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Acronychal Risings in Babylonian Planetary Theory

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The Astronomical Diaries (ADT), and a few known collections for individual planets, contain observations of five synodic phenomena of superior planets: heliacal rising (Γ), first station (Φ), acronychal rising (Θ), second station (Ψ), and heliacal setting (Ω). A date is given for each, in the case of Γ often both an observed date and a 'true' or 'ideal' date on which the rising is considered to have occurred even if it was not observed, as due to clouds, found by a measurement of the interval in degrees of time between the rising of the planet and the rising of the sun. However, location is recorded differently for each class of phenomena. Heliacal risings and settings, Γ and Ω, are located by zodiacal sign, or by beginning or end of zodiacal sign. In some cases Γ contains a measured distance from a nearby 'normal' (standard) star or planet, for conjunctions of planets with stars or with each other were considered ominous. But it does not appear that measurements of distances from stars at Γ were used to establish location more precisely than by zodiacal sign, and distances from planets cannot be used to establish location. First and second stations, Φ and Ψ, usually contain a measured distance from a normal star, presumably to determine when the planet was stationary, but sometimes only a location by zodiacal sign. Acronychal rising Θ contains no location at all. It could have been assumed that the planet was in the zodiacal sign opposite the sun, but no location for the sun is given in the Diaries. (It is curious that acronychal risings were observed at all since there are no omens associated with them. Yet an acronychal rising of Jupiter appears already in the second earliest known Diary, ADT -567, and one may wonder why.) Sometimes observations contain the remark 'not observed' (nu pap), which presumably indicates an inference of the date and location from nearby preceding or following observations.

The dates and longitudes of the same phenomena reported in the Diaries are computed in the ephemerides to a degree of precision exceeding the observations in the Diaries. The ephemerides are published in ACT, and the further analysis in HAMA is essential. In a recent study of Babylonian planetary theory (Swerdlow, 1998), I set out a method by which the parameters of the ephemerides can be derived from synodic times between phenomena, recoverable from the dates in the Diaries, and locations no more precise than by zodiacal sign. The method depends upon a constant difference between synodic time and synodic arc, $\Delta T - \Delta\lambda = C$, found in the ephemerides of all planets except Venus, which allows $\Delta\lambda$ to be found from ΔT. With the exception of Θ and Ψ of Mars, which occur in the retrograde arc, the ephemerides use the same functions for computing all phenomena of the superior planets by complete synodic arcs and synodic times even though the synodic arcs and times between consecutive heliacal risings or settings, which depend upon the latitude and brightness of the planet and the inclination of the ecliptic to the horizon, differ from those between consecutive first or second stations, which depend only upon the longitude, and thus the speed, primarily of the planet and secondarily of the sun. The ephemerides use methods based upon a variation of speed, of the planet alone but treated as of the phenomenon itself, specifying that the phenomenon moves through synodic arc $\Delta\lambda$ in synodic time ΔT, in the case of System A as a function of longitude, in the case of System B as a function of the number of the phenomenon in the ACT period, which is nevertheless close to a function of longitude for Saturn and Jupiter although the departures for Mars are considerable.

Because of the inaccuracy of the observations in the Diaries in the dates of phenomena, frequently amounting to 2 or 3 days or even more, it is not clear just how the maximum and minimum limits of the synodic time, upon which the parameters depend, were selected. In our study, we provided figures (2.2–2.4) comparing synodic times from (1) the computed functions of the ephemerides, (2) the observations recorded in the Diaries and other collections, and (3) modern computations of heliacal risings, using for this purpose a computer program of our own, intended to duplicate Tuckerman's tables (1962), and P. V. Neugebauer's visibility tables (1938), throwing together, we fear, too much disparate information in the same figures. Further, we had noted that A. Aaboe (1958, 247–51) had found a nearly perfect agreement of System A for Mars with synodic arcs derived from the longitudes of oppositions, which are close to acronychal risings, but we did not investigate the matter further and assumed that, with the exception of Mars, for which dates of acronychal risings were preferable, dates of heliacal risings were used to establish synodic times since, through the use of rising times to determine the 'true' date, they appear to have been the most carefully observed. We have now carried out computations for all three superior planets, something we should have done in the first place, of (1) synodic times from oppositions as an approximation and control of acronychal risings, (2) synodic times from a provisional computation of acronychal risings, and (3) synodic arcs from first stations, all of which we compare with the functions in the ephemerides.

The results are shown in Figs. 1–3 A and B. In A, Δt, the excess of the total synodic time ΔT in tithis (τ) over 12 months $= 6{,}0^\tau$ (for Mars 24 months $= 12{,}0^\tau$) is graphed against longitude λ by zodiacal sign. The solid lines are the computed System A function, the broken lines the linear zigzag of System B with small squares showing λ for each Δt, for Jupiter from ACT 620a and for Mars from ACT 510 extended by computation; for Saturn λ falls nearly on the zigzag and is not illustrated. The open circles show Δt between heliacal risings taken from our original figures (2.2–2.4) and the filled circles Δt between true oppositions computed from the same program. I have computed oppositions as a control of acronychal risings and because it is first necessary to compute opposition in order to compute acronychal rising. The computation of acronychal risings is explained in the Appendix. In B we show by filled circles synodic times Δt from acronychal risings and by open circles synodic arcs $\Delta\lambda$ from first station, the scale for which is on the right of the graph. The points are located at the beginning of each Δt and $\Delta\lambda$. We begin with the year -200 (-202 for Mars) and compute the following number of synodic periods: Saturn 29, Jupiter 34, Mars 38. The longitudes are converted to the Babylonian zodiac by adding 6°, following P. Huber's (1958) conclusion that the difference for -100 is about 4;30°; any inaccuracy, surely no more than 1° or 2°, affects only the placement of λ and not Δt or

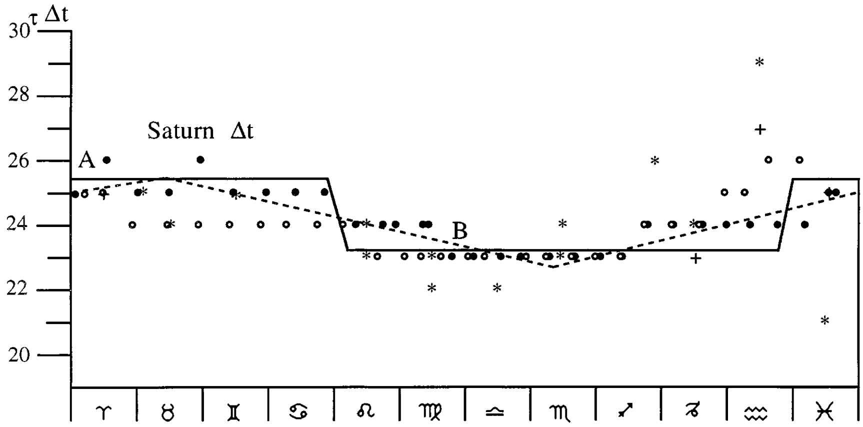

Fig. 1A. Saturn. Computed Δt: Heliacal Rising (○), Opposition (●)
Observed Δt: Heliacal Rising (∗), Acronycal Rising (+)

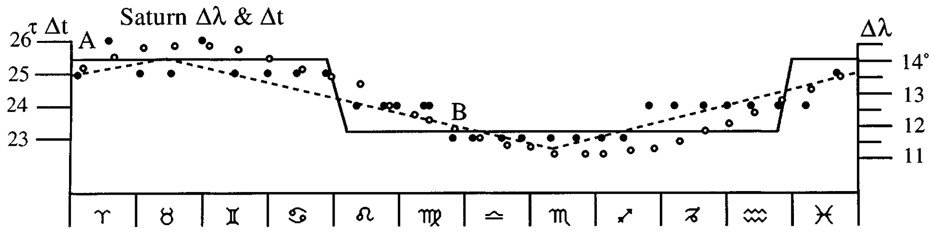

Fig. 1B. Saturn. Computed Δt and $\Delta\lambda$: Acronychal Rising (●), First Station (○)

$\Delta\lambda$. The synodic time in days is converted to tithis by $\Delta t^{\tau} = (30/29;31,50)\Delta T^{\mathrm{d}} - 6,0^{\tau}$ (for Mars $12,0^{\tau}$) rounded to integers; on account of the rounding, there are irregularities in Δt of $\pm 1^{\tau}$, which we have corrected where interpolation is secure. $\Delta\lambda$ in B is graphed to a precision of $1/18°$ for Saturn and Jupiter and $1/3°$ for Mars, and thus follows a more continuous curve than the integer steps of Δt. We also show in A Δt from the dates of observations in the Diaries and other collections, those from heliacal risings indicated by * and from acronychal risings, of which there are not many, by +. Those located by zodiacal sign are placed in the middle of the sign; a few located by beginning of sign are placed at 5° and a few located by end of sign at 25°. It can be seen from these figures, and from the figures (2.2–2.4) in our study, that these are not particularly accurate.

Since our principal interest is the limits of the functions, which were used to derive the parameters of the ephemerides, the minimum and maximum synodic times and arcs, Δt_{m} and $\Delta t_{\mathrm{M}}, \Delta\lambda_{\mathrm{m}}$ and $\Delta\lambda_{\mathrm{M}}$, are shown in Table 1 for Systems A and B to minutes, along with what we believe to be the original assumption for the limits of System A of Mars, which is obviously rounded for $\Delta\lambda$. These are followed by modern computation for heliacal rising Γ, opposition O, acronychal rising Θ, and first station Φ. Δt is given to the nearest integer and $\Delta\lambda$ is given as computed, although the true limits, which may differ slightly, may not have turned up in our computations. Not all of these synodic times and arcs are recoverable from the Diaries or are strictly observable. Thus, for Γ Δt is recoverable from dates, but not $\Delta\lambda$ since location is imprecise. O is, as mentioned,

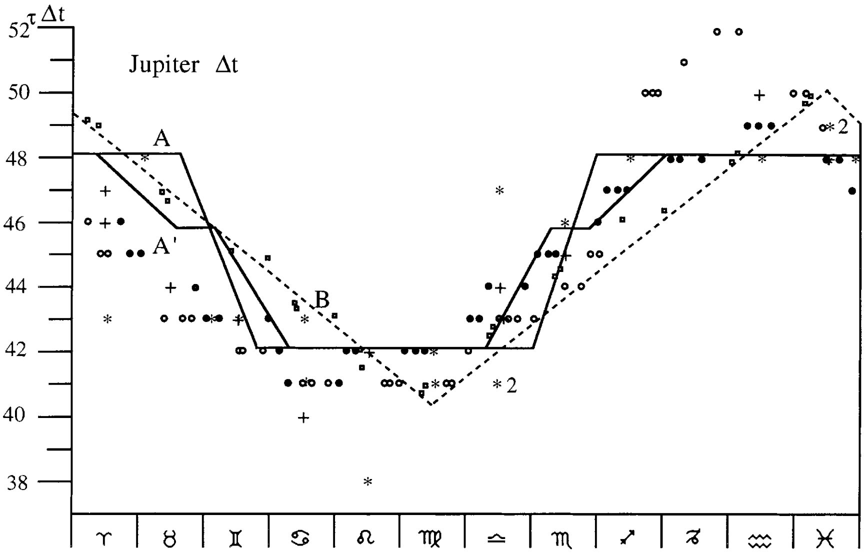

Fig. 2A. Jupiter. Computed Δt: Heliacal Rising (○), Opposition (●)
Observed Δt: Heliacal Rising (∗), Acronycal Rising (+)

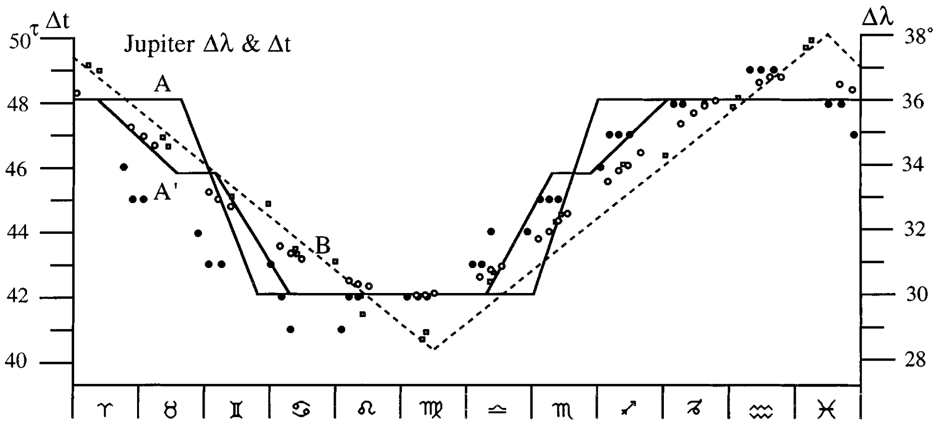

Fig. 2B. Jupiter. Computed Δt and $\Delta\lambda$: Acronychal Rising (●), First Station (○)

intended as a control on the computation of Θ and was not, and could not be, observed. For ΘΔt is recoverable from dates but not $\Delta\lambda$ since no locations are given. For Φ both dates and distances from normal stars were recorded in the Diaries, so that if longitudes could be determined to a precision of, say, ±1° from such distances, $\Delta\lambda$ is recoverable. However, Δt derived from dates of Φ is insecure since the planet moves from not at all to at most ±0;5° in ±3 or ±4 days around station. Still, dates are given, as insecure as they may be.

Consider first the relation of the phenomena to the systems for each planet. For Saturn, Δt of all phenomena agree with the limits of both systems rounded to integers,

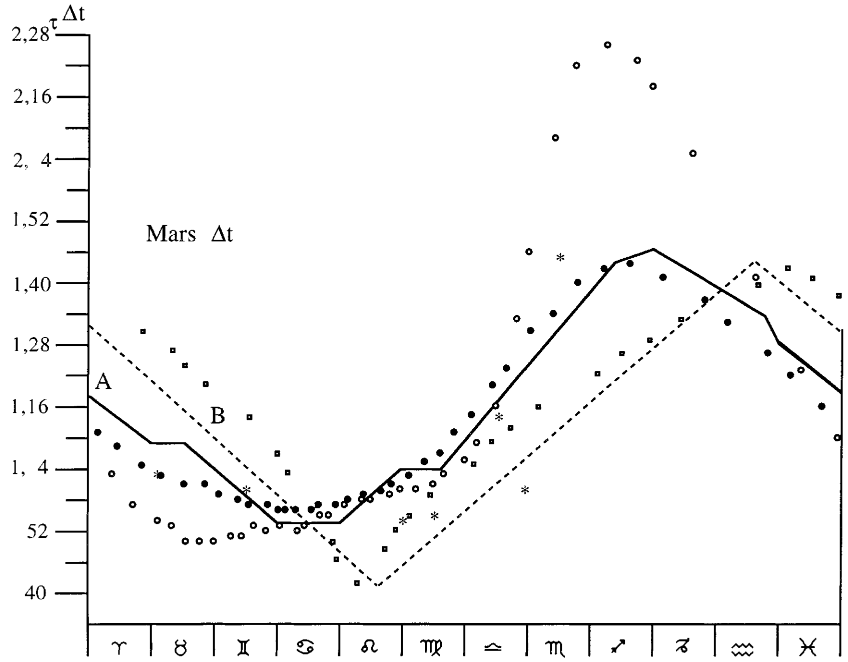

Fig. 3A. Mars. Computed Δt: Heliacal Rising (○), Opposition (●)
Observed Δt: Heliacal Rising (∗)

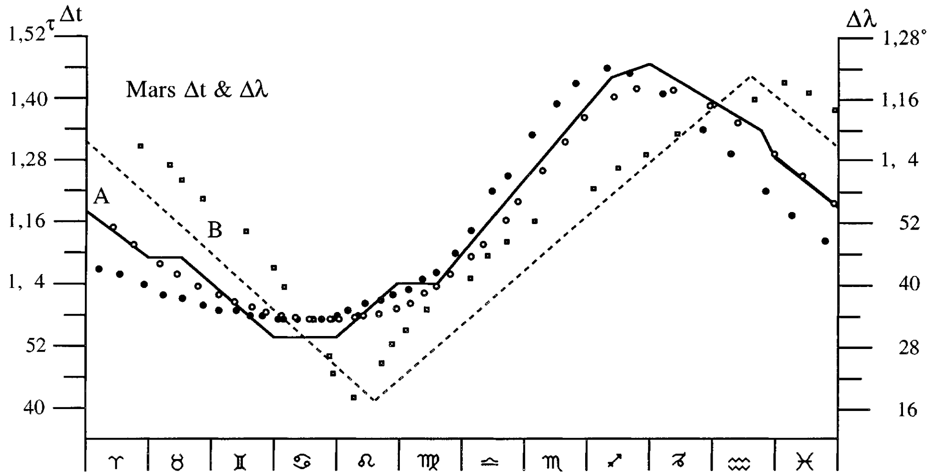

Fig. 3B. Mars. Computed Δt and $\Delta\lambda$: Acronychal Rising (●), First Station (○)

and as can be seen in the figures, Δt of Ο and Θ are identical and $\Delta\lambda$ of Φ tracks System B very nicely. It is of interest that all known ephemerides for Saturn are System B, while System A is known only from procedure texts and longitude templates. For Jupiter, Δt_M of Γ is clearly beyond both systems, Δt of Ο and Θ are identical, and the limits of Ο,

Table 1. Limits of synodic time and arc

Planet		Δt_m	Δt_M	$\Delta\lambda_m$	$\Delta\lambda_M$
Saturn	A	23;10$^\tau$	25;31$^\tau$	11;43°	14; 4°
	B	22;41	25;32	11;14	14; 5
Mod.	Γ	23	26	11;13	14;10
	O	23	26	11; 8	14;28
	Θ	23	26	11; 8	14;23
	Φ	23	26	11; 8	14;25
Jupiter	A	42; 5	48; 5	30	36
	B	40;20	50; 7	28;16	38; 2
Mod.	Γ	41	52	30; 7	37;19
	O	41	49	30; 5	36;41
	Θ	41	49	30; 6	36;43
	Φ	41	49	30; 4	36;40
Mars	A	53;37	1,46; 7	30	1,22;30
Orig.	A	54	1,44	30	1,20
	B	40;57	1,43;45	17;19	1,20; 7
Mod.	Γ	50	2,26	30;11	1,49;54
	O	56	1,44	33;50	1,16;56
	Θ	57	1,46	33;22	1,16;26
	Φ	56	1,42	33;22	1,18;18

Θ, and Φ fall $\pm 1^\tau$ between both systems; $\Delta\lambda$ of Φ can be seen to track both systems well except at the limits of B. In the case of Mars, Δt_M of Γ is, as mentioned earlier, far beyond Δt_M of both systems while Δt_m is somewhat low for System A. Considering the large range of Mars's synodic time and arc, the differences of the limits of Δt and $\Delta\lambda$ of O, Θ and Φ are very small and are close to the limits of System A and the maximum of System B; nothing is close to the minimum of System B. The closest fit in following the graph of System A is $\Delta\lambda$ of Φ and Δt of O, of which the latter cannot be observed. In part the fit is due to the finer resolution of $\Delta\lambda$ for Φ in the figures than of Δt to integers, but $\Delta\lambda$ and λ of Φ really do track the functions in the ephemerides quite well for all three planets.

Consider next the information that may be recovered from the observational reports and the methods of deriving parameters described in our study, which depend upon the limits of Δt and $\Delta\lambda$. The parameters of both systems of Saturn may be derived from Δt of Γ, Θ, and Φ; $\Delta\lambda$ of Φ may be used only by adjustment to very nearly the limits in the systems. Both systems of Jupiter may be derived from Δt of Θ and Φ with adjustments of $\pm 1^\tau$, and System A may be derived from $\Delta\lambda$ of Φ with a slight reduction in $\Delta\lambda_M$. The limits of $\Delta\lambda$ of System B are definitely not observed, but computed from $\Delta\lambda = \Delta T - C$. System A of Mars may be derived from Δt of Θ and Φ and $\Delta\lambda$ of Φ provided that adjustments of from 1 to 3 tithis or degrees are made to give the original assumption for the limits, which amounts to rounding the limits of $\Delta\lambda$ to 30° and 1,20°. The latter is also $\Delta\lambda_M$ of System B, of which $\Delta\lambda_m$, and likewise Δt_m, is incompatible with any observation and clearly erroneous. And thus with this one exception, with small adjustments both Θ and Φ can be used to derive the parameters of all three planets. I must admit that these results surprised me because Θ and Φ present problems of observation, while Γ, which appears to be the most carefully observed, can be used only for Saturn, probably not for Jupiter, even with adjustment, and definitely not for Mars. We shall consider the observational conditions of both Θ and Φ.

Unlike reports of heliacal risings in the Diaries, which contain a date, location, and measured rising time, reports of acronychal risings give only a date and no location, with not a clue about how the date was found. In the case of heliacal rising, the planet has been invisible for some time, and the morning it is first seen a measurement is made of the interval of time between the planets being observed—not necessarily crossing the horizon—and the rising of the sun, in order to infer a possible 'true' or 'ideal' date prior to the date of the observation, meaning the date the planet should have been seen to rise had the horizon been clear and an observation made. Some heliacal risings are described as 'high' or 'bright and high', which must mean that the planet was already above the horizon when it was first seen. This is curious because the planet must have crossed the horizon earlier in the morning when the sky was darker, but evidently the rising itself was not observed. The intervals between observed and true dates in the Diaries may reach as high as 20 days for Mars, although for Saturn and Jupiter they are never more than 6 days and are usually less than 3. In the case of Mars, because of its long period of invisibility, from 90 to more than 220 days, its faintness near heliacal rising, between magnitude 2.1 and 1.3, and the effects of twilight, true heliacal risings are difficult to observe, as shown by the long intervals between observed and true dates. By contrast, the magnitude near opposition is between -0.7 and -2.5 so the acronychal rising is more easily visible in twilight.

Acronychal risings are also different because the planet has not been invisible. After heliacal rising, as the elongation of the planet from the sun increases—most of the motion actually belongs to the sun—the planet rises earlier each night. Acronychal rising is the *last* evening the planet is seen to rise, to cross the horizon, after sunset, and its date can only be determined by observing the following night that the planet is already above the horizon when it becomes visible after sunset. But what if the following night is cloudy? When did acronychal rising occur? And because of twilight, the planet could, indeed will, continue to rise after sunset for a few nights but not be visible until it reaches some altitude. In the absence of any information in the Diaries about measuring the interval of time between sunset and the rising, or visibility, of the planet, this appears to be a rather crude observation: the last evening the planet is *seen* to rise after sunset is acronychal rising even if it continues to rise after sunset for a few nights without its rising being seen. Or perhaps some correction is applied of which the Diaries give no information. One can nevertheless see that for Mars, even with this uncertainty of a few days, acronychal risings are preferable for determining synodic times to the still greater uncertainty of many more days in heliacal risings. For Jupiter and Saturn, heliacal risings, with shorter intervals between observed and true dates, would appear to be better defined than acronychal risings although perhaps the darker horizon at acronychal rising made its use preferable.

We next consider $\Delta\lambda$ derived from stations. In order to find $\Delta\lambda$ from measured distances from normal stars at stations, two points must be considered, (1) the accuracy of measurement of distance and (2) the accuracy of fixing zodiacal longitudes of normal stars. I was skeptical of the sufficiency of both of these, but extensive discussions with John Britton in the course of writing this paper has convinced me that my skepticism was too great. My view was influenced principally by my finding the measurements rather inaccurate, by the difficulties of measuring distances at heliacal risings rather than stations, and by the case of Mercury, in which stations were not observed and only Δt derived from dates of heliacal phenomena could be used. I had also assumed that, large errors aside—of which the figures in our study (2.2–2.4 for superior planets, 2.10–2.13 for Mercury) show that there were many—the inherent precision of specifying dates, 1^τ or 2^τ, was better than in measuring distances. John Britton, relying in part on Gerd Graßhoff's (1999) analysis, has convinced me that, again large errors aside and without pressing the statistics too closely, the accuracy of converting measured distances in cubits and fingers to degrees is about 1°, and, once again large errors aside, the accuracy of longitudes of those normal stars for which assigned zodiacal locations are known is also about 1°. One could thus expect longitudes derived from measured distances from normal stars to have a precision of 1° or 2°, comparable to the precision of dates, 1^τ or 2^τ, assuming for both that large errors have been eliminated. The resulting synodic arcs and synodic times would have about the same precision, 1 or 2 degrees or tithis. And as painstaking as the derivation of $\Delta\lambda$ from measured longitudes may be, it would only have to be done around the limits. I still have strong reservations about whether $\Delta\lambda$ was derived from the conversion of such measured distances to longitudes, because of the difficulty of the procedure compared with the use of dates to find Δt, because there is no textual evidence that such conversions were ever made, and because I do not see how the resulting $\Delta\lambda$ could be adjusted to the required limits for Saturn and System B of Jupiter. Nevertheless, it appears that in principle $\Delta\lambda$ could be derived from longitudes with a precision comparable to the derivation of Δt from dates.

Now, whether we consider synodic times derived from dates or synodic arcs derived from longitudes, two points must be made which appear contradictory, but are not. The first is that whatever observations were used to establish limits, they must have been subject to some kind of analysis to eliminate errors, which occur frequently in observations of dates, as shown by the errors of Δt in our figures, and presumably also in observations of longitude. Thus, the observations used to establish the limits must have been either corrected or of better quality than most of the observations in the Diaries. For otherwise the limits of the functions would not be as close to the computed Δt from Θ and $\Delta\lambda$ from Φ as they are (again setting aside $\Delta\lambda$ of System B of Jupiter and the minimum of System B of Mars). How this was done, I do not know. Presumably it involved the analysis of some number of observations, and the resulting Δt or $\Delta\lambda$, in the vicinity of the limits, first to eliminate gross errors, then to decide upon correct limits within the range of ± 1 or ± 2 tithis or degrees still remaining. For this last step, might one suggest the use of simple statistical techniques by taking mean values in the zodiacal signs in which the limits occur?

The second point is that, even with careful selection of the limits, the resulting functions were intended only as approximations to the true behavior of the phenomena. This can be seen in the different limits of Systems A and B of Jupiter, both of which cannot be uniquely correct, and in the differences between Systems A and System B in general. In System A Δt and $\Delta\lambda$ are strictly a function of longitude, but in System B they are a function of the number of the phenomenon in the period. The differences in the correspondence of Δt and $\Delta\lambda$ to longitude are not great for Saturn and Jupiter, but are very large for Mars, which could explain why only a single fragment of a System B ephemeris for Mars is known. Another approximation concerns the relation of the computed functions to the observations themselves, for which the only reasonable check throughout the zodiac must be Δt. The observations contain many large errors that could be identified from the resulting Δt, but most are also in error by a day or two, resulting in similar errors in Δt, and rather than consider almost all the observations wrong and the computed function uniquely correct, the computation may have been considered an approximation to phenomena that were strictly too complex to compute with full accuracy. This also seems a way of reconciling the concurrent use of Systems A and B for Jupiter.

Yet another approximation is that the same functions are used for all phenomena of the superior planets, except Θ and Ψ of Mars, which also raises a question about the use of Θ. Here there is no problem for Saturn and only a small error for Jupiter, but in the case of Mars the errors for Γ reach -30° and -40^τ for the long synodic periods, which seems intolerable. Then, the longitudes of Θ are computed by auxiliary methods rather than by synodic periods; and in the one ephemeris from which there is evidence, ACT 500, the interval of time between Φ and Θ is taken as constant. Thus, taking limits of Δt, and thus $\Delta\lambda$, from Θ, or of $\Delta\lambda$, and thus Δt, from Φ harms Γ, and the method of computation compromises Θ although not Φ. It is possible that the long synodic times of Mars were thought to be the result of late, that is, not normal, first appearances, whether due to observational error or for other reasons, and thus not taken account of in the ephemerides. Still, as these long periods occur every 15 or 17 years, their regular recurrence should have been noticed. Perhaps it was simply thought preferable to take the narrower range of Δt and $\Delta\lambda$ that would fit acronychal risings and stations, rather than the wider range of heliacal risings and settings, even at the cost of introducing errors into dates and longitudes of heliacal risings and settings, for the most serious errors would be only near the maximum limit. I really have no explanation for these problems, which would remain no matter which phenomena were used for the derivations.

The conversion between synodic arc and synodic time in the ephemerides of the superior planets is taken as constant, $\Delta T - \Delta\lambda = C$, or eliminating complete years, $\Delta t - \Delta\lambda = c$, where c is the difference between the mean values of Δt and $\Delta\lambda$. This produces notable errors at heliacal rising of Mars, but is far smaller for all other phenomena. The errors are greatest near, but not necessarily exactly at, the limits of the synodic arc and time, the minimum $(\Delta t - \Delta\lambda)_\mathrm{m}$ and maximum $(\Delta t - \Delta\lambda)_\mathrm{M}$. In Table 2 we give these to 0;30 computed from the values in Table 1 with the constant conversion c to minutes, and in parentheses the errors $(\Delta t - \Delta\lambda) - c$ to 0;30. The changes for Saturn are too small to be significant, since 11 and 12 are just the integers closest to c, but the reduction for $(\Delta t - \Delta\lambda)_\mathrm{M}$ of Jupiter from $+3$ for Γ to 0 for the other phenomena shows that they are more compatible with the constant conversion with small residual errors. The most striking reduction is the large error for Mars in Γ of $+12;30$, reduced greatly for the other phenomena, especially to 0 for Φ. By coincidence, the minimum of Θ is exactly 23;38, but the maximum, with an error of $+6$ shows that c is not a satisfactory conversion, and this is true of all four phenomena of Mercury, as shown in our original study.

Table 2. Errors of constant conversion of synodic time and arc

Planet		$(\Delta t - \Delta\lambda)_m$	c	$(\Delta t - \Delta\lambda)_M$
Saturn	Γ	12 (+0;30)	11;27	12 (+0;30)
	O	12 (+0;30)		11;30 (0)
	Θ	12 (+0;30)		11;30 (0)
	Φ	12 (+0;30)		11;30 (0)
Jupiter	Γ	11 (−1)	12; 5	15 (+3)
	O	11 (−1)		12 (0)
	Θ	11 (−1)		12 (0)
	Φ	11 (−1)		12 (0)
Mars	Γ	20 (−3;30)	23;38	36 (+12;30)
	O	22 (−1;30)		27 (+3;30)
	Θ	23;30 (0)		29;30 (+6)
	Φ	22;30 (−1)		23;30 (0)

We may also investigate the elongations of Θ from the mean sun implicit in the ephemerides as we did for the other phenomena in Part 3 of our study (Swerdlow, 1998, esp. 152-60) using intervals of time δt and longitude $\delta\lambda$ for the subdivision of the synodic arc and time in the intervals $\Phi \rightarrow \Theta \rightarrow \Psi$ from ephemerides (ACT), templates of undated longitudes (DCL), and procedure texts (ACT). It should, however, be noted that these are not consistent and produce different results depending upon the combination in which they are used. Also, the conditions that determine the elongation of Θ are sufficiently constraining that it may not be significant as an independent parameter. First, limits of the total retrograde arc $\delta\lambda$ and time δt for $\Phi \rightarrow \Psi$ without the division by Θ are given in Table 3 from various sources and from modern computation for the same number of synodic periods used earlier: Saturn 29, Jupiter 34, Mars 38. In each case one limit for arc is quite accurate, perhaps derived from two measured distances from a single normal star for each planet (although the one example of this I have found in the Diaries, distances of Jupiter from η Piscium in -70 IV 23 and VIII 20, gives a very erroneous arc of only $-4;25^\circ$ if 1 cubit $= 2;30^\circ$). But the range for Saturn and Jupiter, adjusted in the ratios of the fast and slow zones, is too large, and the range for Mars is far too small; the times are very schematic and cannot be pressed closely. The peculiar values for $\delta\lambda$ of Saturn from ACT 801.4-5 and 802.2-3 result from multiplying a velocity per day by an interval of time. Note that in the modern computation, for Saturn and Jupiter the variation of the arcs is very slight and the shorter times go with the longer arcs.

The method of finding the elongation of Θ is as follows: For the interval of time δt for $\Phi \rightarrow \Theta$ we compute the mean motion of the sun $\delta\lambda_s = \delta t \cdot v_s$, where the mean velocity of the sun $v_s = 6,0^\circ/6;11,4^\tau \approx 0;58,12,38^{\circ/\tau}$. There is little or no distinction of δt for $\Phi \rightarrow \Theta \rightarrow \Psi$ in the slow and fast zones in the procedure texts or ephemerides, although there are variant values—in fact the variation of time of retrogradation in days far exceeds the variation of arc in degrees—which is further evidence of the uncertainty of determining the time of stations. Then, indicating the slow and fast zones by $i = 1$ and $i = 2$, the difference of elongation in time δt is $\delta\eta_i = \delta\lambda_s - \delta\lambda_i$ and the elongation

Table 3. Total retrograde arc and time $\Phi \rightarrow \Psi$

Planet	Source	Ancient			$\delta\lambda_m$ Modern $\delta\lambda_M$	
		$\delta\lambda_m$	$\delta\lambda_M$	δt	δt	δt
Saturn	DCL A	$-6;40°$	$-8°$		$-6;38°$	$-6;58°$
	ACT 801.4–5, 802.2–3	$-7;\ 1,57,30$	$-8;26,20$	$1,52;30^{\tau}$	$142/43^{d}$	$133/34^{d}$
Jupiter	DCL D, ACT 813.2	-10	-12		$-9;48°$	$-10;5°$
	ACT 810.3-6, 813.9	$-8;20$	-10	$2,2^{\tau}$	$123/24^{d}$	$117/18^{d}$
Mars	S in DCL G-J	-15	-18		$-10;0°$	$-19;30°$
	ACT 501, 501a			$\sim 1,23^{\tau}$	$60/61^{d}$	$81/82^{d}$

of Θ is $\eta_{\Theta i} = \eta_{\Phi i} + \delta\eta_i$, where $\eta_{\Phi i}$ is taken from our earlier determination, but here as a positive rather than a negative elongation. Using the same procedure, we may then extend the computation from Θ to Ψ to find the elongation at Ψ by finding $\delta\eta_i$ for $\Theta \rightarrow \Psi$ and then $\eta_{\Psi i} = \eta_{\Theta i} + \delta\eta_i$, which we compare with our earlier determination, given here in the form $6,0° - \eta_{\Psi i}$ so that η_i increases through $\Phi \rightarrow \Theta \rightarrow \Psi$. What follows are examples of various possible computations.

We begin with Jupiter System A, with δt from ACT 813.23,30 and equal divisions of $\delta\lambda$ in the retrograde arc, as in DCL Text D:

Interval	δt	$\delta\lambda_s$	$\delta\lambda_1$	$\delta\lambda_2$	$\delta\eta_1$	$\delta\eta_2$
$\Phi \rightarrow \Theta$	58^{τ}	$56;16°$	$-5°$	$-6°$	$1,1;16°$	$1,2;16°$
$\Theta \rightarrow \Psi$	$1,\ 4$	$1,\ 2;\ 5$	-5	-6	$1,7;\ 5$	$1,8;\ 5$

Then, taking $\eta_{\Phi 1}$ and $\eta_{\Phi 2}$ from our earlier determination:

$\eta_{\Phi 1}$	$\eta_{\Phi 2}$	$\eta_{\Theta 1}$	$\eta_{\Theta 2}$	$\eta_{\Psi 1}$	$\eta_{\Psi 2}$
$1,53;42°$	$1,52;46°$	$2,54;58°$	$2,55;2$	$4,2;3°$	$4,3;7°$

We earlier found for $\eta_{\Psi 1}$ $4,2;4°$ and for $\eta_{\Psi 2}$ $4,3;8°$, the slight difference due to roundings, and here we see that $\eta_{\Theta} \approx 2,55°$, about $5°$ short of mean opposition.

With the same δt but unequal divisions of $\delta\lambda$ in the retrograde arc, as in ACT 813.2 and 814.2, we have:

Interval	δt	$\delta\lambda_s$	$\delta\lambda_1$	$\delta\lambda_2$	$\delta\eta_1$	$\delta\eta_2$
$\Phi \rightarrow \Theta$	58^{τ}	$56;16°$	$-4°$	$-4;48°$	$1,0;16°$	$1,1;\ 4°$
$\Theta \rightarrow \Psi$	$1,\ 4$	$1,\ 2;\ 5$	-6	$-7;12$	$1,8;\ 5$	$1,9;17$

And again with the same values of $\eta_{\Phi 1}$ and $\eta_{\Phi 2}$:

$\eta_{\Phi 1}$	$\eta_{\Phi 2}$	$\eta_{\Theta 1}$	$\eta_{\Theta 2}$	$\eta_{\Psi 1}$	$\eta_{\Psi 2}$
$1,53;42°$	$1,52;46°$	$2,53;58°$	$2,53;50$	$4,2;3°$	$4,3;7°$

Now $\eta_{\Theta} \approx 2,54°$, about $6°$ short of mean opposition.

There is less information on the division of the interval $\Phi \rightarrow \Theta \rightarrow \Psi$ for System A′, but ACT 611 contains $\Phi \rightarrow \Theta$, although with the rather high δt of $1,1^{\tau}$ or $1,2^{\tau}$, and ACT 612 contains $\Theta \rightarrow \Psi$ with the very low δt of 52^{τ} or 53^{τ}. Since the two texts are not consistent in dates or longitudes of Θ, they cannot be used together and so we shall consider only $\Phi \rightarrow \Theta$ in ACT 611, taking the slow and fast arcs 1 and 3 and omitting the equal transitional arcs 2 and 4:

Interval	δt	$\delta\lambda_s$	$\delta\lambda_1$	$\delta\lambda_3$	$\delta\eta_1$	$\delta\eta_3$
$\Phi \to \Theta$	62^τ	$1,0;9^\circ$	-4°	$-4;48^\circ$	$1,4;9^\circ$	$1,4;57^\circ$

And taking η_Φ for System A′,

$\eta_{\Phi 1}$	$\eta_{\Phi 3}$	$\eta_{\Theta 1}$	$\eta_{\Theta 3}$
$1,53;35^\circ$	$1,52;37^\circ$	$2,57;44^\circ$	$2,57;34^\circ$

Now $\eta_\Theta \approx 2,58^\circ$, only 2° short of mean opposition. Because of the high value of δt, I am particularly suspicious of this result.

For Saturn System A, we take δt and $\delta\lambda$ as given or implied in ACT 801.4-5 and 802.2-3, and round to minutes, from which:

Interval	δt	$\delta\lambda_s$	$\delta\lambda_1$	$\delta\lambda_2$	$\delta\eta_1$	$\delta\eta_2$
$\Phi \to \Theta$	$52;30^\tau$	$50;56^\circ$	$-3;42^\circ$	$-4;26^\circ$	$54;38^\circ$	$55;22^\circ$
$\Theta \to \Psi$	1, 0	58;13	$-3;20$	-4	1, 1;33	1, 2;13

Note that, surprisingly, the longer $\delta\lambda$ occurs in the interval with the shorter δt. And taking $\eta_{\Phi 1}$ and $\eta_{\Phi 2}$ from our earlier determination:

$\eta_{\Phi 1}$	$\eta_{\Phi 2}$	$\eta_{\Theta 1}$	$\eta_{\Theta 2}$	$\eta_{\Psi 1}$	$\eta_{\Psi 2}$
$2,2;8^\circ$	$2,1;26^\circ$	$2,56;46^\circ$	$2,56;48^\circ$	$3,58;19^\circ$	$3,59;1^\circ$

Here $\eta_\Theta \approx 2,57^\circ$, about 3° before mean opposition; η_Ψ cannot be compared with our original derivation, for which we used DCL Text A in which $\delta\lambda$ extends through $\Phi \to \Psi$ without Θ and differs from $\delta\lambda$ used here. Note, however, that η_Φ is close to $2,0^\circ$ and η_Ψ is close to $4,0^\circ$, which shows the simple assumptions underlying the treatment of stations and retrogradations.

For Mars, although mean elongations of each phenomenon and mean values of δt in the intervals $\Omega \to \Gamma \to \Phi \to \Omega$ are known from ACT 811a, and $\delta\lambda$ is known for $\Phi \to \Theta$ from the four methods R, S, T, U—and $\Theta \to \Psi$ is also known from S—there is little information on δt in the retrograde arc. In ACT 500 δt for $\Phi \to \Theta$ is a constant, and rather long, $47;55,4^\tau$, and by taking corresponding synodic periods for Seleucid Era 170–187 in ACT 501 and 501a, which is not really secure since the resulting retrograde arcs are far too long, δt for $\Phi \to \Psi$ is $1,22^\tau$, $1,23^\tau$ or $1,24^\tau$, also nearly constant but in excellent agreement with the maximum $\delta t \approx 1,23^\tau$ by modern computation (the minimum $\delta t \approx 1,1^\tau$ is not found in any source). Still, with this information it is possible to find an elongation for Θ, although the result may not be significant. We shall take η_Φ from ACT 811a, δt for $\Phi \to \Theta$ from ACT 500, and $\delta\lambda$ from T and U, as they have the widest range, using only the limits for zones 1 and 4. We thus have:

Interval	δt	$\delta\lambda_s$	$\delta\lambda_1$	$\delta\lambda_4$	$\delta\eta_1$	$\delta\eta_4$
$\Phi \to \Theta$	$47;55,4^\tau$	$46;27^\circ$	$-7;30^\circ$	-6°	$53;59^\circ$	$52;29^\circ$

Since the mean value, the only one known, $\eta_\Phi = 2,0^\circ$,

$$\eta_{\Theta 1} = 2,53;59^\circ, \quad \eta_{\Theta 4} = 2,52;29^\circ,$$

and acronychal rising is about 6° or $7;30^\circ$ before mean opposition.

We may also investigate the implied elongation of Ψ using only $\delta\lambda_M = -18^\circ$ from S and $\delta t \approx 1,23^\tau$. The results are:

Interval	δt	$\delta\lambda_s$	$\delta\lambda_M$	δ_η	η_Φ	η_Ψ
$\Phi \to \Theta$	$1,23^\tau$	1,20;31°	−18°	1,38;31°	2,0°	3,38;31°

η_Ψ is about 21;30° from symmetry to η_Φ at 4,0°. The reason is that η_Φ is too small, for correctly it extends from about 2,9° near aphelion to 2,26° near perihelion, the smaller elongations thus corresponding to the longer $\delta\lambda$ and δt. Hence, if we take $\eta_\Phi \approx 2{,}9°$ and $\delta\eta \approx 1{,}39°$, $\eta_\Psi \approx 3{,}48°$, only 3° from symmetry at 3,51°. More accurate modern parameters produce still closer symmetry of η_Φ and η_Ψ around aphelion and perihelion, as is to be expected.

In summation, we have found that acronychal rising for each superior planet falls short of mean opposition, $\eta = 180°$, by the following amounts, which are only approximate:

Saturn: 3° Jupiter: 5°,6°,2° Mars: 6°,7;30°

I do not know whether *any* of these results is significant—they cannot all be significant and yet other computations can produce different results—except for showing that acronychal rising occurs at a mean elongation less than 180°. (The true elongations by modern computation, given in the Appendix, are larger and, for Mars, highly variable.) The reason is that the elongation of Θ is determined by the elongation of Φ, perhaps also the elongation of Ψ, and the length and division of the retrograde arc and time, some of which seem to be chosen as convenient values of strictly unknown quantities, perhaps with the condition for Θ that its elongation be less than 180°. Nevertheless, the elongation of Θ could be used to establish a longitude for the phenomenon λ_p by addition to the computed mean longitude of the sun λ_s, that is, $\lambda_p = \lambda_s + \eta_\Theta$, where η_Θ may differ for each planet or be taken as some conventional value for all three. In this way, an acronychal rising, just as a heliacal rising, with a known elongation, assumed to be fixed, could be used to find an epoch. The subdivisions of the synodic arc could then be used to set the initial longitudes of the remaining phenomena in an ephemeris, just as if heliacal rising were used as the primary phenomenon.

Appendix: computation of acronychal risings

For the rising of a planet to be visible, the sun must be below the horizon a minimal distance, called the *arcus visionis*, which is illustrated in Figure 4 for both heliacal and acronychal risings of a superior planet. The observer is at O and the planet P is rising at the eastern horizon. In a heliacal rising, which takes place before sunrise, the sun S_Γ

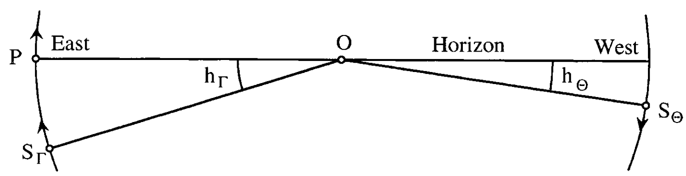

Fig. 4. Heliacal and acronychal *arcus visionis*

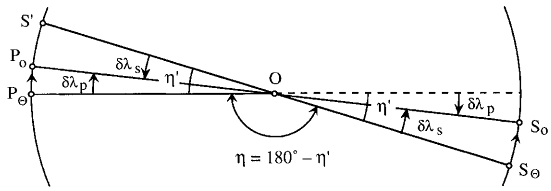

Fig. 5. Elongation of acronychal rising and opposition

is below the eastern horizon by a distance h_Γ, and in an acronychal rising, which takes place after sunset, the sun S_Θ is below the western horizon by a distance h_Θ. The arcus visionis h_Θ is less than h_Γ because (1) the planet is brighter at acronychal rising, near opposition, than at heliacal rising, near conjunction, and (2) the eastern horizon is darker in the evening, when the sun is below the western horizon, than in the morning, when the sun is at an equal distance below the eastern horizon.

The relation of elongation at acronychal rising and opposition is shown in Figure 5, which is not a horizon diagram. The planet at acronychal rising is P_Θ, the true sun is S_Θ, and the projection of the true sun to the 'antisolar point', for which the elongation of the planet at acronychal rising is to be computed, is S'. If we let elongation $P_\Theta S' = \eta'$, then the elongation from the true sun $P_\Theta S_\Theta = \eta = 180^\circ - \eta'$. We may then find η' using the method described by Neugebauer (HAMA, 234ff.) and van der Waerden (1942) for heliacal risings and settings, which is in fact Ptolemy's method and, as is worth noting, is still used in tables for heliacal phases without essential change in over eighteen hundred years. To reach opposition, which follows acronychal rising, the sun moves a distance $\delta\lambda_\mathrm{s}$ to S_o and the planet moves retrograde a distance $\delta\lambda_\mathrm{p}$ to P_o; it can be seen that $\delta\lambda_\mathrm{s} + \delta\lambda_\mathrm{p} = \eta'$ and always $\delta\lambda_\mathrm{s} \gg \delta\lambda_\mathrm{p}$.

Figure 6 shows the configuration for acronychal rising at the eastern horizon for both positive and negative latitudes of the planet. The ecliptic intersects the horizon at the horoscopus H with the horizon angle ν, and the sun, which is below the western horizon, is projected to the antisolar point S', *above* the eastern horizon by the *arcus*

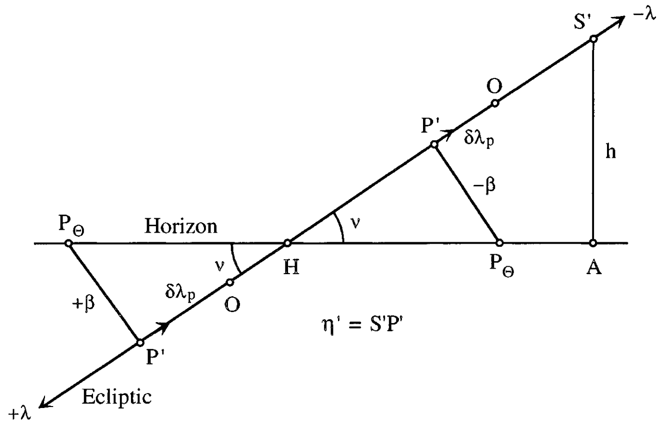

Fig. 6. Computation of elongation of acronychal rising

visionis $h = S'A$. The planet P_Θ, shown at the horizon with positive latitude $+\beta$ and negative latitude $-\beta$, is projected to ecliptic longitude at P'. Since the arcs are small, we may use plane triangles. In triangle $S'AH$, $S'H = h/\sin\nu$, and in triangle $HP'P_\Theta$, $P'H = \beta/\tan\nu$. Thus, for both positive and negative latitudes of the planet, the elongation at acronychal rising from the antisolar point, $S'P' = \eta'$, and the interval of time δt between opposition and acronychal rising are given by

$$\eta' = \frac{h}{\sin\nu} + \frac{\beta}{\tan\nu}, \tag{1}$$

$$\delta t = \frac{\eta'}{v_\eta}, \tag{2}$$

where v_η is the velocity of elongation, found from the velocities of the sun and planet, $v_\eta = v_\mathrm{s} - v_\mathrm{p}$; since the planet is moving retrograde $v_\eta > v_\mathrm{s}$. Given the dates of two oppositions T_1 and T_2, the synodic time between oppositions $\Delta T_\mathrm{o} = T_2 - T_1$; and with the intervals δt_1 and δt_2 from opposition to acronychal rising, the synodic time between acronychal risings ΔT_Θ is

$$\Delta T_\Theta = (T_2 - \delta t_2) - (T_1 - \delta t_1) = \Delta T_\mathrm{o} + (\delta t_1 - \delta t_2). \tag{3}$$

There are various approximations in this method. One is the use of plane for spherical triangles. A second is that the longitude of H, used for finding ν, is taken as the longitude of opposition, which actually lies at a lesser longitude than P', say, at O, since the planet is moving retrograde. One can compensate for this by an iterative computation, first finding ν for $\lambda_\mathrm{H} = \lambda_\mathrm{o}$ and computing η' and δt from (1) and (2). Then compute the motion of P' from $\delta\lambda_\mathrm{p} = \delta t \cdot v_\mathrm{p}$, and $\lambda'_\mathrm{p} = \lambda_\mathrm{o} - \delta\lambda_\mathrm{p}$, noting that $\delta\lambda_\mathrm{p} < 0$. Next find $P'H = \beta/\tan\nu$, observing the sign of β, and then $\lambda_\mathrm{H} \approx \lambda'_\mathrm{p} - P'H$. The computation of η' and δt is then repeated with the better value of ν for the corrected λ_H. However, the iteration makes virtually no difference for computing synodic times from acronychal risings. Hence, in computing Δt in the figures, we have taken ν for $\lambda_\mathrm{H} = \lambda_\mathrm{o}$ without further correction. This approximation is not safe for dates of heliacal risings and settings, particularly of Mars, and van der Waerden (1942) provides a table for making the correction from the longitude of the sun.

A more important question concerns h, the *arcus visionis* for acronychal rising, for which there are, to my knowledge, no established values. For heliacal rising and setting, the modern values of h are those of Schoch, which are the basis of the tables of P.V. Neugebauer (1938) and van der Waerden (1942). Those for the superior planets are:

Saturn	Jupiter	Mars
Γ 13° Ω 10°	Γ 9° Ω 7.5°	Γ 13° Ω 10°

The values for Θ must be less, but by how much? From P.V. Neugebauer (1932), the range of magnitudes at Γ and Θ is:

	m	Γ	M	m	Θ	M
Saturn	1.6		1.1	1.2		0.6
Jupiter	−1.2		−1.8	−2		−2.7
Mars	2.1		1.3	−0.7		−2.5

Since the planets, Mars in particular, are brighter near Θ than near Γ, and since for the same value of h the eastern horizon is darker after sunset than before sunrise, I have used $h = 8^\circ$ for Saturn and $h = 6^\circ$ for Jupiter and Mars. It is possible that h is low for Saturn and high for Jupiter, and no single value would seem to be adequate to the wide variation of brightness of Mars.

As for the effect of h, we have computed Δt for eight synodic periods of Mars, just over one rotation of phenomena through the zodiac, for h from 12° to 0° in 2° steps for the latitude of Babylon, 32.5°. We find that, although h can produce intervals between opposition and acronychal rising of as much as -14 days for $h = 12^\circ$, its effect on Δt between acronychal risings is far smaller. Thus, to take the maximum synodic time in tithis over $12{,}0^\tau$, which shows the greatest change due to h:

Opp.	$h = 12^\circ$	10	8	6	4	2	0
$1{,}44^\tau$	1,44	1,44	1,45	1,46	1,47	1,48	1,48

Three synodic times vary by 2, 3 by 1, and 1 by 0 due to h, although there are differences of up to -6 from Δt at opposition due to latitude β. Hence, even for the large range of synodic times of Mars, it does not matter greatly what value of h is used. From checking $h = 10^\circ, 8^\circ, 6^\circ, 4^\circ$ for the greatest and least synodic times, we have found a variation due to h of about -0.3^d to $+0.3^d$ for Saturn and $+0.7^d$ to -0.1^d for Jupiter, although other synodic periods may show greater variation. Heliacal risings and settings are more sensitive, particularly at higher geographical latitudes, and the entire method of computing heliacal and acronychal phenomena is not exactly secure. Finally, it can show only what could have been observed, not what was actually observed.

It is of interest to examine the range of elongation η', interval of time δt, and motion of the planet $\delta\lambda$ between opposition and acronychal rising, and of the change dt in the synodic time Δt. The limits that we have found in computing Δt for the figures are:

	η'_m	η'_M	δt_m	δt_M	$\delta\lambda_m$	$\delta\lambda_M$	dt_m	dt_M
Saturn	8.5°	10.5°	8.1^d	10.1^d	0.62°	0.76°	-0.2^d	$+0.3^d$
Jupiter	6.3	8.4	5.8	7.3	0.7	0.98	-0.4	$+0.6$
Mars	1.4	11.3	1.1	7.9	0.3	3.1	-5.3	$+3.4$

Note that Saturn has the longest interval δt, but the least change dt in Δt, and the limits of dt do not correspond to the limits of δt, but occur where δt is changing the most rapidly. The locations of the limits are also of interest. The minimum of η', δt, and $\delta\lambda$ for Jupiter are near aphelion and the maximum near perihelion, while for Mars these are reversed. For Saturn the predominant effect is latitude, and the minimum values are (roughly) near the northern limit and the maximum near the southern.

To summarize what we have done: The synodic times from acronychal risings in Figures 1–3 B are computed from formulas (1–3) with $h = 8^\circ$ for Saturn and $h = 6^\circ$ for Jupiter and Mars, taking ν from Table 6 in van der Waerden (1942) for the latitude of Babylon, 32.5°, and assuming that $\lambda_H = \lambda_o$ without adjustment. The range of $\delta\lambda_p$ between opposition and acronychal rising for the values of h we have used is, as above: Saturn, 0.6°–0.8°, Jupiter, 0.7°–1°, Mars, 0.3°–3°, all positive, and the large values for Mars occur in the shortest synodic arcs. Rather than compute these individually, we have advanced longitudes 1° for Saturn and Jupiter and 2° for Mars from opposition. Anything more refined would not be noticeable in the scale of the figures.

Acknowledgment. I wish to thank John Britton for invaluable advice in letters while writing this paper, above all concerning evidence for the use of stations and the method of computing acronychal risings, which he suggested to me in the first place.

Corrections to Swerdlow, 1998:
p. 12 l. 35: for stand read stands.
p. 124 l. 10: for 0 to from 10 read 0 to 10.
p. 163 l. 26: for 27;43 $\approx$ 28 read $-$27;43 $\approx -$28.
p. 195 Table 1.5: Saturn: $\Pi = 4{,}16$ $\Upsilon = 4{,}25$.
p. 199 Table 2.2A. l. 21: for $-$124 VI 12 read $-$123 VI 12; Per = 1.
p. 202 Table 2.3: l. 12: Ω $-$663: $\Delta t = 48^{\mathrm{r}}$; correct Fig. 2.4 accordingly.
p. 203 Table 2.4: Saturn: $\Pi = 4{,}16$.
p. 206 Table 2.7A: l. 3: for -306 read -308; l. 5: after ♒ omit 'd'.
Bibliography p. 242: Thompson, R.C.: for Ninevah read Nineveh.
add: Bottéro, J. 1992. *Mesopotamia. Writing, Reasoning, and the Gods*. Chicago.
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DCL Aaboe, A., A.J. Sachs. Some Dateless Computed Lists of Longitudes of Characteristic Planetary Phenomena from the Late Babylonian Period. *Journal of Cuneiform Studies* 20 (1966), 1–33
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