by minimisation of the wavelet transform entropy. The accuracy of this new technique is confirmed in a numerical example and by applying it to ambient vibration measurements of a bridge excited by traffic.

Keywords Wavelet transform, Modal parameters, Ambient vibration transform entropy, Random decrement technique

1 Introduction

Modal analysis is an important tool in a wide variety of engineering applications including structural vibrations and acoustics. The information obtained by it describes the natural frequencies and damping associated with structural modes of a system. It can be used then to determine properties of the system such as safe operating condition, dynamic response behaviour, material damage or to conduct active vibration control of the system [2, 3, 6]. In general, the experimental identification of structural modes of vibration is carried out by measuring the input, or the excitation, of the structure under test and the resulting response due to this input. There are situations, however, where controlled excitation cannot be used. For example, if the structure to be tested is in operation, applying any kind of external force may cause undesirable effect. Other examples are mechanical or civil engineering systems such as off-shore structures, bridges, [2, 6], towers [3], tall buildings and aircraft in flight which are under ambient vibrations and the excitation is not available. To identify the modal parameters only acceleration responses of the system are available, measured under ambient conditions. System identification using ambient vibration measurements presents a challenge, requiring the use of special identification techniques, which can deal with very small magnitudes of ambient vibration contaminated by noise. In such cases, a promising technique is the use of the random decrement technique, [3, 4], which is a special averaging procedure to obtain, free responses.

The aim of this paper is to show how the wavelet analysis of the free response of a system allows the estimation of modal parameters. The paper is organised as follows. The theoretical background of the wavelet transform with its properties is presented in Sec. 2. In Sect. 3, we minimise the entropy of the wavelet transform to obtain a parameter which improves the time and frequency resolutions. It is shown in Sect. 4 that the modulus and phase of the wavelet transform are directly related to eigenfrequencies and damping coefficients of vibrating systems. Further, it is shown that the mode uncoupling is possible by choosing appropriate parameters of the wavelet function. In Sect. 5, we reconstruct the signal by the inverse wavelet transform and separate each mode. The efficiency of the wavelet transform in modal analysis is

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demonstrated in a numerical model example and in the analysis of experimental results, which are presented in Sect. 6. The brief conclusions are draw in Sect. 7.

2
Theoretical background of the wavelet transform

The Fourier transform represents a decomposition of a function into a linear combination of harmonics weighted by Fourier coefficients. This decomposition does not give any local information about the function due to the infinite nature of the trigonometric functions used in the analysis. The Fourier analysis is not effective when used on nonstationary signals because it does not provide frequency-content information localized in time; the frequency component of a signal can be known but its location in time is not known. Most real-world signals, however, exhibit nonstationary characteristics and therefore the Fourier transform is not adequate in the time-out frequency-domain.

A localized decomposition can be obtained using the wavelet transform (WT). The wavelet transform of a signal $x(t)$ is a time-scale decomposition obtained by dilating and translating along the time axis using a chosen analyzing function. The continuous wavelet transform of a signal $x(t)$ is, s. [1,8, 9],

$$(W_\psi x)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^*(\frac{t-b}{a}) dt, \quad (1)$$

where $\psi(t)$ is an analysing function, called the mother wavelet, a is the dilatation or scale parameter defining the analysing window stretching, and b is the translation parameter localising the wavelet function in the time-domain. The WT represents the correlation between the signal $x(t)$ and a scaled version of the function $\psi(t)$, and the idea of the WT is to decompose a signal $x(t)$ into wavelet coefficients $(W_\psi x)(a,b)$ by using the basis of wavelet functions.

Since the mother wavelet is localized in both time-and frequency-domains, the WT displays the time-evolution of the frequency components of a signal. The wavelet is compared to a section at the beginning of a signal. A number is calculated, showing the degree of correlation between the wavelet and the signal section. The wavelet is then shifted right, and the process is repeated until the whole signal is covered. The wavelet is scaled and the previous process is repeated for all scales.

Any function $\psi(t)$ can be used as a mother wavelet when it satisfies the admissibility condition, [1, 8, 9],

$$0 < c_\psi = \int_{-\infty}^{+\infty} \frac{|\Psi(f)|^2}{|f|} df < \infty, \quad (2)$$

where $\psi(f)$ is the Fourier transform of $\psi(t)$.The mother wavelet must be also a window function which tends to zero at $t \to \pm\infty$. If one assumes a fast decay, that is, the values of $\psi(t)$ are negligible outside the interval (t_{min}, t_{max}), the wavelet transform becomes local. The wavelet transform can be inverted and the signal $x(t)$ recovered, s. [1, 8, 9],

$$x(t) = \frac{1}{c_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (W_\psi x)(a,b) \frac{1}{\sqrt{a}} \psi^*(\frac{t-b}{a}) \frac{da db}{a^2}. \quad (3)$$

Since the wavelet transform is a linear representation of a signal, it follows that the WT of P signals has the following property:

$$(W_\psi \sum_{i=1}^{P} x_i)(a,b) = \sum_{i=1}^{P} (W_\psi x_i)(a,b). \quad (4)$$

This property is convenient for the analysis of multi-component signals.

An alternative formulation of the WT can be obtained by transforming both the signal $x(t)$ and the wavelet function $\psi(t)$ in the frequency domain

$$(W_\psi x)(a,b) = \sqrt{a} \int_{-\infty}^{+\infty} X(f) \Psi^*(af) e^{j2\pi f\, b} df, \quad (5)$$

where $X(f)$ is the Fourier transform of $x(t)$, and $a\psi^*(af)e^{j2\pi f\, b}$ is the Fourier transform of $\psi^*(\frac{t-b}{a})$. The discrete wavelet transform can be calculated numerically as

$$W(m,n) = \sqrt{m\Delta a}\sum_n X(f_n)\Psi^*(m\Delta a f_n)e^{j2\pi f_n n\Delta b}, \qquad (6)$$

where f_n is the discrete frequency and Δa and Δb are increments of dilatation and translation parameters. The discrete wavelet transform can be implemented with the help of the fast Fourier transform (FFT) algorithm, by generating dilated wavelets and the signal in the frequency-domain, before transforming the results back to the time-domain using the inverse FFT.

A number of different analyzing functions have been used in the wavelet analysis. One of the most known and widely used is the Morlet wavelet defined in the time-domain as

$$\psi(t) = e^{j\omega_o t}e^{-t^2/2}, \qquad (7)$$

where ω_o is the wavelet frequency. The dilated version of the Fourier transform is

$$\Psi(a\omega) = \sqrt{2\pi}e^{-\frac{1}{2}(a\omega-\omega_o)^2}. \qquad (8)$$

In practice, the value of ω_o is chosen as $\omega_o = 2\pi\sqrt{2\log(2)}$, which meets approximately the requirements given by condition (2). Note that $\psi(a\omega)$ is maximum at the central frequency $\omega_c = \omega_o/a$, and the Morlet wavelet can be viewed as a linear bandpass filter whose bandwidth is proportional to $1/a$ or to the central frequency. Thus, the value of the dilatation parameter a, at which the wavelet filter is focused on the wavelet frequency, can be determined from $a = \omega_o/\omega_c$. For a given value of the dilatation parameter a, the spectrum of the Morlet wavelet has a fixed bandwidth. If the analysed frequency is high, the dilatation parameter becomes small, and the spectrum of the Morlet wavelet function is wide. There is then a bad spectral resolution. In many cases, a shifted Morlet wavelet can be used. Thus, instead of ω in Eq. (8), the difference $(\omega - \omega_h)$ is applied, where ω_h is the shift frequency, [8]. The frequency of the analysis can then be changed arbitrarily, giving better frequency-resolution but worse time-resolution. An alternative is proposed in Sec. 3, using a modified Molet wavelet.

In summary, the wavelet transform analyzes an arbitrary signal $x(t)$ only locally, at windows defined by a wavelet function. The WT decomposes then the signal into various components at different time-windows and frequency bands. The location of the time-window is controlled by the translation parameter b, while the length of the frequency band is controlled by the dilatation parameter a. Hence, one can examine the signal at different time-windows and frequency bands by controlling the translation and dilatation.

3
The modified Morlet wavelet and the wavelet transform entropy

An alternative to the choice presented in Sec. 2 is to modify the Morlet wavelet function, introducing a parameter N which controls the shape of the basic wavelet ; this parameter balances the time-resolution and the frequency-resolution of the Morlet wavelet. The modified Morlet wavelet function used in this paper is

$$\psi(t) = e^{j\omega_o t}e^{-t^2/N}, \qquad (9)$$

with $N > 0$. The dilated version of its Fourier transform is

$$\psi(a\omega) = \sqrt{N\pi}e^{-\frac{N}{4}(a\omega-\omega_O)^2} \qquad (10)$$

The wavelet filter central frequency is $\omega_c = \omega_o/a$ and gives then a relation between the scale parameter a and the central frequency of the modified Morlet wavelet.

An increased value of N gives a narrower spectrum, allowing a better resolution of closely spaced modes, but at the expense of the time-resolution: it will increase the frequency-resolution but decrease the time-resolution. So, there always exists an optimal N that has the best time-frequency resolution for a certain signal localized in the time-frequency plane. This modified Morlet wavelet function offers a better compromise in terms of localization, in both time and frequency for a signal, than the traditionally Morlet wavelet function.

The optimal value of N is obtained by minimizing the entropy, [7], of the wavelet transform introduced in this section.

We assume that the signal $x(t)$ is given by sampled values $\{ x(q) \}$, $q = 1, 2, \ldots, Q$. The total energy of this sequence is $\sum_q |x(q)|^2$ and the values

$$p(q) = \frac{|x(q)|^2}{\sum_q |x(q)|^2}, \tag{11}$$

give the probability distribution of the signal's energy in the time-domain. Furthermore, in the wavelet multiresolution analysis of the time series $\{ x(q) \}$, the energy for each scale a_i is

$$E_{ai} = \sum_j |W(a_i, b_j)|^2, \tag{12}$$

Energy E_{ai} is then obtained with a set of wavelet coefficients over a number of translations b_j, given a particular scale a_i. As a consequence, the total energy can be obtained by

$$P_{ai} = E_{ai}/E_{total}.$$

Then the normalized values,

$$p_{ai} = E_{total} E_{ai},$$

which represent the relative wavelet energy for $i = 1, 2, .., M$, define within the scale the probability distribution of the energy. Clearly,

$$\sum_i p_{ai} = 1$$

and the distribution $\{p_{ai}\}$ can be considered as a time-scale density. Following the definition of entropy given in [7], we define the time-varying wavelet entropy as

$$WE = -\sum_i p_{ai} \log p_{ai}, \tag{13}$$

which measures the degree of disorder or unpredictability of energy in each wavelet transform. The Shannon entropy, [7], gives a useful criterion for analyzing and comparing these distributions, since it provides a measure of the information of any distribution. Using the modified Morlet wavelet, there exists an optimal value of the parameter N obtained by the minimization of the wavelet entropy. Simulated and experimental signals will be used in Sec. 6 to verify the validity of the wavelet entropy.

4
Application of the wavelet transform to modulated signals

Consider the case of a signal $x(t)$ modulated in amplitude

$$x(t) = A(t) \cos[\phi(t)]. \tag{14}$$

If $x(t)$ is assumed to be asymptotic, the WT of $x(t)$ can be obtained by means of asymptotic techniques and can be expressed as, [9],

$$(W_\psi x)(a, b) = \frac{\sqrt{a}}{2} A(b) \Psi^*[a\phi\,'(b)] e^{j\phi(b)}, \tag{15}$$

the prime indicating a derivative. The dilatation parameter can be calculated in order to maximize $\Psi^*[a\phi'(b)]$, that is using the Morlet wavelet (or modified Molet wavelet) for the dilatation $a(b) = \omega_o/\phi'(b)$. The maximum of the wavelet transform amplitude is essentially concentrated in the neighbourhood of a curve given by $a(b)$. This curve is called the ridge of the wavelet transform, [8,9].

Consider now the free response of a viscously damped single-DOF, system

$$x(t) = Be^{-\zeta\omega_n t}\cos(\omega_d t + \chi_o), \tag{16}$$

with B denoting the residue magnitude, ω_n-the undamped natural frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ the damped natural frequency, ζ-the viscous damping ratio and the product $\sigma = \omega_n\zeta$ called the rate of decay. If the system is underdamped, that is, if the damping ratio is smaller than unit, the signal $x(t)$ can be considered asymptotic. Therefore, the results obtained previously can be used, considering:

$$A(t) = Be^{-\zeta\omega_n t}, \tag{17}$$

$$\phi(t) = \omega_d t + \chi_o \Rightarrow \phi'(t) = \omega_d. \tag{18}$$

The wavelet transform of the damped sinusoid is

$$(W_\psi x)(a,b) = \frac{\sqrt{a}}{2} Be^{-\zeta\omega_n b}\Psi^*(a\omega_d)e^{j(\omega_d b+\chi_o)}. \tag{19}$$

For a fixed value a_o of the dilatation parameter, the wavelet transform modulus is

$$\left|(W_\psi x)(a_o,b)\right| = \frac{\sqrt{a_o}}{2} Be^{-\zeta\omega_n b}\left|\Psi^*(a_o,\omega_d)\right|, \tag{20}$$

and applying the logarithm to this function we obtain

$$\ln\left|(W_\psi x)(a_o,b)\right| = -\zeta\omega_n b + \ln(\frac{\sqrt{a_o}}{2} B\left|\Psi^*(a_o,\omega_d)\right|). \tag{21}$$

Thus, the decay rate σ of the signal can be estimated from the slope of the straight line of the logarithm of the wavelet transform modulus. Note that if the Morlet (or modified Morlet) wavelet is used, the dilatation parameter is related to the damped natural frequency: $a_o = \omega_o/\omega_d$. The wavelet transform phase is given by

$$\mathrm{Arg}[W_\psi(a_o,b)] = \omega_d b + \chi_o \Rightarrow \frac{d}{db}\mathrm{Arg}[W_\psi(a_o,b)] = \omega_d, \tag{22}$$

and the plot of $\frac{d}{db}\mathrm{Arg}[W_\psi(a_o,b)]$ should be constant in time and equal to the damped natural frequency ω_d. Once the decay rate and damped natural frequency have been estimated, it is possible to identify both the natural frequency and damping ratio of the vibrating system.

The damping ratio and frequency estimation procedures, based on the wavelet transform presented above, can be extended to multi-DOF systems by selecting the right value of the dilatation parameter corresponding to the mode of interest. As shown in Sec. 2, the WT is a signal decomposition procedure working as a filter in the time-frequency (or scale) domain. Thus, it offers a possible means of uncoupling vibration modes if the value of the dilatation parameter is correct. Since the analyzing wavelet function has compact support in the time-and frequency-domain, the WT for P multicomponent signals is

$$(W_\psi \sum_{i=1}^{P} x_i)(a,b) = \frac{1}{\sqrt{a}}\sum_{i=1}^{P}\int_{t-a\Delta t_\psi}^{t+a\Delta t_\psi} x_i(t)\Psi^*(\frac{t-b}{a})dt, \tag{23}$$

and

$$(W_\psi \sum_{i=1}^{P} x_i)(a,b) = \sqrt{a}\sum_{i=1}^{P}\int_{f_i-(\Delta f_\psi/a)}^{f_i+(\Delta f_\psi/a)} X_i(f)\Psi^*(af)e^{j2\pi f b}df, \tag{24}$$

where Δt_ψ and Δf_ψ are the duration and bandwidth of the wavelet function. This shows the filtering action of the wavelet transform, where the wavelet analyzing function for each signal i is peaked at frequency f_i. In other words, it is assumed that the function $\psi^*(af)$ is vanishing outside the interval $[f_i - \Delta f_\psi/a, f_i + \Delta f_\psi/a]$.
Consider now the free response of a P-DOF system

$$x(t) = \sum_{i=1}^{P} B_i e^{-\zeta_i \omega_{ni} t} \cos(\omega_{di} t + \chi_{oi}), \tag{25}$$

where B_i is the residue magnitude,ζ_i is the damping ratio, ω_{ni} the undamped natural frequency and ω_{di} the damped natural frequency associated to the i-th mode. From Eq. (19), the wavelet transform of the multi-DOF system is

$$(W_\psi \sum_{i=1}^{P} x_i)(a,b) = \frac{\sqrt{a}}{2} \sum_{i=1}^{P} B_i e^{-\zeta_i \omega_{ni} b} \Psi^*(a\omega_{di}) e^{j(\omega_{di} b + \chi_{oi})}. \tag{26}$$

The wavelet transform is a signal decomposition procedure working as a filter in the time-frequency domain : it analyzes a signal only locally, at windows defined by the wavelet. Thus, a multi-DOF system can be decoupled into single-DOF components. For a fixed value of the dilatation parameter, $a = a_i$, which maximizes $\Psi^*(a\omega_d)$, only the mode associated with a_i gives a relevant contribution in the wavelet transform, while all the other terms are negligible. Thus, the wavelet transform of each separated mode $i = 1, 2, .., P$ becomes

$$(W_\psi x_i)(a_i, b) = \frac{\sqrt{a_i}}{2} B_i e^{-\zeta_i \omega_{ni} b} \Psi^*(a_i \omega_{di}) e^{j(\omega_{di} b + \chi_{oi})}. \tag{27}$$

Clearly, the wavelet transform offers a decoupling of multi-DOF systems into single modes. However, Eq. (27) is true under the assumption of vanishing $\psi^*(a_i\omega_{di})$ outside the interval

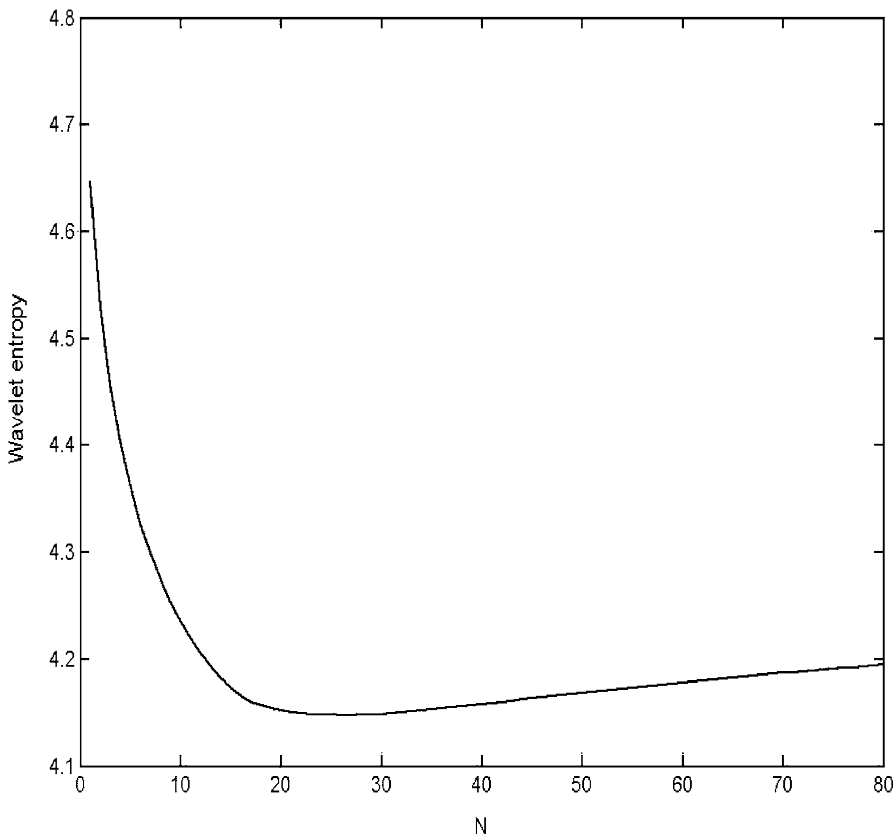

Fig. 1. Variations of the wavelet entropy for the 3 DOF system

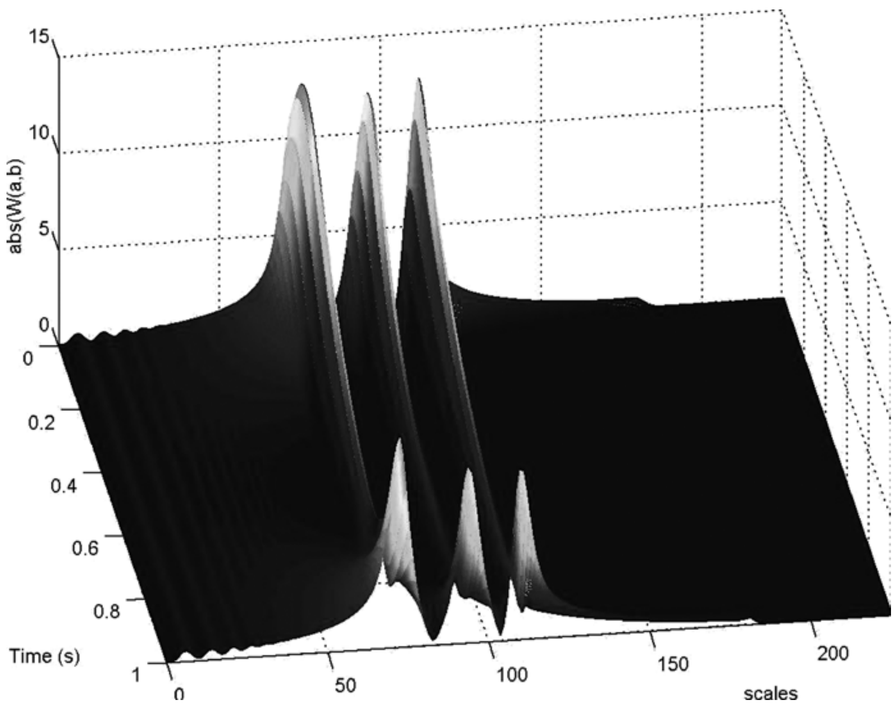

Fig. 2. Amplitude of the wavelet transform for the 3 DOF system

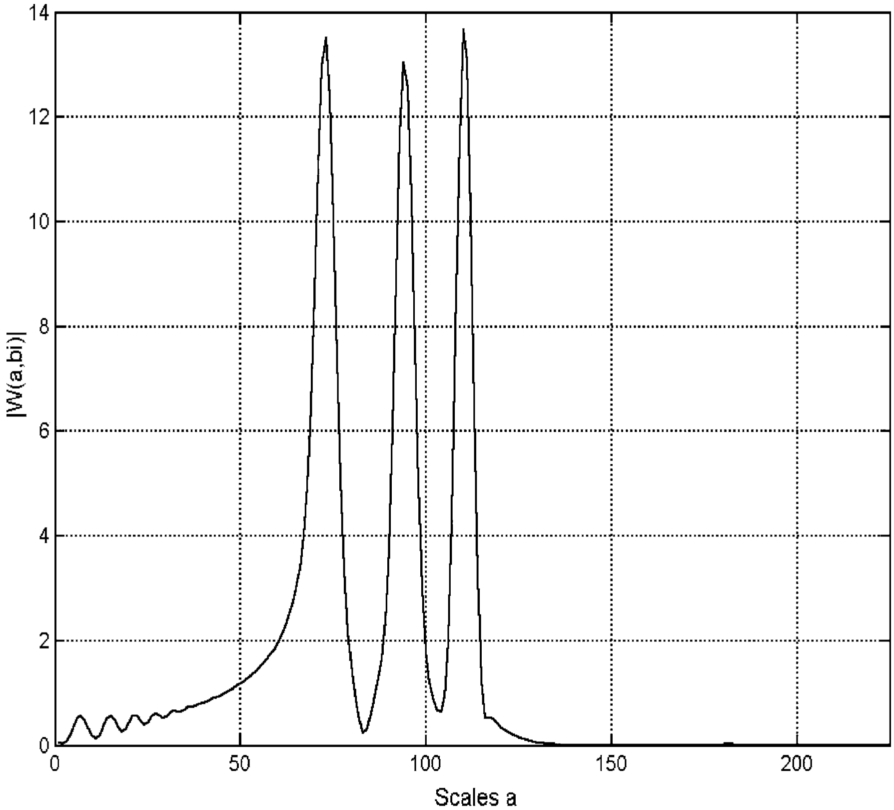

Fig. 3. Determination of dilatation parameters for the 3 DOF system

$[f_i - \Delta f_\psi / a_i,\ f_i + \Delta f_\psi / a_i]$, that is, if none of the other frequencies of the system, except f_i and more likely neither f_{i-1} nor f_{i+1}, belongs to this interval. The resolution of the wavelet transform using the modified Morlet wavelet (9), with an optimal value of N, is good enough to separate the i-th mode from the neighbouring modes.

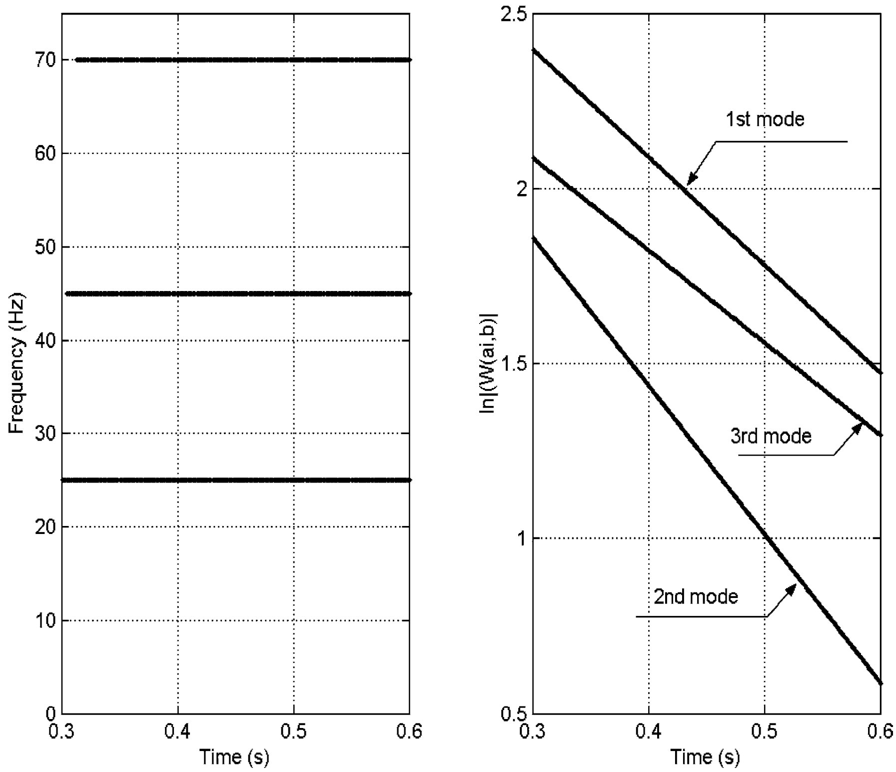

Fig. 4. Instantaneous frequencies and wavelet transform envelopes for the 3 DOF system

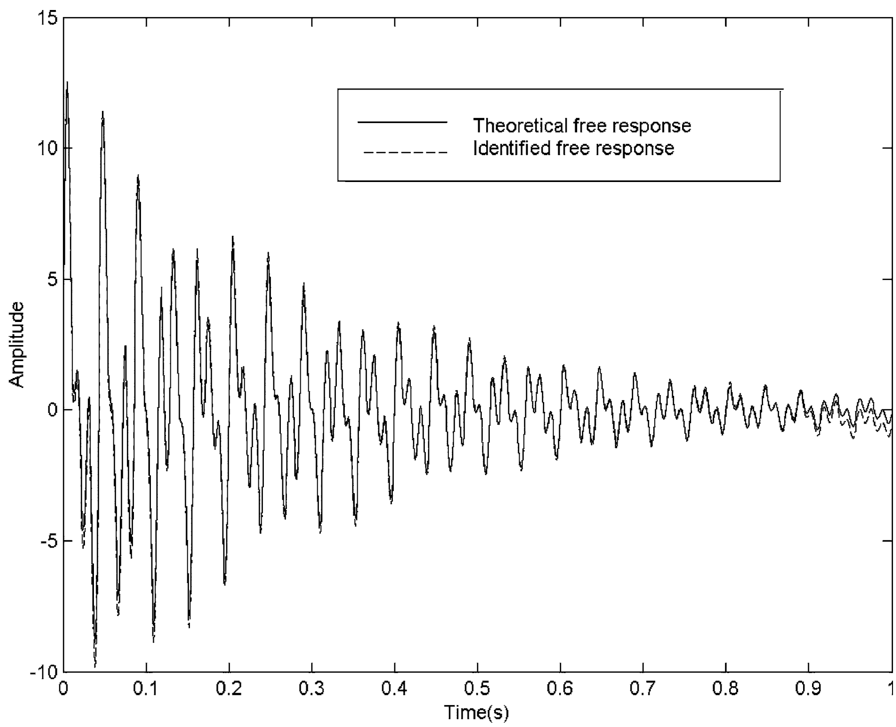

Fig. 5. Comparison between the theoretical free response and the identified response using the inverse wavelet transform for the 3 DOF system

Using (27) associated with (21) and (22), it is possible to follow the amplitude and the phase variations in the time-domain of each modal component, and to estimate the corresponding damping ratio and natural damped frequency associated to the isolated mode. We obtain for each i-th mode

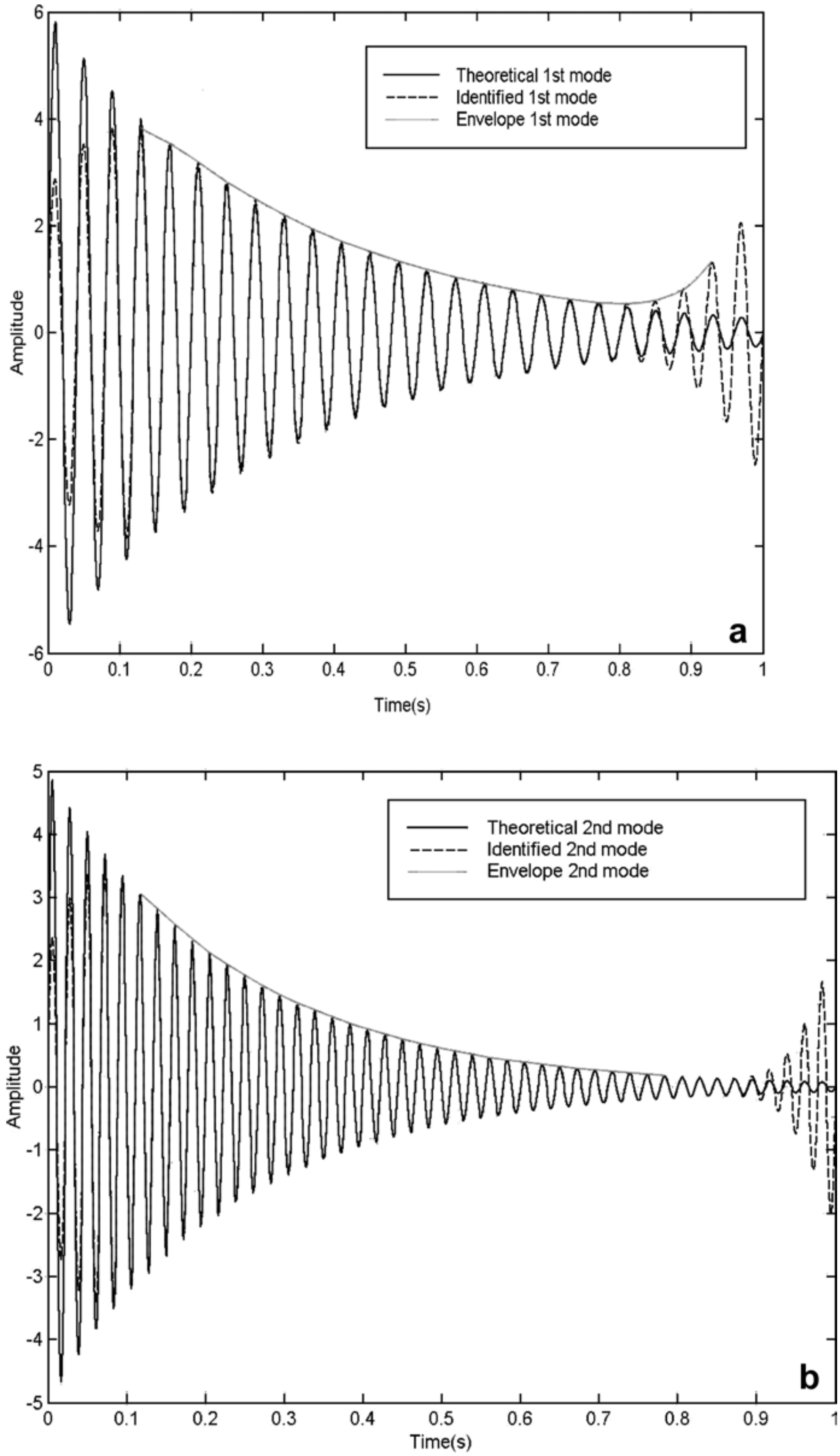

Fig. 6. Comparison between theoretical mode and identified mode using the inverse wavelet transform for the 3 DOF system : (a) 1st mode; (b) 2nd mode; (c) 3rd mode

$$\ln\left|(W_\psi x_i)(a_i, b)\right| = -\zeta_i \omega_{ni} b + \left(\ln \frac{\sqrt{a_i}}{2} B\left|\Psi^*(a_i \omega_{di})\right|\right), \tag{28}$$

$$\operatorname{Arg}[W_\psi(a_i, b)] = \omega_{di} b + \chi_{oi} \Rightarrow \frac{d}{db}\operatorname{Arg}[W_\psi(a_i, b)] = \omega_{di}. \tag{29}$$

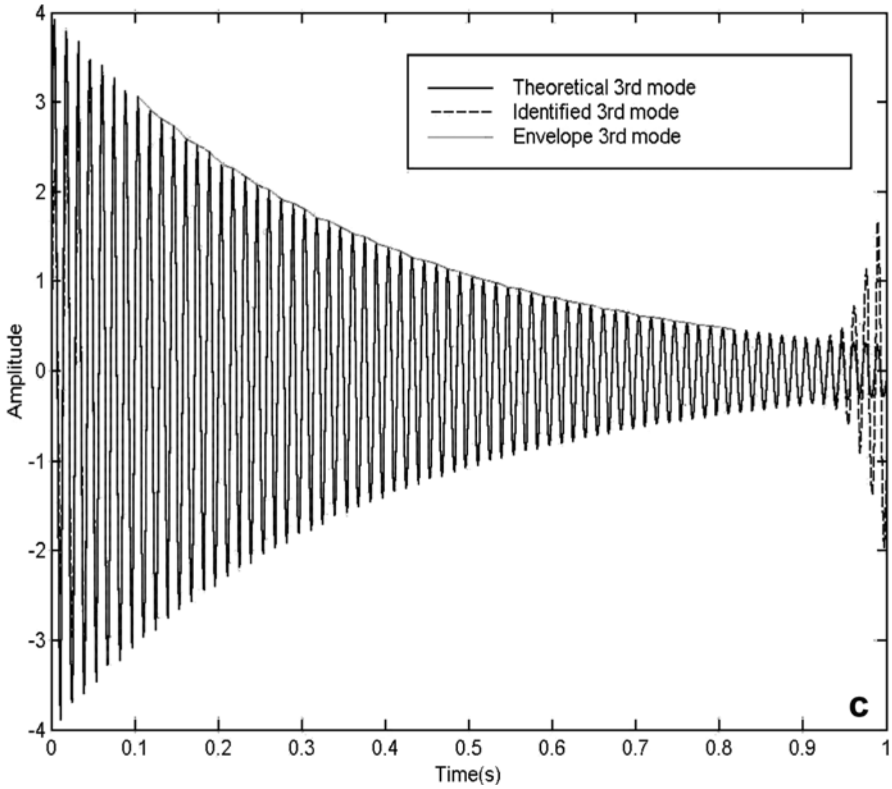

Fig. 6. Contd.

Table 1. Three-DOF system simulation results

Theoretical frequency (Hz)	Estimated frequency (Hz)	Theoretical damping ratio	Estimated damping ratio using Eq. (28)	Estimated damping ratio using Eq. (32)
25	24,99	0,02	0,0196	0,02
45	44,99	0,015	0,015	0,0149
70	70	0,006	0,006	0,0059

This technique requires a previous choice of the value of the dilatation parameter a_i corresponding to the analyzed mode. The resolution of the wavelet transform depends on the value of this scale parameter, thus, the choice of the analyzing wavelet is important.

5 Free response recovery using the inverse wavelet transform

The reconstruction signal $x(t)$ given by Eq. (3), which considers the inverse wavelet transform, can be simplified using the reconstruction formula of Morlet, [9], which only requires a single integration. The formula is

$$x(t) = \frac{1}{k_\psi} \int_0^{+\infty} (W_\psi x)(a, b) \frac{da}{a}, \qquad (30)$$

where k_ψ is a constant given by

$$k_\psi = \int_0^{+\infty} \frac{\Psi(f)^*}{f} df.$$

The free response of a P-DOF system is recovered as

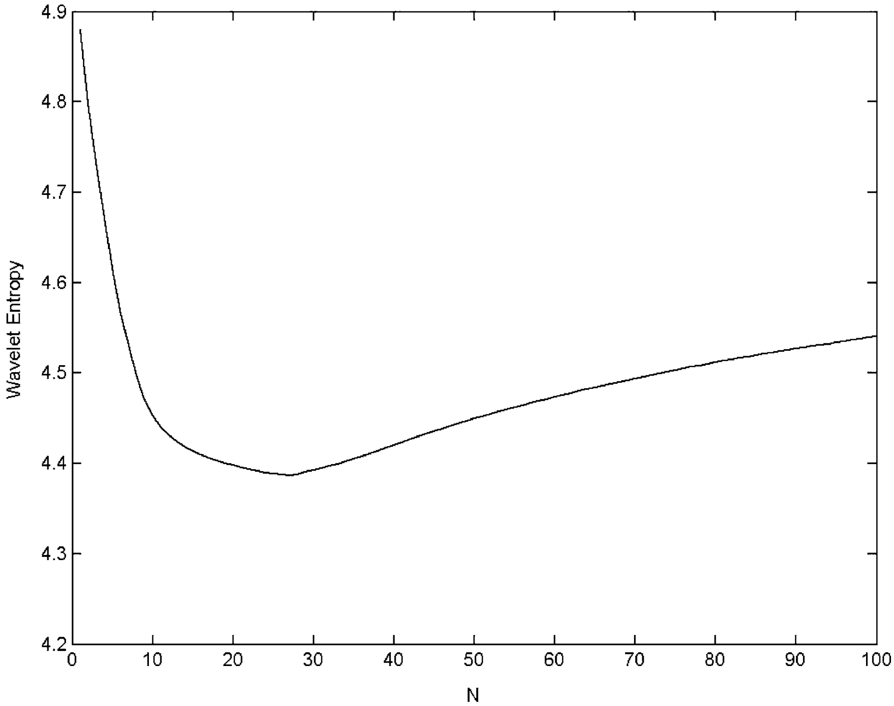

Fig. 7. Variations of the wavelet entropy for the Z24 bridge

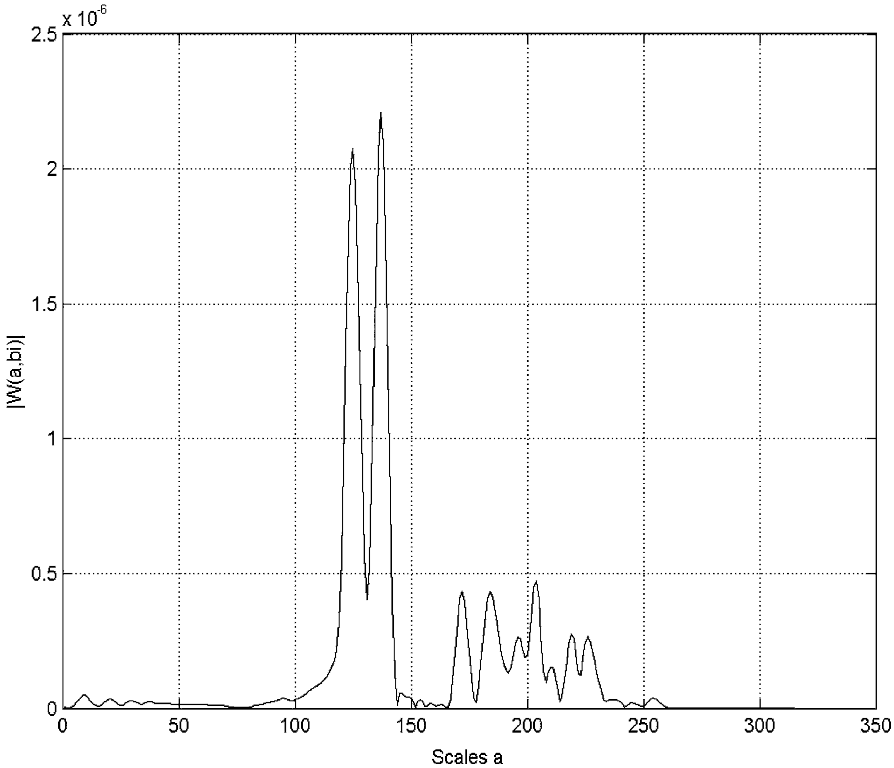

Fig. 8. Determination of dilatation parameters for the Z24 bridge

$$x(t) = \sum_{i=1}^{P} \frac{1}{k_{\Psi_i}} \int_{t-a\Delta t_\psi}^{t+a\Delta t_\psi} (W_\psi x_i)(a,b) \frac{da}{a}, \tag{31}$$

where the constant k_{ψ_i} is associated with the i-th mode by

$$k_{\psi_i} = \int_{f_i-(\Delta f_\psi/a)}^{f_i+(\Delta f_\psi/a)} \frac{\Psi(f)^*}{f}.$$

The free response of a single mode is then

$$x_i(t) = \frac{1}{k_{\Psi_i}} \int_{t-a\Delta t_\psi}^{t+a\Delta t_\psi} (W_\psi x_i)(a,b) \frac{da}{a}. \tag{32}$$

The damping estimation procedure is now straightforward; the free response for a single mode is obtained from the inverse wavelet transform by using Eq. (32), and the damping coefficient is then estimated as the slope of the semi-logarithmic plot of the free response function envelope.

6 Applications

6.1 Simulated results

To prove the effectiveness of the identification procedure based on the WT technique, the free response of a three-DOF system is analyzed. The parameters of the system are chosen as follows : $f_1 = 25$ Hz; $f_2 = 45$ Hz; $f_3 = 70$ Hz; $\zeta_1 = 0,02; \zeta_2 = 0,015$ and $\zeta_3 = 0,006$. The analyzing function is the modified Morlet wavelet, Eq. (9), and the parameter N is obtained by minimization of the wavelet entropy, s. Eq. (13). Increasing N from 2 to 80 and calculating the wavelet entropy of the coefficients, we obtain the relationship between N and the wavelet entropy, as shown in Fig. 1. There exists a minimal value of the wavelet entropy when $N = 25$, which means that $N = 25$ is the optimal value of N. The amplitude of the wavelet transform is given in Fig. 2, and the three modes corresponding to the simulated signal are visible. The dilatation parameters a_i for each eigenmode can be obtained from Fig. 3, and damped natural frequencies can be then estimated by deriving the phase of the wavelet transform. comp. Eq. (29). The decay rate of the envelope for each mode is then calculated from the slope of the straight line of the logarithm of the wavelet transform modulus Eq. 28. Natural frequencies and damping ratios can be then derived from Fig. 4. Figure 5 shows a comparison between the theoretical signal (solid line) and the recovered signal from the inverse wavelet transform, Eq. (30), (dashed line). These two curves show a good agreement. The wavelet reconstruction formula (32) is used to recover separated modes and the values of damping coefficients are estimated from the decaying envelopes of the recovered single modes as shown in Fig.6. Note that these curves show good agreement, apart from the derivations at the beginning and end of the data due to truncation effects.

The results in Table 1 show the accuracy of the technique in estimating natural frequencies and damping ratios.

6.2 Application to real data measurements

The WT identification technique presented above was applied to the analysis of acceleration responses of a real engineering structure, the Z24 bridge between Bern and Zurich in Switzerland. The bridge was excited by ambient forces, which are essentially due to traffic. These inputs could evidently not be measured, so only acceleration data were available. A full description of the test set-up, equipment disposition and bridge geometry can be found in [6].

The wavelet transform estimation technique operates on the free response of the analysed system. A well-established method to convert random responses of a structure to free decay responses is the random decrement technique, [4]. Its basic concept is that the acceleration response $y(t)$ measured on the structure can be decomposed into free vibration component and forced vibration component. The free vibration component contains an impulse or step response, while the forced vibration component represents response to the random load. The free vibration component can be obtained by a special averaging procedure of measurements,

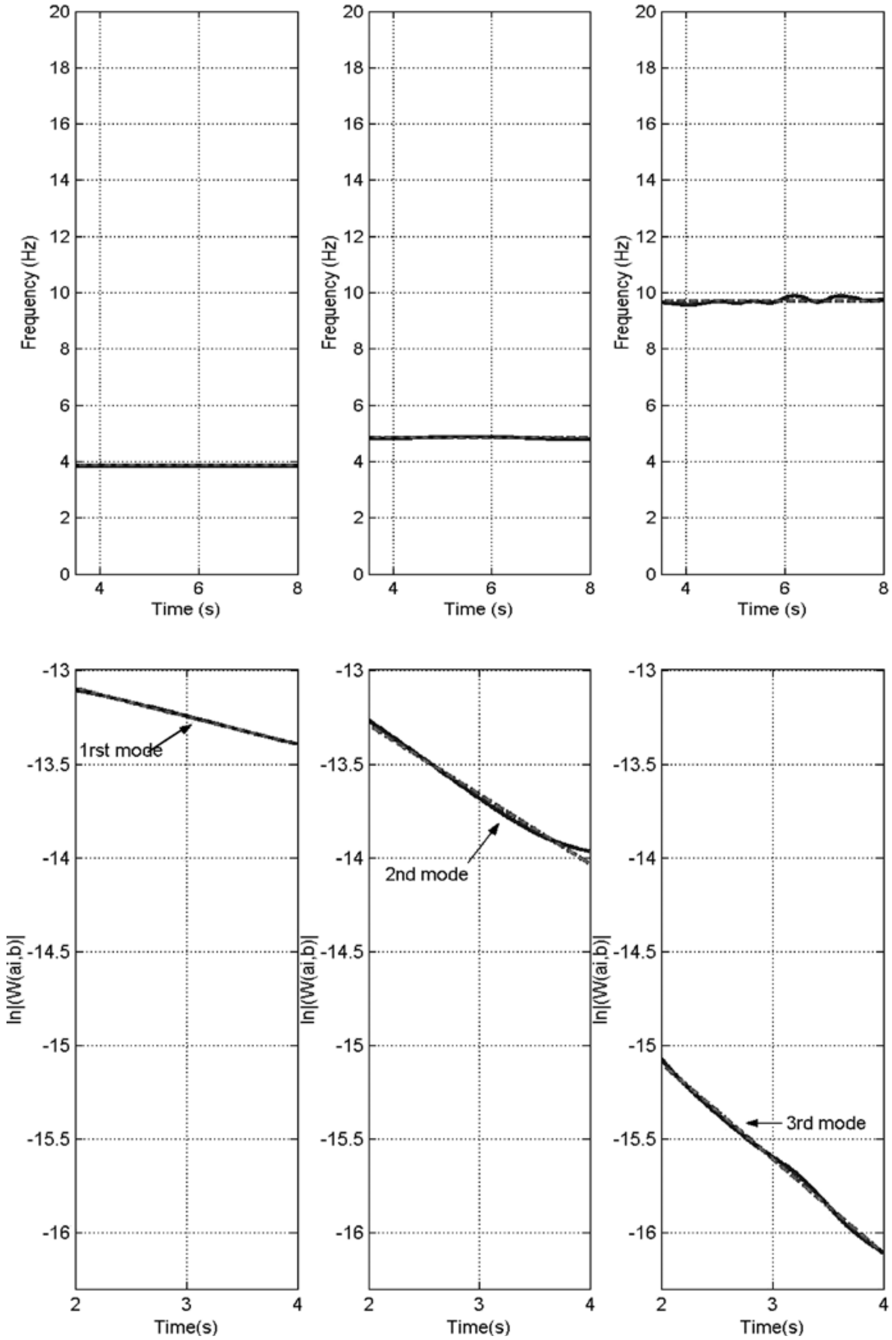

Fig. 9. Instantaneous frequencies and wavelet transform envelopes for the Z24 bridge

which removes the random part, leaving its deterministic part. The randomdec signal (or free response) $x(\tau)$ of the measured signal is defined by, [4]

$$\mathrm{x}(\tau) = \frac{1}{\mathrm{Ns}} \sum_{\mathrm{m}=1}^{\mathrm{Ns}} \mathrm{y}(\mathrm{t_m} + \tau), \tag{33}$$

where *Ns* is the number of time samples averaged, τ is the free response time-length, and t_m are determined from original data as all the time instants satisfying initial conditions; they are taken as those of zero values crossing the time axis with a positive slope on the acceleration-time history record. We have thus a very simple condition

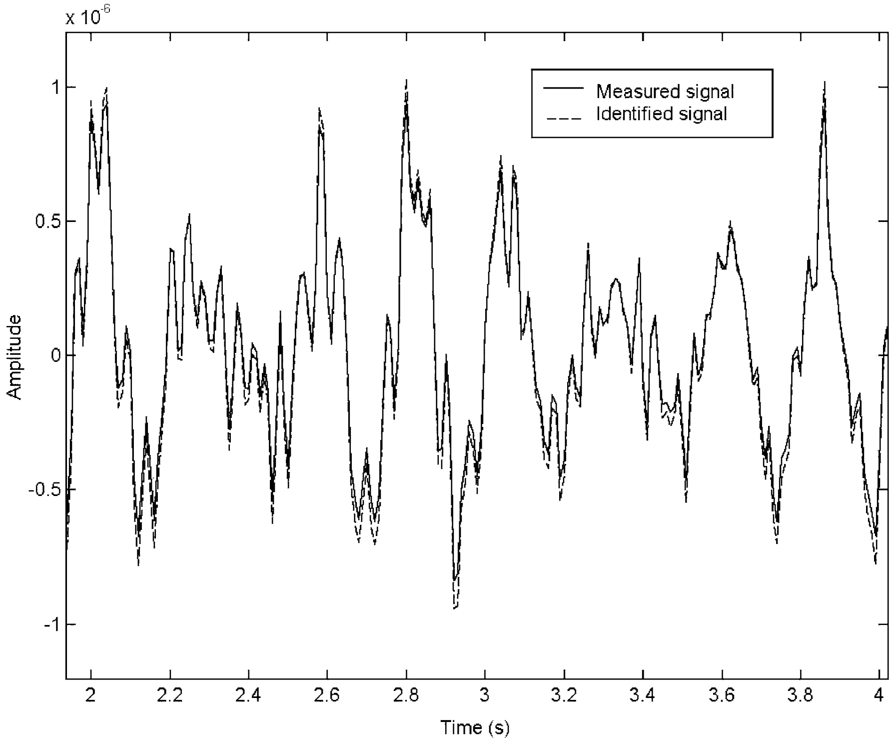

Fig. 10. Comparison between the theoretical free response and the identified response using the inverse wavelet transform for the Z24 bridge

$$\mathrm{t_m} \quad : \quad \mathrm{y(t_m)} = 0 \quad \text{and} \quad \left(\frac{\mathrm{dy}}{\mathrm{dt}}\right)_{\mathrm{t=t_m}} > 0. \tag{34}$$

The total length of the randomdec signal was 4096 points. Once the free responses of the structure have been estimated, the wavelet transform identification procedure is applied. First, the optimal value of N is obtained, using the minimization of the wavelet entropy. From Fig. 7, we obtain $N = 27$. The dilatation parameter a_i for each eigenmode is obtained from Fig. 8. Figure 9 shows three examples of damped natural frequencies and decay rates, natural frequencies and damping coefficients are then estimated from these plots using a linear interpolation procedure. A comparison between randomdec signal (solid line) and recovered signal (dashed line) using the inverse wavelet transform is given in Fig. 10; these two curves show a good agreement. The wavelet reconstruction formula is used to recover separated modes and to estimate damping coefficients from envelopes of these curves, which are plotted in Figs. 11(a–c), for three first modes. The estimated modal parameters for the seven first modes of the Z24 bridge are shown in Table 2.

The natural frequencies obtained using the wavelet transform displayed a good match with those found using other techniques operating in the time-or frequency-domain, [6]. Damping ratios estimation is always crucial when lightly damped structures are considered, however, damping coefficients estimated using the amplitude of the wavelet transform (28) and the recovered modes (32) give satisfactory results.

7
Conclusion

An approach to estimate modal parameters in the time-domain from the output data only, using the wavelet transform has been presented. The results obtained on the numerical simulation underline the accuracy of the wavelet transform method in estimating both natural frequencies and damping ratios. The wavelet transform method is well suitable for the analysis of mechanical systems excited by random forces and can be applied to real data. Note that also other methods based on the spectral decomposition of a transition matrix, [6], have been used to obtain modal parameters from the output measurements. A comparison of these methods

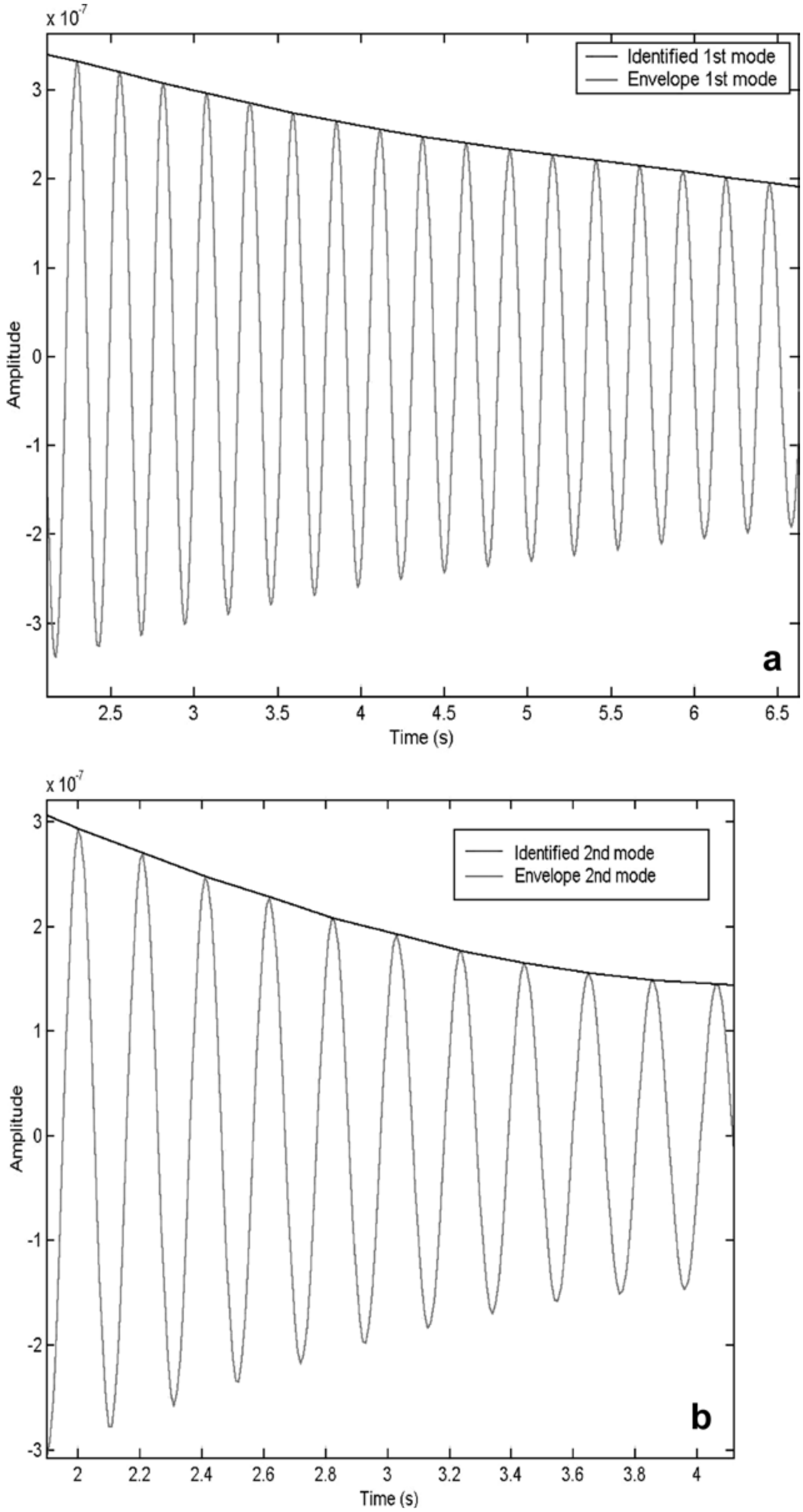

Fig. 11. Identified mode and its envelope using the inverse wavelet transform for the Z24 bridge : (a) 1st mode; (b) 2nd mode; (c) 3rd mode

with the wavelet transform method presented here is under investigation. Further work will be directed to modal parameter identification in nonlinear systems using the wavelet transform. The problem of neighbouring natural frequencies, which is related to the scale parameter, is also under investigation.

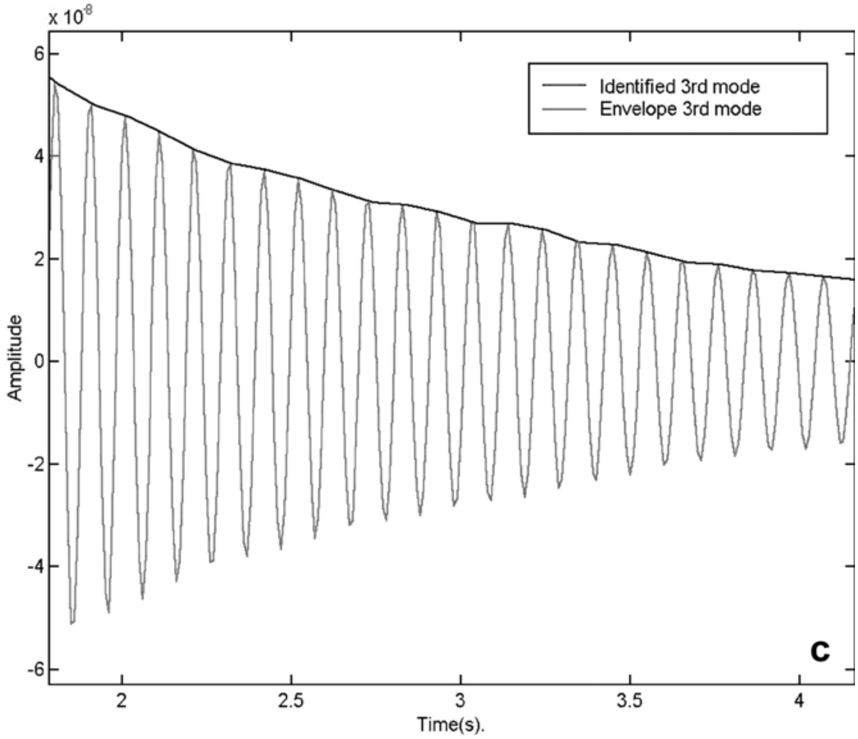

Fig. 11. Contd.

Table 2. Estimated modal parameters of the Z24 bridge

Frequency (Hz)	Damping ratio using Eq. (28)	Damping ratio using Eq. (32)
3,85	0,57	0,61
4,93	1,82	1,84
9,53	2,31	2,42
10,63	1,88	2,07
12,95	1,72	1,87
18,59	2,43	2,67
20,62	2,15	2,28

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