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Interaction of a screw dislocation with a notch in a piezoelectric bi-material

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Summary The electro-elastic interaction of a screw dislocation and a notch in a piezoelectric bi-material is analyzed. The electro-elastic fields induced by the dislocation are derived using the conformal mapping and the image-dislocation approach, where the solution for a piezoelectric bi-material without a notch is used as a base. The stress and the electric displacement intensity factors of the notch and the image force on the dislocation are given explicitly. We find that intensity factors are expressed in terms of the effective material constants, while the radial component of the image force is independent of the notch angle and the angular position of the dislocation in the polar coordinate system. Numerical results for the image force are provided for the use when one of the two media is purely elastic. They illustrate the behavior of the dislocation in the neighborhood of the notch.

Keywords Piezoelectricity, Bi-material, Screw dislocation, Intensity factor, Image force, Notch, Electro-elasticity

1 Introduction

The study of the interaction of dislocations with cracks, inhomogeneities, boundaries or interfaces is important as a means for understanding the physical behavior of materials. This is due to the fact that even in absence of external stress, all mentioned material substructures may provide certain preferred places where dislocations accumulate. Furthermore, such accumulation of dislocations leads to crack initiation at these places. On the other hand, dislocations in the vicinity of a crack can retard or enhance crack propagation [1]. In the past decades, much effort has been devoted to this subject for materials showing no piezoelectric effects. With the wide application of piezoelectric materials, such as piezoceramics, III–V and II–VI compounds or wurtzite **GaN** in engineering, such topic also attracts increasing attention. Recently, some investigations have been made for a screw dislocation interacting with the boundaries of a half-plane, [2], and an infinite long cylinder with general cross-section, [3], the interfaces of a bi-material [4], inhomogeneities [5, 6], and various kinds of cracks [7–12], where piezoelectric effects were involved.

Most of the aforementioned studies of the dislocation-crack interaction problem are confined to a *single-phase* piezoelectric medium, [7–11]. In this paper the problem of interest is the interaction between a notch (wedge crack) and a screw dislocation in two bonded dissimilar piezoelectric materials. Notches often exist in piezoelectric devices such as transducers, delay lines, and surface-acoustic-wave (SAW) components, [13]. Electro-elastic field intensification usually occurs in the vicinity of a notch. The major concern is to reveal the influence of piezoelectricity on the dislocation-notch interaction. The conformal mapping technique in

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conjunction with the image dislocation method (IDM) is adopted here to find the explicit closed-form solution to the problem. The two important quantities, intensity factors (IF) and image force on the dislocation, are given explicitly. The results of the present paper can be reduced to those of several special cases available in the literature.

2
Formulation of the problem

The problem considered is shown in Fig. 1a. A two-phase (Bimaterial) infinite piezoelectric medium contains a notch whose angle is 2β. Assume that both material 1 and material 2 are transversely isotopic piezoelectric solids with respect to the poling direction normal to the (x, y)-plane. The two surfaces of the notch are free of traction and electric charge. A screw dislocation $\mathbf{b} = [b_3, b_\phi]^{\mathrm{T}}$ with dislocation line parallel to the poling direction is situated at z_0 in medium 1, where b_3 and b_ϕ represent the classical elastic displacement jump and the electric potential jump, respectively. Under such circumstances, the out-of-plane elastic displacement w and the electric potential ϕ depend only on the coordinates x and y, such that

$$w = w(x, y), \quad \phi = \phi(x, y) \ .$$

According to [2], for a screw dislocation in an infinite homogeneous piezoelectric medium, the governing differential equations for w and ϕ are

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = 0, \quad e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0 \ , \tag{1}$$

where $\nabla^2 = \partial/\partial x^2 + \partial/\partial y^2$ is the 2-D laplacian operator; c_{44}, e_{15} and ε_{11} are the elastic, piezoelectric and dielectric constants, respectively. Since

$$c_{44}\varepsilon_{11} + (e_{15})^2 \neq 0 \ ,$$

Eqs. (1) can be separated into

$$\nabla^2 w = 0, \quad \nabla^2\phi = 0 \ . \tag{2}$$

The anti-plane stresses σ_{zk} and the in-plane electric displacement $D_k (k = x, y)$ are related to w and ϕ by

$$\sigma_{zk} = c_{44}\frac{\partial w}{\partial k} + e_{15}\frac{\partial \phi}{\partial k}, \quad D_k = e_{15}\frac{\partial w}{\partial k} - \varepsilon_{11}\frac{\partial \phi}{\partial k} \ . \tag{3}$$

For simplicity, we introduce the generalized displacement vector $\mathbf{u} = [w, \phi]^{\mathrm{T}}$, the generalized stress vector $\mathbf{t}_k = [\sigma_{zk}, D_k]^{\mathrm{T}}$ and the material constant matrix

$$\mathbf{G} = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix} \ . \tag{4}$$

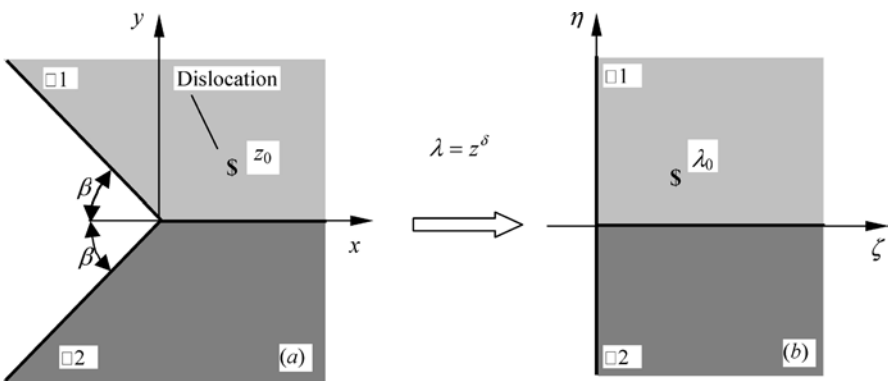

Fig. 1. **a** A screw dislocation in a piezoelectric bi-material with a notch; **b** The plane after the mapping $\lambda = z^\delta$

Consequently Eqs. (2) and (3) can be expressed as

$$\nabla^2 \mathbf{u} = 0 \ , \tag{5}$$

and

$$\mathbf{t}_k = \mathbf{G}\frac{\partial \mathbf{u}}{\partial k}, \quad k = x, y \ . \tag{6}$$

The general solution to Eq. (5) takes the form

$$\mathbf{u}(x, y) = \mathrm{Im}\left[f_w(z), f_\phi(z)\right]^{\mathrm{T}} = \mathrm{Im}\ \mathbf{f}(z) \ , \tag{7}$$

in which $\mathbf{f}(z)$ is an analytical function vector of the complex variable $z = x + iy$, where $i = \sqrt{-1}$, and Im stands for the imaginary part. Substituting Eq. (7) into Eq. (6) yields

$$\mathbf{t}_y + i\mathbf{t}_x = \mathbf{G}\mathbf{f}'(\mathbf{z}) \ , \tag{8}$$

where the prime denotes the derivative with respect to the argument.

The boundary conditions of the considered problem are

$$\oint_\ell \mathrm{d}\mathbf{u}^{(1)} = \mathbf{b}, \quad \oint_\ell \mathbf{t}_y^{(1)}\mathrm{d}x - \mathbf{t}_x^{(1)}\mathrm{d}y = 0 \ , \tag{9}$$

$$\mathbf{u}^{(1)} = \mathbf{u}^{(2)}, \quad \mathbf{t}_y^{(1)} = \mathbf{t}_y^{(2)}, \qquad 0 \le x \le \infty \ , \tag{10}$$

$$\mathbf{t}_x^{(1)} n_x^{(1)} + \mathbf{t}_y^{(1)} n_y^{(1)} = 0, \quad y = -x \tan\beta \ , \tag{11}$$

$$\mathbf{t}_x^{(2)} n_x^{(2)} + \mathbf{t}_y^{(2)} n_y^{(2)} = 0, \quad y = x \tan\beta \ . \tag{12}$$

In Eqs. (9)–(12), the superscripts (1) and (2) denote the quantities corresponding to material 1 and 2, respectively. The first equation in (9) presents the discontinuity conditions of the elastic displacement and electric potential, while the second states the equilibrium conditions of the stresses and charges around the dislocation core, where ℓ is any contour surrounding the dislocation core. From the statement above, it can be noted that all what we need to do is to determine the unknown functions $\mathbf{f}^{(j)}(z)$, $j = 1, 2$.

3
Solution to the problem

In order to obtain the solution of the problem stated in the previous section, we introduce the following mapping function:

$$\lambda = z^\delta, \quad \delta = \frac{\pi}{2(\pi - \beta)}, \quad \frac{1}{2} \le \delta \le 1 \ , \tag{13}$$

which transforms the regions occupied by material 1 and material 2 in the z-plane into the upper and lower quarter planes in the λ-plane, and the point z_0 in the z-plane into the point $\lambda_0 = z_0^\delta$ in the λ-plane, as shown in Fig. 1. The boundary conditions given in Eqs. (10)–(12) become

$$\mathbf{u}^{(1)} = \mathbf{u}^{(2)}, \quad \mathbf{t}_\eta^{(1)} = \mathbf{t}_\eta^{(2)}, \quad \eta = 0, \quad 0 \le \zeta \le \infty \ , \tag{14}$$

$$\mathbf{t}_\zeta^{(1)} = 0, \quad \zeta = 0 \ , \tag{15}$$

$$\mathbf{t}_\zeta^{(2)} = 0, \quad \zeta = 0 \ . \tag{16}$$

Assume that the unknown functions $\mathbf{f}^{(j)}$ have the following forms in the mapped plane

$$\mathbf{f}^{(j)}(\lambda) = \mathbf{f}_b^{(j)}(\lambda) + \mathbf{f}_m^{(j)}(\lambda) \ , \tag{17}$$

where $\mathbf{f}_b^{(j)}(\lambda)$ denote the functions corresponding to the solutions for an infinite bi-material subjected to a screw dislocation, while $\mathbf{f}_m^{(j)}(\lambda)$ are the modified functions associated with the perturbed fields due to the free boundary $\zeta = 0$. By making use of the standard analytical continuation procedure, the functions $\mathbf{f}_b^{(j)}(\lambda)$, which have satisfied the conditions (9) and (14), are, [4],

$$\mathbf{f}_b^{(1)}(\lambda) = \frac{1}{2\pi}\left[\mathbf{I}\ln(\lambda - \lambda_0) + \Pi_{12}\ln(\lambda - \bar{\lambda}_0)\right]\mathbf{b} \ , \tag{18}$$

$$\mathbf{f}_b^{(2)}(\lambda) = \frac{1}{2\pi}(\mathbf{I} - \Pi_{12})\mathbf{b}\ln(\lambda - \lambda_0) \ , \tag{19}$$

where the overbar denotes the complex conjugate, $\mathbf{I}$ is a 2×2 unit matrix. The matrix Π_{12} is

$$\Pi_{12} = \left(\mathbf{G}^{(2)} + \mathbf{G}^{(1)}\right)^{-1}\left(\mathbf{G}^{(2)} - \mathbf{G}^{(1)}\right) \ . \tag{20}$$

From the well-known image dislocation method, [14], follows that if two image screw dislocations with $-\mathbf{b}$ and $-\Pi_{12}\mathbf{b}$, respectively, are placed at the points $-\bar{\lambda}_0$ and $-\lambda_0$ in the infinite bi-material, then the condition (15) will be satisfied. Referring to (18), we obtain

$$\mathbf{f}_m^{(1)}(\lambda) = -\frac{1}{2\pi}\left[\mathbf{I}\ln(\lambda + \bar{\lambda}_0) + \Pi_{12}\ln(\lambda + \lambda_0)\right]\mathbf{b} \ . \tag{21}$$

Similarly, one image screw dislocation with $-(\mathbf{I} - \Pi_{12})\mathbf{b}$ should be placed at the point $-\bar{\lambda}_0$ to satisfy the condition (16). Thus we have

$$\mathbf{f}_m^{(2)}(\lambda) = -\frac{1}{2\pi}(\mathbf{I} - \Pi_{12})\mathbf{b}\ln(\lambda + \bar{\lambda}_0) \ . \tag{22}$$

Substituting Eqs. (18), (19), (21) and (22) into Eq. (17) and using the transform function in Eqs. (13), the functions $\mathbf{f}^{(j)}(z)$ in the z-plane can be written as

$$\mathbf{f}^{(1)}(z) = \frac{1}{2\pi}\left\{\mathbf{I}\left[\ln\left(z^\delta - z_0^\delta\right) - \ln\left(z^\delta + \bar{z}_0^\delta\right)\right] + \Pi_{12}\left[\ln\left(z^\delta - \bar{z}_0^\delta\right) - \ln\left(z^\delta + z_0^\delta\right)\right]\right\}\mathbf{b} \ , \tag{23}$$

$$\mathbf{f}^{(2)}(z) = \frac{1}{2\pi}(\mathbf{I} - \Pi_{12})\mathbf{b}\left[\ln\left(z^\delta - z_0^\delta\right) - \ln\left(z^\delta + \bar{z}_0^\delta\right)\right] \ . \tag{24}$$

With Eqs. (23) and (24) in hand, the electro-elastic fields produced by the screw dislocation $\mathbf{b}$ can be calculated in terms of Eqs. (7) and (8). For the stresses and electric displacements, one has

$$\mathbf{t}_y^{(1)} + i\mathbf{t}_x^{(1)} = \frac{1}{2\pi}\delta\mathbf{G}^{(1)}\left\{\mathbf{I}\left[\frac{1}{z^\delta - z_0^\delta} - \frac{1}{z^\delta + \bar{z}_0^\delta}\right] + \Pi_{12}\left[\frac{1}{z^\delta - \bar{z}_0^\delta} - \frac{1}{z^\delta + z_0^\delta}\right]\right\}z^{\delta-1}\mathbf{b} \ , \tag{25}$$

$$\mathbf{t}_y^{(2)} + i\mathbf{t}_x^{(2)} = \frac{1}{2\pi}\delta\mathbf{G}^{(2)}(\mathbf{I} - \Pi_{12})\mathbf{b}\left(\frac{1}{z^\delta - z_0^\delta} - \frac{1}{z^\delta + \bar{z}_0^\delta}\right)z^{\delta-1}\mathbf{b} \ . \tag{26}$$

4
Electro-elastic fields near a notch and intensity factors

In this section, our attention will focus on the electro-elastic fields near the tip of the notch where local electro-elastic field intensification usually occurs. It can be found from Eqs. (25) and (26) that the stresses and electric displacements exhibit the singularity $1/z^{1-\delta}$ in the vicinity of a notch. Analogous to a slit crack, the amplitudes of the singular fields can be characterized by the stress intensity factor (SIF) and the electric displacement intensity factor (EDIF). These IFs are defined by

$$\mathbf{K} = \begin{bmatrix} K_{III} \\ K_D \end{bmatrix} = \lim_{z\to 0}(2\pi z)^{1-\delta}\mathbf{t}_y^{(j)}(z) = -\frac{\delta}{(2\pi)^{\delta}}\mathbf{G}^{\mathrm{eff}}\mathbf{b}\left(\frac{1}{z_0^{\delta}} + \frac{1}{\bar{z}_0^{\delta}}\right), \quad j = 1, 2 \ , \tag{27}$$

where K_{III} is the classical SIF, while K_D is the EDIF. The matrix $\mathbf{G}^{\mathrm{eff}}$ is

$$\mathbf{G}^{\mathrm{eff}} = 2\mathbf{G}^{(2)}\left(\mathbf{G}^{(2)} + \mathbf{G}^{(1)}\right)^{-1}\mathbf{G}^{(1)} = 2\mathbf{G}^{(1)}\left(\mathbf{G}^{(2)} + \mathbf{G}^{(1)}\right)^{-1}\mathbf{G}^{(2)} \ , \tag{28}$$

which is called the effective material constant matrix. It can be seen from Eq. (27) that the IFs induced by a dislocation are expressed in terms of the effective material constants, as if the dislocation were in a homogeneous piezoelectric medium with material constant $\mathbf{G}_{\mathrm{eff}}$. In addition, it is worth noting that no matter which material the dislocation is in, it will produce the same IFs, for the two conjugate points z_0 and $\bar{z}_0$. The above-mentioned phenomena are identical to that of Ref. [15] for the purely elastic case. In the polar coordinate system, $z_0 = x_0 + iy_0 = r_0 e^{i_0}$. Thus, Eq. (27) can be rewritten as

$$\mathbf{K} = -\frac{2\delta}{(2\pi r_0)^{\delta}}\mathbf{G}^{\mathrm{eff}}\mathbf{b}\cos\delta\theta_0 \ . \tag{29}$$

5
Image force on a dislocation

The image force acting on a dislocation is a configuration force. It is used to judge equilibrium positions and motion of a dislocation. According to the generalized Peach-Koehler formula derived in [2] for piezoelectric materials, we have

$$F_x - iF_y = \mathbf{b}^{\mathrm{T}}\left[\mathbf{t}_{yp}^{(1)}(z_0) + i\mathbf{t}_{xp}^{(1)}(z_0)\right] \ , \tag{30}$$

where $\mathbf{t}_{kp}^{(1)}(z_0)(k = x, y)$ are the perturbed stresses and electric displacements at z_0 due to the interface and the notch. These perturbed stresses and electric displacements are obtained by subtracting $\mathbf{t}_{k\infty}^{(1)}(z_0)$ for a screw dislocation in infinite piezoelectric medium with $\mathbf{G}^{(1)}$ from the corresponding $\mathbf{t}_k^{(1)}(z_0)$.

Omitting some details, $\mathbf{t}_{kp}^{(1)}(z_0)$ is

$$\mathbf{t}_{yp}^{(1)}(z_0) + i\mathbf{t}_{xp}^{(1)}(z_0) = \frac{1}{2\pi}\mathbf{G}^{(1)}\left[\left(\frac{\delta-1}{2z_0} - \frac{\delta z_0^{\delta-1}}{z_0^{\delta} + \bar{z}_0^{\delta}}\right)\mathbf{I} + \left(\frac{1}{z_0^{\delta} - \bar{z}_0^{\delta}} - \frac{1}{z_0^{\delta} + z_0^{\delta}}\right)\delta z_0^{\delta-1}\Pi_{12}\right]\mathbf{b} \ . \tag{31}$$

Insertion of Eq. (31) into Eq. (30) yields

$$F_x - iF_y = \frac{1}{2\pi}\mathbf{b}^{\mathrm{T}}\mathbf{G}^{(1)}\left[\left(\frac{\delta-1}{2z_0} - \frac{\delta z_0^{\delta-1}}{z_0^{\delta} + \bar{z}_0^{\delta}}\right)\mathbf{I} + \left(\frac{1}{z_0^{\delta} - \bar{z}_0^{\delta}} - \frac{1}{z_0^{\delta} + z_0^{\delta}}\right)\delta z_0^{\delta-1}\Pi_{12}\right]\mathbf{b} \ . \tag{32}$$

The first term of Eq. (32) is due to the notch, and the second is induced by the material mismatch. When the polar coordinate system is used, the two components of the image force become

$$\begin{aligned} F_x &= -\frac{1}{4\pi r_0}\mathbf{b}^{\mathrm{T}}\mathbf{G}^{(1)}\left\{\left[(1-\delta)\cos\theta_0 + \frac{\cos(1-\delta)\theta_0}{\cos\delta\theta_0}\delta\right]\mathbf{I} + \delta\left[\cos\theta_0 + \frac{\sin(1-\delta)\theta_0}{\sin\delta\theta_0}\right]\Pi_{12}\right\}\mathbf{b}, \\ F_y &= -\frac{1}{4\pi r_0}\mathbf{b}^{\mathrm{T}}\mathbf{G}^{(1)}\left\{\left[(1-\delta)\sin\theta_0 + \frac{\sin(1-\delta)\theta_0}{\cos\delta\theta_0}\delta\right]\mathbf{I} + \delta\left[\sin\theta_0 - \frac{\cos(1-\delta)\theta_0}{\sin\delta\theta_0}\right]\Pi_{12}\right\}\mathbf{b} \ . \end{aligned} \tag{33}$$

By virtue of Eqs. (33), the image force components along the radial and angular directions are obtained as follows:

$$F_r = -\frac{1}{4\pi r_0}\mathbf{b}^{\mathrm{T}}\mathbf{G}^{(1)}\mathbf{b},$$
$$F_\theta = \frac{\delta}{4\pi r_0}\mathbf{b}^{\mathrm{T}}\mathbf{G}^{(1)}(\mathbf{I}\tan\delta\theta_0 + \Pi_{12}\cot\delta\theta_0)\mathbf{b} \quad . \tag{34}$$

The first equality above shows that F_r is not dependent on the angular position θ_0 of the dislocation, neither the notch angle and the properties of material 2.

6
Example and discussion

In order to have a better understanding of the influence of piezoelectricity on the dislocation-notch interaction, we assume that medium 2 is purely elastic, i.e. $e_{15}^{(2)} = \varepsilon_{11}^{(2)} = 0$, and medium 1 is piezoelectric. In addition, the electric potential jump b_ϕ is taken as zero. For this case, the IFs and the image force become

$$K_{\mathrm{III}} = -\frac{4\delta}{(2\pi r_0)^\delta}\frac{1+m_e^{(1)}}{1+\alpha+m_e^{(1)}}c_{44}^{(2)}b_3\cos\delta\theta_0, \quad K_D = 0 \quad , \tag{35}$$

$$F_r = -\frac{1}{4\pi r_0}c_{44}^{(1)}b_3^2,$$
$$F_\theta = \frac{\delta}{4\pi r_0}c_{44}^{(1)}b_3^2\left\{\tan\delta\theta_0 + \frac{\left[1+2m_e^{(1)}\right]c_{44}^{(2)} - \left[1+m_e^{(1)}\right]c_{44}^{(1)}}{\left[1+2m_e^{(1)}\right]c_{44}^{(2)} + \left[1+m_e^{(1)}\right]c_{44}^{(1)}}\cot\delta\theta_0\right\} \quad , \tag{36}$$

where

$$\alpha = \frac{c_{44}^{(2)}}{c_{44}^{(1)}}, \quad m_e^{(1)} = \frac{\left(e_{15}^{(1)}\right)^2}{\left(c_{44}^{(1)}\varepsilon_{11}^{(1)}\right)} \quad , \tag{37}$$

being the electrical–mechanical coupling factor of the piezoelectric medium. It can be clearly seen from the first of Eqs. (35) that the SIF arising from the screw dislocation is always negative. This means that a screw dislocation is able to retard crack initiation from the notch if the SIF due to the applied electro-mechanical loads is positive. Moreover, one can also conclude that the existence of piezoelectric coupling effect increases the magnitude of the SIF generated by a dislocation because

$$\frac{\left[1+m_e^{(1)}\right]}{\left[1+m_e^{(1)}+\alpha\right]} - \frac{1}{(1+\alpha)} > 0 \quad .$$

Next, the angular component F_θ of the image force is calculated by making use of Eq. (36). Here, PZT-5 piezoceramic is chosen as the material 1. Its electro-elastic constants are, [2]:

$$c_{44}^{(1)} = 3.53\times 10^{10}\,\mathrm{N/m^2}, \quad e_{15}^{(1)} = 17\,\mathrm{C/m^2}, \quad \varepsilon_{11}^{(1)} = 151\times 10^{-10}\,\mathrm{C/Vm} \quad ,$$

where N is the force in N, C is the charge in C, V is the electric potential in V and m is the length in m.

Figure 2 shows the variations of F_θ with the angular position θ_0 of the dislocation for differential elastic modulus ratios $c_{44}^{(2)}/c_{44}^{(1)}$ when $\beta = 30°$. It can be seen from Fig. 2 that the influence of $c_{44}^{(2)}/c_{44}^{(1)}$ on F_θ decreases with increasing θ_0. In particular, the effect of $c_{44}^{(2)}/c_{44}^{(1)}$ on F_θ gradually vanishes and the magnitude of F_θ approaches positive infinite as the dislocation approaches the surface of the notch, which arises from the notch-screening effect. The influ-

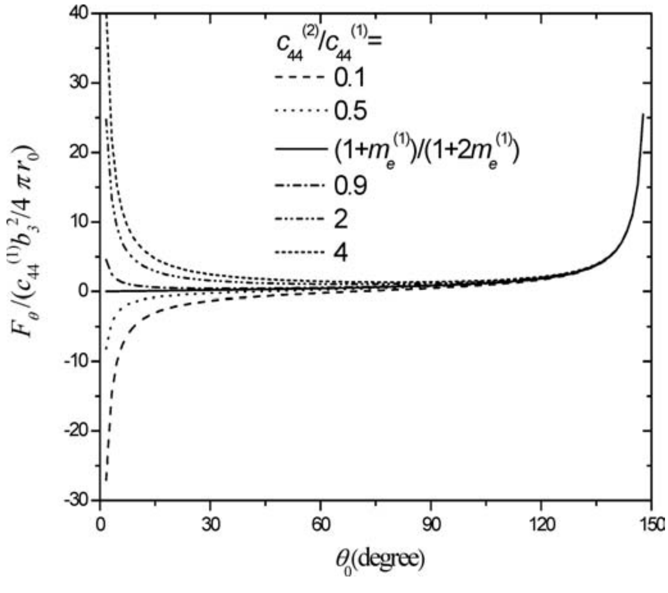

Fig. 2. Variation of the image force F_θ versus the angular position θ_0 of the dislocation for different elastic modulus ratios $c_{44}^{(2)}/c_{44}^{(1)}$ when $\beta = 30°$

ence of $c_{44}^{(2)}/c_{44}^{(1)}$ on F_θ is significant for the case of the dislocation being near the interface. When $c_{44}^{(2)}/c_{44}^{(1)} < [1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$, F_θ is negative in the vicinity of the interface. This means that the dislocation has a movement tendency toward the interface. When $c_{44}^{(2)}/c_{44}^{(1)} > [1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$, F_θ acting on the dislocation near the interface is positive. This implies the interface will repel the dislocation. In addition, when $c_{44}^{(2)}/c_{44}^{(1)} > [1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$, F_θ is larger than zero for arbitrary θ_0. But this is no longer true when $c_{44}^{(2)}/c_{44}^{(1)} < [1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$. In other words, $c_{44}^{(2)}/c_{44}^{(1)}$ has a critical value at $[1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$. Above this value, the F_θ curve is concave, and below exhibits an inflection point. If only purely elastic case is considered, namely $m_e^{(1)} = 0$, then the aforementioned critical value becomes unity. Figure 3 further shows the influence of piezoelectricity on the image force on the dislocation.

From the above discussion, the following points can be drawn that: (1) the dislocation has no equilibrium position; (2) the surface of the notch is always attractive to the dislocation; (3) the dislocation near the interface is attracted or repelled by the interface depending on that $c_{44}^{(2)}/c_{44}^{(1)}$ is smaller or larger than $[1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$; (4) when $c_{44}^{(2)}/c_{44}^{(1)} < [1 + m_e^{(1)}]/[1 + 2m_e^{(1)}]$, the tip of the notch may be a place where dislocations prefer to pile up.

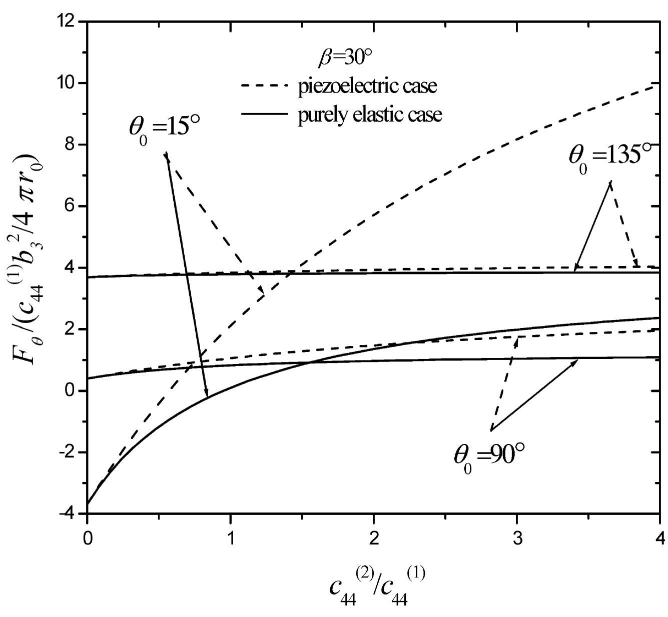

Fig. 3. Variation of the image force F_θ with respect to $c_{44}^{(2)}/c_{44}^{(1)}$ for $\theta_0 = 15°, 90°, 135°$ when $\beta = 30°$

7
Concluding remarks

A screw dislocation interacting with a notch in a piezoelectric bi-material has been studied in this paper. The main findings are that the presence of piezoelectric coupling effect amplifies the magnitude of the SIF, and *enable the material with a lower elastic modulus to repel screw dislocations located in the material with a higher elastic modulus*. The latter conclusion is basically different from that obtained in [16] for purely elastic case. We find that the radial image force is independent of the angular position θ_0 of the dislocation, the notch angle and the properties of another material. This is also one new result.

Moreover, the results of several particular cases reported in the literature can be obtained straightforwardly from our results. For example, when β vanishes, Eqs. (29) and (33) become

$$\mathbf{K} = -\frac{1}{\sqrt{2\pi r_0}} \mathbf{G}^{\mathrm{eff}} \mathbf{b} \cos\frac{\theta_0}{2} \ , \tag{38}$$

$$\begin{aligned} F_x &= -\frac{1}{4\pi r_0} \mathbf{b}^{\mathrm{T}} \mathbf{G}^{\mathrm{eff}} \mathbf{b} \cos^2\frac{\theta_0}{2}, \\ F_y &= -\frac{1}{4\pi r_0} \left[\frac{1}{2} \mathbf{b}^{\mathrm{T}} \mathbf{G}^{\mathrm{eff}} \mathbf{b} \left(\sin\theta_0 + \tan\frac{\theta_0}{2} \right) - \mathbf{b}^{\mathrm{T}} \mathbf{G}^{(1)} \boldsymbol{\Pi}_{12} \mathbf{b} \sec\theta_0 \right] \ , \end{aligned} \tag{39}$$

The above are in agreement with the results given in [12] for the interaction of a screw with a semi-infinite crack in two dissimilar piezoelectric media. However, the derivation procedure in [12] is relatively complicated. When material 1 and material 2 have the same electro-elastic properties, i.e. $\mathbf{G}^{(2)} = \mathbf{G}^{(1)} = \mathbf{G}$, Eqs. (29) and (33) reduce to ones given in [8] and [10] for a single-phase piezoelectric medium containing a screw dislocation and a wedge crack (notch).

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