



# Pipe flow: a gateway to turbulence

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Received: 14 August 2020 / Published online: 2 October 2020  
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## Abstract

Pipe flow has been a challenge that gave rise to investigations on turbulence—long before turbulence was discerned as a research problem in its own right. The discharge of water from elevated reservoirs through long conduits such as for the fountains at Versailles suggested investigations about the resistance in relation to the different diameters and lengths of the pipes as well as the speed of flow. Despite numerous measurements of hydraulic engineers, the data could not be reproduced by a commonly accepted formula, not to mention a theoretical derivation. The resistance of air flow in long pipes for the supply of blast furnaces or mine air appeared even more inaccessible to rational elaboration. In the nineteenth century, it became gradually clear that there were two modes of pipe flow, laminar and turbulent. While the former could be accommodated under the roof of hydrodynamic theory, the latter proved elusive. When the wealth of turbulent pipe flow data in smooth tubes was displayed as a function of the Reynolds number, the empirically observed friction factor served as a guide for the search of a fundamental law about turbulent skin friction. By 1930, a logarithmic “wall law” seemed to resolve this quest. Yet pipe flow has not been exhausted as a research subject. It still ranks high on the agenda of turbulence research—both the transition from laminar to turbulent flow and fully developed turbulence at very large Reynolds numbers.

## Abbreviations

DMA	Deutsches Museum, Archiv, München
GALCIT	Guggenheim Aeronautical Laboratory of the California Institute of Technology
GOAR	Göttinger Archiv des Deutschen Zentrums für Luft- und Raumfahrt
MPGA	Max-Planck-Gesellschaft, Archiv, Berlin
NACA	National Advisory Committee for Aeronautics

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Communicated by Olivier Darrigol.

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## 1 Introduction

Turbulence emerged as a research field in its own right only in the twentieth century. Turbulent pipe flow, on the other side, has been observed much earlier by practitioners concerned with the design of water conduits. Pipe flow became the subject of hydraulics, a science as old as civilization. For centuries, pipe flow and hydraulics were almost synonymous. The word “hydraulics” amalgamates the Greek words for water (hydor) and pipe (aulos). For our ancestors in antiquity, hydraulics involved practical experience with pipe flow in conduits for water supply and in sophisticated devices such as water clocks or water organs (Rouse and Ince 1957, Chapters 1–3). Prior to the seventeenth century, however, there is no evidence of systematic pipe flow investigations, not to speak of turbulence.

There are other empirical gateways to turbulence, such as the wake of bodies moving through water and air, jets, atmospheric turbulence, etc., but pipe flow deserves particular interest as the oldest gateway to turbulence—followed and accompanied by open channel flow—because of its long-standing practical importance. The history of pipe flow sheds light on the gradual evolution of an engineering concern into a fundamental question of science. My narrative leads from the earliest pipe flow measurements for the water art at Versailles in the late seventeenth century (2) via efforts of hydraulic experts in the late eighteenth and early nineteenth century (3 and 4) and new challenges arising with gaseous fluids in long pipes (5) to the establishment of the Darcy–Weisbach equation—with a friction factor as subject of further investigation (6). The following sections focus on the growing awareness of two modes of flow, laminar and turbulent, that culminated (but not originated) in the landmark paper of Osborne Reynolds (7) which in turn enabled the display of the friction factor as a function of the Reynolds number and the establishment of “Blasius’ law” for turbulent pipe flow (8). The failure of contemporary theoretical attempts to determine a critical Reynolds number for the onset of turbulence was proclaimed in 1921 as the “turbulence problem” (9). By the same time, “Blasius’ law” became the starting point for theories about turbulent skin friction. It launched a rivalry between Ludwig Prandtl and Theodore von Kármán for a universal “wall law” and a series of new pipe flow experiments at high Reynolds numbers (10). Although the quest for a basic understanding of wall-bounded turbulence appeared concluded by 1930 by a logarithmic law that was confirmed by the measurements, the achievement became rather a new beginning than an end of pipe flow investigations as I briefly sketch in an outlook on later developments (11). Experiments and theories on pipe flow still rank high on the agenda of turbulence research. The unbroken topicality of pipe flow over several centuries suggests more general conclusions, particularly concerning the historiography and epistemology of turbulence and other research with a long history between science and engineering (12).

## 2 The conduits at Versailles

The first published record of pipe flow measurements concerned the requirements for the water art at Versailles and other royal gardens of the “Sun King”, Louis XIV, such as the height of fountain jets driven from the discharge of elevated water reservoirs. Under these premises, hydraulics was put on the agenda of the newly founded royal academy in Paris (Blay 1986). Two years after its foundation, on 11 July 1668, the academicians Jean Picard and Edme Mariotte were charged with a study of “the force of running water for pressing and moving”<sup>1</sup>—a task that led to a number of experimental investigations on the relation of water discharge through pipes from reservoirs at different heights and through circular holes of various diameter. The results were published in academy memoirs and summarized in a *Traité du Mouvement des Eaux et des Autres Corps Fluides* (Mariotte 1686). Not least because of this work, published posthumously by his academic colleague Philippe de La Hire, Mariotte is regarded as “the father of the experimental method in France” (Rouse and Ince 1957, p. 63).

The pipe flow measurements at Versailles were not a one-time event, nor were Mariotte and Picard the only academicians concerned with this investigation. In the 1660s and 1670s, Versailles and its water supply system was a giant construction site under permanent academic surveillance with regard to the levelling of the canals, pipes and reservoirs for the fountains in the park (Descamps 2003). It involved outstanding scholars such as Christiaan Huygens and Ole Rømer as well as less known academicians like Claude Antoine Couplet, for whom Versailles presented the opportunity to develop his hydraulic skills with pipe flow measurements until he rose to prominence as the academy’s permanent treasurer (de Fontenelle 1722). In 1696, Couplet’s son, Pierre, also became a member of the academy—and a collaborator of his father on the pipe flow experiments at Versailles (Heyman 1976). Many years later, on 22 March 1732, he presented to the academy a comprehensive report about these investigations (Couplet 1932). The “rules” of pipe flow, as reported in Mariotte’s treatise, were derived from measurements that did not regard the lengths of the conduit as an important variable. Couplet junior criticized the earliest pipe flow measurements at Versailles because they could not answer the crucial question about the resistance of the water in the conduits:

This is a question which despite its importance has hardly been clarified; its solution seems to depend on the knowledge of the friction of the water in the pipes in relation to the different diameters, lengths and speed of the water.<sup>2</sup>

Couplet argued that only a great number of experiments could provide this knowledge and that Versailles was the perfect site to perform these measurements. By the early eighteenth century, water supply systems did not yet involve large pipes made from cast iron like those at Versailles, where only the best was good enough to achieve the King’s demands. Couplet’s experiments focused on the discharge of

<sup>1</sup> “de la force des eaux courantes à presser et à mouvoir” (Blay 1986, p. 93).

<sup>2</sup> “C’est une question qui, toute importante qu’elle est, n’a point encore été éclaircie, et dont il paroît que la solution dépend de la connoissance du frottement des eaux dans leurs conduites par rapport à différents diamètres et longueurs différentes, et par rapport à leurs différentes vitesses” (Couplet 1932, p. 114).

water through five pipes made of cast iron that varied in length from 296 to 2340 toises (1 toise=6 feet=1.83 m) and in diameter from 4 to 18 pouces (1 pouce=1 inch=2.54 cm). Through these long pipes, the discharge was quite different from what was to be expected according to Mariotte's rules. Couplet attributed this discrepancy to the friction of the water against the inner wall of the pipes:

It is astonishing that this friction of the water against the walls of this pipe caused a decrease in the flow of water about 30 times greater than the quantity of water which came out through this pipe.<sup>3</sup>

The conduits included elbows where trapped air disrupted the water column in the pipes. In the case of the longest conduit, it could take more than ten days until the water from the reservoir was discharged. Couplet did not arrive at a formula for pipe flow friction, but the conclusion from his measurements was obvious: Friction retarded the flow of water in the pipes considerably—the smaller the pipe diameter and the longer the conduit and the larger the flow speed, the more so. Elbows and bends added to the friction.

Couplet's measurements were regarded important enough to be included 5 years later in Bernard Forest de Bélidor's famous *Architecture hydraulique* (Bélidor 1737, Livre IV, Chapitre II). They deserved particular interest also because of their location at Versailles, "perhaps the only place of the world where one finds everything one could wish in order to perform experiments of that sort we are discussing here".<sup>4</sup> He paid tribute to Couplet by extensive quotes from his academy memoir, including a copper engraving that showed the profile of the five conduits at which the measurements had been performed (Fig. 1).

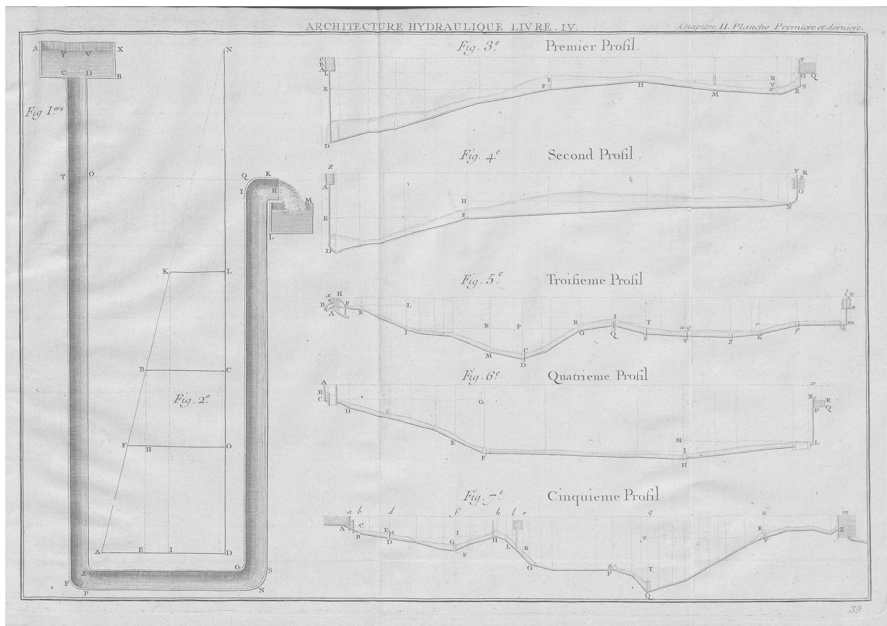
### 3 Pipe flow accounts in the late eighteenth century

As soon as metallic pipes became used more broadly for the water supply of cities, pipe flow measurements on long conduits were no longer confined to royal gardens. In 1771, Charles Bossut, professor of mathematics at the École du Génie in Mézières, published a treatise on hydrodynamics where he included the results of his measurements from conduits used for the water supply of Mézières (Bossut 1771, chapter VI). By and large, he agreed with Couplet's findings, so that he allowed himself to proceed one step further to "form a general and sufficient practical idea of the loss of speed that water makes in pipes, either straight or curvilinear".<sup>5</sup> Instead of a formula for the

<sup>3</sup> "Il est étonnant que ce frottement de l'eau contre les parois de ce tuyau ait causé une diminution d'écoulement d'eau environ 30 fois plus grande que la quantité d'eau qui est sortie par cette conduite" (Couplet 1932, p. 148).

<sup>4</sup> "...depuis que M. Couplet a donné dans les Mémoires de l'Académie Royale des Sciences, année 1732, un détail bien circonstancié de toutes les opérations qu'il a faites autrefois avec M. son père et M. Villard, sur la dépense des tuyaux de conduite qui amènent l'eau dans les réservoirs de Versailles, qui est peut-être le seul endroit au monde où l'on trouve tout ce que l'on peut désirer, pour faire d'expériences de la nature de celles dont nous parlons" (Bélidor 1737, p. 273).

<sup>5</sup> "En combinant nos expériences avec celles de M. Couplet, on se formera une idée générale et suffisante dans la pratique, de la perte de vitesse que l'eau fait dans les tuyaux de conduite, soit rectilignes, soit curvilignes" (Bossut 1771, p. 159).



**Fig. 1** The five conduits at Versailles used for Couplet's pipe flow experiments (Belidor 1737, Livre IV, Chapitre II, plate I)

friction, he provided tables for the presentation of his results from which a user might choose the case closest to the one he wants to treat.

As a member of the Paris academy, Bossut's expert knowledge was requested also in other hydraulic projects. In 1776, for example, he witnessed water raising trials with the famous machine de Marly. This giant machine pumped water via intermediate reservoirs up to an aqueduct 162 m above the level of the Seine from where it was conducted to Versailles. The plan was to dispose of the intermediate reservoirs and pump the water in a single step up to the aqueduct. However, the pipes used for this purpose proved inadequate. These are matters, Bossut wrote to explain the impotence of his expertise in this affair, "whose knowledge is based on practical trial and error and which can only be imperfectly appreciated by theory and reasoning".<sup>6</sup>

Bossut's pipe flow measurements called for more extensive investigations about the friction in long conduits. "I devoured the part of Abbé Bossut's Hydrodynamics, which deals with the movement of water", Pierre du Buat introduced his *Principes d'hydraulique, vérifiés par un grand nombre d'expériences faites par ordre du gouvernement*. He regarded the pipe flow data as "the key to hydraulics". Du Buat reasoned that the flow on an inclined river bed reached a uniform speed only because the accelerating force of gravity was balanced by friction, and

<sup>6</sup> "...dont la connaissance tient à un tâtonnement pratique et qui ne peuvent être qu'imparfaitement appréciés par la théorie et le raisonnement". Quoted in Brandstetter (2006, p. 186). See also Barbet (1907, pp. 141–142).

that the movement of water in a pipe had a great analogy with the uniform course of a river bed, since in both cases gravity was the motor, and the resistance of the bed the moderator. I therefore made use, in order to make a formula of uniform movement, of the experiments of Abbé Bossut on the pipes, and even of those he had made on dummy canals, although one datum was missing, which was the depth of the current. This is how I composed the work that I gave to the public, under the title of Principles of Hydraulics, in the year 1779.<sup>7</sup>

As indicated by the title of du Buat's treatise, his investigations were commissioned by the government, more precisely by the war ministry which also placed at his disposal two officers from the Royal Engineering Corps for performing the experiments. Unlike Bossut's pipes which had only a "mediocre" slope, du Buat's experiments comprised very small and very large inclinations. The slope was calculated by dividing the length  $l$  of the conduit by the height  $h$  of the reservoir from which the water was discharged through the pipes. Furthermore, du Buat and his officers experimented with pipes bent in different ways, comparing the results with those of straight pipes of the same diameter and length. In his treatise, du Buat reported the results of 56 experiments displayed in tables with five entries for each measurement: the length of the pipes, the height of the reservoir, the slope, the experimentally measured velocity (derived from the discharge divided by the cross section of the pipe) and the computed velocity calculated from his formula (du Buat 1786, p. xxv and pp. 71–74).

#### 4 Early pipe flow formulae

Du Buat's formula did not distinguish between pipe flow and open channel flow. His basic assumption for both kinds of flow was that equilibrium was established by a balance of the gravitational force and friction as the retarding force. The former was related to the slope  $b = l/h$ , and the latter to a quantity  $r = a/p$  ("rayon"), where  $a$  is the cross section and  $p$  the wetted perimeter. Underlying these assumptions was a particular concept of fluid coherence that imagined a kind of gearing between the fluid molecules (Darrigol 2005, chapter 3.1.2). This view resulted in a retarding force proportional to the average velocity of the fluid molecules, at a rate which was itself proportional to the velocity. Therefore, the friction would be proportional to the square of the velocity  $u^2$  multiplied by  $r$ . Du Buat finally arrived at the following formula for the flow velocity (du Buat 1786, p. 63):

$$u = \frac{297(\sqrt{r} - 0, 1)}{\sqrt{b} - \ln\sqrt{b+1}, 6} - 0, 3(\sqrt{r} - 0, 1).$$

<sup>7</sup> "que le mouvement de l'eau dans un tuyau de conduite avait une grande analogie avec le cours uniforme d'un lit de rivière, puisque de part et d'autre la pesanteur était le moteur, et la résistance du lit le modérateur. Je me servis donc, pour faire une formule du mouvement uniforme, des expériences de M. l'abbé Bossut sur les tuyaux de conduite, et même de celles qu'il avait faites sur des canaux factices, quoiqu'il y manquât une donnée, qui était la profondeur du courant. C'est ainsi que je composai l'ouvrage que j'ai donné au public, sous le titre de Principes d'Hydraulique, en l'année 1779" (du Buat 1786, pp. xvi–xx).

The units were Parisian foot. The formula was not derived from a fundamental theory but rather amalgamated theoretical speculation with experimental measurements.

Du Buat's account on pipe flow marked the beginning of an era in the history of fluid mechanics in which experiments challenged pre-conceived theoretical concepts about the nature of fluid resistance (Darrigol 2005, pp. 221–222). Pioneers of ideal flow theory in the eighteenth century, such as Johann and Daniel Bernoulli (father and son) or Leonhard Euler, contributed little to elucidate pipe flow friction. Euler's effort in this regard was largely ignored—and would have been doomed to failure if it had been scrutinized by hydraulic engineers (Bistafa 2015). Both Bossut's *Traité élémentaire d'hydrodynamique* and du Buat's *Principes d'Hydraulique*, however, were widely appreciated as valuable contributions. They encouraged further investigations on pipe flow, which were translated into German. “This book certainly is one of the most useful and best ones that has appeared for some time in hydraulics”, Reinhard Woltmann, a German pioneer in hydraulic engineering, praised du Buat's treatise. “Theory and experience go together step by step, and it testifies on almost every page to the skill, diligence and love of truth of the author”. The “almost” alluded to du Buat's speculations on the nature of friction. “I confess that I don't like the author's imagination on the resistance due to friction”, Woltmann qualified his praise. He missed a more detailed consideration of the movement of the water particles close to the wall where, in his view, “they become hindered by collisions, deviated into other directions and into an eddying motion whereby the entire flow is decelerated”. Furthermore, Woltmann was unimpressed by du Buat's formula with which the experimental data were fitted. By means of arbitrary quantities, “Mr. Buat has modified his formula until it fitted precisely all trials”.<sup>8</sup>

Another German expert on hydraulics, Johann Albert Eytelwein, who had started his career in the Prussian administration of constructions and co-founded in 1799 the Berlin Building Academy (Bauakademie), also paid tribute to du Buat in a study “On the motion of water in conduit pipes”.<sup>9</sup> He praised the survey of Couplet's, Bossut's and du Buat's pipe flow measurements and the effort to arrive at a “general expression” for these results, but he, too, did not like du Buat's formula—even though for other reasons than Woltmann. He regarded the formula as largely useless “because of its complicated form”.<sup>10</sup> Eytelwein derived from the same experiments the following formula:

<sup>8</sup> “Dieses Buch ist gewiss eines der nützlichsten und besten, die seit einiger Zeit in der Hydraulik erschienen sind. Theorie und Erfahrung gehen in demselben Schritt für Schritt beisammen; und es zeugt von der Geschicklichkeit, dem Fleiße und der Wahrheitsliebe des H. Verf. fast auf allen Seiten. [...] Ich gestehe, dass mir des H. Verf. Vorstellungsart über den Widerstand wegen des Reibens nicht gefällt [...] Dagegen stoßen die Teilchen an, werden aufgehalten, in andere Richtungen und mancherlei wirbelnde Bewegung gebracht, wodurch der ganze Strom verzögert wird [...] hat Hr. Buat seine Formel so lange abgeändert bis sie auf alle Versuche genau passte” (Woltmann 1791, pp. 142, 152, 162). On Woltmann's career see Rühlmann (1885, pp. 280–283).

<sup>9</sup> “Von der Bewegung des Wassers in Röhrenleitungen” (Eytelwein 1801, 9. Kapitel). On Eytelwein's career, see Rühlmann (1885, pp. 284–289).

<sup>10</sup> “Herr du Buat hatte das große Verdienst, zuerst einen allgemeinen Ausdruck mitgeteilt zu haben, welcher mit den bekannten Erfahrungen übereinstimmt, und der bloß den Fehler hat, dass er wegen seiner verwickelten Form, nur mit vieler Weitläufigkeit Anwendung findet” (Eytelwein 1801, p. 216). Du Buat's formula is discussed in Rouse and Ince (1957, pp. 131–132).

$$u = 6,42 \sqrt{\frac{50dh}{l + 50d}},$$

where  $u$  is the mean flow velocity in the conduit of pipes,  $d$  and  $l$  the diameter and length of the conduit, and  $h$  the pressure head, i.e., the height between the reservoir and the lower end of the conduit of pipes; all length units are in “rheinisch” feet. The discharge  $M$  is given by the product of the cross section of the pipe times the velocity

$$M = \frac{1}{4} \pi d^2 u = 5.04 d^2 \sqrt{\frac{50dh}{l + 50d}}.$$

Eytelwein’s pipe flow formula did not fit the data as well as du Buat’s formula—but had the virtue of being useful for determining the dimensions of pipes for given purposes. It could be used to solve engineering problems such as: “How large has to be the diameter of a straight conduit pipe with a length of 100 feet at a pressure head of 5 feet in order to discharge a half cubic feet per second?”<sup>11</sup>

In 1804, Gaspard de Prony, director of the *École des Ponts et Chaussées* and perhaps the most prominent representative of hydraulic engineering in France, made a major step forward by suggesting a formula which combined pipe flow and open channel flow. Prony adopted the view of Charles Coulomb who concluded from experiments about fluid coherence that there are two forces involved in fluid friction: one due to the mutual interaction of the molecules of the fluid, and the other due to surface irregularities at the bounding wall. The former resulted in a friction proportional to the flow velocity, the latter implied an inertial retardation proportional to the square of the velocity (Darrigol 2005, pp. 104–106). Prony, therefore, attempted to account for the experimental data by the ansatz

$$\frac{gdh}{4l} = au + bu^2,$$

where  $g$  is the gravitational acceleration and  $a, b$  are constants to be determined from measurements. Like Eytelwein, Prony relied on the pipe flow experiments of Couplet, Bossut and Du Buat. He employed the newly introduced metric system. Altogether, the set of data comprised the results of 51 experiments with conduits of tin and cast iron pipes that differed considerably in diameter (from 2.7 to 49 cm) and length (from 9.7 to 2280 m). For the determination of  $a$  and  $b$ , Prony adopted a method developed by Pierre-Simon Laplace in celestial mechanics for matching observational data to mathematical expressions. As a result, Prony’s *Recherches physico-mathématiques sur la théorie des eaux courantes* represented empirical hydraulic knowledge combined with sophisticated mathematical elaboration like no other contemporary account (de Prony 1804).

Prony’s formula dominated hydraulic engineering for several decades. The two terms proportional to  $u$  and  $u^2$  seem to foreshadow the distinction of laminar and

<sup>11</sup> “Wie groß wird man den Durchmesser einer graden 100 Fuß langen Röhrenleitung bei einer Druckhöhe von 5 Fuß annehmen müssen, damit solche in jeder Sekunde einen halben Kubikfuß Wasser liefert?” Inserting  $M = \frac{1}{2}$ ,  $h = 5$ ,  $l = 100$  yields an equation for  $d$  with the result  $d = 0,341$  feet (Eytelwein 1801, p. 223).



turbulent flow, but this is a retrospective interpretation. The notion of viscosity as introduced by Claude-Louis Navier in the 1820s in the fundamental equations of flow did not enter pipe flow formulae before the second half of the nineteenth century (Darrigol 2005, chapter 3).

## 5 Air flow in long conduits of pipes

While Prony's formula made the pipe flow of water accessible to hydraulic engineering, the flow of air in conduits of pipes remained mysterious—although the latter was of comparable practical relevance in the Industrial Revolution. In order to provide mine air or operate blast furnaces, air had to be pumped through conduits of pipes with different diameter and length. In 1797, Joseph Baader, an engineer at the Bavarian court who had spent his formative years in England and Scotland, relayed the following experience of the English “iron master” John Wilkinson about the procurement of air for a blast furnace:

He [Wilkinson] intended to use the steep slope of a stream for the operation of a blast furnace 5000 feet away. He built a large overshot water wheel with a cylindrical blowing engine and installed an air conduit with 12 inches wide pipes of cast iron straight towards the furnace. When the machinery was completed and the water-wheel set in motion for the first time, compressed air escaped to the astonishment of all present through the most minute openings and through the valve of the machine itself, while at the other end of the conduit of pipes there was not the slightest motion of air as was checked by the flame of a candle! Then one carefully sealed all openings and fastened the valve so that it would withstand the pressure of the compressed air. When the water wheel was set in motion again, it turned around slower and slower until it stood still entirely. Although the air inside the blowing engine was now compressed to the extent that its elasticity balanced the whole applied force there was not the slightest current of air at the other end of the conduit. Naturally suspicion was raised that the conduit was accidentally blocked at some point. In order to check this hypothesis, a cat was put inside the entrance of the pipe which was then closed; but the cat happily walked out from the other end and therefore had met no obstruction!<sup>12</sup>

<sup>12</sup> “Er gerieth auf den Einfall, einen Bach mit einem starken Gefälle zur Betreibung eines Hohofens zu benützen, der 5000 Fuß, (ohngefähr eine englische Meile) von der Stelle entfernt war. In dieser Absicht baute er ein großes überschlächtiges Rad mit einer vollständigen Zylindermaschine, und führte eine Windleitung von 12 Zoll weiten gegossenen eisernen Röhren von der Maschine gerade nach dem Ofen. Als nun die ganze Anlage vollendet war, und man das erstemal Wasser aufs Rad schlug, zeigte sich zum großen Erstaunen aller Gegenwärtigen, dass die zusammengepreßte Luft durch die kleinsten Öffnungen und Fugen, vorzüglich aber durch ein mit Gewicht beschwertes Ventil (Wastevalve) an der Maschine selbst entwichte, indes aus der Öffnung am entfernten Ende der Röhrenleitung durch ein vorgehaltenes Licht nicht einmal die geringste Bewegung zu bemerken war!—Man verstopfte hierauf alle Fugen auf das sorgfältigste, und beschwerte das Ventil nach und nach mit soviel Gewicht, dass die verdichtete Luft solches gar nicht mehr zu heben vermögend war, und das Rad, bey vollem Aufschlagwasser, sich immer langsamer und langsamer bewegte, bis es endlich ganz stille stand. Allein obwohl nunmehr die Luft in der Maschine offenbar auf einen so hohen Grad verdichtet war, dass ihre Elastizität der ganzen vorhandenen Kraft das Gleichgewicht

According to Baader's report, Wilkinson further investigated the mysterious blockade of air flow by drilling holes at a distance of 30 feet from another in the pipes. Thus, Wilkinson observed how the escape of air through these holes became weaker and weaker further away from the blowing engine, until at a distance of 600 feet only a faint breath of air was noticed. Baader found it regrettable that the pressure at the drill holes was not measured by barometer tubes. Thus, one had missed the opportunity to uncover the law according to which the resistance of a mass of moving air in a long conduit of pipes increases with the length of the conduit (Baader 1797, p. 167). He reported about Wilkinson's experience again at another opportunity (Baader 1805, pp. IX–XI), but measurements on the flow of air in long pipes were performed only several years later. In July 1811, the *Journal de Physique* reported about experiments of air flow in a 447.5 m long iron pipe with 25 cm diameter. In this case, no blockade was observed, although the air flow was caused by only a tiny fraction of the power of Wilkinson's blowing engine (Lehot et al. 1811).

In 1819, the friction of the air flow became the subject of more systematic measurements at a hospital in Paris where the new technology of gaslight had been introduced 2 years before. Pierre Simon Girard, a former collaborator of Prony and an expert on hydraulics, used the opportunity to measure the flow of gas driven by a gasometer through a 623-m-long conduit of pipes with a diameter of 81 mm. By comparing the flow of coal gas with that of air, both driven by the same pressure from the same gasometer, Girard ascertained that gaseous fluids move through the pipes more or less independent of their compressibility (“indépendamment du plus ou moins d'élasticité dont ils peuvent être doués”). From this observation, he concluded that even incompressible fluids should obey the same law of friction:

Therefore, these different phenomena of gas motion are exactly the same as those of incompressible liquids, from which we may conclude that the same formulae must serve to calculate the movement of both.<sup>13</sup>

Accordingly, Girard applied Prony's formula derived from experiments on the flow of water to the flow of gases:

$$\frac{gdhp}{4p'l} = au + bu^2,$$

where  $p$  and  $p'$  are the specific weights of water and gas, respectively, and  $h$  the height of the water column that drives the gas through the conduit of pipes with length  $l$  and diameter  $d$ . Different gases would be matched by this formula with

Footnote 12 continued

hielt, es war doch an dem entfernten Ende der Windleitung noch nicht der schwächste Luftzug zu spüren. Natürlicherweise entstand jetzt der Verdacht, dass die Röhrenstrecke an irgend einer Stelle durch einen Zufall verstopft wäre, und um diese Hypothese zu prüfen, steckte man in die Mündung der Windleitung bey der Maschine eine lebende Katze, welche, nachdem ihr der Rückweg verschlossen ward, nach einiger Zeit an dem andern offenen Ende (von welchem das enge Blaserrohr abgenommen war) glücklich herauskam, folglich die ganze Röhrenleitung ohne Widerstand durchlaufen hatte!” (Baader 1797, pp. 165–166).

<sup>13</sup> “Or, ces divers phénomènes du mouvement des gaz sont exactement les mêmes que ceux du mouvement linéaire des liquides incompressibles, d'où il est permis de conclure que les mêmes formules doivent servir à calculer le mouvement des uns et des autres” (Girard 1821, p. 144).

different coefficients  $a$  and  $b$ , but for very long pipes the term proportional to  $u$  seemed negligible, and then the coefficients  $b$  differed only little.

The experiments with the pipes of the Paris gaslight did not involve very large flow speeds like those encountered in the pipes of blowing engines for blast furnaces or mine air. In order to fill this gap, Jean-François d'Aubuisson de Voisins, chief engineer of the royal mining corps, performed experiments on the flow in large pipes used to provide air for an iron mine in the department of l'Ariège. Twenty-meter-long pipes made of tin with a diameter of 10 cm were joined together to a 387-m-long conduit. The pressure in the pipes was measured at drill holes 40 m apart by manometers (double-bent glass tubes filled with mercury). In the course of several years, d'Aubuisson concluded from a great number of measurements that indeed "the resistance is proportional to the square of the velocity", as Girard already had assumed for larger pipes, so that only a single coefficient remained to be determined from the experiments. In the same vein, his results corroborated Eytelwein's formula which, as d'Aubuisson noticed, comes out as the same as that of Prony as long as the length of the conduit is very large compared to the diameter (d'Aubuisson de Voisins 1828, p. 444).

## 6 The Darcy–Weisbach equation

Besides the experiments on air flow in long pipes, d'Aubuisson was involved in the design of a new water supply system of Toulouse.<sup>14</sup> When he reviewed the contemporary knowledge on hydraulics as far as it was "useful for engineers", he divided his account in one part on hydraulics proper ("L'hydraulique proprement dite") and another part on air measurements ("L'aérométrie") (d'Aubuisson de Voisins 1834). Hence, he assigned pipe flow of water and air to separate chapters, but he argued that the same forces were responsible for the flow retardation in both cases:

Since the resistance comes from the action of the walls, it will be proportional to their extent, i.e., to the length of the pipe and the perimeter of the cross section [...] On the other hand, the larger the cross section, the more the resistance of the walls will be distributed over a greater number of molecules and therefore will affect each of them and the total mass less, and will therefore be inversely related to this number, hence inversely proportional to the cross section.<sup>15</sup>

Prony's and Eytelwein's formulae could be regarded as the mathematical expression of this view. They provided the theoretical basis from which in most practical cases the required quantities of a conduit of pipes could be calculated—for water as well as for gases. Yet the friction of fluids in pipe flow remained a major challenge to bring theory in line with practice. In 1839, Gotthilf Hagen, a professor of hydraulic

<sup>14</sup> Notice Nécrologique sur M. d'Aubuisson de Voisins in: *Annales des Mines*, 4ème série, tome 11, 1847, pp. 667, available at <http://www.annales.org/archives/x/aubuisson.html#annales>.

<sup>15</sup> "Puisque la résistance provient de l'action des parois, elle sera proportionnelle à leur étendue, c'est à-dire, à la longueur de la conduite et au périmètre de la section [...]. D'un autre côté, plus la section sera grande, et plus la résistance des parois se distribuera sur un plus grand nombre de molécules; par conséquent elle affectera moins chacune d'elles et la masse totale : elle sera donc en raison inverse de ce nombre, lequel dépend de la grandeur de la section" (d'Aubuisson de Voisins 1834, p. 171).

engineering at the Berlin Bauakademie, raised “some doubts” about Prony’s and Eytelwein’s formulae. The influence of temperature, for example, “is completely ignored in these hydraulic formulae” (Hagen 1839, p. 423). He performed a series of experiments on the flow of water at different temperatures through a narrow, mean and larger pipe with diameters of 2.6 mm, 4 mm and 5.9 mm, respectively. Like others before him, Hagen noticed a dependency of the pipe friction on temperature, but he postponed the detailed study of this influence (Hagen 1855). For the time being, he derived from his experiments a formula which would later be called after him and Jean Léonard Marie Poiseuille, the law of laminar pipe flow.<sup>16</sup> Hagen’s textbook reputation is confined to the “Hagen-Poiseuille law” only, but for his contemporaries he was an authority for the entire field of hydraulic engineering. He authored a voluminous *Handbuch der Wasserbaukunst* and served as an expert adviser for water works and other building projects in Prussia. In his pipe flow studies, he did not content himself with establishing a formula for laminar flow, but also observed the transition to turbulent flow:

When I let the water flow freely in the air, for smaller pressure heads the jet had a permanent shape, and near the pipe it looked like a solid piece of glass; but as soon as the velocity, by stronger pressure, exceeded the given limit, the jet started to fluctuate and the efflux was no longer uniform and occurred by pulses.<sup>17</sup>

When Hagen investigated a few years later the influence of temperature more closely, he added saw dust to the water in order to visualize these “two kinds of motion”:

I observed that for small pressures the saw dust propagated only in the direction of the tube, whereas for strong pressures it shot from one side to another and often assumed an eddying motion.<sup>18</sup>

Thus, Hagen’s pipe flow investigations came close to discoveries that are ascribed to Osborne Reynolds 30 years later (Rouse and Ince 1957, p. 160; Darrigol 2005, p. 256).

Another German hydraulic engineer who paid great efforts to improve Prony’s and Eytelwein’s pipe flow formulae was Julius Weisbach. In 1845, he published the first of three volumes of a textbook on mechanics, explicitly for the use of engineers, with an extensive section on the “dynamics of fluid bodies”. With regard to pipe flow, he added to the 51 measurements of Couplet, Bossut and du Buat one experiment of an

<sup>16</sup> For a discussion of Hagen’s and Poiseuille’s experiments, see Sutura and Skalak (1993) and Darrigol (2005, chapter 3.7).

<sup>17</sup> “Ließ ich nämlich das Wasser frei in die Luft ausströmen, so bildete der Strahl bei kleineren Druckhöhen eine unveränderte Form, und er hatte in der Nähe der Röhre das Ansehen eines festen Glasstabes; sobald aber bei stärkerem Drucke die Geschwindigkeit die bezeichnete Grenze überstieg, so fing er an zu schwanken und der Ausfluss geschah nicht mehr gleichförmig, sondern stoßweise” (Hagen 1839, p. 423), translated in Darrigol (2005, p. 244).

<sup>18</sup> “Indem ich durch dieselbe Röhre zugleich mit dem Wasser auch Sägespähne hindurchtreiben ließ, so bemerkte ich, dass dieselben bei geringem Drucke nur in der Richtung der Röhre fortschritten, bei starkem Drucke dagegen, von der einen Seite zur andern geschleudert wurden, und oft in wirbelnde Bewegung geriethen” (Hagen 1855, p. 81), translated in Rouse and Ince (1957, p. 159) and Darrigol (2005, p. 245).

hydraulic engineer at Grenoble (Émile Gueymard) and 11 of his own by which the range of flow velocities in the pipes was considerably increased towards higher speeds. Weisbach arrived at the formula

$$h = f \frac{l}{d} \frac{u^2}{2g},$$

with the friction factor  $f = a + \frac{b}{\sqrt{u}}$ ; he determined  $a$  and  $b$  by the method of least squares. This formula, Weisbach concluded, fits the measurements “much more precisely” than the formulae by Prony and Eytelwein (Weisbach 1845, p. 433).

New pipe flow measurements were regarded imperative also in the course of other hydraulic projects such as for the water supply at Dijon implemented in 1840 by Henry Darcy, a young engineer from the Corps des Ponts et Chaussées. *Les fontaines publiques de la ville de Dijon*, as Darcy titled the treatise about his accomplishment (Darcy 1856), represented one of the most modern water supply systems by this time. “The execution of the work on the Dijon water supply gave me the opportunity to recognize the need to engage in the *Recherches* I am publishing today”, Henry Darcy introduced a subsequent treatise.<sup>19</sup> Darcy’s memoirs are celebrated as landmarks in the history of hydraulics and secured him a place in the hall of fame of this discipline (Rouse and Ince 1957, pp. 169–172; Freeze 1994; Brown 2002a; Gisonni 2003; Hager 2003; Bobeck 2006; Simmons 2008).

The opportunity to perform pipe flow experiments on an heretofore unprecedented scale came when Darcy was charged in 1850 with the supervision of the Paris water supply. As he recalled in his treatise about the Dijon fountains:

My original thought was to seek the means to verify the exactness of the laws that are generally accepted today. However, it did not take me long to recognize that I did not have in hand the elements necessary to resolve such a basic question. I had to wait to take up this question until circumstances permitted. I was able to take up these earlier thoughts when I was called to Paris as Director of Municipal [Water] Service. I then had at my disposal all the equipment necessary to carry out the experiments that I had thought about for so long.<sup>20</sup>

In his experiments, Darcy wished to clarify in particular the influence of the pipe wall roughness and diameter on the flow resistance. He measured the flow profile across the cross section of a pipe by an improved Pitot tube (Brown 2003) and compared the flow of water through 100-m-long conduits of pipes of different materials and diameters between 1 and 50 cm. Furthermore, he employed large hydraulic heads in order to

<sup>19</sup> “L’exécution des travaux relatifs à la fourniture d’eau de Dijon m’a donné lieu de reconnaître la nécessité de me livrer aux *Recherches* que je publie aujourd’hui” (Darcy 1857, p. V).

<sup>20</sup> “Ma première pensée avait été de chercher dans ces expériences les moyens de vérifier l’exactitude des lois admises jusqu’à ce jour; mais je n’ai pas tardé à reconnaître que je n’avais pas sous la main les éléments nécessaires pour résoudre une si grave question, et j’ai attendu que les circonstances me permettent de songer à la reprendre. Je donnai suite à ma pensée première lorsque je fus appelé à Paris, comme directeur du service municipal. J’avais alors à ma disposition tous les appareils indispensables pour procéder aux expériences auxquelles je songeais depuis si longtemps” (Darcy 1856, p. 376). English translation in Bobeck (2006, p. 1003).

achieve large flow velocities. A good deal of his data such as those from old and rough pipes could not be reconciled with Prony's formula. Darcy arrived at a formula for the head loss which contained a linear and a quadratic term in the velocity, like that of Prony, but with coefficients that displayed a marked dependency on the pipe diameter. Furthermore, Darcy noticed that beyond a certain velocity the linear term could be neglected.<sup>21</sup> On the other hand, for pipes with very small diameter and flow velocity, the quadratic term could be neglected. "M. Poiseuille and I arrived at this expression by totally different experiments and circumstances", Darcy remarked about this case which we now call laminar flow. Darcy contented himself with the remark that "it seems that two laws appear in the phenomenon produced by the flow of water in the pipes and that these laws, at least in the diameters that I have used, merge at velocities of 9–10 cm [per second]".<sup>22</sup>

For other pipes, in particular those with rough walls, Darcy's formula reduced to an expression proportional to the square of the velocity where the resistance could be expressed by a friction factor—similar to the one published a few years earlier by Weisbach (whose work seems to have been unknown to Darcy). Unlike Weisbach's friction factor (see above), however, Darcy's factor showed a functional dependence on the pipe diameter:  $f = a + \frac{b}{d}$  (Darcy 1857, p. 117). Like Weisbach, Darcy determined the coefficients ( $a$ ,  $b$ ) by the method of least squares from the experimental measurements. Other features (pipe roughness, viscosity of the fluid) were not made explicit.

It was left to another hydraulic engineer in the USA, John Thomas Fanning, to adjust Weisbach's expression to Darcy's better data and to establish what became known as "Darcy-Weisbach equation" (Brown 2002b). The modern understanding of this law had to await the similarity theory as applied by Heinrich Blasius to empirical pipe flow data (see below). In the same vein, Darcy has been praised for "the first accurate measurements of turbulent pipe velocity distributions" and for providing "the very first evidence of the existence of the fluid boundary layer" (Simmons 2008, p. 1028). Both ascriptions need some qualification: Darcy used erroneous assumptions to derive from his measurements the velocity profiles (Gisonni 2003, p. 34); although he located the origin for the wall friction in "the small liquid band in contact with the walls" where "the fluid elements are agitated in vortical motions" ("la petite couche liquide en contact avec les parois...les éléments de cette couche sont animés de mouvements giratoires"), he made no attempt to determine the features of this layer (Darcy 1857, p. 10).

Darcy's pipe flow measurements became the prototype to which other hydraulic engineers referred in the design of water supply systems elsewhere. After Darcy's death in 1858, his assistant Henri Bazin extended these investigations to the study of open channel flow where the walls seemed to exert a similar flow retardation as in

<sup>21</sup> "...qu'à partir d'une certaine vitesse la valeur du débit ne semble point affectée par la suppression du premier terme" (Darcy 1857, p. 120).

<sup>22</sup> "...nous sommes parvenus, M. Poiseuille et moi, à cette expression, au moyen d'expériences faites dans des circonstances tout à fait différentes.... il semble que deux lois apparaissent dans le phénomène produit par l'écoulement de l'eau dans les tuyaux de conduite et que ces lois, au moins dans les diamètres que j'ai employés, viennent se souder vers les vitesses de 9 à 10 centimètres" (Darcy 1857, pp. 213–215).

closed pipes.<sup>23</sup> The results of Darcy's and Bazin's experiments launched a new quest for a general flow formula (Hager 1994).

In 1874, the Union of German Architectural and Engineering Associations solicited a review on "measurements concerning the pressure head loss in the flow of water through conduits of cast iron pipes of various pipe width, flow velocity and under the influence of successive modification of the inner surfaces". The motivation for this review came from the uncertainty "which of the numerous formulae is to be taken as the basis for any new installations as being closest to the truth". As a result of this request, new pipe flow measurements in several German cities (Danzig, Bonn, Wiesbaden, Hamburg) were performed and analysed with regard to various formulae. By and large, the validity of Darcy's formula was found to agree best for new pipes, even for pipe diameters as large as 122 cm, but no formula could account for older pipes with rough inner surfaces.<sup>24</sup>

## 7 The riddle of two different manners of flow

By the 1860s, the experiments of hydraulic engineers like Hagen, Weisbach, Darcy and others had abundantly made clear that the flow in very narrow pipes was fundamentally different from that in large pipes. The former had been analysed by Hagen and Poiseuille whose law could be derived from the Navier–Stokes equations. In the contemporary parlance of the year 1860, capillary flow represented "the only form of experiment for which the complete theory could be derived from those equations of motions", whereas the experimental results of pipe flow in larger pipes could only be used to derive "practical approximation formulae but not for comparison with a mathematical theory".<sup>25</sup>

The failure of fundamental hydrodynamic theory to cope with this problem became notorious. In 1872, Adhémar Barré de Saint-Venant, the outstanding expert on theoretical mechanics among the French academicians, regarded this problem as "a true enigma".<sup>26</sup> His pupil Joseph Boussinesq echoed this view: "Fluids move in two different manners, according to whether they flow in very narrow tubes or in spaces having sections comparable to those of pipes and open channels". Thus, Boussinesq intro-

<sup>23</sup> "L'influence capitale qu'exerce la nature de la paroi était incontestable en principe après les expériences faites sur les tuyaux de conduite par M. Darcy; elle se trouve donc démontrée d'une manière encore plus frappante pour les canaux découverts" (Darcy and Bazin 1865, p. 79).

<sup>24</sup> "Welche Erfahrungen, bezw. Messungen liegen vor über den Druckhöhenverlust beim Durchfluss von Wasser durch gusseiserne Rohrleitungen für verschiedene Rohrweiten, verschiedene Geschwindigkeiten und unter dem Einfluss der successiven Oberflächenveränderung im Innern? ...welche der zahlreichen Formeln, als der Wahrheit am nächsten kommend, bei etwaigen Neuanlagen zu Grunde zu legen sei" (Iben 1880, p. 1).

<sup>25</sup> "Indessen ist bisher nur eine einzige Form des Experimentes gefunden worden, für welche die vollständige Theorie aus jenen Bewegungsgleichungen hergeleitet werden kann, es ist das die Bewegung einer Flüssigkeit in sehr engen und sehr langen Röhren. [...] so dass man die unter solchen Bedingungen gewonnenen Ergebnisse zwar zur Ableitung praktischer Approximationsformeln, aber nicht zur Vergleichung mit der mathematischen Theorie gebrauchen kann" (Helmholtz and Piotrowski 1860, pp. 607–608). See also Darrigol (2005, chapter 3.7).

<sup>26</sup> "une véritable énigme". Rapport sur un mémoire de M. Boussinesq, présenté le 28 Octobre 1872, in Boussinesq (1877, p. III).

duced a treatise on the flow of water. In the latter case, one had to take into account “the eddying agitation” that extends from the walls into the interior of the fluid and propagates very fast or even abruptly from point to point.<sup>27</sup> Boussinesq merged Saint-Venant’s theoretical views with the engineering knowledge about pipe and channel flow as described in the treatise of Darcy and Bazin. From “the eddying agitation”, he derived the concept of an eddy viscosity which would play a fundamental role in theories of fully developed turbulence in the twentieth century.

Another effort to bring the two different manners of flow under a common theoretical roof was published a few years later by Osborne Reynolds, an engineering professor at Manchester with a tendency towards research problems that have “both a practical and a philosophical aspect”. This was how he described his objective for “An Experimental Investigation of the Circumstances which determine whether the Motion of Water shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channels” (Reynolds 1883)—a landmark paper in the history of turbulence. Reynolds has often been credited for achievements which were made by others. Hagen visualized the transition to turbulence many years before Reynolds (see above). Another part of the “Reynolds legend” (Darrigol 2005, p. 255) concerns the rise of similarity theory. Reynolds did not claim priority for the study of the scaling invariance of the Navier–Stokes equation. “I had no intention whatever of laying down the conditions of dynamical similarity”, Reynolds wrote in 1883 in a letter to Gabriel Stokes [quoted in Darrigol (2005), p. 256]. The late rise of the “Reynolds number” as a key concept in fluid mechanics—it received this name only in 1908 (Rott 1990)—further hints at the haphazard use of similarity theory. Reynolds’s achievements have been analysed in great detail and need no further review (Launder and Jackson 2011; Darrigol 2005, chapter 6.5).

But it seems appropriate to focus on some of his findings once more from the perspective of pipe flow. What we now call “Reynolds number” was introduced as a quantity for comparing different pipe flows. Reynolds displayed the loss of pressure in a pipe in a double-logarithmic plot as a function of the flow velocity in order to expose the common features of different pipe flows. It came as a surprise [“beyond my original intention” (Reynolds 1883, p. 977)] that he was able to display both Poiseuille’s and Darcy’s data in the same diagram as straight lines with different “Log Slopes of Pressure” that intersected at a critical velocity. In a subsequent paper published 12 years later, Reynolds expressed this critical velocity as  $K\mu/D\rho$ , where  $D$  is the pipe diameter,  $\mu$  the viscosity and  $\rho$  the density of the fluid, and  $K$  “a numerical constant, the value of which according to my experiments, and, as I was able to show, to all the experiments by Poiseuille and Darcy, is for pipes of circular section between 1900 and 2000” (Reynolds 1895, p. 125). Thus, the divide between the two manners of flow, “steady direct” and “sinuous or eddying” in Reynolds’s parlance, was for

<sup>27</sup> “Les fluides se meuvent, de deux manières différentes, suivant, qu’ils coulent dans des tubes très-étroits ou dans des espaces ayant des sections comparables à celles des tuyaux de conduite ou des canaux découverts. [...] Au reste, cette production d’une agitation tourbillonnaire au sein de toute masse fluide qui s’écoule à travers des sections d’une certaine étendue [...] Il faut donc...regarder les vitesses vraies, à l’intérieur d’un fluide qui s’écoule, comme rapidement ou même brusquement variables d’un point à l’autre, capables, en un mot, de produire des frottements d’un tout autre ordre de grandeur que dans le cas de mouvements continus...(Boussinesq 1877, pp. 1, 6–7). See also Rouse and Ince (1957, p. 203) and Darrigol (2005, p. 235).



the first time quantified and subjected under a common theoretical roof. However, it took many years until Reynolds's achievement became "a marker for the direction of research in Engineering Fluid Mechanics for the next century" (Jackson and Launder 2007, p. 20). Contemporary hydraulic engineers hardly recognized Reynolds' papers from 1883 and 1895 as key contributions to their field. A review on pipe and channel flow from 1897 pointed to the work of Hagen and others who had investigated the two different manners of flow much earlier: "Reynolds has therefore been anticipated five or six times". Furthermore, there were experimental pipe flow data "adverse to Reynolds' theory, which therefore may be rejected" (Knibbs 1897, pp. 326, 343).

## 8 Blasius' law for turbulent pipe friction

In 1903, two American engineers, Augustus V. Saph and Ernst W. Schoder, presented a joint doctoral thesis to the Cornell University about pipe flow measurements of heretofore unsurpassed extent and precision. In hundreds of experiments with different types of pipes of different diameters, they measured the resistance of the flow of water at different temperatures (thus modifying viscosity like Hagen and others). They displayed the result of their measurements as logarithmic plots for the head loss per unit length as a function of the flow velocity. The plots for different pipes could be grouped as parallel straight lines with two distinct slopes representing the two manners of pipe flow. Above a critical velocity, the slope corresponded to smooth pipes "very nearly" to a head loss  $h$  per length  $l$  according to

$$\frac{h}{l} \sim \frac{u^{1.75}}{d^{1.25}}.$$

Saph and Schoder cited Reynolds only for having demonstrated the change in the character of pipe flow at a critical velocity. They did not express their own results in terms of the Reynolds number (Saph and Schoder 1903; Steen and Brutsaert 2017).

Reynolds' papers found more interest among physicists. Arnold Sommerfeld, theoretical physicist at Munich University, introduced in 1908 the name "Reynolds number" (Rott 1990) in a study that aimed at a determination of the transition to turbulence in the plane flow between two walls moving in opposite directions (plane Couette flow) (Eckert 2010). By the same time, the experimental physicist Wilhelm Wien at Würzburg University asked a doctoral student to investigate the occurrence of turbulence phenomena more closely in gases instead of water, with a particular interest in the range of the Hagen–Poiseuille law. Wien had published in 1900 a textbook on hydrodynamics where he lamented about the gap between theory and practice in this area: "Here the actual processes are so inconsistent with the theoretical conclusions that engineering developed a special way of treating hydrodynamic tasks, which is usually called hydraulics".<sup>28</sup> The experiments of Wien's doctoral student clearly showed

<sup>28</sup> "Hier stimmen die thatsächlichen Vorgänge mit den theoretischen Folgerungen vielfach so ungenügend überein, dass die Technik sich für ihre Zwecke eine besondere Behandlungsweise hydrodynamischer Aufgaben, die meistens den Namen Hydraulik führt, zurechtgemacht hat". (Wien 1900, p. III).

“that the critical value of Reynolds is also decisive for the beginning of turbulent air flow through narrow pipes”.<sup>29</sup>

The next step was made by Heinrich Blasius at the Berlin Research Laboratory for Hydraulics and Naval Architecture (Königliche Versuchsanstalt für Wasserbau und Schiffbau). Blasius was prepared for this investigation like few others (Hager 2003). He had finished his studies at Göttingen University as Ludwig Prandtl’s first doctoral student with an elaboration of Prandtl’s boundary layer concept (Eckert 2017b). In 1907/1908, he reviewed the contemporary knowledge on turbulence in a seminar (Eckert 2019, pp. 42–44, 58–59, 127–136). When he moved in 1908 from the academic environment in Göttingen to the engineering laboratory in Berlin, he was eager to demonstrate his expert knowledge in practical applications. He became aware that hydraulics offered many opportunities for similarity considerations. Even after 30 years “the law of Reynolds” had hardly penetrated engineering practice, he noted in the first of several same-titled papers on “The law of similarity in frictional processes”. When the motion was determined by frictional and inertial forces like in pipe flow, the law of similarity implied that the friction coefficient was a function of the Reynolds number  $\frac{ud}{\nu}$ , where  $\nu$  is the kinematic viscosity of the fluid. Besides the friction in pipe flow, the similarity law provided the basis for wind tunnel tests like those in Prandtl’s new Göttingen Airship Model Research Institute (Luftschiff-Modellversuchsanstalt). “There is then no doubt that the introduction of the law in practice is necessary”, Blasius concluded this paper.<sup>30</sup>

In November 1911, he presented the same message to a conference on aeronautics in Göttingen—with the focus on the flow of air instead of water. He concluded from recent measurements about the head loss of air flow in pipes that “these experiments carried out with atmospheric air yielded the same coefficient for the same Reynolds number as the series of experiments with water mentioned above”. When he was asked in the discussion about the difference between laminar and turbulent flow he presented the Darcy–Weisbach formula for the head loss

$$h = \lambda \frac{l}{d} \frac{u^2}{2g}$$

with the friction factor for laminar flow

$$\lambda = 64 \left( \frac{ud}{\nu} \right)^{-1}$$

and for turbulent flow

$$\lambda = 0.3164 \left( \frac{ud}{\nu} \right)^{-1/4}.$$

<sup>29</sup> “Bei Versuchen mit Strömungen der Luft durch enge Röhren hat sich ergeben, dass der kritische Wert von Reynolds auch hier maßgebend ist für den Beginn turbulenter Strömung” (Ruckes 1908, p. 1020).

<sup>30</sup> “Es ist danach kein Zweifel, daß die Einführung des Gesetzes in die Praxis notwendig ist” (Blasius 1911, p. 1177).

“The transition from one law to the other takes place at values of  $\frac{ud}{v}$  between 2000 and 3000”.<sup>31</sup> The “other” law for turbulent pipe friction became known as “Blasius’ law”.

For the extensive presentation of his results, Blasius chose the Association of German Engineers (Verein Deutscher Ingenieure) as his audience (Blasius 1912a, 1913). In all of his publications, he left no doubt about the importance of experiments because the functional dependence of the friction coefficient  $\lambda$  on the Reynolds number did not result from similarity considerations alone; the supposed formulae had to be matched to the experimental data. One-third of his final report was dedicated to tables and diagrams with the results of pipe flow measurements. “The most careful and extensive tests on pressure loss in smooth pipes were carried out by the American engineers Saph and Schoder”, Blasius acknowledged these experiments as his main source and inspiration. He called his expression for the turbulent friction coefficient the “improved Saph-Schoder formula”.<sup>32</sup> But he also performed himself measurements on the pipe flow of water at different temperatures in order to demonstrate how the data from different experiments collapsed when displayed as a function of the Reynolds number (Fig. 2). Furthermore, he attempted to trace the origin of discrepancies between measurements of Reynolds and Saph-Schoder. “I therefore checked Reynolds’ experiments and found that lead pipes have the same resistance as the Saph-Schoder brass pipes. I therefore suspect that Reynolds has a systematic error in the measurements”.<sup>33</sup>

Blasius explicitly confined the validity of his turbulent friction factor to completely smooth pipes. For pipes with a rough inner surface, he concluded from the available data that the friction factor could no longer be expressed in terms of the Reynolds number alone. He introduced a parameter  $\varepsilon/d$ , where  $\varepsilon$  is the size of the bumps that protrude out of the surface, but he abstained from deriving a formula. He left the roughness issue with the statement “that in any case no pure power law is possible” for the formula of the friction factor.<sup>34</sup>

An attempt in this regard was made subsequently by Richard von Mises (Siegmond-Schultze 2018). A few years later, he made himself a name as the founder of the *Zeitschrift für Angewandte Mathematik und Mechanik (ZAMM)* (Siegmond-Schultze 2020), but at first he regarded himself as a mechanical engineer (Maschinenbau-Ingenieur)—or so he signed in September 1913 the *Elemente der Technischen Hydromechanik* that showcased his familiarity with pipe flow measurements (von Mises 1914). From Darcy’s measurements with rough pipes, he derived the friction factor as

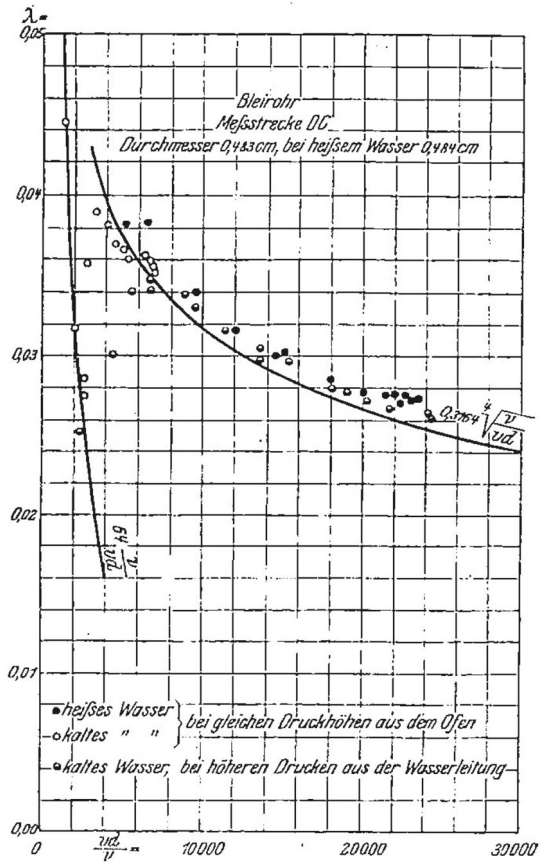
<sup>31</sup> “Diese mit atmosphärischer Luft angestellten Versuche ergaben für gleiche Reynoldssche Zahl denselben Koeffizienten, wie die oben erwähnten Versuchsreihen mit Wasser [...] Bei den Werten von  $\frac{ud}{v}$  zwischen 2000 und 3000 findet ein Übergang von dem einen Gesetz zu dem andern statt” (Blasius 1912b, p. 37).

<sup>32</sup> “verbesserte Formel von Saph-Schoder” (Blasius 1913, p. 29), see also (Steen and Brutsaert 2017, p. 576).

<sup>33</sup> “Ich habe infolgedessen die Reynoldsschen Versuche nachgeprüft und gefunden, daß auch Bleirohre denselben Widerstand wie die Saph-Schoderschen Messingrohre haben. Ich vermute demnach bei Reynolds einen systematischen Fehler der Messungen” (Blasius 1913, p. 14).

<sup>34</sup> “...dass jedenfalls kein reines Potenzgesetz in Frage kommt...” (Blasius 1913, p. 21).

**Fig. 2** Blasius's diagram for the friction coefficient of laminar and turbulent pipe flow as a function of the Reynolds number (Blasius 1913, Abb. 13)



$$\lambda = 0.0024 + \sqrt{\frac{k}{r}} + 0.3 \left(\frac{ur}{v}\right)^{-1/2},$$

where  $k$  is an empirically determined “absolute roughness” and  $r$  the radius of the pipe (von Mises 1914, p. 75).

Another report that displayed a wealth of pipe flow data as a function of the Reynolds number was published somewhat later by two British engineers from the National Physical Laboratory (Stanton and Pannell 1914). Apparently unaware of Blasius’ papers, they offered similar diagrams that showed how the data of quite different experiments with the flow of water and air collapsed when displayed as a function of the Reynolds number. Among hydraulic engineers, such plots became labeled as Stanton diagrams (Steen and Brutsaert 2017, p. 580).

## 9 The turbulence problem

Transition to turbulence and friction factors that collapsed when expressed as a function of the Reynolds number were not only observed in pipe flow. Blasius also displayed, for example, data about the skin friction of the flow of water along flat plates published in 1908 from towing experiments at a shipbuilding facility (Blasius 1913, p. 39). The pertinence of such experiments for naval architecture was obvious. The rise of aeronautics before World War I suggested similar investigations in wind tunnels. “In view of the practical value of the ability to apply this principle [of similarity] to the prediction of the resistance of aircraft from experiments on models, experimental investigation of the conditions under which similar motions can be produced under practical conditions becomes of considerable importance”, Stanton and Pannell remarked about the beginning of experiments on inclined planes in 1911 at the National Physical Laboratory. By adding colouring matter in streams of air and water, “eddy systems” could be observed at the same Reynolds number (Stanton and Pannell 1914, p. 201). In the wind tunnel of the Göttingen Airship Model Research Institute, Prandtl and his collaborator Carl Wieselsberger measured the drag of spheres; they interpreted the sudden drop of the drag coefficient observed earlier in Gustave Eiffel’s wind tunnel in Paris as a consequence of the transition to turbulence in the boundary layer of the spheres which happens at a certain Reynolds number (Eckert 2017b, pp. 88–91).

Despite the experimental evidence for the transition to turbulence and the empirical laws that described fully developed turbulent flow in terms of the Reynolds number, theory failed to predict critical Reynolds numbers for the onset of turbulence, not to speak of friction coefficients for fully developed turbulence. The most elaborate “Orr–Sommerfeld approach” failed to yield critical Reynolds numbers for plane Couette flow (Eckert 2010). In 1921, Fritz Noether, an applied mathematician and former student of Sommerfeld, dubbed this riddle in a review article as “the turbulence problem” (Noether 1921). Five years later, he offered a mathematical proof that this method was in principle unable to yield a critical limit for the stability of plane flows (Noether 1926). Another theoretical physicist added fuel to the fire when he analysed the stability of axisymmetric pipe flow by the Orr–Sommerfeld approach: there was no critical limit beyond which the laminar Poiseuille profile would become turbulent (Sextl 1927a, b). Thus, Noether’s verdict that “the turbulence problem is still far from a real solution” (Noether 1926, p. 232) was not limited to plane flows but seemed valid also for three-dimensional pipe flow.

While theory was in a deadlock, new pipe flow experiments removed any lingering doubts about the onset of turbulence beyond a critical Reynolds number. Ludwig Schiller, an experimental physicist at Leipzig University, spent several research sojourns in Prandtl’s Göttingen laboratory where he investigated the role of disturbances at the entrance of a pipe for the transition to turbulence. By an appropriately shaped mouth and an inflow that avoided disturbing motions as far as possible, he was able to maintain laminar flow up to a critical Reynolds number  $R = 9600$ . The lowest critical Reynolds number was attained with an eddy inflow at  $R = 1160$ . In a double-logarithmic plot of the friction factor as a function of the Reynolds number, the transition to turbulence was displayed by points that followed for small Reynolds

numbers the Hagen–Poiseuille law  $\sim R^{-1}$  until they sharply jumped to the Blasius law for turbulent flow  $\sim R^{-1/4}$  (Schiller 1921, 1922). In contrast to the theoretically predicted stability, laminar pipe flow became unstable beyond a critical limit, even though this limit could be shifted to very large Reynolds numbers.

## 10 The quest for a “wall law”

With regard to the onset of turbulence, the results of the theorists compared badly with the amount of mathematical sophistication expended in their effort. Fully developed turbulence, by contrast, appeared more accessible thanks to the well-established empirical law of Blasius for pipe flow at high Reynolds numbers. The friction factor for smooth pipes proportional to  $R^{-1/4}$  served as a lead from which the flow velocity could be determined as a function of the distance from the wall.

The first who followed this lead were Prandtl in Göttingen and his master pupil at the Technical University in Aachen, Theodore von Kármán, who became at the same time Prandtl’s fierce rival in their quest to solve the riddles of turbulence (Eckert 2017a, 2018). Another participant in this quest was Johannes Nikuradse, Prandtl’s doctoral student in the early 1920s. Nikuradse recalled that Prandtl had presented on 5 November 1920 at a seminar a “consideration” how the law of Blasius could be used to derive the turbulent velocity distribution across the pipe’s diameter (Nikuradse 1926, pp. 14–15). According to Nikuradse Prandtl combined Blasius’ law with the assumption that the shear stress at the wall does not depend on the radius of the pipe. This argument gave rise to the following derivation:

The pressure loss in a straight pipe (radius  $r$ , length  $l$ ) is related to the shear stress  $\tau$  via

$$\Delta p = \frac{\tau \cdot 2r\pi l}{r^2\pi} = 2\tau \frac{l}{r}.$$

According to Blasius’ law, the pressure loss for turbulent pipe flow is

$$\Delta p = \lambda \cdot \frac{l}{r} \rho \frac{u^2}{2}, \quad \text{with } \lambda = 0.133 \left( \frac{ud}{\nu} \right)^{-1/4}.$$

Eliminating  $\Delta p$  from both equations yields

$$2\tau \frac{l}{r} = 0.133 \left( \frac{ud}{\nu} \right)^{-1/4} \cdot \frac{l}{r} \rho \frac{u^2}{2},$$

or

$$\tau = \text{const.} \left( \frac{ur}{\nu} \right)^{-1/4} \rho \frac{u^2}{2}.$$

Dimensional analysis with  $u = ar^q$  then yields

$$\tau \sim \rho v^{1/4} a^{7/4} r^{(7q-1)/4}.$$

The assumption that the shear stress at the wall does not depend on  $r$  demands  $7q - 1 = 0$ , or  $q = 1/7$ .

Kármán derived the same result independently in an attempt to elaborate a “turbulent boundary layer theory”. He outlined his concept in a five-page letter to Prandtl and asked him “whether you have already published your 1/7 law” so that he could refer to it. He recalled that Prandtl had mentioned this law earlier and therefore was hesitant to publish it before Prandtl. Furthermore, he was curious whether his derivation was identical to that of Prandtl.<sup>35</sup> “Your derivation of the law of the 7th root basically agrees with my derivation, but my way of presenting the underlying assumptions is more physical”, Prandtl answered with regard to the 1/7 law. As far as Kármán’s “turbulent boundary layer theory” was concerned, Prandtl confessed “that you are definitively further advanced”. But Prandtl did not want to hurry with the publication and encouraged Kármán to proceed with his plans. “Ultimately, I can get over it if the precedence of publication has gone over to friendly territory”.<sup>36</sup> So Kármán’s version appeared first (von Kármán 1921).

Pipe flow experiments thus provided the lead pipe theory. The law of Blasius from which the 1/7 law was derived, however, was entirely empirical. When Prandtl was asked by a colleague about the theoretical derivation of Blasius’ law, Prandtl answered: “Whoever finds it will become a famous man”.<sup>37</sup> In 1924 new pipe flow measurements at the Physikalisch-Technische Reichsanstalt (Jakob and Erk 1924) cast doubt about the validity of Blasius’ law at very high Reynolds numbers. In a review of these and other pipe flow measurements, Schiller refrained from his earlier view that Blasius’ law was valid up to  $R = 200,000$  (Schiller 1925, p. 592). Prandtl, too, changed his mind. He generalized the consideration that had led him from the Blasius law to the 1/7 law in such a way that it yielded the velocity distribution for any empirical law of pipe flow friction. He concluded that for Reynolds numbers around 200,000 the velocity profile would be rather  $\sim y^{1/8}$  than  $\sim y^{1/7}$ . In the same paper, he outlined a semi-empirical concept which aimed at an expression for the turbulent shear stress in terms of a length that could be adjusted to the boundary conditions of specific turbulent flows and compared to the mean free path of kinetic gas theory. It became known as the mixing-length approach (Prandtl 1925; Bodenschatz and Eckert 2011, pp. 54–56). But

<sup>35</sup> “Ich möchte Sie jedoch bitten, mir mitzuteilen, ob Sie Ihr 1/7 Gesetz bereits veröffentlicht haben, so dass ich mich darauf beziehen kann, oder dass Sie es demnächst veröffentlichen wollen. Ferner ist meine Ableitung mit Ihrer Ableitung identisch oder nicht?” Kármán to Prandtl, 12 February 1921. GOAR 3684.

<sup>36</sup> “Ihre Ausführungen über das Gesetz der 7. Wurzel decken sich dem Inhalte nach mit meiner Ableitung, doch klingt meine Formulierung der Axiome sehr viel physikalischer [...] In der allgemeinen Formulierung der ‘turbulenten Grenzschichttheorie’ sind Sie entschieden weiter als ich [...] Ich werde dann hinterher schon sehen, wie ich mit meiner anderen Herleitung noch zu meinem Recht komme und werde es schliesslich verschmerzen können, wenn die publizistische Priorität in befreundeten Besitz übergegangen ist”. Prandtl to Kármán, 16 February 1921. MPGA, Abt. III, Rep. 61, Nr. 792.

<sup>37</sup> “Sie fragen nach der theoretischen Ableitung des Blasius’schen Widerstandsgesetzes für Rohre. Wer die findet, der wird dadurch ein berühmter Mann!” Prandtl to Birnbaum, 7 June 1923, MPGA, Abt. III, Rep. 61, Nr. 137.

before he ventured to derive a new formula for the velocity distribution of turbulent pipe flow he solicited new experiments “in smooth and rough pipes at larger Reynolds numbers”, as he confided to Kármán. This was one part of a longer “menu” of his experimental research program on turbulence.<sup>38</sup>

Prandtl’s expert for pipe flow measurements was his former doctoral student Johann Nikuradse. The experiments with smooth pipes at large Reynolds numbers were performed in 1928/1929 at Prandtl’s new Kaiser Wilhelm Institute for Fluid Mechanics (Kaiser-Wilhelm-Institut für Strömungsforschung, founded in 1925 in Göttingen next to the aeronautical model research institute). Nikuradse presented his results in a preliminary report in June 1929 at a conference in Kármán’s Aachen institute (Nikuradse 1930). Kármán regarded Nikuradse’s experiments as a confirmation of his recent theoretical results that suggested a logarithmic velocity distribution and a logarithmic formula for the friction factor (von Kármán 1930, 1931). Prandtl had arrived even earlier at a logarithmic velocity distribution—and dismissed it again because it resulted in a singularity at the wall. But in view of Nikuradse’s measurements he became converted to the logarithmic laws. The singularity was no longer an argument against the logarithmic velocity distribution because, in Nikuradse’s words, “a zone immediately adjacent to the wall, where the flow is laminar, must of course be excluded”.<sup>39</sup>

Despite Kármán’s priority, however, Prandtl was often credited with the discovery of the logarithmic wall laws—to the annoyance of Kármán. The quest for a “universal law of turbulence” had turned into a “first-class rivalry” (von Kármán 1967, pp. 135–138). But disregarding differences of the theoretical approaches and interpretations of their achievements—the basic lesson was the same: at higher Reynolds numbers Blasius’ law lost its validity—and so did the theoretical derivations based on it. The “universal velocity distribution” was logarithmic (Fig. 3):  $\varphi = a + b \log \eta$ , where  $\eta = u_* y / \nu$  is a dimensionless wall distance and  $\varphi = u / u_*$  is a dimensionless velocity, with  $u_* = \sqrt{\tau_0 / \rho}$  as the “friction velocity” and  $\tau_0$  the shear stress at the wall.  $a$  and  $b$  are constants. The friction law was

$$\frac{1}{\sqrt{\lambda}} = A + B \log(Re\sqrt{\lambda}),$$

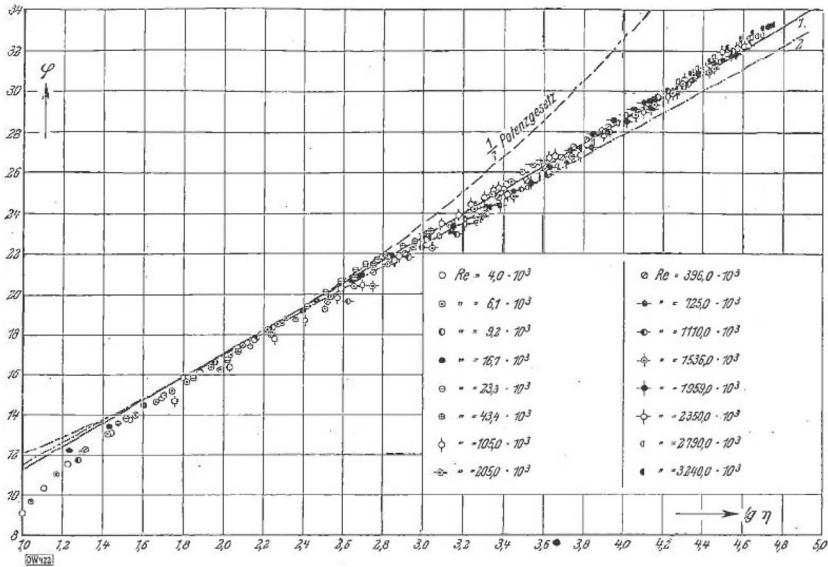
where  $\lambda$  is the friction factor in the Darcy–Weisbach formula,  $Re = \bar{u}d / \nu$  the Reynolds number with  $\bar{u}$  the mean velocity averaged over the diameter  $d$  and  $\nu$  the kinematic viscosity;  $A$  and  $B$  are constants (Nikuradse 1932, pp. 15–17, 29–32).

Unlike the derivation of the  $y^{1/7}$ -formula from Blasius’ law by means of dimensional analysis, the elaboration of the “universal” wall law involved more fundamental theoretical considerations based on Prandtl’s mixing-length concept and Kármán’s similarity assumptions (Darrigol 2005, Chapter 7.3.9; Bodenschatz and Eckert 2011, Chapter 2.6; Leonard and Peters 2011, Chapter 3.2).

<sup>38</sup> “Die Geschwindigkeitsverteilungen in glatten und rauhen Röhren sollen mit grösseren Reynoldsschen Zahlen wiederholt werden. Über die Vorgänge an Wandvorsprüngen, Rauigkeiten usw. sollen Einzeluntersuchungen gemacht werden. Die Vorgänge bei der Entstehung der Wirbel aus der Grenzschicht bei der Anfahrt der Körper wird [sic] studiert ... Das ist ein langer Speisezetteln...” Prandtl to Kármán, 26 July 1928. MPG, Abt. III, Rep. 61, Nr. 792.

<sup>39</sup> “eine Zone unmittelbar an der Wand, wo laminare Strömung herrscht, muss natürlich ausgeschlossen bleiben” (Nikuradse 1932, p. 26).





**Fig. 3** Nikuradse's diagram for the wall law, displayed as a semi-logarithmic plot of the dimensionless velocity as a function of the dimensionless wall distance (Nikuradse 1932, Fig. 24)

Nikuradse also performed a series of pipe flow experiments in order to measure the effect of coarse and fine roughnesses for all Reynolds numbers (Nikuradse 1933). As it turned out, even the extension to rough pipes was amenable to theoretical considerations (Figs. 4, 5). Prandtl regarded these results as the successful conclusion of the development sparked by Blasius' law 20 years ago. "The whole pipe flow problem has thus found a very comprehensive solution by combining a few empirical values with theoretical conclusions", he summarized this achievement in 1933 in a review article on "Recent Results of Turbulence Research".<sup>40</sup>

Kármán, too, regarded the quest for the universal wall law as concluded. He moved by this time permanently to the USA and reviewed the achieved results under the title "Turbulence and Skin Friction" in the *Journal of the Aeronautical Sciences*. In Kármán's version, a constant was introduced that was regarded more universal than those in Prandtl's theory (von Kármán 1934, p. 10). It became widely known in the theory of turbulence as Kármán's constant.

The "comprehensive solution" achieved between 1930 and 1934 by Kármán, Nikuradse and Prandtl became the standard for subsequent presentations of turbulent pipe flow—with slightly different emphasis concerning the universality and the underlying assumptions. Prandtl's review in the *Zeitschrift des Vereines Deutscher Ingenieure* was widely circulated in English translation by the National Advisory Committee for Aeronautics (NACA) of the USA as a *NACA Technical Memorandum* (Prandtl 1933b). In 1945, Hunter Rouse provided a concise elaboration of the "Kármán-Prandtl velocity equations" and the "Kármán-Prandtl resistance equations" for turbulent pipe flow

<sup>40</sup> "Das ganze Rohrproblem hat hiermit auf Grund einer Verknüpfung von wenigen Erfahrungswerten mit theoretischen Schlüssen eine sehr umfassende Lösung gefunden" (Prandtl 1933a, p. 112).

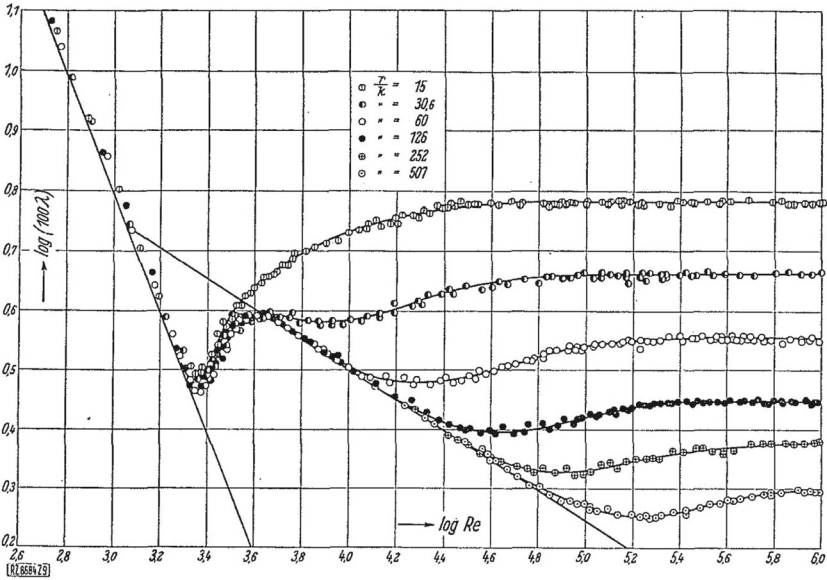


Fig. 4 Nikuradse's diagram for the friction factor as a function of the Reynolds number and the roughness number defined by  $r/k$ , where  $k$  is the absolute size of the grains that roughened the inner surface of the pipe with radius  $r$  (Nikuradse 1933, Fig. 9)

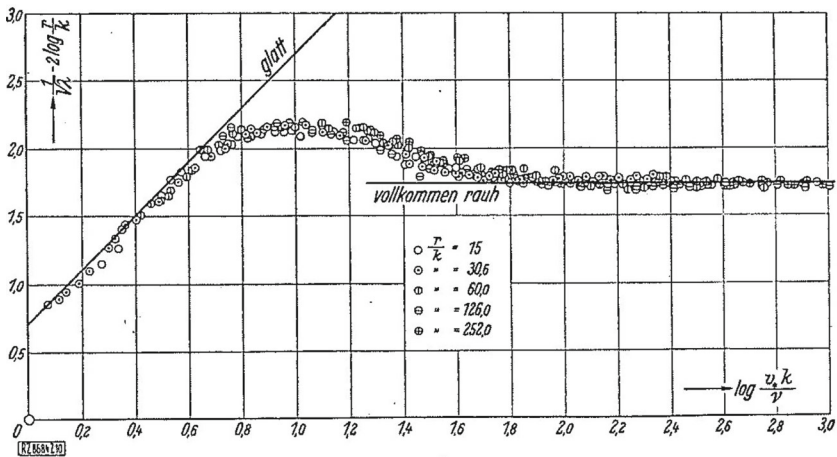


Fig. 5 With increasing roughness, the friction factor was found to become independent of the Reynolds number:  $\frac{1}{\sqrt{\lambda}} = 1.74 + 2 \log \frac{r}{k}$  (Nikuradse 1933, Fig. 11)

in his *Elementary Mechanics of Fluids*—a textbook classic for several decades after WW II (Rouse 1946, Sec. 33 and 34).

## 11 Later developments

Yet the story does not end with the Kármán–Nikuradse–Prandtl solution. For practical purposes, Nikuradse’s uniformly roughened pipes have to be distinguished from industrially produced pipes with randomly sized bumps protruding from the inner surface. These and other qualifications resulted in a variety of equations and charts for the determination of turbulent pipe flow friction factors (“Colebrook–White equation”, “Moody chart”, etc.). Even more than 300 years after the discharge measurements of the conduits at Versailles, turbulent pipe flow remained a major engineering concern (Rennels and Hudson 2012, Chapter 8).

From a more fundamental perspective, the Kármán–Nikuradse–Prandtl solution may be regarded rather as a beginning than a conclusion. It turned pipe flow into a major challenge for turbulence research. The rise of statistical theories of turbulence in the 1930s directed the interest of researchers to the study of turbulent velocity fluctuations which had become amenable to experimental investigation by hot wire measurements in wind tunnels (Eckert 2008). “The assumption of similar flow patterns is identical with the assumption of constant correlation between the components of the velocity fluctuations”, Kármán pointed to the significance of such measurements from which he hoped to corroborate his derivation of the wall law (von Kármán 1934, p. 8). While his collaborator at the Guggenheim Aeronautical Laboratory of the California Institute of Technology (GALCIT), Frank Wattendorf, published first results of such measurements of turbulent air flow in a rectangular channel (Wattendorf 1936), Kármán further elaborated the statistical theory of turbulence so that also the energy balance of turbulent channel flow could be computed from the velocity fluctuations (von Kármán 1937).

For a number of years, the investigation of wind tunnel turbulence was given preference because of its obvious significance for aeronautical applications, but channel and pipe flow was not entirely ignored. In 1949, John Laufer, another collaborator at the GALCIT, reviewed hot-wire measurements from a “two-dimensional wind tunnel” where the air was blown through a 23-foot-long rectangular channel with a cross section of an aspect ratio 12:1. They demonstrated that the hot-wire technique was well enough developed to provide consistent results for the velocity fluctuations across the turbulent flow field (Laufer 1950). Subsequently, Laufer joined the aerodynamics group at the National Bureau of Standards under Hugh Dryden where he applied the hot-wire technique on the air flow in a 16-foot-long seamless brass tube with an inner diameter of 9.72 inches at two Reynolds numbers, 50,000 and 500,000, corresponding to flow speeds of 10 and 100 feet per second. The hot-wire probe allowed precise measurements of velocity fluctuations parallel and perpendicular to the pipe from which the pertinent quantities of the statistical theory of turbulence could be determined (Reynolds stresses, correlation functions, spectrum, turbulent energy production, diffusion and dissipation). It was found, for example, that the edge of the so-called laminar sublayer (which separates the wall surface from the turbulent flow field) played a major role for the transfer of kinetic energy into the turbulent region. These and other findings nourished the view that fully developed turbulent air flow in large pipes provided “a very useful medium in which to study the structure of turbulence in shear flow” (Laufer 1954, p. 18).

Henceforth, pipe flow was regarded more generally as a gateway to the study of more fundamental problems of fully developed turbulence. In 1961, an international conference in Marseille reviewed the state of the art and the perspectives for future research on turbulence; Laufer's measurements were considered as ground-breaking (Hinze 1962, p. 130). Throughout the subsequent decades, pipe flow remained a preferred subject on the agenda of turbulence research—from a theoretical vantage point as well as experimentally (Kim 2012). “A critical review of the literature relative to turbulent shear flow would be quite voluminous”, the authors of a paper on “Turbulent Flow in Smooth and Rough Pipes” introduced their study already 10 years after the Marseille conference (Townes et al. 1972, p. 353). The facilities for turbulent pipe flow investigations also assumed larger dimensions and enabled greater spatial resolution. In the aforementioned study, measurements were reported from air flows in a 70-foot-long and 11.883-inch-wide pipe where the test section could be traversed by the hot-wire probe in movements as small as 0.001 inches (Townes et al. 1972, p. 354).

In the 1990s, a new facility at the Department of Mechanical and Aerospace Engineering of Princeton University employed high-pressure air as a working fluid in order to extend the range of Reynolds numbers by an order of magnitude beyond those of Nikuradse's measurements from 1932. The results from the Princeton Superpipe, as it became famous among turbulence researchers, cast doubt about the previously assumed scaling laws for fully developed turbulent flow in smooth pipes. They suggested a new friction factor relation—similar to Prandtl's formula, but with different constants and an additional term for the near-wall velocity profile (Zagarola and Smits 1998). Yet this did not exhaust the desire for new pipe flow facilities. When the Italian Air Force in 2006 presented to the University of Bologna a subterranean tunnel complex used during World War II for airplane construction, the tunnels became the site for a research laboratory dedicated to precision measurements of turbulent pipe flows at high Reynolds numbers, the Centre for International Cooperation in Long Pipe Experiments, CICLoPE (Talamelli et al. 2014). Another recent addition to the list of pipe flow facilities for investigating fully developed turbulence is the Cottbus large pipe test facility at Brandenburg University of Technology Cottbus-Senftenberg, CoLaPipe (König et al. 2014).

The rise of computational fluid dynamics in the late twentieth century further sparked research on turbulence. With the advent of powerful computers, direct numerical simulation (DNS) matured far enough to compute turbulent pipe flows at high Reynolds numbers. By 2008, a DNS computation of turbulent pipe flow was conducted on  $6.3 \cdot 10^8$  grid points at a Reynolds number of 44,000. Such large-scale computations enable comparison with theoretical predictions and experimental measurements such as from the Princeton Superpipe (Wu and Moin 2008).

These recent developments underscore the continuity of pipe flow investigations up to the present time as a gateway to the study of fully developed turbulence. The same continuity may be observed with investigations of the onset of turbulence. Schiller's experiments on the “turbulence problem” in the 1920s (see Sect. 9) had clearly demonstrated that transition to turbulence in pipe flow could not be fixed at a critical Reynolds number without taking into account the conditions at the entrance of the pipe. In the early 1950s, Werner Pfenninger, an expert on boundary layer research at the Northrop

Corporation Aircraft Division, conducted a number of experiments in order to explore the limits of laminar air flow in smooth pipes. Inlet disturbances were reduced by damping screens mounted ahead of a long nozzle that served as the entrance to the test tube. In order to minimize external disturbances, the test tube was set up in a bomb shelter. Under such conditions, laminar air flow could be maintained up to Reynolds numbers of 100,000 (Pfenninger 1961). Other high-precision experiments on the air flow in smooth pipes focused on the transition zone where “slugs” and “puffs” signalled an intermittently turbulent flow (Wynanski and Champagne 1973). Like fully developed turbulence, the transition to turbulence in pipe flow remained a persistent challenge—both from an experimental and from a theoretical perspective (Darbyshire and Mullin 1995; Kerswell 2005; Eckhardt et al. 2007; Mullin 2011; Eckhardt 2018).

## 12 Conclusion

As pointed out in the Introduction, pipe flow was not the only gateway to turbulence. Other incentives came from open channel flow and from the drag of bodies such as projectiles, vessels in towing channels or airship models in wind tunnels. The “wall law” applied to all kinds of “wall-bounded flows”. Although such parlance became shop talk among turbulence researchers only in the twentieth century, it was not exceptional for du Buat, Prony, Hagen, Darcy, Bazin and their kind to draw parallels between pipe and channel flow. The focus on pipe flow in this study, therefore, is not meant to exclude other flows as less important. Yet pipe flow deserves a particular scrutiny. Its history extends further back in time than that of most other subjects of scientific inquiry. The means and methods of experimental measurements and theoretical analysis have changed over the centuries, but the basic questions remained the same: What are the physical mechanisms that cause the flow retardation? How are the involved quantities (pipe diameter, pipe length, flow velocity, surface roughness, etc.) related to another? What is the structure of flow within the pipe? With these questions, pipe flow became a gateway to research on turbulence—even before turbulence was singled out as the culprit for the unsolved problems.

Hence, pipe flow serves also as a guide to the roots from which turbulence research developed before it was conducted under this label. The conduits at Versailles, the water supply of cities from elevated reservoirs, as well as the gas flow in pipes used for mine air, blast furnaces or gas light remind us of the social context behind such studies from the era of the “Sun King” through the Industrial Revolution to the modern period. Investigations on turbulent flow in newly established research facilities in the twentieth century tend to obscure the utterly practical origins of turbulence research—even though the rise of aeronautics added to the practical context of pipe flow studies which were now often conducted with the hot-wire technology developed for wind tunnel measurements.

From an epistemological perspective, pipe flow investigations, like most researches on turbulence, elude a characterization as mere engineering knowledge. Nor do they fit well with established views of scientific development such as Kuhn’s structure of scientific revolutions. From an historiographic vantage point, the close ties between science and engineering as observed with pipe flow studies over the centuries call for an

amalgamation of the history of science with the history of technology. Like the history of pipe flow, the history of turbulence at large may not be told from one of these two fields without taking into account the other. Turbulence, even in its specific guise as turbulent pipe flow, is not only an unsolved problem between physics and engineering, but also a challenge for the history of science and technology that deserves closer study.

**Acknowledgements** I thank the Deutsche Forschungsgemeinschaft for funding earlier research projects on the history of aerodynamics and the biography of Ludwig Prandtl that sparked my interest in the history of turbulence. My special thanks go to the Forschungsinstitut of the Deutsches Museum in Munich for providing an institutional home for my studies.

**Funding** Open Access funding enabled and organized by Projekt DEAL.

## Compliance with ethical standards

**Conflict of interest** I declare that I have no conflict of interest.

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