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# Analytical thermo-elastodynamic solutions for a nonhomogeneous transversely isotropic hollow sphere

**H. J. Ding, H. M. Wang, W. Q. Chen**



**Summary** The spherically symmetric dynamic thermoelastic problem for a special nonhomogeneous transversely isotropic elastic hollow sphere is formulated by introduction of a dependent variable and separation of variables technique. The derived solution can be degenerated into that for a homogeneous transversely isotropic hollow sphere, a nonhomogeneous isotropic hollow sphere or a solid sphere. The present method, allow to avoid integral transforms, is suited for a hollow sphere of arbitrary thickness subjected to arbitrary spherical symmetric thermal and mechanical loads, and is convenient in dealing with different boundary conditions of dynamic thermoelasticity . The numerical calculation involved is easy to be performed and its results are also presented.

**Keywords** Separation of variables, Dynamics, Transverse isotropy, Thermoelasticity, Hollow sphere

## 1 Introduction

Thermally excited mechanical response of structures is of increasing interest in engineering science, and much work has been done on dynamic thermoelastic problems. An exact, closed-form solution has been obtained for the dynamic problem of a sudden temperature change at the surface of a spherical cavity in an infinite solid, [1]. Under internal thermal shock, the thermal stress-wave propagation in an arbitrary thick-walled spherical shell was discussed in [2]. Dynamic thermal stress response in a hollow sphere, which is subjected to arbitrary spherically symmetric temperature fields, has also been investigated in [3]. The technique is based on the integral theorem of a hyperbolic initial value problem, together with the construction of image temperature fields in the regions outside the actual body. The dynamic thermal stress responses in a uniformly heated, homogeneous isotropic hollow sphere and a solid sphere as well as in a homogeneous transversely isotropic hollow sphere were solved by the ray theory, [4–6]. For a transversely isotropic solid sphere, the thermal stress concentration effects were discussed in [7]. The above analyses are restricted to homogeneous materials. As for nonhomogeneous materials, little work has been done to the author's knowledge. The transient thermal stresses in a nonhomogeneous spherically isotropic elastic medium with a spherical cavity and an exponential distribution of heat sources, were obtained in [8].

Dynamic thermoelastic problems are usually solved using the Laplace transform technique, [1, 2, 4–8]. However, this method encounters the difficulty of inverse transforms in some special cases. The ray theory is a good tool to complete the Laplace inversion. However, for a very thin spherical shell it needs a large number of rays and, hence, becomes impractical, [9]. The present method allows to avoid integral transforms, although it is restricted to a special

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type of inhomogeneity. A new dependent variable is introduced to rewrite the governing equation, the boundary conditions as well as the initial conditions. Then the thermal and mechanical loads are treated as the inhomogeneous part in the boundary conditions, and a special function is introduced to transform the inhomogeneous boundary conditions to homogeneous ones. By using the orthogonal expansion technique, the equation is derived with respect to the time variable from which the solution for the dynamic thermoelastic problem of a nonhomogeneous transversely isotropic hollow sphere is easily obtained.

## 2
## Mathematical formulations of the problem



Transient response of a thermally shocked transversely isotropic solid sphere has been considered in [6]. If a spherical coordinate system $(r, \theta, \varphi)$ with the origin at the center of the sphere is used for the spherically symmetric problem, only the radial displacement $u_r = u_r(r, t)$ is nonzero. The strain-displacement relations are thus

$$\gamma_{rr} = \frac{\partial u_r}{\partial r}, \quad \gamma_{\theta\theta} = \gamma_{\varphi\varphi} = \frac{u_r}{r}, \quad \gamma_{r\theta} = \gamma_{\theta\varphi} = \gamma_{\varphi r} = 0 \ , \tag{1}$$

where $\gamma_{ij}(i, j = r, \theta, \varphi)$ are the strain components. The constitutive relations are, [6]

$$\begin{aligned}\gamma_{rr} &= \frac{1}{E_r}\sigma_{rr} - 2\frac{\upsilon_{r\theta}}{E_r}\sigma_{\theta\theta} + \alpha_r T(r, t), \\ \gamma_{\theta\theta} &= -\frac{\upsilon_{r\theta}}{E_r}\sigma_{rr} + \frac{1 - \upsilon_{\theta\varphi}}{E_\theta}\sigma_{\theta\theta} + \alpha_\theta T(r, t) \ ,\end{aligned} \tag{2}$$

where $\sigma_{rr}$ and $\sigma_{\theta\theta}$ are stress components and $E_r$ and $E_\theta$ are the Young's moduli of $r$ and $\theta$ directions, respectively; $\upsilon_{r\theta}$ is the Poisson's ratio which characterizes the ratio of contraction in $r$ direction due to extension in $\theta$ direction and $\upsilon_{\theta\varphi}$ is Poisson's ratio which characterizes the ratio of contraction in $\theta$ direction due to extension in $\varphi$ direction, respectively; $\alpha_r$ and $\alpha_\theta$ are the coefficients of linear thermal expansion in $r$ and $\theta$ directions, respectively; $T(r, t)$ is the temperature change. Equation (2) can also be rewritten as

$$\begin{aligned}\sigma_{rr} &= c_{11}\gamma_{rr} + 2c_{12}\gamma_{\theta\theta} - \beta_1 T(r, t), \\ \sigma_{\theta\theta} &= c_{12}\gamma_{rr} + (c_{22} + c_{23})\gamma_{\theta\theta} - \beta_2 T(r, t) \ ,\end{aligned} \tag{3}$$

where $c_{ij}(i, j = 1, 2, 3)$ are elastic constants and $\beta_i(i = 1, 2)$ are stress-temperature constants, which can be expressed in terms of $c_{ij}(i, j = 1, 2, 3)$ and $\alpha_i(i = r, \theta)$

$$\beta_1 = c_{11}\alpha_r + 2c_{12}\alpha_\theta, \quad \beta_2 = c_{12}\alpha_r + (c_{22} + c_{23})\alpha_\theta \ . \tag{4}$$

The equation of motion is

$$\frac{\partial \sigma_{rr}}{\partial r} + 2\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\frac{\partial^2 u_r}{\partial t^2} \ , \tag{5}$$

where $\rho$ is the mass density. In this study, we assume that the nonhomogenity of the material is characterized by the special laws

$$c_{ij} = \left(\frac{r}{b}\right)^{2N} A_{ij}, \quad \rho = \left(\frac{r}{b}\right)^{2N} \rho_0 \ . \tag{6}$$

Here, $b$, $A_{ij}$ and $\rho_0$ are known constants, and $N$ is a rational number. The coefficients of linear thermal expansion $\alpha_i$ are constant. From Eqs. (1), (3) and (6), we obtain

$$\begin{aligned}\sigma_{rr} &= \left(\frac{r}{b}\right)^{2N}\left[A_{11}\frac{\partial u_r}{\partial r} + 2A_{12}\frac{u_r}{r} - B_1 T(r, t)\right], \\ \sigma_{\theta\theta} &= \left(\frac{r}{b}\right)^{2N}\left[A_{12}\frac{\partial u_r}{\partial r} + (A_{22} + A_{23})\frac{u_r}{r} - B_2 T(r, t)\right] \ ,\end{aligned} \tag{7}$$

where

$$B_1 = A_{11}\alpha_r + 2A_{12}\alpha_\theta, \quad B_2 = A_{12}\alpha_r + (A_{22} + A_{23})\alpha_\theta \quad . \tag{8}$$

Substitution Eqs. (7) into Eq. (5), gives the following governing equation:

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{2(N+1)}{r}\frac{\partial u_r}{\partial r} - \frac{\mu_1^2}{r^2}u_r = \frac{1}{c_L^2}\frac{\partial^2 u_r}{\partial t^2} + g(r,t) \quad , \tag{9}$$

where



$$\begin{aligned} &\mu_1^2 = 2\frac{A_{22} + A_{23} - (2N+1)A_{12}}{A_{11}}, \quad c_L = \sqrt{\frac{A_{11}}{\rho_0}}, \\ &g(r,t) = 2\frac{(N+1)B_1 - B_2}{A_{11}}\frac{T(r,t)}{r} + \frac{B_1}{A_{11}}\frac{\partial T(r,t)}{\partial r} \quad . \end{aligned} \tag{10}$$

The boundary conditions are

$$\begin{aligned} r = a: &\quad s^{2N}\left[A_{11}\frac{\partial u_r}{\partial r} + 2A_{12}\frac{u_r}{r} - B_1 T(a,t)\right] = p_a(t), \\ r = b: &\quad A_{11}\frac{\partial u_r}{\partial r} + 2A_{12}\frac{u_r}{r} - B_1 T(b,t) = p_b(t) \quad , \end{aligned} \tag{11}$$

where $a$ and $b$ are the inner and outer radii of the spherical shell, respectively, $s = a/b$ is the inner-to-outer radius ratio and $p_a(t)$ and $p_b(t)$ are the prescribed pressures on the internal and external surfaces, respectively. The initial conditions ($t = 0$) are

$$u_r(r,0) = u_0(r), \quad \dot{u}_r(r,0) = v_0(r) \quad , \tag{12}$$

where a dot over the quantity denotes its partial derivative with respect to $t$, and $u_0(r)$ and $v_0(r)$ are known functions.

## 3
## Analytical solution

Firstly, a new dependent variable $w(r,t)$ is introduced as

$$u_r(r,t) = r^{-\left(N+\frac{1}{2}\right)}w(r,t) \quad . \tag{13}$$

Then Eqs. (9), (11) and (12) become

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{\mu^2}{r^2}w = \frac{1}{c_L^2}\frac{\partial^2 w}{\partial t^2} + g_1(r,t) \quad , \tag{14}$$

$$\begin{aligned} r = a: &\quad \frac{\partial w}{\partial r} + h\frac{w}{r} = p_1(t), \\ r = b: &\quad \frac{\partial w}{\partial r} + h\frac{w}{r} = p_2(t) \quad , \end{aligned} \tag{15}$$

$$w(r,0) = u_1(r), \quad \dot{w}(r,0) = v_1(r) \quad , \tag{16}$$

where

$$\begin{aligned} &\mu = \sqrt{\mu_1^2 + \left(N + \frac{1}{2}\right)^2}, \quad g_1(r,t) = r^{N+\frac{1}{2}}g(r,t), \quad h = \frac{2A_{12}}{A_{11}} - (N + \frac{1}{2}), \\ &p_1(t) = a^{N+\frac{1}{2}}\frac{\left[B_1 T(a,t) + \frac{p_1(t)}{s^{2N}}\right]}{A_{11}}, \quad p_2(t) = b^{N+\frac{1}{2}}\frac{[B_1 T(b,t) + p_2(t)]}{A_{11}}, \\ &u_1(r) = r^{N+\frac{1}{2}}u_0(r), \quad v_1(r) = r^{N+\frac{1}{2}}v_0(r) \quad . \end{aligned} \tag{17}$$

We transform now the inhomogeneous boundary conditions into the homogeneous ones by taking

$$w(r,t) = w_1(r,t) + w_2(r,t) \ . \tag{18}$$

Here, $w_2(r,t)$ satisfies the inhomogeneous boundary conditions, which can be taken as

$$w_2(r,t) = d_1(r-a)^m p_2(t) + d_2(r-b)^m p_1(t) \ , \tag{19}$$

where



$$d_1 = \frac{b^{1-m}}{m(1-s)^{m-1} + h(1-s)^m}, \quad d_2 = \frac{b^{1-m}}{m(s-1)^{m-1} + h(s-1)^m/s} \ , \tag{20}$$

in which $m \geq 2$ is an arbitrary integer that satisfies

$$[m(1-s)^{m-1} + h(1-s)^m][m(s-1)^{m-1} + \frac{h(s-1)^m}{s} \neq 0] \ , \tag{21}$$

because of $s \neq 1$.

Substituting Eq. (18) into Eqs. (14)–(16), gives

$$\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r} - \frac{\mu^2}{r^2} w_1 = \frac{1}{c_L^2}\frac{\partial^2 w_1}{\partial t^2} + g_2(r,t) \ , \tag{22}$$

$$\begin{aligned} r = a: \quad & \frac{\partial w_1}{\partial r} + h\frac{w_1}{r} = 0, \\ r = b: \quad & \frac{\partial w_1}{\partial r} + h\frac{w_1}{r} = 0 \ , \end{aligned} \tag{23}$$

$$w_1(r,0) = u_2(r), \quad \dot{w}_1(r,0) = v_2(r) \ , \tag{24}$$

where

$$\begin{aligned} & g_2(r,t) = g_1(r,t) + \frac{1}{c_L^2}\frac{\partial w_2(r,t)}{\partial t^2} + \frac{\mu^2}{r^2} w_2(r,t) - \frac{1}{r}\frac{\partial w_2(r,t)}{\partial r} - \frac{\partial^2 w_2(r,t)}{\partial r^2}, \\ & u_2(r) = u_1(r) - w_2(r,0), \quad v_2(r) = v_1(r) - \dot{w}_2(r,0) \ . \end{aligned} \tag{25}$$

By using the standard separation of variables technique, the solution of Eq. (22) can be assumed in the following form

$$w_1(r,t) = \sum_i R_i(r) F_i(t) \ . \tag{26}$$

Here, $F_i(t)$ are unknown functions of $t$, and $R_i(r)$ is given by

$$R_i(r) = \mathrm{J}_\mu(k_i r)\mathrm{Y}(\mu, k_i, a) - \mathrm{Y}_\mu(k_i r)\mathrm{J}(\mu, k_i, a) \ , \tag{27}$$

in which $\mathrm{J}_\mu(k_i r)$ and $\mathrm{Y}_\mu(k_i r)$ are Bessel functions of the first and second kind and of the order $\mu$, respectively. Quantities $k_i$, arranged in an ascending order, are a series of positive roots of the following eigenequation:

$$\mathrm{J}(\mu, k_i, a)\mathrm{Y}(\mu, k_i, b) - \mathrm{J}(\mu, k_i, b)\mathrm{Y}(\mu, k_i, a) = 0 \ , \tag{28}$$

where

$$\mathrm{J}(\mu, k_i, r) = \frac{\mathrm{d}\mathrm{J}_\mu(k_i r)}{\mathrm{d}r} + h\frac{\mathrm{J}_\mu(k_i r)}{r}, \quad \mathrm{Y}(\mu, k_i, r) = \frac{\mathrm{d}\mathrm{Y}_\mu(k_i r)}{\mathrm{d}r} + h\frac{\mathrm{Y}_\mu(k_i r)}{r} \quad . \tag{29}$$

It can be shown that $w_1(r,t)$ given in Eq. (26) satisfies the homogeneous boundary conditions as shown in Eqs. (23).

Substitution Eq. (26) into Eq. (22) gives

$$-c_L^2 \sum_i k_i^2 F_i(t) R_i(r) = \sum_i R_i(r) \frac{\mathrm{d}^2 F_i(t)}{\mathrm{d}t^2} + c_L^2 g_2(r,t) \quad . \tag{30}$$



By virtue of the orthogonal property of Bessel functions, it is easy to verify the following equation

$$\int_a^b r R_i(r) R_j(r) \mathrm{d}r = N_i \delta_{ij} \quad , \tag{31}$$

where $\delta_{ij}$ is the Kronecker delta and

$$N_i = \frac{1}{2k_i^2}\left\{ b^2 \left[\frac{\mathrm{d}R_i(b)}{\mathrm{d}r}\right]^2 - a^2 \left[\frac{\mathrm{d}R_i(a)}{\mathrm{d}r}\right]^2 + k_i^2\left[b^2 R_i^2(b) - a^2 R_i^2(a)\right] - \mu^2\left[R_i^2(b) - R_i^2(a)\right] \right\} \quad . \tag{32}$$

In the above equation, we denote

$$\frac{\mathrm{d}R_i(a)}{\mathrm{d}r} = \left.\frac{\mathrm{d}R_i(r)}{\mathrm{d}r}\right|_{r=a}, \quad \frac{\mathrm{d}R_i(b)}{\mathrm{d}r} = \left.\frac{\mathrm{d}R_i(r)}{\mathrm{d}r}\right|_{r=b} \quad .$$

Utilizing Eq. (31), we can derive the following equation from Eq. (30):

$$\frac{\mathrm{d}^2 F_i(t)}{\mathrm{d}t^2} + \omega_i^2 F_i(t) = q_i(t) \quad , \tag{33}$$

where

$$\omega_i = k_i c_L, \quad q_i(t) = -\frac{c_L^2}{N_i}\int_a^b r g_2(r,t) R_i(r) \mathrm{d}r \quad . \tag{34}$$

The solution of Eq. (33) is

$$F_i(t) = G_{1i}\cos\omega_i t + G_{2i}\sin\omega_i t + \frac{1}{\omega_i}\int_0^t q_i(\tau)\sin\omega_i(t-\tau)\mathrm{d}\tau \quad , \tag{35}$$

where

$$G_{1i} = \frac{1}{N_i}\int_a^b r u_2(r) R_i(r)\mathrm{d}r, \quad G_{2i} = \frac{1}{N_i\omega_i}\int_a^b r v_2(r) R_i(r)\mathrm{d}r \quad . \tag{36}$$

Finally, the radial displacement is then obtained as follows:

$$u_r(r,t) = r^{-\left(N+\frac{1}{2}\right)}\left[w_1(r,t) + w_2(r,t)\right] \quad . \tag{37}$$

## 4
## Some particular cases

### 4.1
### Isotropic material

If

$$A_{11} = A_{22} = \frac{E(1-\upsilon)}{k}, \quad A_{12} = \frac{E\upsilon}{k}, \quad k = (1+\upsilon)(1-2\upsilon), \quad \alpha_\theta = \alpha_r = \alpha \ , \tag{38}$$



where $E$ and $\upsilon$ are Young's modulus and Poisson's ratio, respectively, the solution obtained above degenerates to that of a nonhomogeneous isotropic hollow sphere for the corresponding dynamic thermoelastic problem.

### 4.2
### Homogeneous material

If $N = 0$, the solution degenerates to that of a homogeneous transversely isotropic hollow sphere for the corresponding dynamic thermoelastic problem. Further, if the material constants satisfy Eqs. (38), the solution becomes that of a homogeneous isotropic hollow sphere. If a homogeneous isotropic solid sphere ($a = 0$) is considered, we can set

$$p_a(t) = 0, \quad p_1(t) = 0, \quad \mathrm{J}(\mu, k_i, a) = 0, \quad \mathrm{Y}(\mu, k_i, a) = 1$$

in the relevant formulations to obtain the corresponding solution.

### 4.3
### Elastodynamic solution

If the temperature change $T(r, t) = 0$, the solution becomes that of a nonhomogeneous transversely isotropic hollow sphere for the elastodynamic problem.

### 4.4
### Fixed boundary conditions

For a hollow sphere fixed at the inner surface, instead of Eq. (11), we have

$$r = a: \quad u_r(a, t) = 0$$

$$r = b: \quad A_{11}\frac{\partial u_r}{\partial r} + 2A_{12}\frac{u_r}{r} - B_1 T(b, t) = p_b(t) \ .$$

Consequently, we have the following equations, instead of Eqs. (15) and (23):

$$\begin{aligned} r = a: &\quad w(a, t) = 0, & r = a: &\quad w_1(a, t) = 0, \\ r = b: &\quad \frac{\partial w}{\partial r} + h\frac{w}{r} = p_2(t), & r = b: &\quad \frac{\partial w_1}{\partial r} + h\frac{w_1}{r} = 0 \ . \end{aligned}$$

Then, we can set

$$p_a(t) = 0, \quad p_1(t) = 0, \quad \mathrm{J}(\mu, k_i, a) = \mathrm{J}_\mu(k_i a), \quad \mathrm{Y}(\mu, k_i, a) = \mathrm{Y}_\mu(k_i a)$$

in the relevant formulations to obtain the solution of the dynamic thermoelastic problem for a nonhomogeneous transversely isotropic hollow sphere with fixed internal surface.

## 5
## Numerical results and discussions

In order to examine the present solution, we first consider the dynamic response in a homogeneous isotropic hollow sphere due to an instantaneous constant internal radial pressure and the dynamic thermal response in a uniformly heated, homogeneous isotropic hollow sphere. The results agree well with those obtained in [9] and [4], and hence the validation of the solution is clarified.

In the following, we will discuss the dynamic thermal response in a nonhomogeneous transversely isotropic hollow sphere. Suppose the spherical shell is subjected to the following load:

$$p_a(t) = 0, \quad p_b(t) = 0, \quad T(r,t) = T_0 H(t) \ , \tag{39}$$

where $T_0$ is a prescribed temperature change and $H()$ denotes the Heaviside step function. The material constants are listed in the following:

$$\frac{A_{22}}{A_{11}} = 3.0, \quad \frac{A_{12}}{A_{11}} = \frac{A_{23}}{A_{11}} = 1.2857, \quad \frac{\alpha_\theta}{\alpha_r} = 2.0 \ . \tag{40}$$



Further, we will take $\alpha_r T_0 = 1.0 \times 10^{-4}$, $u_0(r) = 0$, $v_0(r) = 0$. In the presentation of the results, the following nondimensional quantities are employed:

$$t^* = \frac{c_L}{b-a} t, \quad \xi = \frac{r-a}{b-a}, \quad \sigma_i^* = \frac{\sigma_{ii}}{\sigma_0}, \quad (i = r, \theta) \ , \tag{41}$$

where

$$\sigma_0 = \alpha_r (A_{11} + 2A_{12}) T_0 \tag{42}$$

Figures 1 and 2 show the dynamic response of the hoop stress $\sigma_\theta^*$ at the inner and outer surfaces in a uniformly heated, nonhomogeneous transversely isotropic hollow sphere ($s = 0.2$) for different values of $N$. We can see that the peak values of the dynamic stress at the internal surface decrease with the increase of $N$, while the peak values of the dynamic stress at the outer surface vary slightly with $N$. Figure 3 depicts the dynamic response of the radial stress $\sigma_r^*$ at different locations $\xi$ in the $r$ direction for $N = 0.5$ and $s = 0.5$. We find that the radial thermal stress vary significantly with the position. The distributions of the hoop stress $\sigma_\theta^*$ at different times $t^*$ for $N = 1.0$ and $s = 0.5$ are also depicted in Fig. 4.

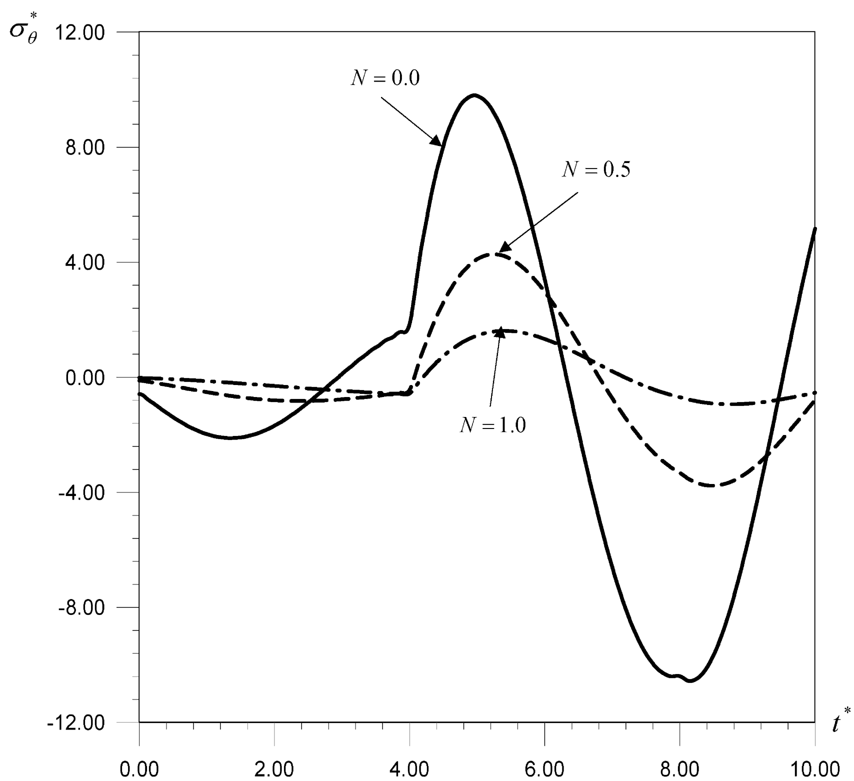


**Fig. 1.** Histories of dynamic stresses $\sigma_\theta^*$ at the inner surface for different $N$ ($s = 0.2$)

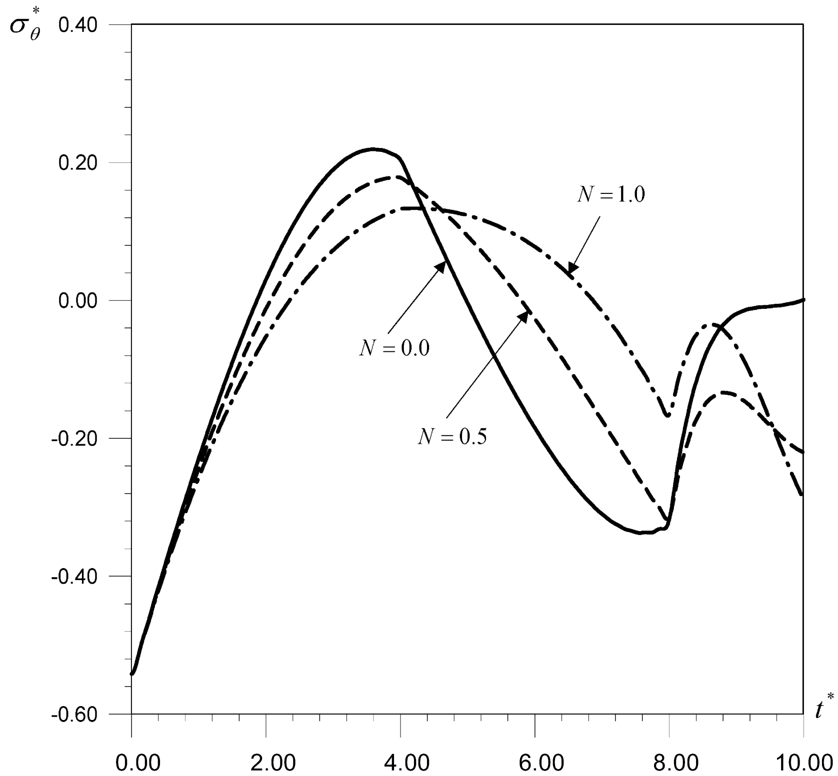


**Fig. 2.** Histories of dynamic stresses $\sigma_\theta^*$ at the outer surface for different $N$ ($s = 0.2$)

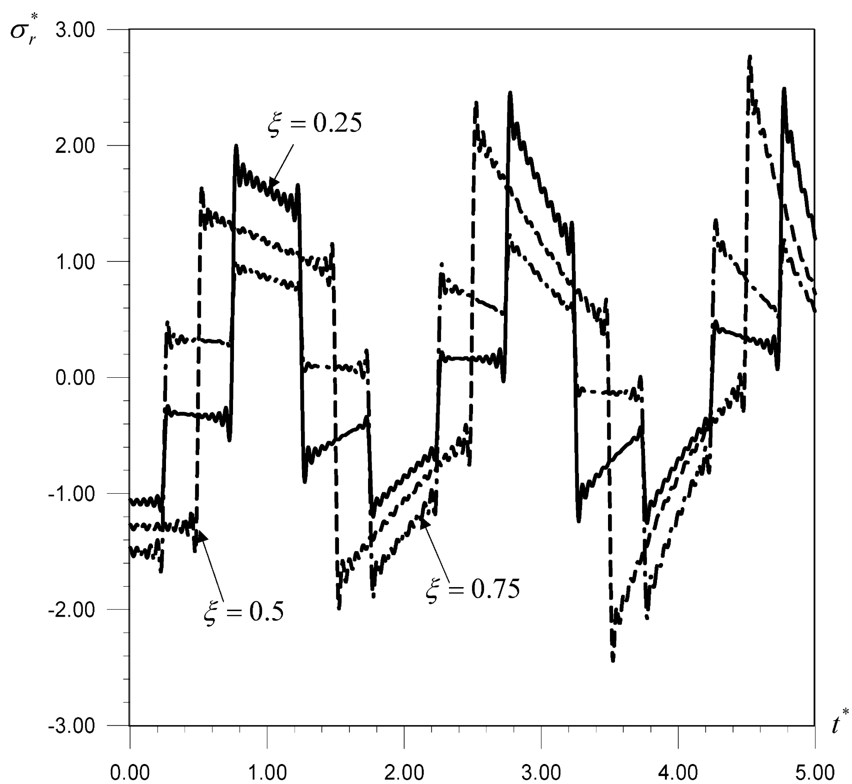


**Fig. 3.** Histories of dynamic stresses $\sigma_r^*$ for different locations $\xi$ ($N = 0.5$ and $s = 0.5$)

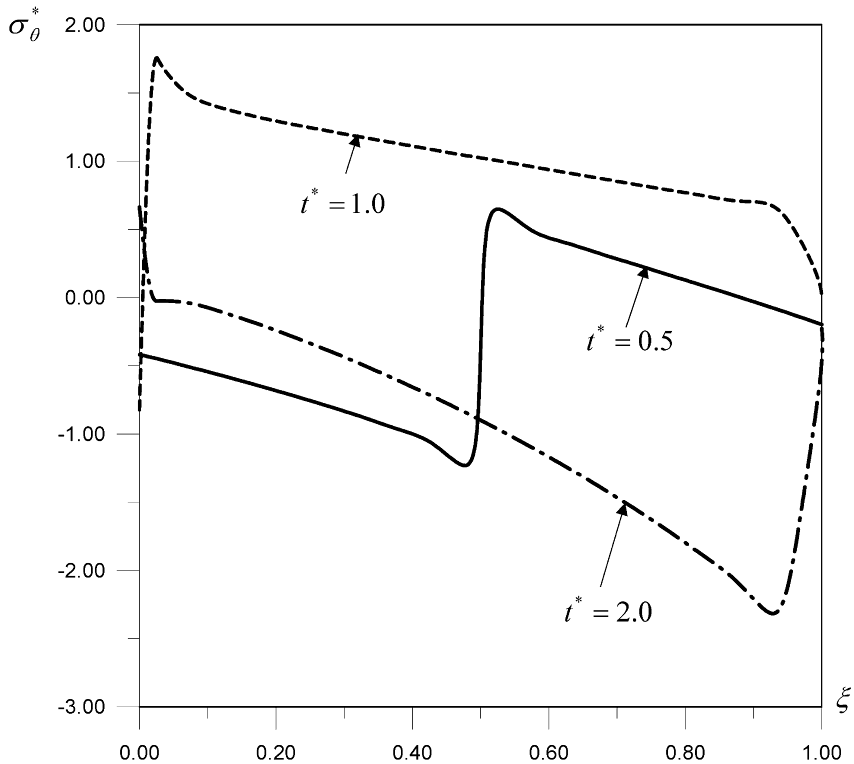




**Fig. 4.** Distributions of dynamic stresses $\sigma_\theta^*$ for different times $t^*$ ($N = 1.0$ and $s = 0.5$)

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