Arch. Hist. Exact Sci. (2012) 66:265–294
DOI 10.1007/s00407-012-0094-9

# Further adventures of the Rome 1594 Arabic redaction of Euclid's *Elements*

**Gregg De Young**

Received: 14 November 2011 / Published online: 18 April 2012


**Abstract** This article takes up the adventure of the Arabic version of the *Elements* published in Rome at the Typographia Medicea in 1594 at the point where the first installment (Cassinet, *Revue française d'histoire du livre* 78–79:5–51, 1993) ended. In this new installment of the adventure, we situate the Rome edition within a stemma of connected Arabic copies spanning some four centuries. We show that the text of the Rome edition was typeset from Biblioteca Medicea Laurenziana, Or. 20 and that Or. 20 in turn was copied from Or. 50. We show that a manuscript (Tehran, Sipahsalar 540) was later copied from the printed edition and that still later the text of the Rome edition was reissued in lithograph form in Fez. The story is important because the Rome edition influenced the development of non-Euclidean geometry in European mathematics and because the mistaken identity of its author has led to persistent errors in the history of mathematics.

**Keywords** Euclid's *Elements* · Medici Press

## 1 Introduction

The 6th/13th century was an age of *Taḥrīr* writing in the exact sciences.[1] This century saw the production of at least three different *Taḥrīr* of Euclid's *Elements*—by Muḥyi

[1] The Arabic term *taḥrīr* is often rendered in English as redaction. It implies an editing, usually not so much in terms of re-arranging or altering the mathematical content but in terms of improving the style or presentation of earlier Arabic versions of the text. Sometimes it also has the connotation of bringing into

Communicated by : Menso Folkerts.

G. De Young (✉)
Department of Mathematics and Actuarial Science, The American University in Cairo, Cairo, Egypt
e-mail: gdeyoung@aucegypt.edu

al-Dīn al-Maghribī, by the most prolific *taḥrīr* writer of them all, Naṣīr al-Dīn al-Ṭūsī, and by a still anonymous author who, apparently inspired by al-Ṭūsī, completed his work at the very end of the century. These efforts are often considered to embody the summation of four centuries of study and discussion of Euclid's text, and the works composed during this century mark perhaps the culmination or fullest elaboration of these discussions, although assuredly not the end of interesting debate and speculation about mathematical questions.

Thaer (1934) was one of the first historians to call attention to the fact that among the many manuscripts of the Arabic redaction of Euclid's *Elements* attributed to Naṣīr al-Dīn al-Ṭūsī there were a few that differed somewhat from the others. These few included only books I–XIII rather that the usual 15 books (omitting the apocryphal books XIV and XV which were traditionally attributed to Hypsicles) and contained a demonstration of the perennially problematic parallels postulate different from that in the majority of manuscripts. They were typically designated as the "longer" and "shorter" versions and both were assumed to be the work of al-Ṭūsī—the "shorter" being a further or secondary editing of the "longer" version. The differences in their demonstrations of the parallels postulate were intriguing, though. More than 40 years ago Rosenfeld (1966) and Sabra (1968, p. 15) independently raised doubts whether these two versions actually represented the work of the same person. It was Sabra (1969, p. 18) who argued that the date of completion (698 AH/AD 1298) given in the colophon of one of the surviving manuscripts of the "shorter" or "thirteen book" version was inconsistent with al-Ṭūsī's known date of death (674 AH/AD 1274).

The shorter or "thirteen book" version was printed in Rome in 1594 as part of the Arabic publishing program of the Medici Oriental Press (Typographia Medicea). This press was set up by Ferdinand de Medici (1549–1609), Grand Duke of Tuscany, with Giovan Battista Raimondi (ca. 1536–1614) as its director. The precise origins of the press are somewhat shadowy.[2] Although Ferdinand may have also entertained the hope of personal aggrandizement and perhaps pecuniary gain through the press, his primary publicly stated purpose was religious. In his directions to Raimondi, the Duke

> ordered first the printing of the Holy Bible in Arabic . . . and all Catholic books on Scripture that could be recovered in that language to serve the Arab Christians in the East and the Granadans in Spain. And he ordered the printing of all available Arabic books on permissible human sciences which had no religious content to introduce the art of printing to the Mahomedan community so that by the same means knowledge of the Mahomedans' errors and of the truth of the Christian faith could gradually get through to them (Jones 1994, p. 97).

Footnote 1 continued
the text supplementary materials, such as alternate demonstrations, or discussions of the more difficult mathematical points of the text. Most of the *taḥrīr* in the exact sciences were based on Arabic translations of Greek treatises that had originally been made in the second/eighth or third /ninth centuries at a time when Arabic lacked a consistent and standardized vocabulary and rhetorical style for expressing mathematical concepts. This genre represents only one of several—there were also corrections, additions, commentaries, extracts, abridgements and more being produced in a steady stream even into the thirteenth century. These various genres typically represent overlapping categories which shade into one another rather than indicating rigidly defined domains.

[2] Jones (1994, p. 97) gives 1584 as the date of inception, while Cassinet (1993, p. 5) suggests the date 1586.

Based on these instructions, Raimondi constructed a very ambitious publication plan that would have included some 80 titles. Unfortunately, the press of immediate activities and duties gradually drew Ferdinand's interests away from the press and his financial contributions became less and less. Eventually, Raimondi himself purchased the press from Ferdinand, but without financial resources he was unable to complete more than a tiny fraction of what he had once intended.

This publishing program represented one of the earliest attempts to print Arabic in Europe. That Euclid should have been chosen as one of the Arabic texts to be printed by the press only serves to remind us of the remarkable role that the *Elements* has played in the intellectual history of Mediterranean and European cultures. By the time this Arabic version was issued by the press the *Elements* had already been printed numerous times in Europe.[3] This was the only Arabic print edition to be published prior to the nineteenth century and the only printed Arabic edition to be produced in Europe. Inevitably, it became the Arabic treatise that was most widely studied by European mathematicians and historians of mathematics over the past centuries.

The treatise published by the Medici Press was not, however, a version of one of the original translations into Arabic. It was, rather, a reworking of Euclid's text—one of several such redactions produced during the thirteenth century. It has been suggested that a secondary motive for printing an Arabic version of the *Elements* was to provide additional material to scholars who wished to learn Arabic or improve their linguistic skills. But if this goal were to be usefully realized, it would have been better to print an actual Arabic translation from the Greek, which could then have been compared either with the Greek text or one of the Latin translations to introduce the needed technical vocabulary and to familiarize the student with basic grammatical structures.[4] A second goal might have been to expose the intellectuals of the Muslim East to the content of the *Elements*. For what purpose? Ferdinand's stated goal was to introduce the Islamic world to printing and so show them the value of European learning and woo them to Christianity. But if this were indeed the motivation, it seems a strangely misplaced hope since there was a long-standing and prolific Arabic manuscript tradition featuring discussions of Euclid going back almost to the time of the first translations some 750 years earlier. This printed edition would be unlikely to impress the eastern Mediterranean scholars with European mathematical superiority.

Whatever its motivation, the initial print run of the Arabic *Elements* was an astonishing 3000 copies. This was exceeded only by the 3500 print run for the bilingual Arabic–Latin Gospels printed in 1590. Clearly Raimondi and the press administrators must have expected a considerable market from the Arabic-speaking Middle East for this first venture into printing Arabic mathematics.[5] The records of the press also attest

[3] The first Latin printed Euclid (the Campanus version) was published by Ratdolt in 1482. A Greek edition (by Simon Grynaeus) was printed in 1533. Italian, German, French, and English versions of the *Elements* had also been printed by the 1590s (Heath 1956, I, pp. 97–113).

[4] The most popular printed Latin editions also were not based on the primary transmission of Euclid but on reworking of the text by Campanus and others. Hence, it would have been a delicate or even dauntingly difficult problem to make direct comparisons between the Arabic and the Latin printed versions of the Euclidean text.

[5] Even in nineteenth century Britain, the typical print run for popular geometry textbooks seems rarely to have exceeded 1000 copies (Ackerberg-Hastings 2002, p. 43).

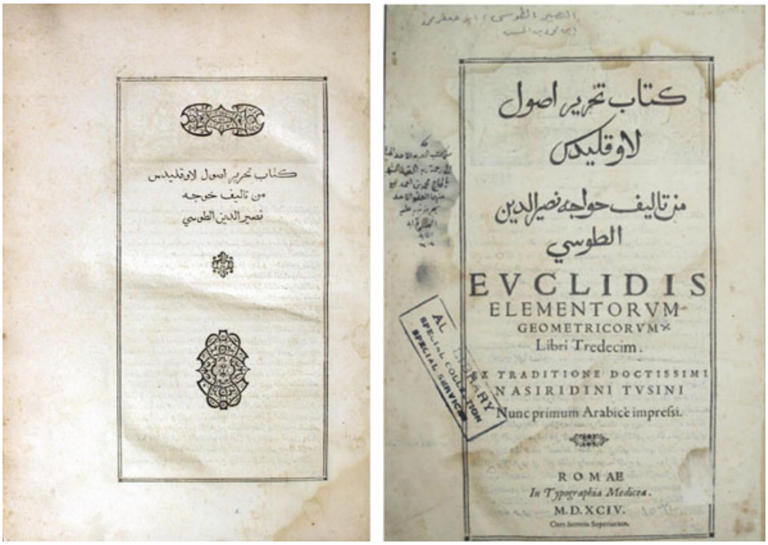


**Fig. 1** That the Rome 1594 edition targeted two different audiences seems clear even from the title page. Some copies were issued with only an Arabic title page containing nothing more than an author and title statement (*left*). Image courtesy History of Science Collections, University of Oklahoma Libraries. Other copies were issued with a bilingual Arabic and Latin title page (*right*). The Latin portion of the page contains not only author and title statement but also names the printing press and states the date of publication. Image courtesy of the Rare Books and Special Collections Library, The American University in Cairo

that these expectations were not met—only a little more than 1000 copies had been sold when the press ceased operations Jones (1994, p. 108). Despite the personal pious hopes of Ferdinand, the printed edition had almost no impact on the Eastern world, either religiously or mathematically. Most copies were apparently bought by European scholars.

## 2 Disentangling genuine and Pseudo-Ṭūsī *Taḥrīr*

Until Rosenfeld and Sabra began to question the ascription of the "shorter" version to al-Ṭūsī, most historians had regarded it as a somewhat edited form of the "longer" (and genuine) version of al-Ṭūsī's treatise. They did so in good faith, misled by the ascription to Naṣīr al-Dīn al-Ṭūsī on the title page of the Rome edition (Fig. 1).[6] And since earlier historians tended to focus their study mainly on the demonstration of the parallels postulate it contained, they considered only a small part of the treatise. Since there are two demonstrations of the parallels postulate found in the genuine works of al-Ṭūsī and they differ somewhat from one another, it was easy to assume that the demonstration in the printed Rome edition, which differs from both genuine

6 Although some have suggested that the author must be Ṣadr al-Dīn al-Ṭūsī, son of Naṣīr al-Dīn and his successor as head of the Marāgha observatory, there is at present no explicit evidence to support this hypothesis. I prefer to denominate the author simply as Pseudo-Ṭūsī.

demonstrations in significant ways, although it in part also seems inspired by these demonstrations, could also be the work of al-Ṭūsī.[7]

If we examine the texts closely, however, we must conclude that if the Rome edition represents an editing of the original version of al-Ṭūsī, then that editing must have been very extensive indeed. For example, the Rome edition contains an introductory discussion not found in al-Ṭūsī's treatise and inserts into the definitions of book V paraphrases of al-Jawharī's attempt to demonstrate Euclid's definitions 5 and 7 and the converse of 5 (De Young 1997, 2009a). Moreover, nearly every definition and every proposition enunciation has been reformulated. One example, the enunciation of proposition I, 2, must suffice to illustrate these rhetorical differences:

T = We want to extend from a given point a line equal to a bounded line.[8]
PT = We are to add to any specified point a straight line equal to a bounded straight line on condition of the two of them being in a single plane.[9]

And when we examine the demonstrations carefully, we find sometimes substantial differences in formulation between the two.[10] Al-Ṭūsī had included numerous mathematical notes and alternative demonstrations, many of them derived from an earlier commentary on the *Elements* by Ibn al-Haytham, *Kitāb fī ḥall shukūk kitāb uqlīdis* (De Young 2009b). None of the notes and alternative demonstrations based on Ibn al-Haytham appear in the Rome edition. The few notes that occur are mainly structural and editorial remarks such as notes on how the translation version of al-Ḥajjāj differed from the edition of Thābit ibn Qurrah. Some of these notes also appear in al-Ṭūsī's *Taḥrīr*, but they are formulated differently, and sometimes contain information not present in the Pseudo-Ṭūsī text. A few editorial notes not found in al-Ṭūsī also appear in the Rome edition. More striking, though, are the many references to preceding propositions found within the text of the Rome edition. Such references were not part of the text of al-Ṭūsī, although one often finds that some references have been added either interlinearly or in the margins of manuscripts of the genuine *Taḥrīr*, presumably by later readers. There are also occasional differences in ordering of definitions or numbering of propositions between the traditional Byzantine Greek text and the Arabic version of al-Ṭūsī.[11] The re-orderings found in the Rome edition

7 Jaouiche discussed the details of these demonstrations (1986, pp. 99–111) and also provided a French translation of the Arabic texts (1986, pp. 201–226, 233–241). Rosenfeld (1980, pp. 74–80) gives one of the more comprehensive discussions of these demonstrations in English.

8 London, British Library, ms. add. 23387, fol. 5a:

نريد أَن نخرج من نقطة مفروضة خطا مسَاويا لخط محدود.

9 Pseudo-Ṭūsī, *Kitāb taḥrīr uṣūl li-uqlīdis* (Rome, 1594), p. 9:

لنَا أَن نضيف إِلَى أَي نقطة مفروضة كَانت خطا مستقيما مسَاويا لخط مستقيم محدود من شرط كونهمَا في سطح وَاحد.

10 See, for example, the different attempts to demonstrate Euclid's fifth postulate, as edited by Sabra (1959).

11 These differences appear already in the Arabic primary transmission documents. Based on these differences in ordering and on differences in technical vocabulary, (DeYoung 1984; De Young 2004, pp. 314–318) and Engroff (1980) divided the testimonia into Group A and Group B. Although it has been suggested that these two primary textual traditions may derive from the earliest translations of the *Elements* into Arabic,

**Table 1** An example of alteration in proposition ordering in book VI

| Greek | Ṭūsī | Rome |
|---|---|---|
| VI, 9 | VI, 12 | VI, 11 |
| VI, 10 | VI, 13 | VI, 12 |
| VI, 11 | VI, 10 | VI, 10 |
| VI, 12 | VI, 11 | VI, 10 Porism |
| VI, 13 | VI, 9 | VI, 9 |

The proposition numbers in the Greek column represent the actual number of the propositions as recorded in the manuscripts. The table shows that the ninth proposition in the Greek text is the twelfth proposition in al-Ṭūsī's *Taḥrīr*, and the same proposition is the eleventh in the Pseudo-Ṭūsī text. The difference between al-Ṭūsī and Pseudo-Ṭūsī is that the latter has converted the equivalent of Greek proposition VI, 12 (which is proposition VI, 11 in al-Ṭūsī's text) into a porism attached to the previous proposition. This change was introduced, we are informed, because al-Ḥajjāj, the early translator of the Greek text into Arabic, had not included the demonstration as an independent proposition (Pseudo-Ṭūsī 1594, 144)

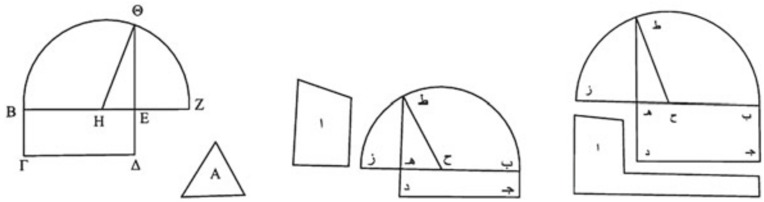


**Fig. 2** Diagrams for *Element*, II, 14. *Left* the diagram edited from Bologna, Biblioteca Comunale, 19, fool. 74v. Greek manuscripts typically show the given figure as a triangle. *Center* the diagram from *Taḥrīr Kitāb Uqlidis* of al-Ṭūsī edited from Cairo, Dār al-Kutub, Ṭal'at riyāḍa 107, fol. 34a. The Arabic tradition sometimes shows the given figure as a triangle but more often as a square or rectangle. In this manuscript, the copyist uses an irregular quadrilateral. *Right* the diagram edited from the Rome 1594 printed edition, p. 62. The Pseudo-Ṭūsī tradition typically uses an unusual elongated shape to represent the given figure

sometimes differ from both the Byzantine transmission and from al-Ṭūsī's version. One example, taken from book VI, is outlined in Table 1.

Finally, there are sometimes significant differences in construction of diagrams between al-Ṭūsī's genuine *Taḥrīr* and the Rome edition. For example, the diagram for *Elements* II, 14 has distinctly different characteristics in different transmission streams (Fig. 2). Also, diagrams representing numbers, which were traditionally constructed using line segments, are constructed in the printed Pseudo-Ṭūsī *Taḥrīr* using columns of dots to represent the discrete magnitudes (Fig. 3).[12]

Footnote 11 continued
this speculation has been difficult to demonstrate because only a few brief quotations ascribed to the earlier translation work of al-Ḥajjāj ibn Yūsuf ibn Maṭar have survived, embedded within other versions or commentaries (De Young 1991; De Young 2003a). Al-Ṭūsī usually follows the formulation and ordering of Group A primary manuscripts.

[12] Although these differences in diagram construction are helpful diagnostic devices, they are not completely reliable. We know at least one copy of the genuine *Taḥrīr* of al-Ṭūsī that also constructs the diagrams of books VII–IX using columns of dots (Biblioteca Medicea Laurenziana, or. 26). At present I am unable to determine whether this use of dots represents an independent experiment by a copyist or whether it might have been influenced by the diagram conventions of the Pseudo-Ṭūsī tradition.

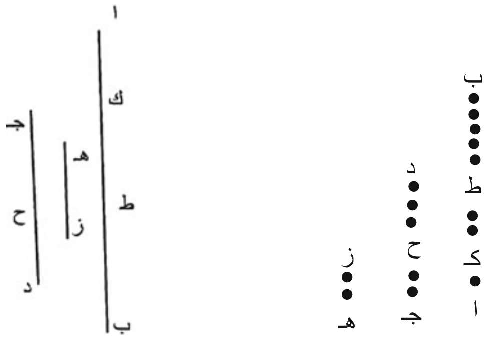


**Fig. 3** Diagrams for *Elements*, VII, 1 from the Pseudo-Ṭūsī tradition. *Left* the diagram edited from Or. 50, fol. 81a. Use of line segments to represent discrete magnitudes is consistent with the traditional approach to diagramming numbers in books VII–IX. *Right* the diagram edited from Or. 20, fol. 149a. Use of columns of points to represent discrete magnitudes (numbers) is typical of this manuscript and later works dependent upon it

These differences strongly suggest that the Rome 1594 printed edition does not represent simply a re-editing of al-Ṭūsī's treatise but an independent work. At the same time, it is not completely independent of the earlier work, since it incorporates some distinctive technical vocabulary from al-Ṭūsī's redaction.

## 3 Laurentian library manuscripts Or. 50 and Or. 20

We have long known that there were two manuscript copies of the "shorter" version of the *Taḥrīr* in the Biblioteca Medicea Laurenziana in Florence. These manuscripts, once thought to be the only extant copies of the text, were identified by Murdoch (1971, p. 454) as Or. 2 and Or. 51. The manuscripts are currently classed as Or. 20 and Or. 50 (Sabra 1968, p. 15). In the four decades since Murdoch wrote his summary, a few other copies have been identified. I shall discuss them further later in this article. It is impossible to estimate how many additional manuscripts may also exist, still incorrectly cataloged under the name of al-Ṭūsī. Cassinet (1993, p. 6), following Taha (1986), who had studied three manuscripts attributed to al-Ṭūsī in the Biblioteca Medicea Laurenziana, argued that Or. 50 rather than Or. 282 (a copy of the genuine al-Ṭūsī *Taḥrīr*), was the source for the text of the Rome 1594 printed edition.[13] He was correct in the sense that Or. 50 is a copy of the "shorter" (Pseudo-Ṭūsī) version that was printed. But closer examination of Or. 20 and Or. 50 shows definitively that it was mainly from Or. 20 that the compositors worked when they typeset the Rome edition. Moreover, it is now clear that the text of Or. 20 (but not its diagrams) was copied from Or. 50.

### 3.1 Manuscript characteristics

Manuscript Or. 50 is complete in 205 folios. Its colophon gives the date of composition of the text as 698 AH and the date of its copying as 969 AH/AD 1561. The text is

[13] Taha's study is not available to me. Thus, the question why he chose these copies—two incomplete copies of the genuine al-Ṭūsī *Taḥrīr* and a complete copy of the Pseudo-Ṭūsī *Taḥrīr* — for his study remains unanswered.

copied with 20 lines per page. Definitions and axioms are written in a single block of text. Sometimes the beginning of a new definition is indicated by a line over the first word, a very typical format for an Arabic manuscript in the Euclidean tradition. Proposition numbers are given only in the outer margins using the traditional *abjad* system. Diagrams are also placed in the outer margins beside each demonstration. In general, these diagrams are constructed in the traditional forms used in the Arabic primary transmission manuscripts. We note especially that the diagrams of books VII–IX are constructed using line segments.[14] The copyist has inserted guide words in the lower margin at the end of each folio to aid the transition to the next folio. Corrections, apparently in the hand of the copyist, have been placed in the margins. Someone, whether the copyist or a reader is not clear, has inserted in the upper margin of each page an indication of the book (indicated by the Arabic letter qāf) and the propositions (indicated by the Arabic letter shīn) found on that page. These book and proposition numbers are given using Arabic-script numerals. At the end of the manuscript, following a blank folio, are six scraps of paper containing notes of various kinds, written in a different hand. Some appear to be calligraphic practice, while others appear to contain some geometrical content.

Manuscript Or. 20 is now incomplete. At present, the text fills 262 folios and breaks off at the end of folio 262 before the completion of proposition X, 95.[15] Since the manuscript is now incomplete, there is no colophon to provide information on the date of copying.[16] The Arabic text of Or. 20 was copied with 24 lines per page. Definitions and axioms are written in a single block of text at the beginning of the various books, except in the case of book V, where later definitions each begin on a new line. This formatting pattern is continued in the definitions of book VII and of book X. Propositions are given a verbal heading written in larger script, which is usually centered on an independent line and takes the form “al-shakl” followed by an ordinal number. Propositions are also numbered in the margin using the traditional Arabic *abjad* system. For unknown reasons, not every proposition is numbered. When no number is given, the proposition is simply labeled “al-shakl,” the term used for a proposition within the Euclidean tradition. Each proposition’s enunciation is written in larger script, separated from the text of the heading by two blank lines and is followed by a blank line. Porisms and other comments are also separated from the preceding text by a blank line. There are occasional corrections, apparently in the hand of the copyist, placed in the margins. Their intended location in the text is usually indicated by symbols (such as a pair of crossed lines) above the line.

When included, diagrams are placed in the outer margins beside the demonstration of the proposition. These diagrams often appear relatively small in relation to the text. Many propositions, however, lack diagrams. Diagrams are missing from proposition I, 38 to the end of the book (including the misplaced folios from book I that now lie at

[14] These diagrams thus constitute a notable exception to the style of the remainder of the Pseudo-Ṭūsī manuscripts described earlier.

[15] 56 folios from books I–III are now disordered. Their correct order is: 1–8, 13–16, 9–12, 49–56, 17–48. From folio 57 onward, the ordering is correct.

[16] As I shall argue below, Or. 20 was copied from Or. 50. Since the colophon of Or. 50 indicates that the date of its copying was 969 AH, this provides us at least a *terminus post quem* for the copying of Or. 20.

the beginning of book III). Diagrams are also missing from VI, 18 and 20. Again, we find no diagrams for propositions VII, 33–IX, 36. There are diagrams for propositions X, 1–8 but thereafter more and more propositions have either no diagrams or only rough sketches of diagrams rather than the more carefully constructed diagrams found earlier in the manuscript.

Following the text there are several folios containing geometrical diagrams. Folios 263 and 272–273 contain diagrams apparently from the *Spherics* of Menelaus. The diagram points are labeled in Greek and the proposition numbers are also indicated with Greek alphanumeric notation beside each diagram. Folio 263, containing diagrams 1–9, has been bound into the treatise upside down. The diagrams on folio 272r are inverted relative to the diagrams on folios 272v–273v. The diagram numbering in this section is somewhat confusing, since several diagrams have been labeled with the same number—presumably they represent diagrams for different parts of each proposition. Folios 264r–271r contain diagrams apparently from the *Conic Sections* of Apollonius. At the head of the collection the name "Āblūnyūs" is written in Arabic script. The collection is further subdivided into seven books labeled in Arabic book two through book seven. Although the diagram points are labeled with Arabic letters, the corresponding proposition numbers are given in Latin script numerals, beginning anew in each section. The diagrams of book seven are followed by thirty diagrams from an unidentified treatise (folio 271r). These diagrams are separated from the diagrams attributed to Apollonius by a double ruled horizontal line across the page.

## 3.2 Or. 20 copied from Or. 50

A careful examination of the manuscripts indicates that the text of Or. 20 was copied from Or. 50.[17] The relationship between the two manuscripts is shown perhaps most clearly in the copying of proposition IV, 3. The original copy of Or. 20 contains a lacuna corresponding to the text between the last word of folio 45a and the first word of folio 46b in Or. 50. Apparently the copyist accidentally turned two leaves of Or. 50 and failed to notice his error until he reached the beginning of proposition IV, 5 (second line of Or. 50, folio 47a) and realized that he had not yet copied proposition 4. He canceled the incorrectly copied section and replaced it with the correct copy in the margins of two folios (81a–82a). In doing so, the diagrams for propositions 3 and 4 were omitted since the text left no space in the margin for their insertion. Since we can still read the original text of Or. 20, we can see that the text from Or. 50, folio 46b as originally copied had been reproduced correctly by the copyist. We cannot know at this distance in time what prompted the copyist to cancel the text and recopy the offending section rather than simply adding the omitted material in the margin, which would have been a more common copyist response to such an accidental lacuna. But in several other places as well the copyist of Or. 20 preferred to cancel and replace portions of text rather than merely correct his errors.

[17] It is tempting to speculate that the copyist was Patriarch Nehemias himself (see Sect. 4, below), who provided many Arabic manuscripts to the Medici Press. The paleographic evidence so far is too scanty to reach a definite conclusion.

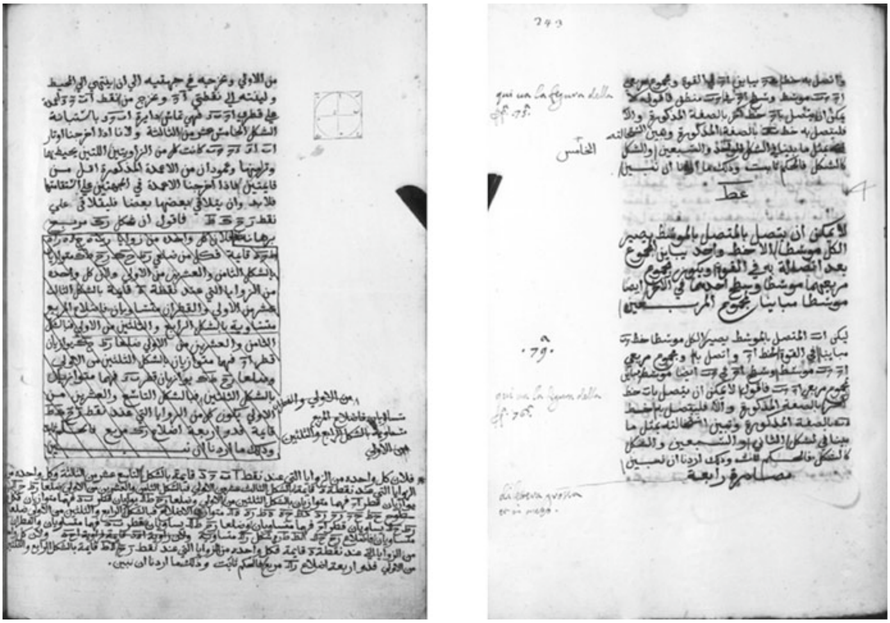

**Fig. 4** *Left* Orientali 20, folio 83b, with the demonstration of proposition IV, 7. The original text has been canceled by the copyist and replaced with alternative text in the margin. The alternative text served as the template when typesetting the Rome 1594 edition. The canceled text had been correctly copied from Orientali 50, folio 48a. The underscored words near the center of the canceled section appeared in the margin of Or. 50. The copyist has inserted them into the text, but indicates uncertainty about their placement by underscoring. Vertical strokes indicating the end of typeset lines are visible in several places, for example, in the line immediately above the canceled section. *Right* Orientali 20, folio 243a. Italian notes to the compositors in the margin indicate placement of diagrams and indicate the use of larger font for the heading at the bottom of the page. The numeral 4 placed in the right margin marks the beginning of the fourth of the eight panes printed on a single sheet. Images courtesy Biblioteca Medicea Laurenziana

Another lengthy passage has been canceled from proposition VIII, 4 and replaced by a shorter text in the margin (folios 160a–161a). In this case, however, the explanation is not a copyist error, since the text from Or. 50, folio 90b–91a was copied correctly in the canceled section. It appears that the copyist, once he had completed his copying of this demonstration, realized that the result could be demonstrated more concisely. It is this more succinct version found in the margin that was used in the printed Rome edition (pp. 189–190). A similar alternate formulation has replaced part of the demonstration of proposition IV, 8 (folio 63b). This demonstration also was copied correctly from Or. 50 but later canceled and an alternative demonstration was written in the margin, again in the hand of the copyist. Translations of these passages are given in the Appendix. These alterations must have been made after the copying was completed but before the typesetting was done because in each case the copying of the canceled section follows very closely the wording of Or. 50 but the typesetting follows the alternate formulation written in the margins.

Similar examples of canceled and replaced passages, although smaller in extent, occur in propositions III, 34; IV, 7; IV, 9; VII, 22; IX, 32; X, 7; X, 16; X, 24; X, 25; X, 26;

X, 44; X, 45. These passages are different from the typical corrections that occur in the margins of so many manuscripts, including this one. In these examples, parts of the text—some more extensive than others—are very clearly and definitively canceled and replaced by alternate text in the margins (Fig. 4). Sometimes the cancelation is so heavy that it is almost impossible to decipher the original text.

In proposition III, 34, the main difference between the canceled passage and the replacement passage is whether the justification for the demonstration (which relies on the Pythagorean Theorem) should refer to proposition I, 46 or proposition I, 47. This is a legitimate debate, since some Arabic versions—most notably those associated with the translation of al-Ḥajjāj—omit *Elements* I, 45. Thus, the famous theorem attributed to Pythagoras becomes number 46.[18] Whether the differences in this reference are indicators of personal preference in this debate is unclear because the Rome 1594 edition places the Pythagorean Theorem as number 47, so if only for reasons of internal consistency the correction of the reference was a positive contribution. It is not so immediately clear, though, why the entire passage had to be canceled and recopied. In other propositions where a similar correction is made, for example, X, 44 and 45, it is only the proposition number itself that is corrected.

In proposition IV, 7, the copyist inserted two sections which had been placed in the margin of Or. 50 (folio 48a), apparently as corrections to the text. Since no indication of precisely where they were intended to be placed is provided in Or. 50, the copyist had to make assumptions. We find that he underlined the second insertion in his copy, perhaps to indicate some uncertainty as to its placement. Then the entire demonstration was canceled and replaced by another formulation (folio 63b). In a similar fashion, a section of the original demonstration of proposition VII, 22 has been canceled and replaced by another formulation (folio 152a).

In the case of proposition IV, 9, we find a section of text in the margin of Or. 20 (folio 84b). This material corresponds to material in the margin of Or. 50 (folio 49b). It seems likely that the copyist was unsure where the material should be inserted, but felt it was important to include it in his copy. In a similar way, in propositions V, 7, X, 7 and X, 18 the copyist of Or. 20 repeated material from the margin of Or. 50. But since he did not know where to insert the text, he simply placed it in boxes in the margins of Or. 20.

We find that the copyist has canceled part of the enunciation of three propositions near the beginning of book X and has re-written them in the margin. In proposition X, 24, the first of these examples, the re-written material (folio 207b) appears to be exactly the same as the original material which was itself copied correctly from Or. 50 (folio 114b). I suggest that this "correction" probably resulted from a mistaken action by the copyist. In the next proposition (X, 25), we find a very similar correction in which the last part of the enunciation of the proposition was canceled and replaced by new text (folio 208a). This new text is exactly the same as the original version except that a reference to proposition 23 has been removed and the copyist has incorrectly written *al-ṭūl* instead of *al-aṭwal* although the term is correctly given in the original copy. Again in proposition X, 26, the copyist has (mistakenly?) introduced a word —

[18] This omission is noted both by al-Ṭūsī (De Young 2003b, p. 135) and by the author of the Rome 1594 edition (p. 44).

it appears to be *ḥīna* (the word is not found in Or. 50) into the enunciation. And once again, rather than simply cancel the incorrect word, he canceled the entire last half of the enunciation and rewrote it in the margin (folio 208b).

### 3.3 Puzzling features of the copying process

Although the evidence adduced so far seems strongly to point to the direct copying of the text of Or. 50 into Or. 20, there are still some puzzles associated with that transmission hypothesis. Perhaps the most puzzling feature of the transition from Or. 50 to Or. 20 are the differences in diagram construction between the two manuscripts. We have already noted the distinctive difference, for example, in the diagram for proposition II, 14 (Fig. 2). In Or. 50, the given shape is drawn as a quadrilateral, while Or. 20 has the classic irregular elongated figure with which we are familiar from the printed Rome edition.[19] This striking difference in construction can function as a convenient diagnostic to identify manuscripts related to the Pseudo-Ṭūsī tradition. A second useful diagnostic is the choice of dots rather than line segments to represent discrete magnitudes (numbers) in book VII (Fig. 3)—a pattern found also in books VII–X of the printed Rome edition.[20] In fact, the differences between Or. 50 and Or. 20 appear already in the diagrams illustrating the definition of the circle and its components at the beginning of book I. In Or. 50, there are only three diagrams, but in Or. 20 there are four. (Or. 50 does not include a diagram to illustrate the diameter of the circle.)

A second puzzling feature about the diagrams of Or. 20 is that several diagrams have been constructed only to be canceled and redrawn again. Often these canceled diagrams represent errors by the copyist rather than errors in construction of the diagrams themselves. For example, on folio 63b (end of proposition III, 12), the copyist mistakenly repeated diagram number 13 instead of diagram number 14. A parallel situation appears on folios 67b and 68a, as well as folios 70b and 71a, when once again, diagrams have been copied or drawn in the wrong margin and then canceled and replaced by correct diagrams. In the case of propositions V, 12 and VI, 7, we can only say that the copyist apparently made a mistake. One has the impression that these diagrams are being constructed based on the text and that sometimes the draughtsman did not clearly understand the text. But this observation merely begs the question — if the text is being copied from Or. 50, why were not the diagrams also copied?[21]

19 The diagram for this proposition has undergone a considerable evolution. The Byzantine tradition regularly constructs the given shape as a triangle. The Arabic transmission sometimes retains the triangle, but more often replaces is with a quadrilateral.

20 I have found a similar technique using dots in a manuscript of the genuine *Taḥrīr* of al-Ṭūsī, Laurentiana, Or. 26. The manuscript is unfortunately incomplete—it breaks off abruptly in the middle of the folio 123b after three lines of book VIII. There is no colophon to provide a date but the script appears to be relatively recent. In the case of proposition VII, 1, its diagram is exactly like that found in Or. 20, except that its structure is inverted, so that instead of reading from bottom to top, one reads from top to bottom. Although this is a striking coincidence, there is no consistent pattern of numerical values to indicate that the Or. 26 manuscript might be the source for the apparent innovation in Or. 20.

21 Placement of the diagrams in the margins of the manuscript leaves the question of when they were constructed somewhat more ambiguous. Diagrams placed within the text more often contain clues that help to establish the sequencing of text and diagram construction. If the text completely surrounds the diagrams,

A third puzzling feature of manuscript Or. 20, is that so many propositions lack diagrams. Even when there is no extant diagram in Or. 20, the intended placement of diagrams has been indicated, apparently by the compositor preparing the typesetting, using the appropriate proposition number. But when there were no diagrams in the manuscript Or. 20, where did the printer or compositor find the diagrams that were commissioned for inclusion in the printed edition? Of course, there is always the manuscript Or. 50. But its diagrams typically differ structurally, as well as in their proportions and their orientation, from those found in Or. 20 and in the printed edition. At the moment, our questions have no easy answer.

## 4 Arabic manuscripts and the Medici Press

Where did the Medici press get the Arabic manuscripts needed to prepare its printed editions? The announced publishing program of the Medici press necessitated access to Arabic source material in manuscript form. I have already outlined some of the evidence indicating that the Rome 1594 edition of Euclid was typeset from Arabic manuscript Or. 20, which is now in the Biblioteca Medicea Laurenziana in Florence. In this section, I outline how this and other manuscripts used in the publishing program of the press were acquired.

Current evidence suggests that they were acquired from the exiled Jacobite Patriarch of Antioch, Ighnāṭyus Niʿmat Allāh (died ca. 1590), whom Latin sources called Patriarch Ignatius or Nehemias.[22] Ignatius appears to have been an exceptionally learned polymath who, in addition to his ecclesiastical position as head of the Jacobite Christians of Asia Minor, served as personal physician to the Muslim ruler of Diyarbakir because of his widely recognized skills in Islamic medical lore. Members of the local Muslim religious elite were not happy with the influential court position held by the Patriarch and stirred up public feeling against him. Apparently in the hope of quelling the controversy and at the same time protecting his physician, the ruler, during one of his regular evening séances, took off his own turban and placed it on Ignatius's head while declaring that he had converted to Islam. Niʿmat Allah was scarcely in a position to refuse the "honor" bestowed by the ruler's gesture, but the episode turned the Christian community against him, and he was forced to abdicate his ecclesiastical position (1576).

When he departed from Antioch into self-imposed exile, the former Patriarch took with him his considerable collection of Arabic manuscripts. With his manuscripts, he

Footnote 21 continued
so that the space for the diagram takes the same basic shape as the diagram itself, we would conclude that the diagram of the proposition was drawn first, then the remaining text added around. On the other hand, if some of the diagrams impinge on or overlap lines of text, the diagrams were added later in spaces that apparently were too small for the intended diagram (De Young 2012). In both manuscripts Or. 50 and Or. 20, we find that even though placed in the margins, the diagrams occasionally impinge on the neighboring text. Hence, it is probable that these diagrams were added after the text was copied.

22 In this section, I rely primarily on the biographical information published by Toomer (1996, pp. 22–24) and Saliba (2008, pp. 199–212). The main source for information on the career of this colorful scholar is a letter sent by Ignatius near the end of his life to his former Jacobite Christian congregation in Diyar Bakir. This letter has been published in Arabic (ʿAzzō 1933).

sailed to Venice (1577).[23] From there he traveled to Florence, where he probably met Ferdinand, who was already thinking about the possibility of setting up his printing venture, and ultimately made his way to Rome. There he was received with great honor by Pope Gregory XIII, perhaps with the hope that he might be able to help return the Eastern church to papal authority.[24] After some negotiation with Ferdinand, Ignatius reached an agreement under which he would donate his manuscripts to the press.[25] In gratitude for his donation, he was named to its editorial board, awarded a monthly stipend of 25 scudi, and promised free access to his manuscripts for the rest of his life. Today these manuscripts form the core of the Orientali collection of the Biblioteca Medicea Laurenziana in Florence.

## 5 Typesetting the Rome 1594 edition

In this section, I turn to specific evidence supporting my claim that the Rome 1594 edition was typeset from manuscript Or. 20. The primary evidence for the use of Or. 20 is the markings left in the manuscript by the typesetter. These markings are of three basic types which are introduced into the manuscript in successive stages. The first type of marking is a pencil stroke to indicate the beginning of each typeset page. The second type of marking, apparently introduced a little later in the typesetting process, is a pencil stroke to indicate the beginning of each new typeset line of text in the printed edition. The third type of marking are printer notes written in the margins in Italian, usually indicating placement of diagrams, appears only near the end of the manuscript.

### 5.1 Printer instruction markings in Or. 20

No printer markings can be discerned in the first 56 folios of the manuscript. This section of the manuscript corresponds to the portion of the manuscript that is currently disordered, but it is impossible to ascertain whether this fact is anything more than a coincidence.

Beginning with the fourth proposition of book III, someone has inserted into Or. 20 a pencil stroke to mark the beginning of each *page* of text in the printed edition and placed opposite this point in the margin of the manuscript a numeral in Latin script.

[23] The date is certain because he noted at the end of a short mathematical manuscript that he had finished studying the tract while sailing to Venice "in the year 1888 of the Greeks" or 1577 AD. The manuscript in question was a copy of the *Ashkāl al-Ta'sīs* of al-Samarqandī (fl. last half of 6th/13th century), a collection of 35 propositions and summary demonstrations drawn mainly from books I and II of the *Elements* (De Young 2001) in the commentary of Qāḍīzāde al-Rūmī. The manuscript is now in the Biblioteca Medicea Laurenziana, Or. 177. The Patriarch's hand-written note appears on fol. 79b.

[24] Although he was never in a position to effect such an ecclesiastical re-unification, Ignatius did enter Papal service, playing a key role on the panel of mathematical astronomers deliberating on the reform of the calendar which was eventually to produce the modern Gregorian calendar. DellaVida (1948) contains interesting details of the Patriarch's relations with the Vatican, drawn from original documents.

[25] Although Toomer (1996, p. 23) merely reports this donation, Saliba (2008, p. 202) goes on to suggest that the donation was influenced by monetary considerations.

The first of these numbers that we encounter is 2 (folio 57b) and the numbers continue to 8, after which they repeat again from 1.[26] This pattern continues to the end of the manuscript. These marks appear to serve no useful purpose apart from the typesetting process. From this observation, we may deduce that the book was printed with four pages on the one side of each sheet and another four pages on the reverse. The sheet would then have been folded into quarters making eight pages.

Beginning with book III, proposition 10 (which represents the eighth pane of a sheet), the beginning of every *line* of Arabic text in the typesetting has been indicated in the manuscript with a vertical pencil stroke (Fig. 4). This pattern is continued for the remainder of the treatise, superimposed on the first marking pattern. The reason for the introduction of this second marking procedure at this point in the treatise is not clear. We may speculate that this marking was introduced to facilitate checking the proofs against the manuscript for accuracy.

Some diagrams in Or. 20 are numbered. In book III, this numbering occurs usually when there are two or three different cases and is intended to indicate the order in which the diagrams are to appear in the printed text. Beginning from proposition V, 6 each diagram is numbered using the alphanumeric *abjad* values—usually the same as the proposition number. These numbers begin anew in each new book. For reasons still not understood, in book X the numbering changes to use the Latin-script numerals and these numerals often indicate the intended placement of the diagrams in the margins when the actual diagrams are missing. I suspect that these numerals also are inserted by the compositor or typesetter, not a reader of the manuscript. In the earlier books, the diagram numbers often seem correlated to the ordering of diagrams in the printed edition when they differ from the ordering given in the manuscript. For example, the diagrams accompanying the definitions of the parts of a circle in book I are not in the same order in Or. 20 as those found in the printed edition, but the compositor has numbered them (using Latin-script numerals) to show in which order he wished them to appear—and this is just the order we find in the Rome 1594 version. The diagrams illustrating classes of triangles and quadrilaterals as well, although not different in form from those in Or. 50, were printed in a different order from that found in Or. 20. For unknown reasons, the compositor has used the traditional *abjad* numbers to indicate the desired ordering of these diagrams, although everywhere else he used Latin-script numerals.

The clearest indication that the Rome 1594 edition was typeset from Or. 20 lies in the Italian notes found in the margin in book X (beginning with folio 235). Most often these refer to placement of diagrams (*qui va la figura della* xx—where xx is the number of the diagram within the book, not necessarily the proposition number—sometimes a proposition uses the same diagram as the previous proposition). There seems no other explanation than that these Italian notes must be instructions to the compositor.

The manuscript Or. 20 was already in its present incomplete state when the modern cataloging and foliation were done—probably in the eighteenth or nineteenth century.

[26] This implies that the first page to be marked in this way would have been the immediately preceding page, which begins in the middle of proposition III, 2. This seems an unlikely place to introduce a new procedure—the first proposition of book III begins a new page and that would have seemed a more logical beginning point. The beginning point of the new procedure was not indicated in the manuscript, however. Although numerals 2 through 8 are easily read, the form of the initial numeral is not written as a typical Latin-script "1" but appears more like a combination of Roman letters B and A.

This leaves open the question of exactly what was the state of the manuscript when the text was typeset. It must have been more complete than it is today because the typesetter's markings indicate that the fourth frame would have been incomplete if the manuscript ceased at the point it does today. The manuscript Or. 50 is still in complete form, and it is obvious that the typesetters must have had available an exemplar from which they worked, but there are no annotations in Or. 50 similar to those found routinely in Or. 20 that would indicate that Or. 50 was used to complete the printed edition. But if manuscript Or. 20 was complete at the time of typesetting the Rome edition, what has happened to the remainder of the text?

### 5.2 Printing errors

The Dutch Arabist Thomas Erpenius (1584–1624) had complained that one of the first Medici Press publications, an Arabic grammar titled *Ājurrūmiyya* (1592), had been printed "with very elegant lettering but... with omissions here and there not so much of words but of entire sentences" (Jones 1994, p. 91). Erpenius, we should note, was not a completely neutral witness—he had set up a press in his home in Leiden to print Arabic materials, among the first of which was his own popular edition of the *Ājurrūmiyya* (1613). Nevertheless, we should ask ourselves whether his criticism may have had some basis in fact and whether the same criticism may be applicable to the Medici edition of Euclid. As one reads through the text, occasional errors tend to leap out from the page. We find, for example, that the typesetter has incorrectly omitted the letter *kāf* from the word *kullu* in the demonstration of proposition III, 8 (two lines from the bottom of page 69). But even though such typesetting errors occur in the Rome edition, they are no more frequent than proof-reading errors in any modern printed book. And in most cases the correct reading is obvious and so the mistakes are often not transmitted into later copies.

But not all errors in the typeset text are the result of compositor carelessness, as we can tell from comparing the printed text with the manuscript Or. 20. In the enunciation of proposition VII, 1, the typesetter has written *al-awwal*, rather than *al-aqall* and a little later in the same enunciation, he has written *baʿd* instead of the correct *yaʿuddu* (p. 149). The typesetter in this case has correctly reproduced the text of Or. 20 (fol. 143a). These errors represent apparent copying lapses that were introduced between manuscript Or. 50 (fol. 81a) and Or. 20. When we look at the text in manuscript Or. 50, we can understand how these errors might have occurred as the result of orthographical ambiguities in the earlier treatise. Another apparent typesetting error occurs in the numbering of proposition III, 11. In manuscript Or. 20, each proposition is given a title—"al-shakl al-awwal" for example—written out in words. Beside the first line of the proposition enunciation, one usually finds the same number written in traditional *abjad* form. Someone has changed the *abjad* number of this proposition from "yā-alif" to "yā-bā"—that is, from eleven to twelve (fol. 62b). When the Rome edition was typeset, it did not retain the verbal titles found in Or. 20 but used only the *abjad* numerals as headings for propositions. So the printers followed the apparent "correction" in the manuscript and labeled this proposition "yā-bā"—resulting in two propositions being given the same number in the printed edition (pp. 72–73). Again, when one looks at Or. 20, it is clear that this is not a typesetting error but a faithful representation of an error that had crept into the manuscript copy. This example provides one more piece of evidence, if more is needed, that the typesetting of the Rome edition was done using manuscript Or. 20.

### 5.3 Diagrams puzzles

One of the persistent puzzles pertaining to the printing of this *Taḥrīr* is how Raimondi managed to produce such elegant diagrams. The standard presumption is that they were created using woodblock technology, but mixing woodblock with cast type while still obtaining a clear crisp print was not an easy task. Inserting the diagrams into the text rather than placing them in the margins would probably have complicated the typesetting process still further. The results, though, show the care that went into the production process. The quality of the final product reveals the cost expended to produce the edition.

Much more puzzling, of course, is the question where the diagrams in Or. 20 came from. It is obvious that they were not copied from Or. 50, since there are so many basic differences between the diagrams of the two manuscripts. The appearance of the diagrams in the Medici edition indicate a connection with the manuscript Or. 20 (Figs. 2, 3), but many propositions in this manuscript lack diagrams. It appears that the press must have commissioned someone to construct the diagrams, but the models for the printed diagrams have not yet been discovered. One may speculate about the existence of yet another manuscript of the Pseudo-Ṭūsī tradition, but until it is found, we cannot reach a definite resolution to our puzzles.

## 6 From print to manuscript: Sipahsalar 540

Tehran manuscript, Sipahsalar 540,[27] was copied from the printed edition of 1594. The manuscript is complete in 234 folios. We do not know for certain where in the Islamic world the manuscript was copied, but the colophon is dated 1101 AH/AD 1690. By a remarkable historical coincidence, this manuscript has also been the victim of mistaken identity. The copyist of Sipahsalar 540 apparently did not copy the title page of the Rome edition—manuscripts do not have title pages. Without the title page and its inaccurate attribution to al-Ṭūsī, however, the user must attempt to make his own identification of the author and title of the treatise. Thus, someone has written on the first folio, in what appears to be a hand different from that of the copyist, the statement that this is the *Iṣlāḥ* ("Correction") of the *Elements* created by Athīr al-Dīn al-Abharī (died 663 AH/AD 1265). This statement, repeated in the printed catalogs, has misled at least one compiler of a modern bio-bibliography.[28]

[27] I thank Dr. M. Bagheri for his assistance with this manuscript.

[28] In fact, it was this incorrect attribution in the bio-bibliographical literature that first drew the manuscript to my attention. Al-Abharī's work is currently known only in two other manuscript copies: Chester Beatty, Arabic ms. 3424 and Bursa, Hüsein Çelebi 744 (Sezgin 1974, p. 111).

Manuscript Sipahsalar 540 reproduces many of the essential features of the printed edition. Propositions are numbered with the *abjad* numerals centered in a blank line, as in the printed edition. Thus, we know that the copyist is not relying on Or. 20 because in that manuscript proposition headings are written verbally, not in alphanumeric form. Nor can the copyist be following Or. 50 because he includes the alternative formulations inserted by the copyist of Or. 20 and which are incorporated into the Rome 1594 edition.

The diagrams are constructed exactly following the pattern of those in the Rome printed edition, for example, the curiously formed diagram for proposition II, 14 (Fig. 2) and the use of dots to diagram numbers in books VII–IX (Fig. 3) and the diagrams are placed in the same general location within the manuscript as in the printed text. It is also true that the diagrams of the printed edition closely follow the pattern of diagrams found in Or. 20, but since there are many diagrams missing from that manuscript, I argue that it is far more likely that the diagrams in Sipahsalar 540 are constructed on the model of the diagrams in the printed edition.

The copying is not a completely slavish or mindless action, though. Most of the obvious typographical errors in the text or diagrams of the Rome edition, such as those in the demonstration of proposition III, 8 mentioned above, have been corrected.

One of the clearest indications that this manuscript was copied from the Rome edition is found in proposition VI, 1. The proposition is accompanied by a porism in the margin in manuscript Or. 50. This porism was transferred into the text when ms. Or. 20 was copied. Near the end of the porism is the statement "*wa-hādhihi ṣūratuhu*" (and this is its picture), implying that one should find a diagram at that point. Acting on this implication, the person preparing the typesetting has drawn a box in the margin opposite this statement in Or. 20, within which is a plus sign surmounting a star. This sign corresponds to an identical sign indicating placement in the text. Obeying this instruction, the typesetters of the Rome edition left a space in the text for insertion of a diagram. But since the intended diagram would have been the same as the diagram for the proposition itself, no new diagram was commissioned and the space was simply left blank. In Sipahsalar 540, no space has been left for a diagram, but we find at this point a note in the margin, written in the hand of the copyist: *iḥtimāl maḥall shakl* (probable site of a diagram). It seems obvious that the copyist must be looking at the printed Rome edition at this point, since this is the only place where we find an apparently missing diagram for this porism.

In fact, examination of the diagrams leads us to the confident conclusion that Sipahsalar 540 was copied from the Rome printed edition. There can be no other interpretation of the diagrams of books VII–IX with their distinctive construction. Not only does Sipahsalar 540 use the same convention of representing numbers with dots, it consistently uses the same number of dots and the same numerical values as those found in the Rome edition. Furthermore, in the Rome edition, every diagram in book VII is constructed with vertical columns of dots except for the diagram of proposition 28, where the dots are arranged horizontally (p. 182). This unusual horizontal orientation is repeated in the diagram of VII, 28 in Sipahsalar 540 (fol. 96a), and the same number of dots have been used to construct the diagram. And the argument is clinched by the observation that the upside down placement of the term *al-wāḥid* (the unit) in proposition IX, 14 (page 214) in the Rome edition is mirrored in the copy of Sipahsalar

540 (folio 111b). There are no diagrams present in Or. 20 for these propositions, so the inspiration for these distinctive features clearly cannot come from the manuscript.

In addition, the person who inserted the diagrams into the text—it is clear that they were added later in spaces left blank by the copyist because occasionally they invade the already written text (for example, the diagram for proposition I, 30 (fol. 20b) or XI, 23 (fol. 176a))—sometimes rotated the diagram from vertical to horizontal orientation.[29] The rotation of a diagram ninety degrees to the right or left is not entirely unknown within the manuscript tradition. Usually such rotation represents a need for more space than the copyist left for the diagram. On several occasions in this manuscript, however, we find that diagrams together with their letter labels are completely inverted in relation to the text.[30] Such superficial differences in orientation, though, do not undermine the very strong impression that this manuscript was copied from the printed edition.

## 7 From typography to lithography (Fez 1876)

A new printing of this Pseudo-Ṭūsī *Taḥrīr* was made in Fez in 1293/1876 using lithograph technology. As far as I am aware, this was the only time that an Arabic version of the *Elements* was printed in the Maghrib before the twentieth century. There were, of course, manuscript copies not only of the Arabic primary transmission, but also of many classical Arabic Euclidean works from the secondary transmission, including al-Ṭūsī's *Taḥrīr*, present in the great libraries of the Maghrib. And one must wonder, with so many manuscript copies of the genuine *Taḥrīr* available to choose from, why would the printer decide to revive the Rome 1594 edition?

Although this lithograph edition initially looks rather like a manuscript, the presence of a title page is one clue that it is not.[31] Written in Maghribi script, the Fez edition follows almost exactly the same architectural principles as the Rome 1594 edition. It appears that the insertion of a title page was a last minute addition by the publisher. Being somewhat unfamiliar with the conventions of printed books, though, he inserted the title page facing the first page of text. This has the effect of displacing the running header so that, until the problem is corrected on pages 154–155, the second part of the header appears on the right-hand page, while the first part appears on the left-hand page. Another obvious feature is the presence of page numbers—and curiously, these page numbers are only in Latin script rather than Arabic script numerals.[32] (The Rome

29 When a diagram is rotated in this fashion, it is rotated as a unit. Not only are the diagram lines rotated, but also the labels of the diagram points are typically rotated along with the lines themselves. This observation suggests that the diagram, both lines and labels, was considered as a unit.

30 These inverted diagrams occur in propositions II, 7 (fol. 32a), III, 19 (fol. 44b) and VI, 22 (fol. 82a). The reason for this inversion of the diagram relative to its standard orientation is puzzling, since it seems to serve no useful purpose. I have not seen such inversions outside this manuscript.

31 Since lithograph printing requires that the text be written by hand on a stone or plate, it is often said that lithography represents a kind of half-way house between typographic print and the manuscript. It rapidly became the preferred method for reproducing Arabic treatises quickly and cheaply.

32 This Fez edition was published in two volumes. The pagination begins anew in volume II, rather than running continuously.

1594 edition had page numbers in both scripts—Latin script at the bottom of the page and Arabic script at the top, except that the Latin script numbers prior to page 25 are mostly missing.) The treatise is followed by several pages of corrigenda—another feature implying some familiarity with Western printing conventions. (This corrigenda section is in addition to small corrections placed in the margins throughout the treatise.)

Although there are many superficial differences between the two printed editions, there are still several clues that tie the Fez 1293 edition to the earlier printed version. Once again, these clues are found mainly in the diagrams. The diagram for proposition II, 14, for example, is constructed in exactly the same way as that found in the Rome edition (Fig. 2). Similarly, the use of dotted lines rather than line segments to represent numbers in the diagrams of books VII–IX (Fig. 3) mirrors what we have seen in the Rome edition. Not only are the diagrams in the same style, but the same number of dots are regularly used. An even more telling piece of evidence comes from the diagram of proposition VII, 28. In the Rome edition, this diagram is the only one from book VII that is oriented horizontally. It appears in the Fez edition (vol. 1, p. 355) in the same horizontal orientation and with the same number of dots representing each number. Moreover, the insertion of the term "al-wāḥid" upside down in the diagram of proposition IX, 14 (vol. 1, p. 422) mirrors its upside down placement in the diagram found in the Rome edition (p. 214). These observations are consistent with the hypothesis that the Fez edition was based on the earlier Medici edition printed in Rome. But could the Fez edition have been based on Sipahsalar 540? This is not an easy question to answer. Although in each of these versions diagrams are rendered with a certain degree of "artistic license," we can detect a few significant differences. The diagrams for the alternative forms discussed in proposition I, 9, for example, are missing from Sipahsalar 540 but they are present in the Fez edition (vol. 1, p. 29) and are a close approximation of the diagrams found in the Rome edition (p. 15). Thus, I would argue that this Fez edition is also derived from the earlier printed edition of 1594.

## 8 Concluding thoughts

The reader who has followed this tangled tale thus far may well wonder what benefits can be gained from the journey. I propose several areas in which our historical knowledge has been enriched. First, we are now able to offer a fairly secure stemma of the Arabic Pseudo-Ṭūsī transmission as we know it today (Fig. 5).

Second, the repeated copying of the text outlined in this article provides us a rare window into the world of the manuscript copyist. There are few examples currently known in the Arabic mathematical tradition wherein we can with considerable confidence identify both the copy and the original manuscript from which it was copied.[33] Through study and analysis of such cases of direct copying we can trace the changes introduced into successive copies of a treatise and we acquire greater understanding

33 In another likely example of manuscript copying, I suggest that manuscript Tehran, Majlis 200 is a direct or very nearly direct copy from Chester Beatty Library Arabic manuscript 3035.

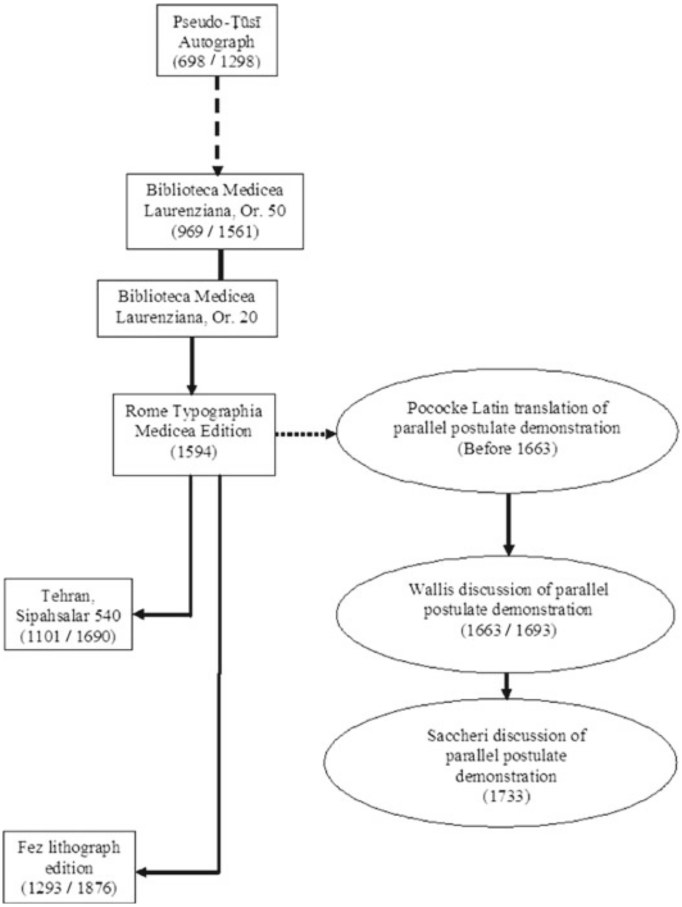


**Fig. 5** Proposed stemma for the known Pseudo-Ṭūsī tradition. Dates of copying/printing are included in parentheses when known. Apart from the Rome printed edition, all dates for the Arabic sources use the Hijra calendar. I have included the Gregorian equivalent to clarify the temporal relations. The dashed line indicates an unknown portion of the transmission. Solid line indicate a direct transmission of the text. The Latin transmission is indicated with ovals rather than rectangles. The dotted line indicates translation from the Arabic (but only the demonstration for the parallel lines postulate)

of the details of the process of reproducing a manuscript. Thanks to such chains of copying, we are able to discern the finer details of how one copy differed from another, providing insight into the copying process itself and allowing us the opportunity to assess the quality of the copying trade. In the case of mathematical texts involving diagrams, we are also able to observe changes to the diagrams by successive draughtsmen.

Third, we obtain some fresh insights into the process by which Arabic manuscripts were converted to printed texts during the early European experiments in typesetting Arabic script. In the present case, the chain of connection extends for at least four centuries and we have both manuscript and print copies produced. And because the

Fez edition was produced using lithograph technology, we also have an opportunity to compare directly the same text printed by typography and by lithography.

But someone may protest that we have so far said almost nothing about the influence of the Rome edition in Europe.[34] As the only printed Arabic version of Euclid's work available to the European scholarly community, the Rome edition was studied far more than any Arabic manuscript. It was not so often studied in its entirety, but usually only with regard to specific mathematical points. From the beginning, the focus of attention has been on its attempted demonstration of Euclid's parallels postulate. Jesuit mathematician, Christoph Clavius (1537–1612), who published the second edition of his Latin version of the *Elements* in 1589,[35] wrote in its introduction (Knobloch 2002, p. 419):

> We learned long ago that the Arabs demonstrated the same principle. [He refers to the parallel lines postulate.] Though I diligently looked for this demonstration [for] a long time, I could not see it because it was not yet translated from the Arab into Latin. Hence, I was obliged to imagine it by myself.

Since Clavius did not read Arabic himself, he searched for a Latin translation. So far as we know, no Latin translation of the Pseudo-Ṭūsī *Taḥrīr* existed at that time.[36] If Clavius had had access to the Arabic text, he could have at least used the diagrams to guide his imagination, and he could also have had the text translated. Although this statement of Clavius leaves the situation somewhat ambiguous, he is more precise in his complaint in the 1612 edition of his collected mathematical works (Knobloch 2002, p. 420):

> I never got permission to read [this demonstration] though I continuously asked the owner of this Arabic Euclid for it.

So Clavius not only knew that an Arabic text existed, he knew where the manuscript was. Saliba (2008, pp. 211–212) suggests that Clavius must be referring in these passages to the manuscripts of the *Elements* owned by Patriarch Nehemias, who was

[34] In one sense, something of the Pseudo-Ṭūsī tradition had been known in Europe long before the Rome edition appeared. Already in the thirteenth century, Rabbi Levi ben Gershon (1288–1344)—known also by his Latinized name, Gersonides—had discussed a demonstration of the parallel lines postulate that is identical to that found in the Pseudo-Ṭūsī *Taḥrīr* (Lévy 1992a,b). There is no indication that the Arabic treatise was ever translated into Hebrew, so this knowledge probably traveled by way of an extract or report embedded in another text. Since the Pseudo-Ṭūsī treatise had been completed, apparently in the territory of what is now Iran, during the lifetime of Gersonides, it is evident that mathematical ideas were capable of quickly traversing the great distance from Iran to southern France even in the medieval period.

[35] This second edition differs from the first edition of 1574. In the first edition, the scholion following proposition I, 28 gives only the demonstration of Proclus. In the second edition, this scholion is divided into two parts. The first gives again the demonstration of Proclus. The newly added second part is essentially the demonstration of the Pseudo-Ṭūsī Arabic edition (Maieru 1978, p. 192). This scholion is repeated in later editions of Clavius's Euclid as well (Clavius 1611, pp. 49–53).

[36] Youschkevitch (1976, p. 183) claimed that the entire treatise was published in Latin translation under the title *Euclides Elementorum libri tredecim studio Nasseredini* (Rome, 1657), but Mercier (1994, p. 213) reports being unable to find any confirming evidence for the existence of such a published translation. The similarity of the putative title to the Latin title used on the bilingual title page of the Rome 1594 Arabic edition suggests that Youshkevitch might be referring to the Rome Arabic edition, but using an incorrect date.

probably in the process of negotiating his agreement with Ferdinand. Or perhaps even after he reached his agreement, he hesitated to share his manuscripts with scholars for fear of damaging sales of the printed volume. In any case, we see that from the time when the manuscripts entered Europe with the hapless Patriarch, some knowledge of their existence and contents was circulating by word of mouth among mathematicians. Thus, by the time the Medici Press issued its printed edition there was an eagerness for this new source of knowledge spreading widely among the mathematicians of Europe. In this fertile intellectual climate, at least one mathematical seed contained in the Arabic text soon took root.

The Arabic text of the Pseudo-Ṭūsī attempt to demonstrate the parallel lines postulate was eventually translated into Latin by Edward Pococke (1604–1691). This translation must have been made before 1663 when John Wallis (1616–1703) discussed the attempt to demonstrate the parallels postulate in his lecture as Savillean Professor of Geometry at Oxford (11 July 1663). But Wallis was apparently not keen to publish his thoughts, so it was thirty years later that this quotation from Pococke became widely available in printed form (Wallis 1693, II, pp. 665–678).[37] Through Wallis, knowledge of this Arabic discussion of Euclid's postulate entered the mainstream of European geometrical discussions. Wallis's discussion of the postulate directly influenced Saccheri (1667–1733) in his famous attack on the parallels postulate in his *Euclides ab omni nævo vindicatus* (1733) (Saccheri 1920, pp. 100–109). It was his discussion and criticism of the approach of Pseudo-Ṭūsī to the parallels postulate (in Scholion III of proposition XXI) that helped to lay the foundation for the modern study of non-Euclidean geometry (Rosenfeld 1980, pp. 97–99). And thus the Pseudo-Ṭūsī attempt to demonstrate the parallel lines postulate entered the mainstream of European discussion under a mistaken identity, although the remainder of the treatise has been largely ignored by historians.

And so our little historical adventure comes to an end. Not all our questions have been answered, leaving the hope that there may yet be future installments to our narrative. For now, the convoluted transmission story of this Arabic Euclidean treatise has demonstrated that many of the traditional historical interpretations about Naṣīr al-Dīn al-Ṭūsī have been built on a false attribution.[38] Moreover, even though our present conclusions rely on a line of evidence that leads back to a single manuscript, Or. 50, that treatise and its descendants have played a continuing part in the history of mathematics, both in the Arabic East and in the Latin West, for more than four centuries.

**Acknowledgements** I am very grateful to the directors of the History of Science Collections of the University of Oklahoma Library, the Rare Books and Special Collection Library of the American University in Cairo, and the Biblioteca Medicea Laurenziana for permission to reproduce images of material held in their respective libraries. Dr. M. Bagheri very generously responded to my queries about several manuscripts from Iranian libraries. All diagrams included in this article have been edited using the free software tool,

[37] Cassinet (1993, p. 16) reproduces a page from Wallis's Latin treatise illustrating his criticism of the supposed al-Ṭūsī demonstration of the parallel lines postulate. This is the first documented publication of a Latin translation of the Pseudo-Ṭūsī and it was apparently based on the printed Rome edition.

[38] Perhaps the most commonly read is still that of Heath (I, 77–82) in his introduction to his English translation of the *Elements*.

*DRaFT*, developed by Ken Saito (Osaka Prefecture University). A summary version of this article was presented at the 5th Joint Meeting of the Canadian Society for History and Philosophy of Mathematics and the British Society for History of Mathematics (Dublin, 2011). I thank the School of Sciences and Engineering of the American University in Cairo for a travel grant to attend this meeting. The referees have provided detailed comments on the text and have saved me from several errors. I am very grateful for their efforts. Needless to say, whatever faults still remain are attributable solely to me. I gratefully acknowledge a Minigrant from the School of Sciences and Engineering, The American University in Cairo, in partial support of research costs.

## Appendix I

The copyist of Or. 20 replaced the demonstration of proposition VIII, 4 with another, shorter, demonstration. This appendix provides an English translation of both the original and the replacement texts. I begin with the original (later canceled) text. I translate the entire proposition, including enunciation, to provide a complete context for the two demonstrations.

> We want to show how we find the smallest numbers according to the ratio of given numbers.
>
> Let the given numbers according to a ratio be A, B, G, D, E, Z and let each one of them be the smallest pair of numbers according to their ratio. Let us take the smallest number which B [and] G measure, according to proposition VII, 34, and let it be T. Let A measure H and D [measure] K according to the amount by which B measures T. Let us take the smallest [number] that numbers E and K measure, according to proposition VII, 34. Now that number is either K or [a number] other than that.
>
> As for the first, namely, that the smallest number which numbers E, K measure is K, let Z measure L according to the amount by which E measures K. Then, because B, G measure T according to the amount by which A [measures] H and D [measures] K and E measures K according to the amount by which Z [measures] L, the ratio of A to H is as the ratio of B to T and the ratio of G to T is as the ratio of D to K.
>
> But the ratio of E to K is as the ratio of Z to L. Then by alternation the ratio of A to B is as the ratio of H to T and the ratio of G to D is as the ratio of T to K.
>
> But the ratio E to Z is as the ratio of K to L, according to proposition VII, 13. Thus, I say that H, T, K, L are the smallest numbers according to the ratios of A, B, G, D, E, Z.
>
> Its proof is that if not, let M, N, S, Ṣ be the smallest numbers according to these ratios. Then because the ratio of A to B is as the ratio of M to N, and the two of them are the smallest pair of numbers according to their ratio, B measures N, according to proposition VII, 20
>
> But the ratio of G to D is as the ratio of N to S, and the two of them are the smallest numbers according to their ratio. G measures N by the amount by which D [measures] S, according to proposition VII, 20.
>
> Now because B, G measure N and T is the smaller of the two numbers that measure it, T measures N, according to proposition VII, 35. Thus, the larger measures the smaller. This is a contradiction. Therefore, the proposition is established.

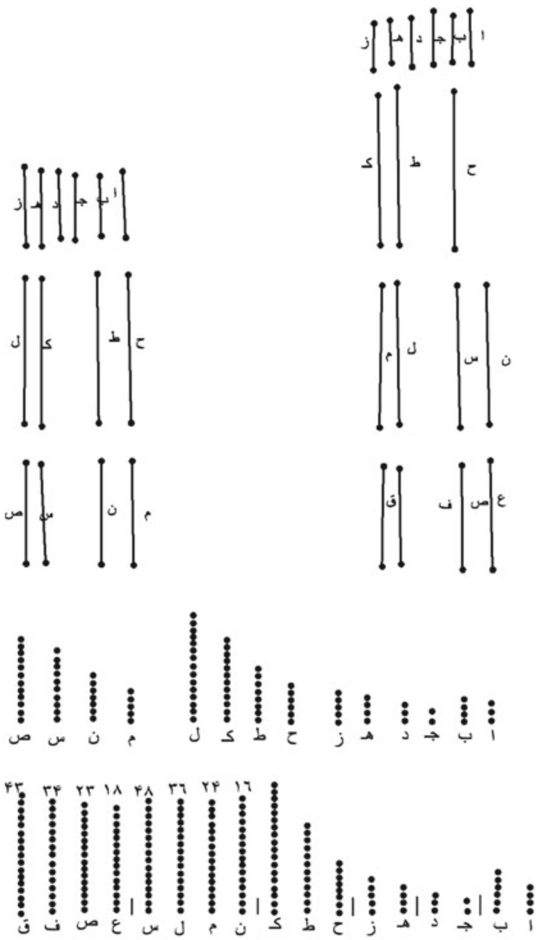

**Fig. 6** Diagrams for *Elements*, VIII, 4. *Above* diagrams as edited from Or. 50, fol. 90b. Left, diagram of part I of the demonstration. *Right* diagram of part II of the demonstration. Or. 50 uses line segments to represent numbers. *Below* diagrams as edited from Rome, 1594. *Top* diagram of part I of the demonstration (p. 189). *Bottom* diagram of part II of the demonstration (p. 190). Note the use of small vertical lines to group sets of numbers in the diagram

Further, as for [the case that] the smallest number that measures E and K is a number other than K, let it be L. Let Z measure M by the amount by which E [measures] L. Then let H measure N and T [measure] S by the amount by which K [measures] L.

Now because A measures H and D [measures] K according to the amount by which each one of numbers B, G [measures] T and H measures N, K [measure] L by the amount by which T measures S.

But Z measures M by the amount by which E measures L. Thus, A, B measure N, S by the amount by which G, D [measure] S, L and E, Z [measure] L, M. Thus, the ratio of A to N is as the ratio of B to S and the ratio of G to S is as the ratio of D to L.

But the ratio of E to L is as the ratio of Z to M. Then by alternation the ratio of A to B is as the ratio of N to S and the ratio of G to D is as the ratio of S to L and the ratio of E to Z is as the ratio of L to M, according to proposition VII, 13.

Now I say that N, S, L, M are the smallest numbers according to the given ratios. If not, let the smallest numbers according to that ratio be numbers O, Ṣ, F, Q. Thus, the ratio of O to S is as the ratio of A to B and the ratio of Ṣ to F is as the ratio of G to D.

But each of the pairs of numbers A, B [and] G, D is the smallest pair of numbers according to their ratio. Thus, the two of them measure each pair of numbers according to their ratio, according to proposition VII, 20. Thus, B, G measure Ṣ.

But T was the smallest number which B, G measure. So T measures Ṣ, according to proposition VII, 35.

But the ratio of T to K is as the ratio of G to D and the ratio of Ṣ to F is as the ratio of G to D. Thus, according to the porism of proposition VII, 4, the ratio of T to K is as the ratio of Ṣ to F.

But T measures Ṣ. Thus, K measures F.

But the ratio of E to Z is as the ratio of F to Q, and the two of them are the smallest number according to their ratio. Thus, E measures Q, according to VII, 20.

But K, E measure F, and L is the smallest number that K, E measure. Thus, L measures F, according to proposition VII, 35. Therefore the larger measures the smaller. This is a contradiction. Thus, the proposition is established. That is what we wanted to show.

This demonstration, copied correctly according to the text of Or. 50 and then canceled by the copyist, is replaced by another, somewhat shorter, demonstration written in the margins of folios 160a–b.

Let the given numbers according to a ratio be A, B, G, D, E, Z and let each one of them be the smallest pair of numbers according to their ratio. Let us take the smallest number that measures B, G, according to proposition VII, 34. Let it be T and let A measure H according to the amount by which B measures T and D [measure] K according to the amount by which G measures T.

Now E either measures K or it does not.

As for the first [case] we make Z measure L according to the amount by which E measures K. Then, because A measures H according to the amount by which B [measures] T, the ratio of A to B is as the ratio of H to T, according to proposition VII, 17. Likewise, the ratio of G to D is as the ratio of T to K and the ratio of E to Z is as the ratio of K to L.

I say that H, T, K, L are the smallest numbers according to the ratios of A, B, G, D, E, Z.

The demonstration is that if not, then let M, N, S, Ṣ be the smallest numbers according to these ratios. Then, because the ratio of A to B is as the ratio of M to N and the two of them, namely A, B, are the smallest pair of numbers according to their ratio, A measures M and B [measures] N, according to proposition VII, 20.

For that reason also G measures N. Then, because B, G measure N, T, which is the smallest [number] that B, G measure, measures N, according to proposition

VII, 35. Therefore, the larger measures the smaller. This is a contradiction. Thus, the proposition is established.

But as for the second [case], namely that E does not measure K, let us take the smallest number that measures E, K, according to proposition VII, 34, and let it be L. And we make T measure M and H [measure] N by amount by which K [measures] L.

But Z measures S according to the amount by which E measures L. Thus, because T measures M according to the amount by which H [measures] N, the ratio of H to T is as the ratio of N to M, according to proposition VII, 17, and the ratio of H to T is as the ratio of A to B. Thus, the ratio of A to B is as the ratio of N to M. Likewise the ratio of G to D is as the ratio of M to L.

But because E measures L according to the amount by which Z measures S, the ratio of E to Z is as the ratio of L to S, according to proposition VII, 17. Therefore, numbers N, M, L, S are according to the ratio of the given numbers, A to B and G to D and E to Z.

I say that N, M, L, S are the smallest numbers according to the ratios of the given [numbers]. But if not, let the smallest numbers according to these ratios be numbers O, Ṣ, F, Q.

Then, because the ratio of O to Ṣ is as the ratio of A to B and A, B are the smallest numbers according to their ratio, B measures Ṣ and likewise G measure Ṣ. Therefore, B, G measure Ṣ.

Now the smallest [number] that measures B, G, measures Ṣ. But the smallest of the numbers that measures B is T. Thus, T measures Ṣ. Therefore, the larger measures the smaller. This is a contradiction. Thus, the proposition is established. This is what we wanted to show.

## Appendix II

In this section, I first translate proposition IV, 7 as found in Orientali 50 and copied into Orientali 20. The demonstration of the proposition was later canceled by the copyist and replaced by another formulation written in the margin of folio 83b (Fig. 4).

[For] any specified circle, we [are] to draw about it a square.

Let the circle be ABGD. We find its center by proposition III, 1. Let it be point E. We connect between point B on its circumference and the center by a straight line and we extend it until it ends at its circumference. Let it end at point D. Let us extend from point E a perpendicular to diameter BD according to proposition I, 11 and let us extend it in both directions until it ends at the circumference at points A [and] G. We extend from points A, B, G, D perpendiculars to the diameters AG, BD. They are tangent to the circle, according to the porism of proposition III, 15. And because we extended chords AB, AD, DG, GB, each of the pair of angles bounded by a pair of chords and two of the mentioned perpendiculars is less than two right angles. Thus, if we extend the perpendiculars rectilinearly in both directions, it is necessary that they meet each other at points Z, H, T, K.

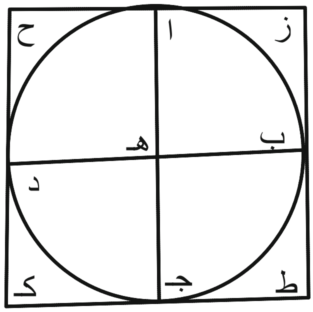


**Fig. 7** Diagram for *Elements*,IV, 7, as edited from Orientali 20, folio 83b. This diagram does not differ from that found in Orientali 50 (fol. 47b) or from that found in the Rome 1594 edition (p. 99).

I say that figure ZK is a square.

Its proof: because each one of angles ZBE, HDE, ZAE, TGE is right, each pair of sides ZT, HK, ZH, TK is parallel to the other, according to proposition I, 28.

But because each of the angles which are at point E is right, according to proposition I, 13, and the two diameters are equal to one another, the sides of the square are equal to one another, according to proposition I, 34.

Now, according to proposition I, 28, sides ZT, HK are parallel to diameter AG. Thus, the two of them are parallel to one another, according to proposition I, 30.

But sides ZH, TK are parallel to diameter BD. Thus, the two of them are parallel to one another, according to proposition [I], 30.

Now, according to proposition I, 29, each of the angles at points Z, H, T, K is right. Therefore, quadrilateral ZK is a square. Thus, the proposition is established.

That is what we wanted to show.

The demonstration has been replaced by a shorter alternative formulation in the margin.

Now because each one of the angles at points A, B, G, D is right, according to proposition III, 19, and each one of the angles which is at point E is right, according to proposition I, 13, sides ZT, HK are parallel to diameter AG. Thus, the two of them are parallel to one another, according to proposition I, 30.

But ZH, TK are parallel to diameter BD. Thus, the two of them are parallel to one another. Therefore, each one of areas HT, HG, ZG, ZD, DT, HE, ET, ZE, EK are parallelograms.

Now, according to proposition I, 34, sides ZT, HK are equal to diameter AG so they are equal to one another and sides ZH, KT are equal to diameter BD so they are equal to one another.

But the two diameters are equal to one another. Thus, sides ZH, HT, TK, KZ in figure ZK are equal to one another.

But because each one of the angles at E is right, each one of the angles at Z, H, T, K is right, according to proposition I, 34. Therefore, quadrilateral ZK is a square. Thus, the proposition is established.

That is what we wanted to show.

## References


Ackerberg-Hastings. 2002. Analysis and synthesis in John Playfair's *Elements of Geometry*, *British Journal for History of Science* 38: 43–72.

Azzō, Y. 1933. Risālat al-baṭriyark Ighnāṭyus Niʻmeh, *Al-Mashriq* 31: 613–623, 730–737, 831–838.

Cassinet, R. 1986. L'aventure de l'édition des Éléments d'Euclide en arabe par la Société Typographique Médicis vers 1594. *Revue française d'histoire du livre* 78–79: 5–51 (1993). An earlier version appeared under the same title in *Cahiers d'histoire des mathématiques de Toulouse* 9.

Clavius, C. 1999. *Operum Mathematicum, Tomus Primus Complectens Commentaria in Euclidis Elementa Geometrica, In Sphærica Theodosij*. Rome (1611). Re-issued in facsimile edition, ed., E. Knobloch, Hildesheim: Olms-Weidmann.

De Young, G. 1984. The Arabic textual traditions of Euclid's *Elements. Historia Mathematica* 11: 147–160. doi:10.1016/S0315-0860(84)80006-3

De Young, G. 1991. New traces of the lost al-Ḥajjāj Arabic translation of Euclid's *Elements. Physis: Rivista Internazionale di Storia della Scienza* 28: 647–666.

De Young, G. 1997. Al-Jawharī's additions to book five of Euclid's elements. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 11: 153–178.

De Young, G. 2001. The *Ashkāl al-Ta'sīs* of al-Samarqandī: A translation and study. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 14: 57–117.

De Young, G. 2003a. The Arabic version of Euclid's elements by al-Ḥajjāj ibn Yūsuf ibn Maṭar: New light on a submerged tradition. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 15: 125–164.

De Young, G. 2003b. The Taḥrīr of Euclid's *Elements* by Naṣīr al-Dīn al-Ṭūsī: Redressing the balance, *Farhang: Quarterly Journal of Humanities and Cultural Studies* 15–16: 117–143.

De Young, G. 2004. The Latin translation of Euclid's *Elements* attributed to Gerard of Cremona in relation to its Arabic antecedents. *Suhayl: Journal for the History of the Exact and Natural Sciences in Islamic Civilization* 4: 311–383.

De Young, G. 2009a. A Persian translation of al-Jawharī's additions to *Elements*, book V. *Tārīkh-e Elm: Iranian Journal for the History of Science* 7: 1–24. doi:10.1177/0278364910396551.

De Young, G. 2009b. The *Taḥrīr Kitāb Uṣūl Uqlīdis* of Naṣīr al-Dīn al-Ṭūsī: its sources. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 18: 1–71

De Young, G. 2012. Mathematical diagrams from manuscript to print: Examples from the Arabic Euclidean transmission. To appear in Synthese. doi:10.1007/s11229-012-0070-6.

Della Vida, G. 1948. *Documenti intorno alle relazioni della chiese orientali con la S. Sede durante il Pontificato di Gregorio XIII*. Città del Vaticano: Biblioteca apostolica vaticana.

Engroff, J. 1980. *The Arabic Tradition of Euclid's Elements*: *Book V*. Harvard University PhD Dissertation, unpublished.

Heath, T. L. 1956. *The thirteen books of Euclid's Elements* (3 Vols). New York: Dover.

Jaouiche, K. 1986. *La Théorie des parallèles en pays d'Islam*. Paris: Vrin.

Jones, R. 1994. The Medici Oriental Press (Rome 1584–1614) and the impact of its Arabic publications on northern Europe. In *The 'Arabick' Interests of the Natural Philosophers in Seventeenth Century England*, ed. G. Russell, 88–108. Leiden: Brill.

Knobloch, E. 2002. Christoph Clavius (1538–1612) and his knowledge of Arabic sources. In *Gesuiti e università in Europa* (secoli XVI–XVIII), eds. G. Brizzi and R. Greci, 403–420. Bologna: CLUEB.

Lévy, T. 1992a. Gersonide, le Pseudo-Ṭūsī, et le postulat des parallèles. Les Mathématiques en Hébreu et leurs sources arabes. *Arabic Sciences and Philosophy* 2: 39–82.

Lévy, T. 1992b. Gersonide, commentateur d'Euclide: Traduction annotée de ses gloses sur les *Éléments*. In *Studies on Gersonides—A Fourteenth Century Jewish Philosopher-Scientist*, ed. G. Freudenthal, 83–147. Leiden: Brill.

Maieru, L. 1978. Il quinto postulato Euclideo in Cristoforo Clavio. *Physis* 20: 191–212.

Mercier, R. 1994. English orientalists and mathematical astronomy. In *The 'Arabick' Interests of the Natural Philosophers in Seventeenth Century England*, ed. G. Russell, 158–223. Leiden: Brill.

Murdoch, J. 1971. Euclid: Transmission of the *Elements*. In *Dictionary of scientific biography*, Vol. 4, ed. C. Gillispie, 437–458. New York: Scribner.

Rosenfeld, B. 1980. *A history of non-Euclidean geometry*. New York: Springer.

Rosenfeld, B., Kubesov, A. K., Sobirov, G. S. 1966. Kto by avtorom rimskogo izdania 'Izlozhania Evklida Nasir ad-Dina at-Tusi' [Who was the author of the Rome edition 'Recension of Euclid by Naṣīr al-Dīn al-Ṭūsī?]. *Voprosy istorii estestuoznaniya i tekhniki* 20: 51–53.

Sabra, A. I. 1959. Burhān Naṣīr al-Dīn al-Ṭūsī ʿalā muṣādarat Uqlīdis al-khāmisa. *Bulletin of the Faculty of Arts of the University of Alexandria* 13: 133–170.

Sabra, A. I. 1968. Thābit ibn Qurra on Euclid's parallels postulate. *Journal of the Warburg and Courtauld Institutes* 31: 12–32.

Sabra, A. I. 1969. Simplicius's proof of Euclid's parallels postulate. *Journal of the Warburg and Courtauld Institutes* 32: 1–24.

Saccheri, G. 1920. *Euclides vindicatus*. G. Halsted, ed. and trans. Chicago: Open Court.

Saliba, G. 2008. Embedding scientific ideas as a mode of science transmission. In *A shared legacy: Islamic science east and west*, eds. E. Calvo, M. Comes, R. Puig, M. Rius, 193–213. Barcelona: University of Barcelona Press.

Sezgin, F. 1974. *Geschichte des arabischen Schrifttums*, *Band V: Mathematik*. Leiden: Brill.

Taha, A. 1986. Sur trois manuscrits arabes du $XIII^e$ siècle: Or. 26, Or. 50 et Or. 282 de la Bibliothèque Laurentienne de Florence. *Cahiers des Mathématiques de Toulouse* 9: 41–50.

Thaer, C. 1934. Die Euklid-Überlieferung durch at-Tusi. *Quellen und Studien zur Geschichte der Mathematik*, *Astronomie und Physik, Abteilung B, Studien* 3: 116–121.

Toomer, G. J. 1996. *Eastern wisedome and learning: the study of arabic in seventeenth-century England*. Oxford: Oxford University Press.

Wallis, J. 1972. De postulato quinto et definitione quinta lib. 6 Euclidis: disceptatio geometrica. In his *Opera mathematica, volumen secundum* [De algebra tractatus : cum variis appendicibus]. Oxford: E Theatro Sheldoniano (1693), 665–678. Hildesheim: Olms-Weidmann. Reprinted.

Youshkevitch, A. P. 1961. Les mathématiques arabes: VIIIe–XVe siècles (M. Cazenave and K. Jaouiche, trans.). Paris: Vrin (1976). Originally published in Russian under the title *Istoriya matematiki v srednie veka*. Moscow.