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Photon buildup factors in some dosimetric materials for heterogeneous radiation sources

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Abstract Effective photon energy absorption (EABF_{eff}) and exposure buildup factors (EBF_{eff}) have been calculated based on the effective energy concept, for some dosimetric materials such as water, polymethyl methacrylate (PMMA), polystyrene, solid water (WT1), RW3 (Goettingen Water 3), and ABS (acrylonitrile butadiene styrene), for MV X-rays and ${}^{60}Co$ gamma rays. Firstly, the equivalent atomic numbers (Z_{eq}) of the given materials have been determined using the effective photon energies (E_{eff}) . Then, the five-parameter geometric progression (G-P) fitting approximation has been used to calculate both EAB-F_{eff} and EBF_{eff} values. Since the G-P fitting parameters are not available for the E_{eff} values of the given materials, a linear interpolation in which a function of the logarithm of the variable is used has been performed, in order to calculate the parameters in each E_{eff} , which will be further used for the determination of $EABF_{eff}$ and EBF_{eff} . In the present paper, water equivalence properties of the given materials are also discussed based on the effective buildup factors. In this study, special emphasis is placed on the calculation of $EABF_{eff}$ and EBF_{eff} values of different materials for photons that are not monoenergetic but heterogeneous in energy, to obtain an initial and prior knowledge of the probable energy and buildup of photons at locations of interest, i.e., to understand whether the real absorbed dose occurs at the surface or somewhere inside the medium of interest.

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Introduction

In the field of medical physics, gamma ray buildup factors indicating photon buildup in a medium represent important parameters used to estimate the distribution of photon flux and radiation dose received by biological molecules (Chilton et al. [1984](#page-9-0)). This is so because the buildup of photons that could give rise to secondary radiation important, for example, in nuclear physics experiments, shielding design or dose estimation, is an important issue that has to be taken into account. For proper shielding and estimation of dose, the buildup factors should be determined for the materials under consideration. When photons penetrate through any material, they interact with the atoms of the target material, thus resulting in their buildup in the medium. This buildup which is due to secondary radiation can be characterized by the ''buildup factor'' (Sidhu et al. [2000](#page-9-0)). Because the medium in which photons interact can be the human body, buildup factors in human tissue are of importance. Studies regarding photon buildup factors in different materials have been widely made using wellknown methods such as G-P fitting, invariant embedding (IE), using generalized feed-forward neural networks (GFFNNs), and Monte Carlo codes such as MCNP (Kurudirek and Ozdemir [2011a,](#page-9-0) [b](#page-9-0); Kurudirek et al. [2011a,](#page-9-0) [b](#page-9-0); Singh et al. [2008,](#page-9-0) [2010](#page-10-0); Manohara et al. [2010](#page-9-0); Sidhu et al. [1999](#page-9-0); Sardari et al. [2009;](#page-9-0) Sardari and Baradaran [2010](#page-9-0); Sardari et al. [2011](#page-9-0); Mann et al. [2012a,](#page-9-0) [b;](#page-9-0) Mann and Sidhu [2012;](#page-9-0) Sakamoto and Tanaka [1988;](#page-9-0) Shimizu [2002](#page-9-0); Shimizu et al. [2004;](#page-9-0) Harima et al. [1986;](#page-9-0) Kucuk [2010](#page-9-0);

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Kurudirek and Topcuoglu [2011\)](#page-9-0). In those studies, the incident photons have been assumed as monoenergetic. However, there are almost no studies on the calculation of $EABF_{\text{eff}}$ and EBF_{eff} values for clinical MV X-rays and 60° Co gamma rays, which are not monoenergetic but heterogeneous in energy. In accelerator laboratories including, for example, LINACs, radiation monitoring (area and personnel) is a challenging task as no instruments are standardized for such high-energy radiation environments. In particular, it is necessary to have a prior knowledge of the probable energy of the photons at locations of interest around such accelerators, for a subsequent evaluation of absorbed dose (Bakshi et al. [2008\)](#page-8-0). In the measurement of hard X-ray radiation of Bremsstrahlung with an extremely broad energy spectrum, X-ray radiation is generally characterized by the effective photon energy determined by an attenuation curve in certain suitable materials (Price [1958](#page-9-0)). For example, water is a commonly used phantom material for dosimetry measurements in radiation therapy since it is the main ingredient of the human body and because it has similar physical properties as soft tissue. Due to some practical restrictions when used in liquid form, solid waterequivalent materials would be more preferable because they can be can be modified in shape and size and therefore easily adjust to any required experimental geometry. To be used as a water phantom, any materials considered should have similar properties as water when exposed to ionizing radiation. This means that the photon interaction parameters such as effective atomic number and effective electron density should be as close as possible to those of water (Khan [2003](#page-9-0); Baldock et al. [2010\)](#page-9-0). In order to validate the calculation algorithms used, it could be reasonable to compare the calculated doses with measured doses in different phantoms (Shivaramu et al. [2001\)](#page-9-0). As a sequel to the author's previous work (Kurudirek 2013), EABF_{eff} and EBFeff values for water and some water-equivalent materials have been calculated for 6 MV, 15 MV X-ray, and 60Co gamma beams. The water-equivalent materials used are PMMA, polystyrene, solid water (WT1), RW3, and ABS. The physical properties of the water-equivalent materials as well as those of water are given elsewhere (Kumar et al. [2010](#page-9-0)).

Materials and methods

Calculation of effective atomic cross section σ_{eff} and effective photon energy E_{eff} .

Equation (1) has been used to calculate the effective atomic cross section of the investigated material:

$$
\sigma_{\text{aeff}} = \frac{(\mu/\rho)_{\text{m}}}{N_{\text{A}} \sum_{i} \frac{w_i}{A_i}} \text{(barns/atom)} \tag{1}
$$

where $(\mu/\rho)_{\text{m}}$ is the mass attenuation coefficient of the sample, w_i is the weight fraction of the element i, and Airefers to the atomic weight.

In some cases, assigning an effective energy to a heterogeneous photon radiation could be useful in calculations (Khan [2003\)](#page-9-0). In order to obtain the effective photon energy (E_{eff}) , the effective atomic cross-sectional values for the materials of the investigated phantoms obtained using Eq. 1 were interpolated using the attenuation cross sections calculated with the WinXCom (Gerward et al. [2001,](#page-9-0) [2004\)](#page-9-0) program as follows (Kurudirek and Celik 2012):

$$
E_{\text{eff}} = \frac{E_1(\log \sigma_2 - \log \sigma) + E_2(\log \sigma - \log \sigma_1)}{\log \sigma_2 - \log \sigma_1} \tag{2}
$$

where σ_1 and σ_2 are total atomic cross sections (barns/ atom) defining the interval in which the effective atomic cross section σ of the considered material lies, and E_1 and $E₂$ are photon energies corresponding to the cross sections σ_1 and σ_2 , respectively.

As an example, in the following, calculation of E_{eff} for a 15-MV photon beam for water $(H₂O)$ is described: The effective atomic cross section of water is 0.3022 barn, which lies between the atomic cross sections 0.3024 and 0.2763 barn corresponding to the energies $E_1 = 5$ MeV and $E_2 = 6$ MeV, respectively. Then, using these values in Eq. (2), a value for E_{eff} of 5.005 MeV is obtained. Variation of the total effective atomic cross section of water with photon energy up to an energy of 15 MeV is shown in Fig. 1 as an example. Detailed information on the calculation of Eeff values for the given materials can be found elsewhere (Kurudirek [2013](#page-9-0)).

Fig. 1 Variation of total effective atomic cross section of water with photon energy up to 15 MeV

Calculation of the equivalent atomic number Z_{eq}

The equivalent atomic number, Z_{eq} , of a material defines the same ratio, $(\mu/\rho)_{\text{Compton}}/(\mu/\rho)_{\text{Total}}$ (R), of that material at a specific energy (E_{eff}) corresponding to the ratio of an element at the same energy. For this purpose, a large attenuation coefficient pool including the Compton partial mass attenuation coefficient, $(\mu/\rho)_{\text{Compton}}$, and the total mass attenuation coefficients, $(\mu/\rho)_{\text{Total}}$, of the elements of Z=4–40 and as well as those for the investigated materials at E_{eff} values has been constructed using the WinXCom computer program (Gerward et al. [2001,](#page-9-0) [2004\)](#page-9-0) (initially developed as XCOM (Berger and Hubbell [1999\)](#page-9-0)). Then, interpolation to calculate Z_{eq} has been done using the following relation (Sidhu et al. [1999\)](#page-9-0):

$$
Z_{eq} = \frac{Z_1(\log R_2 - \log R) + Z_2(\log R - \log R_1)}{\log R_2 - \log R_1}
$$
(3)

where Z_1 and Z_2 are the atomic numbers of the elements corresponding to the ratios R_1 and R_2 , respectively, R is the ratio of Compton mass attenuation coefficient to total mass attenuation coefficient for the given materials at a specific energy.

Let water again be the example, now to illustrate calculation of Z_{eq} at an E_{eff} value of 2.039 MeV, which corresponds to 6 MV X-rays. The ratio $(\mu/\rho)_{\rm Compton}/(\mu/\rho)_{\rm Total}$ of water is 0.9914, which lies between the ratios 0.9909 and 0.9922 corresponding to the elements $Z_1 = 7$ and $Z_2 = 6$, respectively. Then, using these values in Eq. (3) , Z_{eq} is equal to 6.62. The variation of the ratio of incoherent to total mass attenuation coefficients for elements with atomic number Z between 4 and 20 is given in Fig. 2.

Calculation of geometric progression (G-P) fitting parameters at E_{eff} values of the investigated materials

The G-P fitting parameters have been calculated using a similar interpolation procedure as used for the equivalent atomic number. The G-P fitting parameters for the elements present in the ANSI/ANS-6.4.3 [\(1991](#page-8-0)) database have been used for this purpose. However, for these elements the G-P

Fig. 2 Variation of incoherent to total mass attenuation coefficient ratios for various elements $(Z = 4-20)$

fitting parameters at E_{eff} values are not available in the database. Thus, Eq. (4) has been used to interpolate the G-P fitting parameters (Harima [1986](#page-9-0)):

$$
C(E_{\rm eff}) = \frac{C_1(\log E_2 - \log E_{\rm eff}) + C_2(\log E_{\rm eff} - \log E_1)}{\log E_2 - \log E_1}
$$
\n(4)

Then, Eq. (5) has been applied to obtain the G-P fitting buildup factor coefficients of the materials:

$$
C = \frac{C_1(\log Z_2 - \log Z_{\text{eq}}) + C_2(\log Z_{\text{eq}} - \log Z_1)}{\log Z_2 - \log Z_1}
$$
(5)

Where C_1 and C_2 are the values of the G-P fitting coefficients for the elements of atomic numbers Z_1 and Z_2 , respectively, and Z_{eq} is the equivalent atomic number of the material.

Calculation of energy absorption and exposure buildup factors

Finally, energy absorption and exposure photon buildup factors have been calculated using the parameters obtained above with the help of the G-P fitting formula (Eqs. 6–[8\)](#page-3-0) (Harima et al. [1986\)](#page-9-0):

$$
B(E, X) = 1 + \frac{b-1}{K-1}(K^x - 1) \quad \text{for } K \neq 1
$$
 (6)

Table 1 Equivalent atomic numbers of the investigated materials at different effective photon energies corresponding to 6 and 15 MV X-rays, and 60Co gamma rays

Material	E_{eff} –6 MV (MeV)	Z_{eq} –6 MV	E_{eff} -15 MV (MeV)	Z_{eq} -15 MV	E_{eff} - ⁶⁰ Co (MeV)	Z_{eq} - ⁶⁰ Co	
Water	2.039	6.62	5.005	6.60	1.250	6.89	
PMMA	2.000	5.86	4.999	5.85	1.250	6.03	
Polystyrene	1.998	5.29	4.999	5.29	1.250	5.37	
Solid water WT1	1.999	5.99	5.012	5.96	1.219	6.94	
RW3	2.001	5.50	5.003	5.48	1.250	6.01	
ABS	2.001	5.32	5.001	5.32	1.252	5.41	

Table 2 G-P EBF and EABF coefficients of the investigated materials at different effective photon energies

Materials	$E_{\rm eff}$ (MeV)	$G-P$ EBF _{eff} coefficients				G-P EABF _{eff} coefficients					
		b	$\mathcal C$	a	Xk	d	b	\boldsymbol{c}	a	Xk	d
Water	2.039	1.8658	1.1760	-0.0401	13.9297	0.0180	1.8318	1.1671	-0.0376	14.1174	0.0155
PMMA	2.000	1.9051	1.1946	-0.0447	14.0458	0.0215	1.8417	1.1690	-0.0376	14.4798	0.0149
Polystyrene	1.998	1.9376	1.2069	-0.0482	14.2982	0.0256	1.8364	1.1783	-0.0405	14.2548	0.0177
WT1	1.999	1.8986	1.1924	-0.0441	13.9922	0.0207	1.8431	1.1673	-0.0371	14.5270	0.0144
RW3	2.001	1.9249	1.2018	-0.0468	14.2018	0.0240	1.8382	1.1745	-0.0393	14.3399	0.0166
ABS	2.001	1.9353	1.2057	-0.0479	14.2842	0.0253	1.8363	1.1774	-0.0403	14.2666	0.0175
Water	5.005	1.5667	0.9394	0.0177	13.9251	-0.0121	1.5655	0.9370	0.0184	14.1012	-0.0118
PMMA	4.999	1.5783	0.9382	0.0171	14.5673	-0.0108	1.5650	0.9458	0.0144	14.8835	-0.0072
Polystyrene	4.999	1.5918	0.9349	0.0177	14.8185	-0.0102	1.5689	0.9408	0.0161	14.2042	-0.0090
WT1	5.012	1.5753	0.9383	0.0172	14.5167	-0.0109	1.5637	0.9460	0.0143	14.9651	-0.0071
RW3	5.003	1.5868	0.9359	0.0175	14.7236	-0.0104	1.5673	0.9424	0.0155	14.4376	-0.0084
ABS	5.001	1.5908	0.9350	0.0177	14.7980	-0.0102	1.5686	0.9410	0.0160	14.2399	-0.0089
Water	1.250	2.0432	1.3672	-0.0763	14.3606	0.0337	2.0101	1.3466	-0.0720	14.5037	0.0306
PMMA	1.250	2.0904	1.4138	-0.0870	13.7556	0.0419	2.0068	1.3581	-0.0747	14.2597	0.0328
Polystyrene	1.250	2.1320	1.4720	-0.0990	13.6921	0.0520	2.0043	1.3658	-0.0763	14.2038	0.0337
WT1	1.219	2.0506	1.3776	-0.0782	14.3427	0.0344	2.0212	1.3557	-0.0735	14.5359	0.0311
RW3	1.250	2.0915	1.4149	-0.0872	13.7414	0.0421	2.0067	1.3584	-0.0748	14.2540	0.0329
ABS	1.252	2.1289	1.4674	-0.0981	13.6949	0.0513	2.0040	1.3647	-0.0761	14.2076	0.0336

Fig. 3 Energy absorption buildup factor for water in the energy region 0.015–15 MeV at different penetration depths (given in mean free paths (mfp))

$$
B(E, X) = 1 + (b - 1)x \quad \text{for } K = 1 \tag{7}
$$

where

$$
K(E, x) = cx^{a} + d \frac{\tanh(x/X_{k} - 2) - \tanh(-2)}{1 - \tanh(-2)}
$$
(8)
for $x \le 40mfp$

where E is the incident photon energy, x is the penetration depth in mean free paths (mfp), a, b, c, d , and X_k are the G-P fitting parameters, and b also corresponds to the value of buildup factor at 1 mfp.

Fig. 4 Energy absorption buildup factor for water up to 40 mean free paths (mfp) at 0.015, 0.15, 1.5, and 15 MeV

Results and discussion

Effective photon energy

The values of the effective photon energy (E_{eff}) for the investigated water-equivalent materials as well as for water, for different MV X-rays and ${}^{60}Co$ gamma rays, are given in Table [1](#page-2-0). In order to validate the used E_{eff} concept for MV beams, the E_{eff} values of ⁶⁰Co gamma radiation have also been determined. It is well known that after the decay of 60Co photons of two main energies, i.e., 1.173 and

Fig. 5 Energy absorption buildup factor for the investigated materials at different penetration depths (given in mean free paths (mfp) for 6 MV and 15 MV X-rays, and for 60 Co gamma rays

Fig. 6 Exposure buildup factor for the investigated materials at different penetration depths (given in mean free paths (mfp)) for 6 and 15 MV X-rays, and for 60 Co gamma rays

1.332 MeV, with branching ratios 99.97 and 99.99 %, respectively, they are emitted. Thus, the effective photon energy for the ${}^{60}Co$ gamma radiation corresponds to a mean value of 1.25 MeV. Table [1](#page-2-0) shows that except for the WT1 phantom, the E_{eff} values of the given materials for 60Co gamma radiation are close to 1.25 MeV. Greenhouse et al. [\(1967\)](#page-9-0) used the effective photon energy concept based on half-value layers (HVL) for photons of keV energies ranging from 60 to 1,000 keV, for different materials (Greenhouse et al. [1967](#page-9-0)). The validity of the Beer–Lambert law $(I = I_0e^{-\mu x})$, which is generally used to compute photon attenuation coefficients, has already been tested and verified for describing the attenuation of a wide Bremsstrahlung X-ray beam in solid water for 6 MV photons (Midgley et al. [1998](#page-9-0)). The violation of Beer– Lambert law due to photon buildup and its subsequent

Fig. 7 Difference (%) between EABF and EBF values for water in the energy region 0.015–15 MeV for mean free paths up to 40 mfp

Fig. 8 Difference (%) between EABF and EBF values for the investigated materials for 6 MV X-rays and for mean free paths up to 40 mfp

modified version is explained in (Singh et al. [2008\)](#page-9-0). In the previous work of the author, it has been shown that whether one calculates effective atomic numbers for MV X-rays using Auto- Z_{eff} (Taylor et al. [2012](#page-10-0)) or using the method based on effective photon energy that is adopted in the present study, the resulting differences are in general less than 1 % except for water for which the difference is 2 % for 6 MV photons (Kurudirek [2013](#page-9-0)).

Effective photon buildup factors

The Z_{eq} values obtained here for the investigated materials at different E_{eff} values correspond to the 6 and 15 MV X-rays, and the ⁶⁰Co gamma rays are also given in Table [1.](#page-2-0) Additionally, the G-P EABF_{eff} and EABF_{eff} coefficients for the investigated materials are given in Table [2](#page-3-0). It is worth

Fig. 9 Difference (%) between EABF and EBF values for the investigated materials for 15 MV X-rays and for mean free paths up to 40 mfp

Fig. 10 Difference (%) between EABF and EBF values for the investigated materials for ${}^{60}Co$ gamma rays and for mean free paths up to 40 mfp

noting that the word ''effective'' is used due to the polychromatic nature of the photon spectra. The EABF values are shown graphically at fixed penetration depths (Fig. [3\)](#page-3-0) as well as at fixed energy values (Fig. [4\)](#page-3-0) for water as an example. It has to be noted that EABF values for water have been taken from the ANSI database ([1991\)](#page-8-0). It can be observed that the EABF values show variations in the continuous energy region, depending on which of the different photon interaction processes dominates in the different energy regions (Fig. [3](#page-3-0)). At lower energies, the dominant photon interaction process is photoelectric absorption for which the atomic cross section is proportional to $\tau \propto \frac{Z^{4-5}}{E^{7/2}}$. At intermediate energies, the Compton scattering process starts dominating showing a linear

Fig. 11 a, b, c Differences in EABF values relative to water for MV X-rays and ${}^{60}Co$ gamma rays

atomic number dependence, while at higher energies the pair production process dominates with an atomic number dependence, which is in the order of Z^2 . Among these

Fig. 12 a, b, c Differences in EBF values relative to water for MV X-rays and 60Co gamma rays

interactions, the absorption processes (photoelectric absorption and pair/triplet production) result in a complete removal of photons, and this clearly explains the lower values of EABF and EBF at lower and higher energies. In contrast, the highest EABF and EBF values are observed at intermediate energies where Compton scattering dominates. For this process, the photons are not completely removed but their energies are only reduced. Hence, this process results in more multiple scattered photons that lead to an increase in the buildup of photons in the medium. It can be seen that at lower energies, i.e., at 0.015 MeV, the values of EABF remain constant and are close to unity for all penetration depths for water (Fig. [4\)](#page-3-0). At higher energies, however, there is an increase in the EABF and EBF values for water, due to the increase in the number of multiple scattered photons. It can be seen from Figs. [5](#page-4-0) and [6](#page-4-0) that higher EABF_{eff} and EBF_{eff} values are observed for 60 Co gamma rays that have an E_{eff} value of approximately 1.25 MeV. On the other hand, lower EABF_{eff} and EBF_{eff} values are observed for 15 MV X-rays that have an E_{eff} value of about 5 MeV. This can be explained due to the predominance of Compton scattering at 1.25 MeV rather than at 5 MeV for which the main dominant interaction process is pair production. Maximum differences up to 5 % are observed between EABF and EBF values for water (Fig. [7\)](#page-4-0). Figures [8](#page-4-0), [9](#page-5-0), and [10](#page-5-0) show significant differences between EABF_{eff} and EBF_{eff} values for the investigated materials for 6 MV and 15 MV X-rays, and for ${}^{60}Co$ gamma rays. It should be noted that in general, the values for EBF_{eff} are higher than the values for $EABF_{\text{eff}}$ (Figs. [8,](#page-4-0) [9](#page-5-0), and [10](#page-5-0)). The absorption of photons in air is comparable with materials of low Z_{eq} ; hence, the EBF_{eff} values could be higher than the $EABF_{eff}$ values. The Z_{eq} value of air lies between 7 and 8, whereas the Z_{eq} values of water and water-equivalent phantoms are between 5.32 and 6.94. The investigated materials show in general lower values of Z_{eq} when compared to air. Thus, absorption in air contributes much more to EBF_{eff} rather than to $EABF_{\text{eff}}$. Figures [8](#page-4-0), [9,](#page-5-0) and 10 show that the relative difference $(\%)$ between EABF_{eff} and EBF_{eff} is high (up to 37 % for ⁶⁰Co and up to 17 % for 6 MV) in the intermediate energy region (\sim 1.25 MeV for ⁶⁰Co gamma rays, \sim 2 MeV for 6 MV X-rays), due to the build up of scattered photons; the difference is not significant (up to 2 %), however, at higher energies (\sim 5 MeV for 15 MV X-rays), due to the absorption processes resulting in lower values of the buildup factor.

Water equivalence properties

The investigated materials have been studied with respect to the water equivalence in terms of effective buildup factors (Figs. [11](#page-5-0), 12). For 6 MV photons, differences in EABF_{eff} are \lt 4 % and the minimum differences are observed for ABS (Fig. [11a](#page-5-0)). For 15 MV photons, differences in EABF_{eff} are $\langle 3 \, \% \rangle$ and the minimum differences are observed for WT1 up to 10 mfp, and for PMMA

Penetration depth (mfp)	E_{eff} (1.25 MeV)	1.173 MeV	1.332 MeV	Weighted mean energy (MeV) ^a	$%$ Difference ^t
Exposure buildup factor					
$0.5\,$	1.474	1.478	1.477	1.477	$0.2\,$
$\mathbf{1}$	2.043	2.057	2.051	2.054	0.5
\overline{c}	3.396	3.446	3.411	3.428	0.9
\mathfrak{Z}	5.005	5.112	5.023	5.067	1.2
$\overline{4}$	6.849	7.036	6.863	6.949	1.4
5	8.913	9.204	8.912	9.058	1.6
6	11.183	11.601	11.153	11.377	1.7
7	13.644	14.213	13.57	13.891	1.8
$\,$ 8 $\,$	16.283	17.025	16.144	16.584	1.8
10	22.032	23.188	21.702	22.445	1.8
15	38.557	41.091	37.306	39.198	1.6
20	57.42	61.741	54.562	58.151	1.3
$25\,$	78.971	85.522	73.828	79.675	0.9
$30\,$	104.275	113.607	96.2	104.903	0.6
35	132.623	145.126	121.099	133.112	0.4
40	160.081	175.509	144.862	160.185	0.1
Energy absorption buildup factor					
0.5	1.461	1.472	1.451	1.461	$0.0\,$
$\mathbf{1}$	2.01	2.039	1.984	2.011	$0.1\,$
$\sqrt{2}$	3.304	3.387	3.228	3.307	0.1
3	4.832	4.994	4.683	4.838	$0.1\,$
$\overline{4}$	6.573	6.841	6.329	6.585	$0.2\,$
5	8.514	8.914	8.151	8.532	0.2
6	10.643	11.2	10.135	10.667	$0.2\,$
7	12.945	13.687	12.268	12.977	0.3
$\,$ 8 $\,$	15.408	16.361	14.538	15.449	0.3
10	20.768	22.218	19.439	20.828	0.3
15	36.163	39.243	33.312	36.277	0.3
20	53.724	58.893	48.919	53.906	0.3
25	73.65	81.369	66.506	73.937	0.4
30	96.783	107.632	86.81	97.220	0.4
35	122.584	137.09	109.213	123.151	0.5
40	147.849	166.016	130.798	148.406	0.4

Table 3 Comparison of buildup factors of water at the effective photon energy of 60 Co, and 1.173 and 1.332 MeV monoenergetic photons; mfp—mean free path

^a Weighted mean corresponds to the mean energy of the two gamma energies (1.173 and 1.332 MeV) weighted with their corresponding branching ratios of 99.974 and 99.986 %, respectively

^b Difference (%) refers to the relative difference between the buildup factors at E_{eff} and the weighted mean energy

beyond 10 mfp (Fig. [11b](#page-5-0)). Finally, for ${}^{60}Co$ gamma rays, differences in $EABF_{eff}$ are $\lt 5$ % and the PMMA material shows better water equivalency than the other materials (Fig. [11](#page-5-0)c). When it comes to EBFeff, it should be noted that WT1 has better water equivalence properties for 6 MV X-rays, 15 MV X-rays (up to 15 mfp), and ${}^{60}Co$ gamma rays. Differences are $\lt 1$ % between 20 and 30 mfp, while beyond 30 mfp polystyrene seems to be the best choice of material for a water substitute.

Comparison with buildup factors for monoenergetic photons

For the purpose of the comparison, buildup factors for water have been also calculated for photons of 1.173 and 1.332 MeV separately. Then, the weighted mean buildup factor has been obtained by taking into account the branching ratios of the emission of gamma radiation emitted due to the decay of 60Co (99.974 % at 1.173 MeV and 99.986 % at

1.332 MeV). The results have been compared with the buildup factors of water, which were obtained using the effective energy approximation that is adopted in the present work. It can be seen from Table [3](#page-7-0) that the relative differences are in the range of $0.1-1.8$ % for exposure buildup factors and 0.03–0.46 % for energy absorption buildup factors.

Calculation uncertainty

The accuracy of the calculated values of effective photon energy is based on total atomic cross sections, thus on mass attenuation coefficients. The mass attenuation coefficients of the elements have been taken from the WinXCom (Gerward et al. [2001](#page-9-0), [2004](#page-9-0)) database, which is the successor of the XCom (Berger and Hubbell [1999\)](#page-9-0) database. The uncertainties in mass attenuation coefficients have been addressed in the previous reports (Hubbell [1999;](#page-9-0) Chantler [2000](#page-9-0)).

A previous study by Asano and Sakamoto (Asano and Sakamoto 2007) indicated that the results of Monte Carlo calculations agree well with those obtained with the method adopted in the present study, but with slight differences. The reasons behind these differences might be that the ANSI/ ANS database uses the method of moments (Eisenhauer and Simmons [1975](#page-9-0)) with a parallel beam source and the Monte Carlo code EGS4 with an isotropic emission source, and the differences might also be due to the low-energy photon treatment applied in EGS4 for K–X radiation, L–X radiation, and Bremsstrahlung. In another study, it has been shown that different methods such as invariant embedding, G-P fitting, and Monte Carlo simulation agree well for the materials of low atomic numbers (Shimizu et al. 2004). The Z_{eq} values of the materials in the present work cover a range from 5 to 7, which means that they can be considered as low Z materials. It has also been shown that the maximum deviations in exposure buildup factors were 0.5–3, 0.4–9.3, 0.9–42.7, and 0.4–53.2 % for G-P fitting, three-exponential approach, Berger approach, and Taylor approximation, respectively (Harima et al. [1986\)](#page-9-0).

Additionally, it should be noted that a revision has been made to the existing ANSI database to improve it and catch up with the latest developments in fundamental data (Ryman et al. [2008;](#page-9-0) Ruggieri and Sanders [2008](#page-9-0)). These improvements include the issues raised from the lack of self-consistency within the standard (ANSI/ANS-6.4.3. 1991) in that the mass attenuation coefficients tabulated in the standard were not necessarily used in the calculation of the buildup factors, correction factors for coherent scattering, and tissue dose. Moreover, more accurate attenuation coefficient data have become available (Chadwick et al. [2006](#page-9-0)) since the use of the standard database (Ryman et al. [2008](#page-9-0)). One of the other improvements is that photon buildup factors for high atomic number materials have been calculated (Ruggieri and Sanders [2008\)](#page-9-0).

As a summary, photon buildup factors, which have to be taken into account, for example, prior to a radiotherapy application as well as prior to shielding calculations, have been calculated for heterogeneous radiation sources. Dosimetric materials such as water and water-equivalent materials have been considered, to estimate energy absorption as well as exposure buildup factors for MV photons and gamma radiation. The present work could be useful from the point of view that it can provide an initial estimation on how radiation could interact with a target especially in terms of photon buildup factors.

Conclusions

Studies regarding photon buildup factors in different materials have been widely made under the assumption that the incident photons are monoenergetic. In contrast, in the present study, photon buildup factors in water and some water-equivalent phantoms have been calculated for clinical MV X-rays and ${}^{60}Co$ gamma rays that are not monoenergetic, to obtain an initial knowledge of the probable energy and the buildup of photons at locations of interest in water-equivalent media, i.e., to answer the question whether the absorbed dose occurs at the surface or somewhere inside the medium. The present study reports new data on photon buildup factors of some dosimetric materials for different multi-energetic radiation sources. In order to get a rough and quick estimation of photon radiation buildup in terms of exposure and energy absorption, which is important for therapy planning or shielding calculations, the effective photon buildup factors introduced in the present study will be useful. However, for final corrections and/or detailed planning, and to obtain more confirmation and a precise knowledge on the photon buildup factors for multienergetic photons, Monte Carlo calculations should be applied, since they take into account not only all secondary effects but also any specific experimental arrangements.

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