

# **Solar and lunar observations at Istanbul in the 1570s**

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**Abstract** From the early ninth century until about eight centuries later, the Middle East witnessed a series of both simple and systematic astronomical observations for the purpose of testing contemporary astronomical tables and deriving the fundamental solar, lunar, and planetary parameters. Of them, the extensive observations of lunar eclipses available before 1000 AD for testing the ephemeredes computed from the astronomical tables are in a relatively sharp contrast to the twelve lunar observations that are pertained to the four extant accounts of the measurements of the basic parameters of Ptolemaic lunar model. The last of them are Taqī al-Dīn Muhammad b.  $Ma'r\bar{u}f's$  (1526–1585) trio of lunar eclipses observed from Istanbul, Cairo, and Thessalonica in 1576–1577 and documented in chapter 2 of book 5 of his famous work, *Sidrat muntaha al-afkar fī malakūt al-falak al-dawwār (The Lotus Tree in the Seventh Heaven of Reflection)*. In this article, we provide a detailed analysis of the accuracy of his solar (1577–1579) and lunar observations.

# **1 Introduction**

Generally speaking, astronomical observations in the medieval period were made for one of the two purposes:

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- (1) Testing contemporaneous astronomical tables and ephemeris. Some instances of such tests contain interesting cases of reconciling theory and observations.<sup>1</sup>
- (2) The derivation of the fundamental parameters of astronomical theories, most notably the solar, lunar, and planetary orbital elements.

In the early Islamic period, the observation of heavenly phenomena such as planetary conjunctions and solar and lunar eclipses was directed at testing the accuracy of contemporary astronomical tables, the so-called *z¯ıjes*. [2](#page-1-1) The most extensive collection of such observations are reported in Ibn Yūnus's *Hākimī zīj* (d. 1009). For example, from the observation of the conjunction of Jupiter with Regulus on 6 September 864, Habash al-Hāsib (d. after 869) found that the mean longitude of the planet as computed from the tables of the mean motions in the *Mumtahan*  $z\bar{y}$  (complied at Baghdad about 830 under the observational programme ordered by the Abbasid caliph, al-Ma'mūn, who reigned 81[3](#page-1-2)–833) should be decreased by  $0;47^\circ$ .<sup>3</sup> Also, from the observation of the conjunction between Venus and Mars on 22 October 864, he argued that the mean anomaly of Venus as computed from that  $z\bar{y}$  should increase to 4;30 $^{\circ}$  and the mean anomaly of Mars should be reduced by 0;30°.<sup>[4](#page-1-3)</sup> Abū al-Rayḥān al-Bīrūnī's works (973– 1048), most notably his *al-Qānūn al-mas'ūdī*, are an important source for Islamic solar and lunar observations other than eclipses. For instance, he gathers and discusses in detail nearly all of the solar observations made by his Islamic predecessors for the determination of the orbital elements of the sun (eccentricity and direction of the apsi-dal line)<sup>[5](#page-1-4)</sup> and of the obliquity of the ecliptic. He also informs us of the Banū Mūs $\vec{a}$ 's

<span id="page-1-0"></span> $1$  The two interesting cases are Shams al-Dīn Muhammad al-Wābkanawī's (1254?–after 1316) test of the times and longitudes computed from the  $\bar{I}lkh\bar{a}n\bar{i} z\bar{j}$  for four conjunctions of Jupiter with Saturn in 1286 and 1305–1306 against observations and his observation of the annular eclipse of 30 January 1283 (see [Mozaffari 2009,](#page-18-0) [2013a,](#page-18-1) [b,](#page-18-2) pp. 239–240; about the curious situation of annular solar eclipses in the medieval astronomy, especially see [Mozaffari 2014b.](#page-18-3) The *Īlkhānī zīj* is the formal product of the early period of the activities in the Maragha observatory, northwestern Iran, *c*. 1259–1271, under the directorship of Nastr al-Dīn al-Tūsī  $(1201-1274)$ ; about it, see [Kennedy 1956,](#page-18-4) pp. 161–2 and [Samsó et al. 2001](#page-19-0), p. 46) and Abrah[am](#page-18-5) [Zacut's](#page-18-5) [observation](#page-18-5) [of](#page-18-5) [the](#page-18-5) [occultation](#page-18-5) [of](#page-18-5) [Venus](#page-18-5) [by](#page-18-5) [the](#page-18-5) [Moon](#page-18-5) [on](#page-18-5) [24](#page-18-5) [July](#page-18-5) [1476](#page-18-5) [\(see](#page-18-5) Goldstein and Chabás [1999](#page-18-5)). For the previous studies about the role observation and its relation to theory in the medieval astronomy, see, e.g., [Hartner](#page-18-6) [\(1977\)](#page-18-6), [Goldstein](#page-17-0) [\(1985a](#page-17-0), [1988](#page-17-1)), [Saliba](#page-19-1) [\(1994](#page-19-1)). For the astronomers referred to here, see DSB, NDSB, BEA, E1<sub>2</sub>, [Sezgin](#page-19-2) [\(1978\)](#page-19-2), and [Rosenfeld and](#page-19-3) İhsanoğlu [\(2003\)](#page-19-3).

<span id="page-1-1"></span><sup>&</sup>lt;sup>2</sup> On Islamic astronomical tables, see [Kennedy](#page-18-4) [\(1956](#page-18-4)) and [Samsó et al.](#page-19-0) [\(2001](#page-19-0)); these two main sources of the knowledge of Islamic astronomical tables, the so-called *z¯ıjes*, are followed by a new comprehensive survey that is currently prepared by Benno Van Dalen.

<span id="page-1-2"></span><sup>&</sup>lt;sup>3</sup> Ibn Yūnus, L: pp. 108–109; [Caussin de Perceval](#page-17-2) [\(1804\)](#page-17-2), p. 155. The date given in the text is "Wednesday, 30 Rajab 250 Hijra" (= 6 September 864) and "21 Murdād 238 Yazdigird" which is the equivalent of Monday, 5 September 86*9*. The year in the latter date is evidently incorrect and should be 233 (on 5 September 869, Jupiter and Regulus were over 125◦ apart).

<span id="page-1-3"></span><sup>&</sup>lt;sup>4</sup> Ibn Yūnus, pp. 108–109, [Caussin de Perceval](#page-17-2) [\(1804](#page-17-2)), pp. 155, 157. The date given in the text is "Sunday, 6 Ramadan  $250$  H" (=  $11$  October 864) and "7 Mihr  $233$  Y" (= 22 October 864). The first date is in error, simply because 11 October 864 was a *Wednesday*. Moreover, in the report it is stated: "Ahmad b. 'Abd-Allāh [Habash] said: '[...] at daybreak (*tulū' al-fajr*; i.e. the start of the morning twilight), I saw Venus and Mars being close to (associated with) each other (*mutalāsiqayn*) in [the zodiacal sign] Virgo, as if the two were one star"'; at the mentioned time on 22 October 864, the two planets were less than 5' apart while 11 days earlier they were about 6◦ apart. Cf. [Caussin de Perceval](#page-17-2) [\(1804\)](#page-17-2), p. 156.

<span id="page-1-4"></span> $<sup>5</sup>$  These are summarized and analysed in [Mozaffari](#page-18-7) [\(2013c\)](#page-18-7). For the solar meridian altitude observations,</sup> see [Said and Stephenson](#page-19-4) [\(1995](#page-19-4)), [Newton](#page-19-5) [\(1972](#page-19-5)) which deals specifically with the solar data recorded by Bīrūnī.

observation of the lunar maximum latitude and the value about  $4:45°$  they deduced, which is one of the two non-Ptolemaic values for the inclination of lunar orbit from the medieval Islamic period.<sup>6</sup>

In the late Islamic period, post-1000 AD, the number of observational reports significantly decreases, despite the fact that the great Islamic observatories were constructed in the same period.<sup>7</sup> Muhyī al-Dīn al-Maghribī (d. 1283) gives a summary of his own "extensive" observations carried out at the Maragha observatory between 1262 and 1274.[8](#page-2-2) A short while later, al-Kawāshī, a thirteenth-century Yemenite scholar, presents 13 random observations of planetary conjunctions, planetary appulses to stars, and occultations that he made in Yemen and Egypt during 1277–1284.<sup>[9](#page-2-3)</sup> Nothing is known about the details of the observations made at the Samarqand observatory, although it is striking that Ulugh Beg's *Sultānī zīj*, the principal fruit of the astronomical programme conducted there, shows the application of a good number of the unprecedented values for the solar, lunar, and planetary parameters in addition to the incorporation of the now well-known star catalogue into this work.<sup>10</sup> Over one century later, Taqī al-Dīn Muhammad b. Ma'r $\bar{u}f(1526-1585)$  documented the solar and lunar observations made

<sup>9</sup> See [King and Gingerich](#page-18-11) [\(1982](#page-18-11)).

<span id="page-2-0"></span><sup>&</sup>lt;sup>6</sup> Bīrunī, *al-Qānūn al-mas'ūdī* VII.5: [\(1954–6](#page-17-3)), vol. 2, p. 779. The other value is 5;3° as observed by Ibn Yūnus (see [King 1999](#page-18-8), pp. 502–503).

<span id="page-2-1"></span> $^7$  About the observatories founded on in the medieval Islamic period, Sayılı ([1960] [1988](#page-19-6)) is still the only available study, although some of his argumentations and conclusions should be treated with caution; e.g. in the case of the latter period of the Maragha observatory, see some critical remarks in [Mozaffari and Zotti](#page-18-9) [\(2013](#page-18-9)), pp. 61–62.

<span id="page-2-2"></span><sup>8</sup> About Muhyī al-Dīn, especially see [Saliba](#page-19-7) [\(1983](#page-19-7), [1985,](#page-19-8) [1986\)](#page-19-9) and [Mozaffari](#page-18-10) [\(2014a](#page-18-10)). A monograph about his unique contribution to observational and practical astronomy at the Maragha observatory on the basis of a thorough analysis of his documented observations in the *Talkhūs al-majistī* (*Compendium of the* Almagest) is being prepared by one of us (SMM).

<span id="page-2-4"></span><span id="page-2-3"></span> $10$  In his detailed commentary on Ulugh Beg's  $Z_{ij}$ , 'Alī Qūshčī (d. 1474), one of the contributors of this work, says nothing as to the details of the astronomical observations at Samarqand. This source is, of course, invaluable for checking the parameter values deduced from the tables in that  $z\bar{t}j$ . Much less emphasis in the secondary, modern literature has been put on the parameter values underlying the tables in this work; the eccentricity of Venus is a good example: the majority of the early Islamic astronomers including al-Battānī and Ibn Yūnus, influenced by Indian astronomy, took the eccentricity of this planet to be equal to that of the sun (i.e. the earth). Although some astronomers like Bīrūnī and 'Abd-al-Rahmān al-Khāzinī (*fl.* the first half of the twelfth century) kept the Ptolemaic distinction of the eccentricity of Venus from that of the sun, this idea did not disappear completely in the Middle Eastern branch of Islamic astronomy until the foundation of the Maragha observatory, as it can be traced back in some *z¯ıjes* until the mid-thirteenth century (e.g. in Muntakhab al-Dīn al-Yazdī's *Manzūm zīj*, "Versified zīj" written in Yazd, central Iran, *ca.* 1252, f. 46v). Nevertheless, it can be found prevalently in the Western branch of Islamic astronomy (Spain and northwestern Africa) in the latter periods (e.g. in the *zīj* [of](#page-19-10) [Ibn](#page-19-10) al-Bannā of [Marrakech,](#page-19-10) [d.](#page-19-10) [1321;](#page-19-10) [see](#page-19-10) Samsó and Millás [1998](#page-19-10), pp. 265, n. 19, 266) and was transmitted to the late medieval Latin and Jewish astronomy (e.g. cf. [Swerdlow 1977,](#page-19-11) p. 205; [Goldstein 1985b](#page-17-4), p. 113; [Chabás and Goldstein 1994](#page-17-5)[,](#page-18-5) [p.](#page-18-5) [33;](#page-18-5) Goldstein and Chabás [1999](#page-18-5), p. 188; [Goldstein 2003,](#page-18-12) pp. 160–161; [Chabás and Goldstein 2003,](#page-17-6) pp. 253–254; [Chabás](#page-17-7) [2004,](#page-17-7) p. 188; [Chabás and Goldstein 2009](#page-17-8), p. 34). By this idea, the double eccentricity of the planet (i.e. the distance of the equant point from the earth) remain larger than 2 (the radius of the orbit  $= 60$ ). But, the geocentric eccentricity of Venus approximately remains equal to about 1.74 in the past two millennia. The maximum equation of centre of Venus in Ulugh Beg's  $Z\bar{t}j$  (P1: fol. 144r; P2: fol. 161v) is equal to 1;39,19° which corresponds to a double eccentricity of 1.73, in agreement with the value  $0;52$  Qūshčī gives for the half of it (N: pp. 273–4, PN: f. 241v). An analysis of medieval values for the orbital elements of Venus is being prepared by one of us (SMM).

at the Ottoman territory, most notably in the short-lived observatory at Istanbul before its deconstruction, which are the main concern of the present paper. He was not provided with an opportunity and, more important, facilities to deal with the planets, and so his main contribution to observational astronomy was confined to the determination of the solar and lunar parameters.

Let us make another distinction between simple/random and purposed/systematic observations. Al-Kawāshī's observations are typical of the first category, while Taqī al-Dīn and Muhyī al-Dīn's observations fall into the second one, where an astronomer explains quantitatively how he has derived his own parameter values from direct observations, often a problematical task with unexpected difficulties requiring sufficient and reasonable justifications, so that other medieval astronomers show little intention to do so. Ibn Yūnus, for instance, never explains whether and how he derived his non-Ptolemaic parameter values from observations, although it cannot be far from the truth to assume that his new parameter values are actually based upon the data he gathered from his documented observations. For instance, the possibility exists that his non-Ptolemaic values for the radii of the epicycles of the interior planets<sup>[11](#page-3-0)</sup> were the fruit of his own observations of these planets.<sup>12</sup>

For lunar eclipses, a distinction between random and systematic observations is especially relevant. A good number of reports of observations of lunar eclipses survive from early Islamic astronomy, especially in Ibn Yūnus's *Hākimī zū*j. These observations have played a pivotal role in the modern estimation of the rate of the deceleration of the earth's rotation about its axis  $(\Delta T)$ , the difference between terrestrial and universal time).<sup>[13](#page-3-2)</sup> For a medieval astronomer, lunar eclipses were the only means by which the lunar orbital elements in the Ptolemaic model could be determined. In order to measure the size of the lunar epicycle, a trio of lunar eclipses is required. Observations of the moon at quadratures are necessary for determining the eccentricity.

Some of the preserved reports that belong to the early Islamic period appear to be simple observations that, at best, only fulfil the first purpose posited in the beginning of this paper, namely to test available *zijes*, and do not show any clear relation to the second purpose, i.e. the determination of the parameters of the lunar model:<sup>[14](#page-3-3)</sup> Al-Māhānī observed a trio of lunar eclipses in 854–6, but only measured the times of the beginning of eclipses and/or immersions (i.e. first and second contacts), while for

<span id="page-3-0"></span><sup>11</sup> He has the value 22;52 for the radius of the epicycle of Mercury (Ptolemy: 22;30 in the *Almagest* and 22;15 in the *Planetary Hypotheses*) and 43;28 for that of Venus (Ptolemy: 43;10) if the radius of the geocentric orbit of the epicycle centre, the deferent, is taken as 60 arbitrary units. These values are derived from the maximum value for the epicyclic equation of these planets at mean distance as tabulated in Ibn Yūnus's *zīj*, i.e. 22;24° and 46;25°, respectively, for Mercury and Venus (Ibn Yūnus, L: pp. 121, 190, 192; [Caussin de Perceval 1804](#page-17-2), p. 221).

<span id="page-3-1"></span> $12$  He observed some conjunctions of the inferior planets with each other (e.g. the morning of 22 June 985; modern: the evening of 18 June 985), with stars (e.g. Venus and Regulus: one hour after sunset in Cairo on 23 June 990; modern: about 3 h after midnight on 24 June 990), and with the other planets (e.g. Venus and Saturn: half an hour before the sunrise in Cairo on 20 January 988; modern: about two hours before the sunrise in Cairo on the given date); see Ibn Yūnus, *Zīi*, L: pp. 113–114; [Caussin de Perceval](#page-17-2) [\(1804\)](#page-17-2), pp. 179–184.

<span id="page-3-2"></span><sup>&</sup>lt;sup>13</sup> The results of the researches by Prof. F. R. Stephenson and his colleagues on medieval Islamic eclipses reports are summarized in [Stephenson](#page-19-12) [\(1997](#page-19-12)), chapters 12 and 13 and [Steele](#page-19-13) [\(2000](#page-19-13)), chapter 4.

<span id="page-3-3"></span><sup>&</sup>lt;sup>14</sup> What follows is based upon [Stephenson](#page-19-12) [\(1997\)](#page-19-12), pp. 476–493 and [Steele](#page-19-13) [\(2000\)](#page-19-13), pp. 107–124.

the measurement of the radius of the lunar epicycle, it is necessary to determine the times of the middle (maximum phase) of lunar eclipses. Ibn al-Amājūr's observations of five lunar eclipses in a decade from 923 to 933 were mainly directed at testing Habash's *zīi*. Al-Battānī describes only two lunar eclipses that he observed in 883 and 901. By them, he shows the existence of glaring differences in magnitudes and timings of the eclipses between what are computed on the basis of the *Almagest* and what are observed. He also employs them to derive the apparent angular diameter of the moon at mean distance.<sup>[15](#page-4-0)</sup> Nevertheless, from this period, we have three values for the radius of the lunar epicycle; the first two are the Ban $\bar{u}$  M $\bar{u}$ s $\bar{a}$ 's 5;22 and Ibn al-A'lam's  $5;5.^{16}$  $5;5.^{16}$  $5;5.^{16}$  No lunar eclipse is reported from the Banū Mūsā, and their own value for the maximum lunar first inequality is mentioned in a later source, namely the thirteenth-century *Ashrafī* zī*i*, while Bīrūnī has nothing to say about it; nevertheless, his clear evidence of the other lunar observational data from the Banū Mūsā makes it not far from true to accept the validity of this attribution and that their own value for the radius of the epicycle was actually an observational achievement. The same can also be true of Ibn al-A'lam, from whom a non-Ptolemaic table of the lunar equations has survived, though not any observation of a lunar eclipse. The third value is Ibn Yūnus's 5;1 as derived from a maximum lunar first inequality of  $4:48°$  as tabulated in his own *z*ī*j*. Ibn Yūnus observed ten lunar eclipses spread over a period from 979 until 1002; for half of them, the times of the first and last contact are given either directly or with reference to the altitudes of the moon or of some luminous clock stars.

In the late medieval Islamic period, the situation drastically changed, so that the astronomers of this period no longer seem intent on simply presenting the results of their own observations of eclipses for the purpose of testing astronomical tables against the obtained observational data;  $17$  rather, all the twelve lunar observations we

<span id="page-4-0"></span><sup>&</sup>lt;sup>15</sup> Nallino ([1899–1907] [1969\)](#page-18-13), vol. 3, p. 87. After this, al-Battānī concentrates on the determination of the sun's distance to the earth; see [Swerdlow](#page-19-14) [\(1972\)](#page-19-14).

<span id="page-4-1"></span><sup>&</sup>lt;sup>16</sup> The radius of the lunar orbit, the inclined eccentric deferent, is taken as 60 arbitrary units. These two values are derived, respectively, from the maximum values given for the first inequality of the moon by al-Kamālī in his *Ashrafī zīj*, ff. 49r and 229v–230r: 5;8° and 4;51°. Muhyī al-Dīn al-Maghribī adopts Ibn al-A'lam's lunar equations in his first *zīj*, *Tāj al-azyāj* (*Crown of the zījes*), compiled in Damascus before his joining to the Maragha observatory (see [Dorce 2002–3](#page-17-9), p. 203; [2003,](#page-17-10) pp. 127, 184).

<span id="page-4-2"></span><sup>&</sup>lt;sup>17</sup> A main factor appears to be the fair agreement between the computed and observed results, as a good number of such accounts scattered in the late Islamic *zījes* testify; in them, an astronomer explains his computation of the circumstances and parameters of an eclipse and then usually claims that they were in agreement with observation, which can easily be checked by aid of modern data. For example, in his 'Alā'ī *z¯ıj* [preserved in a unique copy in India, Hyderabad, Salar Jung Library, no. H17; see [Dalen](#page-17-11) [\(2004](#page-17-11))] on pp. 32–35, Farīd al-Dīn Abu al-Hasan 'Alī b. 'Abd al-Karīm al-Fahhād of Shirwān or Bākū (both cities now in Azerbaijan, the latter north to the first) presents at length his computation of the parameters of a solar and a lunar eclipse that were to take place, respectively, in the conjunction and opposition about the month Shawwal of the year 571 H/April–May 1176. For the solar eclipse (which occurred on 11 April 1176), he computes the ecliptic longitude at the instant of the apparent conjunction (i.e. the topocentric longitude of the sun and moon in the maximum phase of the eclipse) as  $\lambda \sigma = 2732^\circ$ , the time of mid-eclipse as about *T*  $= 4;40$  h before noon, and its magnitude as 11;46 digits (the diameter of the solar disc is taken as 12 digits). He then states that he observed this eclipse and found its circumstances in agreement with the computed results. It is not precisely known whether the place of observation was Bakū or Shirwān; for the first, the modern values are:  $\lambda_{\sigma} = 27.56^{\circ}$ ,  $T = 8.18$  MLT, and magnitude 0.996. For the lunar eclipse (which occurred on 25 April 1176), he gives the longitude of the moon at the instant of the mid-eclipse as about  $\lambda_{\sigma}$ ° = 222;31°, *T* = 3;53 h after sunset, and magnitude 6;51 digits (the diameter of the lunar disc is taken

have at our disposal from the period in question pertain to the four extant accounts of the lunar measurements surviving from Islamic astronomy. In them, the four Islamic astronomers present their observational data of a trio of lunar eclipses and explain how they have computed their own values for the radius of the lunar epicycle from them:

- (i) Bīrūnī in *al-Qānūn al-mas'ūdī* (*Mas'ūdīc canons*) VII.3: the lunar eclipses of 1003–1004, observed at Ghazna; $^{18}$
- (ii) Muhyī al-Dīn al-Maghribī in *Talkhīs al-majistī* V: the lunar eclipses 7 March 1262, 7 April 1270, and 24 January 1274, observed from Maragha; $^{19}$
- (iii) Jamshīd Ghiyāth al-Dīn al-Kāshī in the prologue of the *Khāqānī zīj*: the lunar eclipses of 1406–7 observed in Kāshān, central Iran; $^{20}$  $^{20}$  $^{20}$  and
- (iv) Taqī al-Dīn Muhammad b. Ma'rūf in Sidrat muntaha al-afkar fī malakūt al-falak *al-daww¯ar* (*The Lotus Tree in the Seventh Heaven of Reflection*) V.2: the lunar eclipses of 1576–1577 observed in Istanbul, Cairo, and Thessalonica.<sup>[21](#page-5-3)</sup>

Bīrūnī and Muhyī al-Dīn determined the value 5;12 for the radius of the lunar epicycle; Kāshī reached the figure about 5;17; Taqī al-Dīn derived the value about 5;24. Of them, only Muhyī al-Dīn and Taqī al-Dīn explain their observations of the moon near quadrature for the sake of determination of the lunar eccentricity in Ptolemaic model; the first derives the value 9 and the latter a value a bit more than 9;46 (radius of orbit  $= 60$ ).

The first three trios have already been studied. Here, Taqī al-Dīn's solar and lunar observations are presented and analysed. The accuracy of his lunar observations is also compared with the precision attained in the three earlier sets of observations of the triple lunar eclipses from the late Islamic period as well as in the extensive observations of the lunar eclipses from both the late medieval European and early Islamic periods.

Footnote 17 continued

as 12 digits); the modern values are:  $\lambda_{\alpha}$  $\phi$  = 221;59°, *T* = 22:34 MLT (sunset: 18:53 MLT), and magnitude 0.673. In both cases, the computed longitudes are of errors of about 1/2◦; the computed magnitude of the solar eclipse and the time of the lunar counterpart are of good accuracy. Such accuracies are not entirely matters of coincidence, since similar instances can be traced back in medieval Islamic astronomy (a notable case may be Wābkanawī's calculation of the circumstances of the annular solar eclipse of 30 January 1283; see note 1 above). Rather, this reflects our lack of knowledge about the quantitative precision of some Islamic *zījes* that were the fruits of undertaking the difficult task of continuous observations and derivations of parameters of Ptolemaic models, and the fact that if Ptolemaic models were quantified anew by the re-measurement of its fundamental parameters, it would be probable to predict eclipses with precisions within an hour, one degree in longitude, and one digit in magnitude. Wābkanawī replaced al-Fahhād's computations and eclipses by his calculation of the solar eclipses of 5 July 1293 and 28 October 1296 (for latitude of Tabriz, northwestern Iran) and the lunar eclipse of 30 May 1295 when he taught al-Fahhād's *Z¯ıj* to Gregory Chioniades (*c*. 1240–1320) who translated it into Greek (see [Pingree 1985](#page-19-15), p. 352f). For a brief review of the other cases of the calculations of circumstances of eclipses, see [Mozaffari](#page-18-14) [\(2013d](#page-18-14)), pp. 313–314.

<span id="page-5-0"></span><sup>&</sup>lt;sup>18</sup> Bīrūnī [\(1954–6](#page-17-3)), vol. 2, pp. 740–743. These eclipses are nos. 07224, 07225, and 07227 in the NASA's Five Millennium Catalog of Lunar Eclipses (hereafter, 5MCLE). For the analysis of Bīrūnī's observations, see [Said and Stephenson](#page-19-16) [\(1997\)](#page-19-16), pp. 45–46; [Stephenson](#page-19-12) [\(1997\)](#page-19-12), pp. 491–492.

<sup>19</sup> [Mozaffari](#page-18-10) [\(2014a\)](#page-18-10), pp. 72–74. The eclipses nos. 07878, 07897, and 07907 in 5MCLE.

<span id="page-5-2"></span><span id="page-5-1"></span><sup>&</sup>lt;sup>20</sup> Kāshī, IO: ff. 4r–6r, P: pp. 24–28. The eclipses nos. 08220, 08221, and 08222 in 5MCLE. See [Mozaffari](#page-18-14) [\(2013d](#page-18-14)), pp. 318–322.

<span id="page-5-3"></span><sup>&</sup>lt;sup>21</sup> Taqī al-Dīn, *Sidrat*, K: ff. 42r–43r. The eclipses nos. 08610, 08611, and 08612 in 5MCLE.

#### **2 Taq¯ı al-D¯ın's observations**

For the present study, we made use of MS Istanbul, Kandilli Observatory Library, no.  $208$ , which is a collection of some works by Taq $\bar{a}$  al-D $\bar{b}$  copied in his own hand; the first treatise in this codex is his *Sidrat*, the first  $z\bar{y}$  (astronomical tables with accom-panying instructions to use them) he composed.<sup>[22](#page-6-0)</sup> As usual in this genre of  $z\bar{y}$ *jes*, in the canons, our author presents the variant topics pertaining to the theoretical, mathematical, and practical astronomy such as the sections on chronology, trigonometry, spherical astronomy, and the methods for the derivation of the fundamental parameters. In the parts related to the sun and moon, detailed accounts of his observations and the instruments applied to them are given, $^{23}$  $^{23}$  $^{23}$  and then he explains how he has derived his own values for the solar and lunar parameters from the date obtained in these obser-vations. In what follows (Sect. [2.1\)](#page-9-0), we first present Taq $\bar{\text{I}}$  al-D $\bar{\text{I}}$ n's solar observations as translated from the original Arabic text, which are also summarized in Table [1,](#page-7-0) together with a brief commentary upon the accuracy of the unprecedented values he achieved for the solar parameters. This is followed by presenting the accounts of his four lunar observations in the same way (Sect. [2.2\)](#page-11-0); the fourth observation is investigated there, but his first three lunar observations, i.e. the trio of lunar eclipses, shall be analysed at length in Sect. [3.](#page-14-0) We number his nine observations in the chronological order and indicate those of the sun by the prefix S, and of the moon by M. M1 (1576) is the earliest documented observation and M4 (1579), the latest. As the contents of this work shows, Taqī al-Dīn's observations are limited only to the two luminaries; as he definitely says in the account of M4, at that time, he had not yet dealt with the stellar observations. He died in 1585 and apparently did not find any opportunity to deal with the stars and planets; moreover, as mentioned earlier, in his later  $z\bar{t}$ *i*, *Khar* $\bar{t}$ *dat*, he strangely returns back to Ulugh Beg's values for the solar and lunar parameters.

Two notes about the dates and a technical astronomical term in the following accounts merit consideration: the Alexander's era mentioned in the reports is in fact the Seleucid or Byzantine era (1 October –311), to which Ptolemy refers as "accord-ing to the Chaldaeans"<sup>[24](#page-6-2)</sup> ("Two-Horned", i.e. Alexander, in Islamic astronomy), in which the years are Julian years of  $3651/4^d$ ; although our author repeatedly make use of the alternative of "the death of Alexander" for this calendar, it has nothing to do with the Philip era which is referred to as "Death of Alexander" (12 November –323) throughout the *Almagest*, and which is used with the Egyptian years of 365<sup>d</sup>. Also, The Hijra date in the first observation of the sun is according to the *civil* reckoning (the epoch 16 July 622), but in the other four solar observations as well as in all of the lunar observations, the Hijra dates are according to the *astronomical* reckoning (the epoch 15 July 622).<sup>25</sup> The terms *sā* '*āt al-bu'd* or *daqā'iq al-bu'd*, literally, "hours/minutes of the distance", as found in all the passages, refer to the interval of time remaining

<sup>22</sup> [King](#page-18-15) [\(2004/5\)](#page-18-15), vol. 1, p. 64.

<span id="page-6-1"></span><span id="page-6-0"></span><sup>&</sup>lt;sup>23</sup> For the illustration of the instruments of the Istanbul observatory, see [Sezgin and Neubauer](#page-19-17) [\(2010\)](#page-19-17), vol. 2, pp. 53–61.

<sup>24</sup> *Almagest* IX.7 and XI.7: [Toomer](#page-19-18) [\(1998](#page-19-18)), pp. 452–3, 541.

<span id="page-6-3"></span><span id="page-6-2"></span><sup>&</sup>lt;sup>25</sup> See B. V. Dalen's entry *Ta'rīkh* (date, chronology) in *E1*<sub>2</sub>, vol. 10, pp. 259, 261.

<span id="page-7-0"></span>



to or passed from the meridian passage/transit of a heavenly body (in the case of the sun: true noon) counted in terms of equal hours or minutes.<sup>[26](#page-9-1)</sup>

#### <span id="page-9-0"></span>**2.1 The solar observations**

- [S1] We have observed the extremal [noon-altitudes of the sun] at the two solstices in the same year. The second [first (?) observation] had been made at true noon (*nisf*) *nahār*; lit. "middle of daylight/midday") on Tuesday, 24 Rabī' al-Awwal [3] [...] in the year 985 of Hijra and  $[11$  Hazīrān  $[9]$ ] 1888 of Alexander's era. After the correcting adjustment for making the true altitude and [i.e., deriving the summer solstice altitude from the noon-altitude on this solstice day by] considering the period/argument *(hissa)* [between noon and the time of occurring the summer solstice], the maximum altitude at the summer solstice was  $72;30,8,29°$ <sup>[27](#page-9-2)</sup>
- [S2] But, the first [second (?) observation] had been made at true noon on Wednesday, the first day *(ghurrat)* of Shawwal [10] in the mentioned year [i.e., 985]. After doing the adjustments, the extremal altitude at the winter solstice was 25;32,20,14◦. [28](#page-9-3)

From Taqī al-Dīn's above statements as well as the precision to which the two values just mentioned are given, it is evidently understood that they are the product of some kind of adjustment. In fact, the solar noon-altitudes in the solstitial days can be representative of the sun's extremal meridian-altitudes, if and only if the solstices take place exactly at true noon. Otherwise, medieval astronomers undertook some methods for extrapolating the extreme solstitial noon-altitudes. Taq $\bar{a}$  al-D $\bar{b}$ n does not explain his adopted method in order to do this, but the practical procedures for such adjustments can be addressed in the works of his predecessors, e.g. in Bīrūnī's *Tahdīd nahayāt* al-amākin.<sup>[29](#page-9-4)</sup> Taqī al-Dīn should have computed in some way the period/argument from true noon on the given dates to the time when the solstice occurred, and then extrapolated the extremal altitude from the rate of change in the sun's declination about the solstices; this is, however, very minor, about  $14$ <sup>"</sup>, and thus, it can be deduced that his observed values for the solar noon-altitude at the summer and winter solstices should not differ too much from 72;30◦ and 25;32,30◦. From these two observations, he derives his own unique value  $\varepsilon = 23.28,54°$  for the obliquity of the ecliptic,<sup>30</sup> which is only about  $-0;1°$  in error, and  $\varphi = 40;58,46°$  for the latitude of Istanbul.

[S3] […] in order to derive the time of the vernal equinox, we installed the instrument having the chords *(dhāt al-awtār)*, and observed the shadow-covering by means of it. Then, [we found that] it took place before true noon (*al-zaw¯al*) on Wednesday, 13 Muharram  $[1]$  in the year 987 of the noble Hijra, 20 Pharmouthi

<sup>26</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 22v.

<span id="page-9-1"></span><sup>&</sup>lt;sup>27</sup> Taqī al-Dīn, *Sidrat*, K: f. 17v (on the right margin).

<span id="page-9-2"></span><sup>&</sup>lt;sup>28</sup> Taqī al-Dīn, *Sidrat*, K: f. 17v (on the right margin).

<span id="page-9-3"></span><sup>&</sup>lt;sup>29</sup> See al-Bīrūnī [\(1967\)](#page-17-12), pp. 61–64; [Kennedy](#page-18-16) [\(1973\)](#page-18-16), pp. 34–38.

<span id="page-9-5"></span><span id="page-9-4"></span><sup>&</sup>lt;sup>30</sup> Taqī al-Dīn, *Sidrat*, K: f. 17v; also, see [King](#page-18-15) [\(2004/5](#page-18-15)), vol. 1, pp. 57, 116, 123, 133, 151.

[8] in the year 2327 Nabonassar, 11  $\bar{A}$ dhār [6] 1890 after the death of Alexander, at 2;34,47 equal hours from true noon ( $s\bar{a}$ ' $\bar{a}t$  bu'd mu'addala).<sup>[31](#page-10-0)</sup>

- [S4] Then, on Saturday, 22 Jumādā al-Ākhira [6] in the year 987 of Hijra, 22 Thoth [1] in the year 2328 Nabonassar,  $15$   $\overline{Ab}$  [11] in the year 1890 from the death of Alexander, we observed the body of the Great Luminary [i.e., the sun] by the armillary sphere some minutes before true noon (*al-zaw¯al*) and by the mural meridional quadrant (*lubna*) at it [i.e., true noon] for the examination of the correctness of the two observations. After the agreement of the two observations by taking into account the time [of the first observation] from true noon in minutes (*daqā'iq al-bu'd*), [we found that] it was in the [ecliptic] sign Virgo 1;21,15◦ at the time of transiting the meridian (*tawassut.*).[32](#page-10-1)
- [S5] After it, on Monday, 23 Rajab [7] in the year 987 of Hijra, the longitude (*mawd. i'*, lit. "position") of the Great Luminary was in the [ecliptic] sign Libra 0;36◦ in the time of passing the meridian as observed by the armillary sphere before true noon and the mural quadrant at it and the correct agreement of the two observations by the adjustment (*ta'dīl*) mentioned earlier. From the [sun's] mean motion known from the New Observations and the derivation of its true daily motion (*al-buht*), it necessitates that the time of the sun's entrance into the head of Libra, the autumnal equinox, in hours and their fractions from true noon (*s¯a'¯at al-bu'd wa kusūrihā*), was 9;22,36h on Sunday, 22 Rajab [987], the 21st of the month of Phaophi  $[2]$  in the year 2328 Nabonassar, 13 Ayl $\bar{u}$  [12] in the year 1890 after the death of Alexander $33$

These three solar observations are related to the determination of the times of equinoxes of 1579 and the position of the sun at an intermediary point, from which the basic parameters of the solar model are derived. The accuracy of Taqī al-Dīn's values for the solar noon-altitudes and times of equinoxes is significant and comparable with that of the outstanding figures of the early Islamic period.[34](#page-10-3) Owing to an error in counting the time between the vernal equinox of this year and Ptolemy's observation of the same equinox in 140, Taqī al-Dīn deduced a value about 365;14,38,34 days for the length of the solar year, which is about  $3 \text{ min}$  too long.<sup>35</sup> Then, having employed the general three-point method, $36$  he obtained the value about 2;0,34 for the solar eccentricity (radius of orbit =  $60$ ; or 0.01675, if the radius of orbit is taken as the unit) and 95;33 $\degree$  for the longitude of its apogee.<sup>37</sup> Taq $\bar{\text{a}}$  al-D $\bar{\text{m}}$ 's documentation of his solar observations makes it possible to compute the true values for the eccentricity of the

<sup>31</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 35r.

<span id="page-10-0"></span> $32$  Taqī al-Dīn, *Sidrat*, K: f. 36r.

<span id="page-10-1"></span><sup>33</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 36r.

<span id="page-10-2"></span><sup>&</sup>lt;sup>34</sup> See [Said and Stephenson](#page-19-4) [\(1995\)](#page-19-4).

<span id="page-10-4"></span><span id="page-10-3"></span><sup>35</sup> Note that this corresponds to a *mean* solar year computed between the two vernal equinoxes in the period from 140 to 1579; the true value for such conception of the solar year in this period is 365;14,32,5 days.

<sup>36</sup> About this, see [Mozaffari](#page-18-7) [\(2013c](#page-18-7)), pp. 323–324.

<span id="page-10-6"></span><span id="page-10-5"></span> $37$  Taqī al-Dīn, *Sidrat*, K: ff. 36r–v (cf. also, [Tekeli 1962,](#page-19-19) [2008\)](#page-19-20). It is somewhat strange that in his later  $z\bar{t}$ *i* (*Kharīdat*, B: ff. 28r–v.), Taqī al-Dīn comes back to Ulugh Beg's value for the solar eccentricity, since the maximum tabular value for the solar equation of centre in this work is given as 1.9315◦, corresponding to  $e \approx 2;1,20$  (see [Mozaffari 2013c](#page-18-7), p. 326, Table 3, no. 13).

earth and the longitude of the solar apogee in a circular orbit, which should be, respectively, 0.01686 and 95;10 $\degree$  for 1579 (in the elliptical orbit: 0.01688 and 95;43 $\degree$ ).<sup>[38](#page-11-1)</sup> His values for the solar orbital elements are highly precise: for the longitude of the solar apogee, he has one of the most accurate values observed in Islamic astronomy.<sup>39</sup> Of course, for the eccentricity, his accuracy had already been reached by Ulugh Beg, the best of what was achieved in late Islamic astronomy, but not repeating the brilliant accuracy of Yahyā b. Abī Mansūr and Bīrūnī with errors, respectively,  $\sim$   $-1 \times 10^{-5}$ and  $+5 \times 10^{-5}$ .<sup>[40](#page-11-3)</sup> It is noteworthy that his value for the eccentricity is remarkably better than that of his Danish contemporary, Tycho Brahe, who derived 0.01792 in 1588 (computed value for a circular orbit: 0.01690; true value in the elliptical orbit:  $0.01688$ .<sup>41</sup>

## <span id="page-11-0"></span>**2.2 The lunar observations**

In what follows, Taqī al-Dīn's reports of his four lunar observations are presented, which are also summarized in Table [2.](#page-12-0) For the lunar eclipses, Taqī al-Dīn uses both types of the description of the date of a lunar eclipse as customary in medieval Islamic treatises: "on the night whose morning was [the day after eclipse]" and "after the meridian passage of the sun on [the preceding day]".

[M1] The first of the triple lunar eclipses we observed in the house of the great master  $[\dots]$  Sa'd al-Milla wa-'l-Dawla wa-'l-Dīn  $[\dots]$ <sup>[42](#page-11-5)</sup> the distance of which from the observatory (*dār al-rasad*) does not make any perceptible difference in seconds [of time]. The time of the middle [i.e., the maximum phase] of the eclipse was 12;3,56 h after the meridian transit of the sun [i.e., true noon] on Sunday, 15 Rajab [7] 984. [...] The moon was eclipsed by 9 digits of its light.<sup>[43](#page-11-6)</sup>

Sa'd al-Dīn Efendī (d. 1599) was one of Taqī al-Dīn supporters, whom he praises in the prologue.[44](#page-11-7) Observations used for the derivation of the mean motions and orbital elements of the sun, moon, and planets should be made in or converted to the local time of a specific meridian. The place of the observatory was representative of the principal longitude of Istanbul, from which its latitude was also measured. Taqī al-D<sub>In</sub> notes that the difference in longitude between Sa'd al-D<sub>In</sub>'s house, where this first observation was made, and the observatory, where the second lunar eclipse was observed, is sufficiently small as to have no undesirable consequence in the use of these observations.

<sup>38</sup> For the technical discussion on this topic, see [Mozaffari](#page-18-7) [\(2013c](#page-18-7)), Part 2.

<span id="page-11-1"></span><sup>39</sup> See [Mozaffari](#page-18-7) [\(2013c](#page-18-7)), p. 399–400.

<span id="page-11-2"></span><sup>40</sup> See [Mozaffari](#page-18-7) [\(2013c](#page-18-7)), p. 393, 397.

<span id="page-11-4"></span><span id="page-11-3"></span><sup>41</sup> See [Brahe 1913–29,](#page-17-13) *Opera Omnia*, vol. 2, pp. 19–28; [Dreyer](#page-17-14) [\(1890\)](#page-17-14), p. 333; [Moesgaard](#page-18-17) [\(1975\)](#page-18-17), pp. 85–89; [Thoren and Christianson](#page-19-21) [\(1990](#page-19-21)), p. 223–224; [Swerdlow](#page-19-22) [\(2010\)](#page-19-22), p. 155.

<sup>&</sup>lt;sup>42</sup> The vacant places only indicate the glorying titles Taqī al-Dīn ascribes to Sa'd al-Dīn Efendī.

<span id="page-11-5"></span><sup>&</sup>lt;sup>43</sup> Taqī al-Dīn, *Sidrat*, K: f. 42r.

<span id="page-11-7"></span><span id="page-11-6"></span><sup>44</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 2v.



<span id="page-12-0"></span>

[M2] [...] the observation of the total lunar eclipse that took place in the night whose morning was Wednesday, 15 Muharram [1] in the year 985 of the noble Hijra. We found the time of its middle with the utmost investigation with excellent masters in the observations by means of the great instruments installed in the new observatory […] to be 9;8,46 after the meridian passage of the sun on Tuesday, 14 Muharram in the mentioned year.

[This was] a total lunar eclipse with a perceptible duration (*makth*, lit. "staying").[45](#page-13-0)

[M3] We were not able to observe the third eclipse, because of the entrance of the clouds. Our excellent brothers from Egypt told us of it and also Dāwūd al-Riy $\bar{a}d\bar{a}$  transmitted it to us with the measurement of [the altitude(?) of the star] Aldebaran [i.e., α Tau]. Then, we converted it to the longitude of Constantinople. Then, the time of its middle was 13;36,36 h after the meridian transit of the sun on Thursday, 14 Rajab [7] in the year 985. So, it occurred in the night whose morning was Friday 15 [Rajab].<sup>[46](#page-13-1)</sup>

A. Ben-Zaken identifies Dāwūd al-Riyādī (the Mathematician) from Thessalonica mentioned in the report of M3 as David Ben-Shushan, a Jewish scholar.<sup>[47](#page-13-2)</sup> He appears to have measured the altitude of the star Aldebaran ( $\alpha$  Tau) at the time of the maximum phase of the eclipse, since Taqī al-Dīn immediately mentions that he converted it to the meridian of Istanbul, and then gives the time of the middle of the eclipse. No information is given on what Taqī al-Dīn believes is the difference in longitude of Istanbul and whatever place in Egypt the observation was made (probably, Cairo?).

For all three eclipses, Taqī al-Dīn reports only the time of the middle of the eclipse. Because the midpoint of an eclipse is difficult to determine directly from observation, it is likely that in all cases he has calculated the midpoint from observations of the times of the beginning and end of either the whole eclipse or the total phase of the eclipse. This suggests that the reports of the eclipse given by Taqī al-Dīn represent observations that have already been through a process of analysis, rather than the original raw data of the observations. A similar conclusion can be drawn from the lack of details about the altitude of Aldebaran in the final report; only the reduced time has been given.

The fourth observation is used for the measurement of the maximum value of the second inequality of the moon and hence its eccentricity in Ptolemy's lunar model. Such observations should fulfil some essential conditions: in the time of the observation, the moon should be near quadrature and have the maximum distance from its mean longitude as well as it should culminate, so that its vertical circle of altitude is perpendicular to the ecliptic, which is to neutralize the effect of the longitudinal component of parallax.<sup>[48](#page-13-3)</sup>

<span id="page-13-0"></span><sup>&</sup>lt;sup>45</sup> Taqī al-Dīn, *Sidrat*, K: ff. 41v, 42v. The report of this eclipse is given both on ff. 41v and 42v; the only extra data in the second report are the perceptible duration of the eclipse

<sup>46</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 42v.

<span id="page-13-1"></span> $47$  [Ben-Zaken](#page-17-15) [\(2010](#page-17-15)), especially pp. 21–24.

<span id="page-13-3"></span><span id="page-13-2"></span><sup>48</sup> See [Neugebauer](#page-18-18) [\(1975\)](#page-18-18), vol. 1, pp. 86–87.

[M4] God rendered those circumstances easy for us in the early morning of Friday, 18;48,46 h after the meridian passage of the sun on Thursday, 21 Shawwal [10] in the year 987 of the noble Hijra. The moon was nearly in the mentioned limits. It was not possible for us to observe it by the armillary sphere neither with the sun, for it being below the horizon, nor with any of the fixed stars, since it was not previously possible for us to record any of them from a reliable observation. Thus, we purposed to observe the moon by the [instrument] having the azimuth and altitude, and we derived the [oblique] ascension (*matali*) [of the moon] at the time of the observation and endeavored to record the procedures with the extreme diligence. [...] Then, the longitude of the moon was 176;27°.<sup>[49](#page-14-1)</sup>

Taqī al-Dīn evidently states that he could not use the armillary sphere at the time of the observation, because he had not yet measured the longitudes of some reference stars trustworthily; the task he apparently never found time to accomplish. He thus adhered to the methods of the spherical astronomy in order to derive the longitude of the moon from its horizontal coordinates as observed by the Altitude-Azimuthal Instrument, which is the same instrument called the Two Quadrants by Mu'ayyad al-Dīn al-'Urdī in his treatise *Fī kayfiyyat al-arṣād*, "On how to make the observations", and constructed by him at the Maragha observatory.<sup>[50](#page-14-2)</sup> The intended time of this observation is when the vertical circle of the altitude of the moon is perpendicular to the ecliptic; this, of course, occurred after sunrise (7:24 MLT) on Friday, 22 Shawwal 987/11 December 1579 (JDN 2298132). Moreover, as regards the other three observations, which shall presently be discussed in the next section, the time Taqī al-Dīn computes is probably in error. As a result, the precise time of this observation cannot be determined with certitude, although it should have been made somewhere between 5:45 MLT (the meridian passage of the moon) and 6:45 MLT (the start of the civil twilight). At 6:49, apparent longitude  $\lambda \approx 176;15^\circ$ ; the allowance has been made for refraction, causing a 4' increase in true longitude  $\lambda$ , ≈ 176;11°.

## <span id="page-14-0"></span>**3 Analysis of the lunar eclipse observations**

In Table [2,](#page-12-0) we analyse the trio of lunar eclipse observations reported by Taqī al-Dīn. It is evident that the times of mid-eclipse that Taq $\bar{a}$  al-D $\bar{b}$ n derived from observations are considerably in error. Indeed, they are consistently earlier than the times of mideclipse computed using modern ephemerides by amounts ranging from just under 40 min to over 50 min. This poor level of accuracy is rather surprising and compares unfavourably with the observation of eclipses by other Islamic astronomers, both from the early and from the late period. For example, among the eclipses reported by al-Battānī, Ibn Yūnus (including also observations from Habash al-Hāsib, al-Māhānī, and the Banū Mūsā), and al-Bīrūnī, no single eclipse timing is in error by more than about 36 min, and the vast majority have errors of less than  $20 \text{ min}$ ,  $51 \text{ irrespective of}$  $51 \text{ irrespective of}$ 

<sup>49</sup> Taq¯ı al-D¯ın, *Sidrat* V.7.8: K: f. 48r.

<span id="page-14-1"></span><sup>50</sup> See [Seemann](#page-19-23) [\(1929](#page-19-23)), pp. 72–81; [Mozaffari and Zotti](#page-18-19) [\(2012](#page-18-19)), p. 403.

<span id="page-14-3"></span><span id="page-14-2"></span><sup>51</sup> [Steele](#page-19-13) [\(2000](#page-19-13)), pp. 112–124.

whether the time was determined using a clepsydra or by the observation of the altitude of either the eclipsed luminary or a fixed star.

Of the late Islamic astronomers, Muhy $\bar{a}$  al-D $\bar{b}$ n has significantly more accurate values for the times of the maximum phase of his trio of lunar eclipses than Taq $\bar{i}$ al-Dīn; indeed, the errors in Muhyī al-Dīn's times do not exceed  $5 \text{min}$ .<sup>[52](#page-15-0)</sup> Muhyī al-Dīn made use of a precise clepsydra which was probably implemented by some mechanical components, and which made it possible for an operator to measure hour and minute separately.<sup>[53](#page-15-1)</sup> By contrast, the accuracy of K $\bar{a}$ sh $\bar{a}$ 's eclipse times is sim-ilar to those of Taqī al-Dīn.<sup>[54](#page-15-2)</sup> Neither Kāshī nor Taqī al-Dīn gives full details of how he determined the times. For example, Taqī al-Dīn does not explicitly mention whether he used his own mechanical clocks, which were seemingly influenced by European sources and models,  $55$  in the observation of his first two lunar eclipses; he only refers to "the great instruments installed in the new observatory".<sup>56</sup> For the last observation, he should have applied the method of spherical astronomy to convert the measured/computed time to the meridian of Constantinople, as his reference to the star Aldebaran gives testimony to it. The use of this method with not-highly precise values for basic parameters, e.g. the geographical latitudes, might partly be responsible for the appearance of such great errors. The other contributing factor might have been the values applied for the difference in longitudes between Cairo/Thessalonica and Istanbul. The support comes from the fact that these lunar eclipses were also of geographical use for Taq $\bar{I}$  al-D $\bar{I}$ n: in *Sidrat* II.4,<sup>[57](#page-15-5)</sup> he explicitly asserts that from his observations of this triple of lunar eclipses, he derived the value 56;39,45<sup>°</sup> for the longitude of Istanbul from the Fortunate Islands; also, in *Sidrat* V.1,<sup>58</sup> where he converts the time of one of the lunar eclipses which Ptolemy observed at Alexandria to the meridian of Istanbul, he takes the meridian of Istanbul equal to  $56;40°$ and that of Alexandria as 61;54◦, and states that the then resulting difference of 5;14<sup>°</sup> in terrestrial longitude between the two cities corresponds to a difference of  $0;20,56<sup>h</sup>$  in local times between the two. However, he presumably discarded the value 56;39,45◦ for the longitude of Istanbul later, since the relevant lines on f. 17v are blacked out and in the geographical table attributed to him,  $59$  the longitudes of Istanbul and Alexandria are given, respectively, as 60◦ and 61;55◦. Note that Istanbul (longitude  $L = 28.57°$  from Greenwich) is actually only about *one* degree west of Alexandria ( $L = 29;55°$ ).

Taqī al-Dīn and al-Kāshī's eclipse timings also compare unfavourably with European astronomers of the time. Regiomontanus and his colleague Bernard Walther at the end of the fifteenth and beginning of the sixteenth centuries observed many eclipses

<sup>52</sup> See above, note 19.

<span id="page-15-1"></span><span id="page-15-0"></span><sup>53</sup> It was probably a Chinese clepsydra brought to the Maragha observatory by Chinese astronomers; see [Mozaffari](#page-18-2) [\(2013b\)](#page-18-2), p. 257; [\(2013d](#page-18-14)), p. 317; [\(2014a](#page-18-10)), p. 103.

<sup>54</sup> See above, note 20.

<span id="page-15-2"></span><sup>&</sup>lt;sup>55</sup> Taqī al-Dīn, *Sidrat*, K: f. 90r; see [Sezgin and Neubauer](#page-19-17) [\(2010](#page-19-17)), vol. 3, pp. 118–122.

<span id="page-15-3"></span><sup>56</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 41v.

<span id="page-15-4"></span><sup>57</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 17v.

<span id="page-15-5"></span><sup>58</sup> Taq¯ı al-D¯ın, *Sidrat*, K: f. 41v.

<span id="page-15-7"></span><span id="page-15-6"></span><sup>59</sup> See [King](#page-18-15) [\(2004/5](#page-18-15)), vol. 1, p. 449–450.

of both the sun and the moon and timed the eclipses with an accuracy of better than 15 min (in many cases, significantly better), and even Copernicus in the middle of the sixteenth century, an astronomer not generally regarded as a particularly accomplished observer, observed the times of eclipses with errors of less than about  $30 \text{min}$ .<sup>60</sup> And towards the end of the sixteenth century, at the same time as Taqī al-Dīn, Tycho Brahe was determining the time of eclipses to an accuracy of about 12 min.<sup>[61](#page-16-1)</sup> Indeed, it is worth noting that two of the three eclipses reported by Taqī al-Dīn were also observed by Tycho: the eclipses of 2/3 April 1577 and 26/27 September 1577. Tycho observed the time of the four phases of the 2/3 April 1577 eclipse, each with an error of about −6,  $+1$ ,  $-3$ , and  $+3$  min respectively, in contrast to Taqī al-Dīn's error of about  $-52$  min for his determination of the time of mid-eclipse. For the eclipse of 26/27 September 1577, Tycho observed the time of the end of totality with an error of about −10 min in contrast to a −45 min error in Taqī al-Dīn's time of mid-eclipse.

Of the three eclipses reported by Taq $\bar{a}$  al-D $\bar{b}$ n, two were total and one partial. Taq $\bar{a}$ al-D<sub>In</sub> gives the magnitude in terms of the decrease in the brightness of the lunar disc, a term that is not encountered in previous Islamic reports. It is not known whether Taqī al-Dīn refers to the eclipsed portion of the lunar diameter or surface; however, the naked-eye estimation of the eclipsed area of the sun and moon or even the measurement of it by aid of medieval optical aids such as camera obscuras or pinhole image devices is difficult, and consequently we assume that Taqī al-Dīn refers to the eclipsed diameter of the moon. However, the computed magnitude of 0.842 for the eclipse is equal to about 10 digits of the lunar diameter and nearly corresponds to the 10 digits of its surface as well, according to the Ptolemaic norm that the angular radius of umbra (i.e. the earth's shadow in the distance of the moon from the earth) is 2.6 times as large as the apparent diameter of the moon.<sup>[62](#page-16-2)</sup> Thus, regardless of whether Taq $\bar{\rm i}$  al-D $\bar{\rm i}$ n refers to the eclipsed diameter or surface of the moon, his measured magnitude is –1 digit in error.

Of the other late Islamic astronomers, Muhy $\bar{a}$  al-D $\bar{n}$  has exceptionally accurate values of the magnitudes for the two partial lunar eclipses he observed at Maragha. He expresses the magnitudes in more fractions than one may expect from the ancient and medieval normal unit of one-twelfth of the diameter of the lunar disc. His values might then have been the results of doing some interpolations after observing the shape of eclipses in dioptra and pinhole image devices available to him at Maragha.<sup>[63](#page-16-3)</sup>

## **4 Conclusion**

Taqī al-Dīn was among a small number of the outstanding figures of Islamic astronomy that show the admirable intentions to give the full accounts of their observations and derivations of parameters. Although, unlike his solar observations, the accuracy of his lunar observations compares unfavourably with both earlier and contemporary astronomers, his work is important for studying the relationship between observation

<sup>60</sup> [Steele](#page-19-13) [\(2000](#page-19-13)), p. 139–150.

<span id="page-16-0"></span><sup>61</sup> [Steele](#page-19-13) [\(2000](#page-19-13)), p. 151–154.

<span id="page-16-1"></span><sup>62</sup> *Almagest* V.14: [Toomer](#page-19-18) [\(1998\)](#page-19-18), p. 254.

<span id="page-16-3"></span><span id="page-16-2"></span><sup>63</sup> See [Mozaffari](#page-18-14) [\(2013d\)](#page-18-14), p. 317; 2014a, p. 73, note 20.

and theory in Islamic astronomy. It is curious that Taqī al-Dīn was among the possessors of the only surviving manuscript of Muhyī al-Dīn's *Talkhīṣ al-majistī*,<sup>[64](#page-17-16)</sup> the work that undoubtedly reflects the acme of observational astronomy in the thirteenth-century Middle East; it is not hard to imagine a probable positive influence that this work might have exercised on Taqī al-Dīn to document his observations and to explain the procedures of derivations of parameter values from them.

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<span id="page-17-16"></span><sup>&</sup>lt;sup>64</sup> His specific signature on f. 1r of the only surviving copy of *Talkhīs* is identical to that found on f. 1r of the Leiden MS. of Ibn Yūnus' *Hākimī zīj* which Taqī al-Dīn possessed of, too. He also left some comment on al-Maghribī's work (e.g. on f. 50v).

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