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Three-pattern decomposition of global atmospheric circulation: part II—dynamical equations of horizontal, meridional and zonal circulations

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Abstract The three-pattern decomposition of global atmospheric circulation (TPDGAC) partitions three-dimensional (3D) atmospheric circulation into horizontal, meridional and zonal components to study the 3D structures of global atmospheric circulation. This paper incorporates the three-pattern decomposition model (TPDM) into primitive equations of atmospheric dynamics and establishes a new set of dynamical equations of the horizontal, meridional and zonal circulations in which the operator properties are studied and energy conservation laws are preserved, as in the primitive equations. The physical significance of the newly established equations is demonstrated. Our findings reveal that the new equations are essentially the 3D vorticity equations of atmosphere and that the time evolution rules of the horizontal, meridional and zonal circulations can be described from the perspective of 3D vorticity evolution. The new set of dynamical equations includes decomposed expressions that can be used to explore the source terms of large-scale atmospheric circulation variations. A simplified model is presented to demonstrate the potential applications of the new equations for studying the dynamics of the Rossby, Hadley and Walker circulations. The model shows that the horizontal air temperature anomaly gradient (ATAG) induces changes in meridional and zonal circulations and promotes the baroclinic evolution of the horizontal circulation. The simplified model also indicates that the absolute vorticity of the horizontal circulation is not conserved, and its changes can be described by changes in the vertical vorticities of the meridional and zonal circulations. Moreover, the thermodynamic equation shows that the induced meridional and zonal circulations and advection transport by the horizontal circulation in turn cause a redistribution of the air temperature. The simplified model reveals the fundamental rules between the evolution of the air temperature and the horizontal, meridional and zonal components of global atmospheric circulation.

Keywords Three-pattern decomposition of global atmospheric circulation · New dynamical equations of horizontal, meridional and zonal circulations · Three-dimensional vorticity equations · Simplified dynamical equations

1 Introduction

The Rossby wave at middle–high latitudes and the Hadley and Walker circulations at low latitudes are significant in the evolution of global atmospheric circulation. The evolution of the ridges and troughs of the Rossby wave provides valuable information for the analysis and prediction of large-scale weather processes in the middle–high latitudes (Rossby 1939; Charney 1947; Charney and Eliassen 1949). In contrast, the Hadley and Walker circulations are critical to the global water vapor transport and energy balance (Oort and Peixóto 1983; Bowman and Cohen 1997; Trenberth and Stepaniak 2003; Hosking et al. 2012). However, these global circulations have clear interdecadal variations under the global warming conditions in recent decades. For

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example, the Hadley circulation has an intensified trend, especially in the Northern Hemisphere in winter (Chen et al. 2002, 2014; Wielicki et al. 2002; Mitas and Clement 2005; Sohn and Park 2010), with its descending branch moving towards the pole (Fu et al. 2006; Lu et al. 2007; Hu and Fu 2007; Nguyen et al. 2013), indicating poleward expansion of the tropical belt and poleward shifting of the subtropical jet stream (Archer and Caldeira 2008; Seidel et al. 2008; Davis and Birner 2013). The evolution characteristics of the Walker circulation are currently under debate. A number of studies have suggested a strengthening trend and a westward shift of the Walker circulation in recent decades (Sohn et al. 2013; L'Heureux et al. 2013; Bayr et al. 2014; McGregor et al. 2014; Ma and Zhou 2016), and these trends affect the distribution pattern of tropical precipitation. However, some studies have indicated that the Walker circulation has been weakening because of anthropogenic forcing (Held and Soden 2006; Vecchi and Soden 2007; Yu and Zwiers 2010; Tokinaga et al. 2012; DiNezio et al. 2013). Such interdecadal variations of the Hadley and Walker circulations may induce a shift in the global climate belt and increase the frequency of extreme weather and climate events, thereby increasing social and economic losses (Karnauskas and Ummenhofer 2014; Ma and Zhou 2016).

On the other hand, the Rossby, Hadley and Walker circulations exhibit complex interactions. For example, according to observations and model simulations, the Rossby wave at middle–high latitudes can propagate to tropical areas and affect tropical weather and climate (Kiladis and Weickmann 1992a, b; Kiladis and Feldstein 1994), whereas the Hadley and Walker circulations at low latitudes are closely related to anomalous behavior of global circulation. Studies on the variations of the Rossby, Hadley and Walker circulations and the interactions between the circulations at low latitudes and middle–high latitudes are of critical importance (Kiladis and Feldstein 1994). Although many efforts have been devoted to the variations in and interactions among these circulations (e.g., Charney 1969; Zhang and Webster 1992; Kiladis and Weickmann 1992a, b; Kiladis and Feldstein 1994), the underlying dynamical mechanisms are not completely understood. Thus, new theories and methods are needed to explore these issues.

On the basis of the basic features of global atmospheric circulation, we defined the horizontal, meridional and zonal circulations, which can be regarded as the global generalization of the Rossby, Hadley and Walker circulations, and then developed a novel three-pattern decomposition of global atmospheric circulation (TPDGAC) (Xu 2001; Hu 2006, 2008; Liu et al. 2008; Deng et al. 2010; Hu et al. 2015, 2017). The TPDGAC method partitions global atmospheric circulation into the horizontal, meridional and zonal circulations within three orthogonal planes to study the three-dimensional (3D) structures and variations of global atmospheric circulation (Hu et al. 2015). This study aims to establish the dynamical equations of the horizontal, meridional and zonal circulations by using the three-pattern decomposition model (TPDM) and primitive equations of atmospheric dynamics, thereby providing new opportunities to study the dynamics of the Rossby, Hadley and Walker circulations under nonuniform global warming.

This paper is organized as follows. The TPDGAC is introduced in Sect. 2. The dynamical equations of the horizontal, meridional and zonal circulations are established and the physical interpretation of the new equations are presented in Sect. 3, and the operator properties and energy conservation law are also studied. In Sect. 4, the basic evolution rules of the horizontal, meridional and zonal circulations are interpreted with a simplified model of the new dynamical equations. Finally, a summary is presented in Sect. 5.

2 Three-pattern decomposition of global atmospheric circulation

According to Hu et al. (2015), the Rossby wave at middle–high latitudes and the Hadley and Walker circulations at low latitudes can be generalized globally, and these generalized circulations are called the horizontal, meridional and zonal circulations. The global atmospheric circulation is then decomposed into three orthogonal components (horizontal, meridional and zonal circulations), and the three-pattern decomposition method is called the TPDGAC (Hu et al. 2015). The main procedures of the method will be introduced in this section.

To resolve the unit inconsistency in calculating the 3D vorticity vector in the pressure coordinates, the spherical σ -coordinate system is used in the TPDGAC. Namely, we assume $\sigma = \frac{p}{P_s}$, where p is the atmospheric pressure and P_s is the pressure at the earth's surface. It should be noted that the unit vector $\vec{i}$ of the σ-coordinate system points from west to east along the latitude circles, $\vec{j}$ points from north to south along the longitudinal direction, and $\vec{k}$ points from the earth's surface to the earth's center (Hu et al. 2015).

If we assume $P_s = 1000hPa$, then the velocities of the atmosphere in the spherical σ-coordinate system are defined as follows:

$$u' = \frac{u}{a},\ v' = \frac{v}{a},\ \dot{\sigma} = \frac{\omega}{P_s}, \tag{1}$$

with the same unit of s^{-1}, and the continuity equation can be represented as follows:

$$\frac{1}{\sin\theta}\frac{\partial u'}{\partial \lambda} + \frac{1}{\sin\theta}\frac{\partial(\sin\theta v')}{\partial\theta} + \frac{\partial\dot{\sigma}}{\partial\sigma} = 0, \tag{2}$$

where a is the earth's radius, λ is the longitude, θ is the colatitude and u, v and ω are the velocities in the pressure coordinate system.

According to the basic features of the Rossby, Hadley and Walker circulations, we define the global horizontal, meridional and zonal circulations as follows:

$$\begin{cases} \vec{V}'_R(\lambda,\theta,\sigma) = u'_R(\lambda,\theta,\sigma)\vec{i} + v'_R(\lambda,\theta,\sigma)\vec{j}, \\ \vec{V}'_H(\lambda,\theta,\sigma) = v'_H(\lambda,\theta,\sigma)\vec{j} + \dot{\sigma}_H(\lambda,\theta,\sigma)\vec{k}, \\ \vec{V}'_W(\lambda,\theta,\sigma) = u'_W(\lambda,\theta,\sigma)\vec{i} + \dot{\sigma}_W(\lambda,\theta,\sigma)\vec{k}. \end{cases} \tag{3}$$

The horizontal circulation $\vec{V}'_R$ has zero vertical velocity, the meridional circulation $\vec{V}'_H$ has zero zonal wind, and the zonal circulation $\vec{V}'_W$ has zero meridional wind. However, the velocity components of $\vec{V}'_R$ are varying with the height, and the components of $\vec{V}'_H$ and $\vec{V}'_W$ are varying with the longitude and colatitude, respectively. They satisfy the following continuity equations, respectively:

$$\begin{cases} \dfrac{1}{\sin\theta}\dfrac{\partial u'_R}{\partial\lambda} + \dfrac{1}{\sin\theta}\dfrac{\partial(\sin\theta v'_R)}{\partial\theta} = 0, \\ \dfrac{1}{\sin\theta}\dfrac{\partial(\sin\theta v'_H)}{\partial\theta} + \dfrac{\partial\dot{\sigma}_H}{\partial\sigma} = 0, \\ \dfrac{1}{\sin\theta}\dfrac{\partial u'_W}{\partial\lambda} + \dfrac{\partial\dot{\sigma}_W}{\partial\sigma} = 0. \end{cases} \tag{4}$$

Equation (4) ensures that the components of $\vec{V}'_R$, $\vec{V}'_H$ and $\vec{V}'_W$ can be quantitatively represented by the stream functions $R(\lambda,\theta,\sigma)$, $H(\lambda,\theta,\sigma)$ and $W(\lambda,\theta,\sigma)$ as follows:

$$\begin{cases} u'_R = -\dfrac{\partial R}{\partial\theta}, \quad v'_R = \dfrac{1}{\sin\theta}\dfrac{\partial R}{\partial\lambda}, \\ v'_H = -\dfrac{\partial H}{\partial\sigma}, \quad \dot{\sigma}_H = \dfrac{1}{\sin\theta}\dfrac{\partial(\sin\theta H)}{\partial\theta}, \\ u'_W = \dfrac{\partial W}{\partial\sigma}, \quad \dot{\sigma}_W = -\dfrac{1}{\sin\theta}\dfrac{\partial W}{\partial\lambda}. \end{cases} \tag{5}$$

For large-scale motions, the global atmospheric circulation can be expressed as the superposition of the horizontal, meridional and zonal circulations defined above. Namely, for $\vec{V}'(\lambda,\theta,\sigma) = u'(\lambda,\theta,\sigma)\vec{i} + v'(\lambda,\theta,\sigma)\vec{j} + \dot{\sigma}(\lambda,\theta,\sigma)\vec{k}$ satisfying the continuity Eq. (2), we have

$$\vec{V}' = \vec{V}'_H + \vec{V}'_W + \vec{V}'_R, \tag{6}$$

with the following components:

$$\begin{cases} u' = u'_W + u'_R = \dfrac{\partial W}{\partial\sigma} - \dfrac{\partial R}{\partial\theta}, \\ v' = v'_R + v'_H = \dfrac{1}{\sin\theta}\dfrac{\partial R}{\partial\lambda} - \dfrac{\partial H}{\partial\sigma}, \\ \dot{\sigma} = \dot{\sigma}_H + \dot{\sigma}_W = \dfrac{1}{\sin\theta}\dfrac{\partial(\sin\theta H)}{\partial\theta} - \dfrac{1}{\sin\theta}\dfrac{\partial W}{\partial\lambda}. \end{cases} \tag{7}$$

We refer to Eq. (6) or Eq. (7) as the TPDM (Hu et al. 2015). However, Eq. (4) cannot ensure the uniqueness of R, H and W because $\vec{V}'_R$, $\vec{V}'_H$ and $\vec{V}'_W$ have three spatial dimensions. The following restriction condition help us to pick up the correct ones (Theorems 1 and 2 in Hu et al. 2015):

$$\frac{1}{\sin\theta}\frac{\partial H}{\partial\lambda} + \frac{1}{\sin\theta}\frac{\partial(W\sin\theta)}{\partial\theta} + \frac{\partial R}{\partial\sigma} = 0. \tag{8}$$

By using Eqs. (7) and (8), we have

$$\Delta_3 R = \zeta_\sigma, \tag{9}$$

$$\frac{\partial H}{\partial\sigma} = \frac{1}{\sin\theta}\frac{\partial R}{\partial\lambda} - v', \tag{10}$$

$$\frac{\partial W}{\partial\sigma} = \frac{\partial R}{\partial\theta} + u', \tag{11}$$

where $\zeta_\sigma = \frac{1}{\sin\theta}\frac{\partial v'}{\partial\lambda} - \frac{1}{\sin\theta}\frac{\partial(u'\sin\theta)}{\partial\theta}$ represents the vertical vorticity of the entire atmospheric layer and $\Delta_3 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{\partial^2}{\partial\sigma^2}$ denotes the 3D Laplacian in the spherical σ-coordinates. The stream functions R, H and W can be obtained by solving Eqs. (9), (10) and (11). Thus the global atmospheric circulation $\vec{V}'$ is decomposed into the three-pattern circulations $\vec{V}'_R$, $\vec{V}'_H$ and $\vec{V}'_W$ using Eq. (5).

3 Dynamical equations of the horizontal, meridional and zonal circulations

On the basis of the TPDM (7) in Sect. 2, we find that the components of $\vec{V}'$ can be rewritten as the following operator equation:

$$\begin{pmatrix} u' \\ v' \\ \dot{\sigma} \end{pmatrix} = \begin{pmatrix} 0 & \frac{\partial}{\partial\sigma} & -\frac{\partial}{\partial\theta} \\ -\frac{\partial}{\partial\sigma} & 0 & \frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} \\ \frac{1}{\sin\theta}\frac{\partial\sin\theta}{\partial\theta} & -\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} & 0 \end{pmatrix} \begin{pmatrix} H \\ W \\ R \end{pmatrix}, \tag{12}$$

with the meaning that the velocities $(u', v', \dot{\sigma})$ can be represented by three stream functions (H, W, R). In this section, by using Eq. (12), the velocity components $(u', v', \dot{\sigma})$ in the primitive equations are replaced with the stream functions (H, W, R) to obtain a new set of dynamical equations of the three-pattern circulations $(\vec{V}'_H, \vec{V}'_W, \vec{V}'_R)$ and the physical meaning of the new equations will be demonstrated.

3.1 Dynamical equations

The primitive equations of atmospheric dynamics can be written as the following operator equation (see Chou 1974, 1983 and the Sect. 2 and 3 of the supplementary material):

$$B'\frac{\partial\phi'}{\partial t} + (N' + L')\phi' = \xi', \tag{13}$$

where $\phi' = (u', v', \dot{\sigma}, \varphi', T)^T$ are the regular variables of atmosphere and $\varphi' = \frac{\varphi}{a}$. $B' = diag\left(a, a, 0, 0, \frac{R_0^2}{ac^2}\right)$ is a diagonal matrix and R_0 is the gas constant for dry air. $L' = diag(\tilde{L}_1', \tilde{L}_1', 0, 0, \tilde{L}_2')$ represents the dissipative terms, and $\xi' = \left(0, 0, 0, 0, \frac{R_0{}^2}{ac^2}\frac{\varepsilon}{c_p}\right)^T$ represents the non-adiabatic heating process. N' is a nonlinear anti-adjoint operator, which describes factors such as the nonlinear advection and convection, Coriolis force, pressure gradient, and so on. It depends on ϕ' with the following form:

$$N' = \begin{bmatrix} a\pi' & aA' & 0 & \frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} & 0 \\ -aA' & a\pi' & 0 & \frac{\partial}{\partial\theta} & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial\sigma} & \frac{R_0}{a\sigma} \\ \frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} & \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta & \frac{\partial}{\partial\sigma} & 0 & 0 \\ 0 & 0 & -\frac{R_0}{a\sigma} & 0 & \frac{R_0{}^2}{ac^2}\pi' \end{bmatrix},$$

where,

$$\pi' = \frac{u'}{\sin\theta}\frac{\partial}{\partial\lambda} + v'\frac{\partial}{\partial\theta} + \dot{\sigma}\frac{\partial}{\partial\sigma},$$

$$A' = 2\Omega\cos\theta + ctg\theta u',$$

$$\tilde{L}_1' = -a\frac{\partial}{\partial\sigma}\nu_1\left(\frac{g\sigma}{R_0\bar{T}}\right)^2\frac{\partial}{\partial\sigma} - a\mu_1\nabla^2,$$

$$\tilde{L}_2' = -\frac{1}{a}\frac{\partial}{\partial\sigma}\nu_2\left(\frac{g\sigma}{R_0\bar{T}}\right)^2\frac{\partial}{\partial\sigma} - \frac{1}{a}\mu_2\nabla^2,$$

$$c^2 = \frac{R_0^2\bar{T}}{g}(\gamma_d - \bar{\gamma}),$$

and

$$\nabla^2 = \frac{1}{a^2\sin^2\theta}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{a^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}.$$

$\overline{T} = \overline{T}(p)$(or $\overline{\varphi'} = \overline{\varphi'}(p)$) represents the time-mean air temperature (or geopotential function) after globally averaged in isobaric surface p, and T (or φ') is the bias of the air temperature (or geopotential function) related to $\overline{T}$ (or $\overline{\varphi'}$). The other variables presented above are conventional. Owing to the above properties, the operator Eq. (13) reveals the natural characteristics of the atmospheric motions more succinctly than the primitive equations in the component form (Chou 1974, 1983).

Next, we use Eq. (13) to establish the dynamical equations of the horizontal, meridional and zonal circulations (represented by $\vec{V}_R'$, $\vec{V}_H'$ and $\vec{V}_W'$, respectively). To have the TPDM to comply with Eq. (13), Eq. (12) is rewritten as follows:

$$\phi' = C\psi = \begin{bmatrix} 0 & \frac{\partial}{\partial\sigma} & -\frac{\partial}{\partial\theta} & 0 & 0 \\ -\frac{\partial}{\partial\sigma} & 0 & \frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} & 0 & 0 \\ \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta & -\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\psi, \tag{14}$$

where $\psi = (H, W, R, \varphi', T)^T$. If we replace ϕ' in Eq. (13) with ψ using Eq. (14) and keep the operator properties of Eq. (13) unchanged, we have

$$CB'C\frac{\partial\psi}{\partial t} + (CN'C + CL'C)\psi = C\xi'. \tag{15}$$

The left-side operator C in every term of Eq. (15) is to preserve the dissipative nature of the primitive equations, which is discussed in detail in Sect. 3.3 of this study and in Sect. 3 of the supplementary material. Because the stream functions H, W and R are used to replace the velocity fields u', v' and $\dot{\sigma}$ in the primitive equations, we call Eq. (15) the dynamical equation of the horizontal, meridional and zonal circulations. Using Eq. (8) and applying the corresponding matrix operations, we easily obtain the components of Eq. (15) as follows (see Sect. 4 of the supplementary material for the specific deducing process):

$$\begin{aligned} &\frac{\partial}{\partial t}\left(-\frac{\partial^2 H}{\partial\sigma^2} + \frac{1}{\sin\theta}\frac{\partial^2 R}{\partial\lambda\partial\sigma}\right) - \frac{\partial}{\partial\sigma}\pi'\frac{\partial H}{\partial\sigma} - \frac{\partial}{\partial\sigma}A'\frac{\partial W}{\partial\sigma} \\ &\quad + \frac{\partial}{\partial\sigma}\left(A'\frac{\partial R}{\partial\theta} + \pi'\frac{1}{\sin\theta}\frac{\partial R}{\partial\lambda}\right) + \frac{\partial}{a\partial\sigma}\tilde{L}_1'\left(\frac{1}{\sin\theta}\frac{\partial R}{\partial\lambda} - \frac{\partial H}{\partial\sigma}\right) \\ &= \frac{R_0}{a^2\sigma}\frac{\partial T}{\partial\theta}, \end{aligned} \tag{16}$$

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{\partial^2 W}{\partial\sigma^2} - \frac{\partial^2 R}{\partial\theta\partial\sigma}\right) - \frac{\partial}{\partial\sigma}A'\frac{\partial H}{\partial\sigma} + \frac{\partial}{\partial\sigma}\pi'\frac{\partial W}{\partial\sigma} \\ &\quad + \frac{\partial}{\partial\sigma}\left(A'\frac{1}{\sin\theta}\frac{\partial R}{\partial\lambda} - \pi'\frac{\partial R}{\partial\theta}\right) + \frac{\partial}{a\partial\sigma}\tilde{L}_1'\left(\frac{\partial W}{\partial\sigma} - \frac{\partial R}{\partial\theta}\right) \\ &= \frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda}, \end{aligned} \tag{17}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(\Delta_3 R) + \frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta A'\frac{\partial H}{\partial\sigma} - \frac{\partial}{\partial\lambda}\pi'\frac{\partial H}{\partial\sigma}\right) \\ &- \frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\pi'\frac{\partial W}{\partial\sigma} + \frac{\partial}{\partial\lambda}A'\frac{\partial W}{\partial\sigma}\right) \\ &+ \frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\pi'\frac{\partial R}{\partial\theta} + \frac{\partial}{\partial\lambda}\pi'\frac{1}{\sin\theta}\frac{\partial R}{\partial\lambda}\right. \\ &\left. -\frac{\partial}{\partial\theta}A'\frac{\partial R}{\partial\lambda} + \frac{\partial}{\partial\lambda}A'\frac{\partial R}{\partial\theta}\right) \\ &+ \frac{1}{a\sin\theta}\frac{\partial}{\partial\lambda}\tilde{L}_1'\left(\frac{1}{\sin\theta}\frac{\partial R}{\partial\lambda} - \frac{\partial H}{\partial\sigma}\right) \\ &- \frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\tilde{L}_1'\left(\frac{\partial W}{\partial\sigma} - \frac{\partial R}{\partial\theta}\right) = 0, \end{aligned} \tag{18}$$

$$\frac{R_0^2}{ac^2}\frac{\partial T}{\partial t}-\frac{R_0}{a\sigma}\left(\frac{1}{\sin\theta}\frac{\partial(\sin\theta H)}{\partial\theta}-\frac{1}{\sin\theta}\frac{\partial W}{\partial\lambda}\right)+\frac{R_0^2}{ac^2}\pi' T+\tilde{L}_2' T=\frac{R_0^2}{ac^2}\frac{\varepsilon}{c_p}. \tag{19}$$

In the above component equations, we see that the continuity equation in primitive equations vanishes because the TPDM (7) naturally satisfies the continuity Eq. (2). Equations (16)–(19) reflect the dynamical evolution rules between the three-pattern circulations (represented by the stream functions H, W and R) and the air temperature anomaly (represented by T), and they can be used to diagnose and predict the variations of the three-pattern circulations directly from the perspective of stream functions instead of from that of the velocity field.

3.2 Physical interpretations: 3D vorticity equations

To identify the physical concepts of the new dynamical equations, we must replace the stream functions H, W and R in Eqs. (16)–(19) with the velocity components of the three-pattern circulations. After a rearrangement of the terms, we obtain the equivalent expressions of Eqs. (16)–(19) as follows (see Sect. 5 of the supplementary material for the specific deducing process):

$$\frac{\partial}{\partial t}\left(\frac{\partial v'_H}{\partial\sigma}+\frac{\partial v'_R}{\partial\sigma}\right)+\pi'\left(\frac{\partial v'_H}{\partial\sigma}+\frac{\partial v'_R}{\partial\sigma}\right)+\nabla_3(v'_H+v'_R)\cdot\frac{\partial\vec{V}'}{\partial\sigma}-(f+2ctg\theta u')\left(\frac{\partial u'_W}{\partial\sigma}+\frac{\partial u'_R}{\partial\sigma}\right)+\frac{\partial}{a\partial\sigma}\tilde{L}_1'(v'_H+v'_R)=\frac{R_0}{a^2\sigma}\frac{\partial T}{\partial\theta}, \tag{20}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial u'_W}{\partial\sigma}+\frac{\partial u'_R}{\partial\sigma}\right)+\pi'\left(\frac{\partial u'_W}{\partial\sigma}+\frac{\partial u'_R}{\partial\sigma}\right)+\nabla_3(u'_W+u'_R)\cdot\frac{\partial\vec{V}'}{\partial\sigma}+(f+ctg\theta u')\left(\frac{\partial v'_H}{\partial\sigma}+\frac{\partial v'_R}{\partial\sigma}\right)+ctg\theta v'\left(\frac{\partial u'_W}{\partial\sigma}+\frac{\partial u'_R}{\partial\sigma}\right)+\frac{\partial}{a\partial\sigma}\tilde{L}_1'(u'_W+u'_R)=\frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda}, \tag{21}$$

$$\frac{\partial}{\partial t}(\Delta_3 R)+\pi'(\Delta_3 R-f)+(\Delta_3 R-f)\left(\frac{1}{\sin\theta}\frac{\partial u'_W}{\partial\lambda}+\frac{1}{\sin\theta}\frac{\partial(v'_H\sin\theta)}{\partial\theta}\right)+\left(\frac{1}{\sin\theta}\frac{\partial(\dot\sigma_H+\dot\sigma_W)}{\partial\lambda}\frac{\partial(v'_R+v'_H)}{\partial\sigma}-\frac{\partial(\dot\sigma_H+\dot\sigma_W)}{\partial\theta}\frac{\partial(u'_W+u'_R)}{\partial\sigma}\right)+\frac{1}{a\sin\theta}\frac{\partial}{\partial\lambda}\tilde{L}_1'(v'_H+v'_R)-\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\tilde{L}_1'(u'_W+u'_R)=0, \tag{22}$$

$$\frac{R_0^2}{ac^2}\frac{\partial T}{\partial t}-\frac{R_0}{a\sigma}(\dot\sigma_H+\dot\sigma_W)+\frac{R_0^2}{ac^2}\pi' T+\tilde{L}_2' T=\frac{R_0^2}{ac^2}\frac{\varepsilon}{c_p}, \tag{23}$$

where $f=2\Omega\cos\theta$, $\nabla_3=\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}\vec{i}+\frac{\partial}{\partial\theta}\vec{j}+\frac{\partial}{\partial\sigma}\vec{k}$.

Furthermore, we can deduce the 3D vorticity vector of the actual velocity vector $\vec{V}'$ in the spherical σ-coordinate system as follows:

$$\nabla\times\vec{V}'=\left(\frac{\partial\dot\sigma}{\partial\theta}-\frac{\partial v'}{\partial\sigma}\right)\vec{i}+\left(\frac{\partial u'}{\partial\sigma}-\frac{1}{\sin\theta}\frac{\partial\dot\sigma}{\partial\lambda}\right)\vec{j}+\left(\frac{1}{\sin\theta}\frac{\partial v'}{\partial\lambda}-\frac{1}{\sin\theta}\frac{\partial(u'\sin\theta)}{\partial\theta}\right)\vec{k}. \tag{24}$$

By the diagnostic analysis, we find that the horizontal shears of vertical velocity $\left(\frac{\partial\dot\sigma}{\partial\theta}\ and\ -\frac{1}{\sin\theta}\frac{\partial\dot\sigma}{\partial\lambda}\right)$ are much smaller than the vertical shears of horizontal velocity ($-\frac{\partial v'}{\partial\sigma}$ and $\frac{\partial u'}{\partial\sigma}$) for the large-scale motions. Then, we can use $-\frac{\partial v'}{\partial\sigma}$ and $\frac{\partial u'}{\partial\sigma}$ to approximate the zonal vorticity and meridional vorticity, respectively. Thus, Eq. (24) can be approximately represented as follows:

$$\nabla\times\vec{V}'=\zeta_\lambda\vec{i}+\zeta_\theta\vec{j}+\zeta_\sigma\vec{k}\cong-\frac{\partial v'}{\partial\sigma}\vec{i}+\frac{\partial u'}{\partial\sigma}\vec{j}+\left(\frac{1}{\sin\theta}\frac{\partial v'}{\partial\lambda}-\frac{1}{\sin\theta}\frac{\partial(u'\sin\theta)}{\partial\theta}\right)\vec{k}, \tag{25}$$

where ζ_λ, ζ_θ and ζ_σ are the zonal, meridional and vertical vorticities, respectively.

Then, we combine Eqs. (20)–(22) with Eq. (25) and make the appropriate deduction (see Sect. 6 of the supplementary material), and Eqs. (20)–(22) can be rewritten as follows:

$$\frac{\partial\zeta_\lambda}{\partial t}=-\pi'\zeta_\lambda-\zeta_\lambda D_\lambda+\frac{1}{\sin\theta}\frac{\partial v'}{\partial\lambda}\frac{\partial u'}{\partial\sigma}-f\xi_\theta-\left(2ctg\theta u'\frac{\partial u'}{\partial\sigma}+ctg\theta v'\frac{\partial v'}{\partial\sigma}\right)+\frac{\partial}{a\partial\sigma}\tilde{L}_1' v'-\frac{R_0}{a^2\sigma}\frac{\partial T}{\partial\theta}, \tag{26}$$

$$\frac{\partial\zeta_\theta}{\partial t}=-\pi'\zeta_\theta-\zeta_\theta D_\theta-\frac{1}{\sin\theta}\frac{\partial u'\sin\theta}{\partial\theta}\frac{\partial v'}{\partial\sigma}+f\zeta_\lambda-ctg\theta v'\frac{\partial u'}{\partial\sigma}-\frac{\partial}{a\partial\sigma}\tilde{L}_1' u'+\frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda}, \tag{27}$$

$$\frac{\partial\zeta_\sigma}{\partial t}=-\pi'\zeta_\sigma-\pi'(-f)-\zeta_\sigma D_\sigma-(-f)D_\sigma+\left(\frac{\partial\dot\sigma}{\partial\theta}\zeta_\theta+\frac{1}{\sin\theta}\frac{\partial\dot\sigma}{\partial\lambda}\zeta_\lambda\right)+\left(\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\tilde{L}_1' u'-\frac{1}{a\sin\theta}\frac{\partial}{\partial\lambda}\tilde{L}_1' v'\right), \tag{28}$$

where $D_\sigma=\frac{1}{\sin\theta}\frac{\partial u'}{\partial\lambda}+\frac{1}{\sin\theta}\frac{\partial v'\sin\theta}{\partial\theta}$ represents the horizontal divergence and $D_\lambda=\frac{1}{\sin\theta}\frac{\partial v'\sin\theta}{\partial\theta}+\frac{\partial\dot\sigma}{\partial\sigma}$ and $D_\theta=\frac{1}{\sin\theta}\frac{\partial u'}{\partial\lambda}+\frac{\partial\dot\sigma}{\partial\sigma}$ represent the divergence in the meridional and zonal planes, respectively. Thus, the newly established dynamical equations (16)–(18) are actually the zonal, meridional and vertical vorticity equations of global atmospheric circulation.

Next, we will provide the physical interpretation for each of the new 3D vorticity equations in the spherical σ-coordinate system. For better understanding, we must remember that the unit vector $\vec{j}$ in the spherical σ-coordinate system points from north to south along the longitudinal direction and $\vec{k}$ points from the earth's surface to the earth's center (Hu et al. 2015).

3.2.1 Vertical vorticity equation

Equation (28) is the vertical vorticity equation of global atmospheric circulation (Holton 2004), and it shows that the rate of the change of the local vertical vorticity is given by the sum of the six terms on the right side. These terms include the advection term of the vertical vorticity, the advection term of the geostrophic vorticity, the divergence term of the vertical vorticity, the divergence term of the geostrophic vorticity, the tilting or twisting term of the horizontal vorticity and the dissipative term (Holton 2004). To facilitate an understanding of the zonal and meridional vorticity equations, we provide a physical explanation of each term on the right side of the vertical vorticity Eq. (28) except for the dissipative term.

The advection term of the vertical vorticity, namely, the first term $-\pi'\zeta_\sigma$ on the right side of Eq. (28), can be expanded as $-\left(\frac{u'}{\sin\theta}\frac{\partial\zeta_\sigma}{\partial\lambda}+v'\frac{\partial\zeta_\sigma}{\partial\theta}+\dot{\sigma}\frac{\partial\zeta_\sigma}{\partial\sigma}\right)$. If the wind flows from west to east ($u'>0$) and there is a stronger anticyclonic vertical vorticity ζ_σ (positive vertical vorticity in the spherical σ-coordinates) in the west than in the east (Fig. 1a), then we have $\frac{\partial\zeta_\sigma}{\partial\lambda}\cong\frac{\Delta\zeta_\sigma}{\Delta\lambda}=\frac{(\zeta_\sigma)_{east}-(\zeta_\sigma)_{west}}{\Delta\lambda}<0$ and $-\frac{u'}{\sin\theta}\frac{\partial\zeta_\sigma}{\partial\lambda}>0$. That is, if there is transport of the vertical vorticity from a high value area to a low value area by the wind, then a positive local vertical vorticity $\left(\frac{\partial\zeta_\sigma}{\partial t}>0\right)$ will be generated.

Because the Coriolis parameter depends only on the colatitude θ, the advection term of the geostrophic vorticity is $-\pi'(-f)=-v'\frac{\partial(-f)}{\partial\theta}$. Because $-f=-2\Omega\cos\theta<0$ (geostrophic vorticity $-f$ is negative in the spherical σ -coordinates), then we have $\frac{\partial(-f)}{\partial\theta}>0$, i.e., the geostrophic vorticity (cyclonic vertical vorticity) in high latitudes is lower (higher) than that in low latitudes in the Northern Hemisphere. If the wind flows from north to south ($v'>0$ in the spherical σ-coordinates) and the geostrophic vorticity is transported from high latitudes to low latitudes, then the cyclonic vertical vorticity will be increased in low latitudes, i.e., there will be a decrease in the local vertical vorticity ($-v'\frac{\partial(-f)}{\partial\theta}<0$, Fig. 1b).

According to Eq. (2), the divergence term of the vertical vorticity $-\zeta_\sigma D_\sigma$ can be rewritten as $\zeta_\sigma\frac{\partial\dot{\sigma}}{\partial\sigma}$. If there is an anticyclonic vertical vorticity ($\zeta_\sigma>0$) and the flow is horizontally convergent, i.e., $D_\sigma<0$ $\left(\frac{\partial\dot{\sigma}}{\partial\sigma}>0\right)$, then $-\zeta_\sigma D_\sigma>0$. Specifically, if the flow is horizontally convergent (the vertical velocity increases in the vertical direction), then the vertical vortex tube will be elongated and narrowed (Fig. 1c), thereby leading to an increase in the local vertical vorticity.

Similarly, the divergence term of the geostrophic vorticity $-(-f)D_\sigma$ can be rewritten as $(-f)\frac{\partial\dot{\sigma}}{\partial\sigma}$. For the geostrophic vorticity $-f<0$ in the spherical σ-coordinates (cyclonic vertical vorticity), if the flow is convergent, i.e., $D_\sigma<0\left(\frac{\partial\dot{\sigma}}{\partial\sigma}>0\right)$, then $-(-f)D_\sigma<0$. That is, if the flow is horizontally convergent (the vertical velocity increases in the vertical direction), then the vertical vortex tube will be elongated and narrowed (Fig. 1d), thereby leading to an increase in the geostrophic vorticity, i.e., decrease in the local vertical vorticity.

The fifth term $\frac{\partial\dot{\sigma}}{\partial\theta}\zeta_\theta+\frac{1}{\sin\theta}\frac{\partial\dot{\sigma}}{\partial\lambda}\zeta_\lambda$ on the right side of Eq. (28) is the tilting or twisting term of the horizontal vorticity. The mechanism of this term is illustrated in Fig. 1e, which shows that if a positive meridional vorticity $\zeta_\theta>0$ occurs and the vertical velocity increases in the colatitude direction $\left(\frac{\partial\dot{\sigma}}{\partial\theta}>0\right)$, then the horizontal vorticity will be tilted, thereby generating a positive vertical vorticity $\left(\frac{\partial\dot{\sigma}}{\partial\theta}\zeta_\theta>0\right)$.

3.2.2 Zonal vorticity equation

Equation (26) is the zonal vorticity equation, which shows that the rate of the change of the local zonal vorticity is given by the sum of the seven terms on the right side. These terms are the advection term of the zonal vorticity, the divergence term of the zonal vorticity, the tilting or twisting term of the vertical vorticity, the Coriolis force term, the curvature effect term, the dissipative term and the meridional air temperature anomaly gradient (ATAG) term. As with the descriptions of Eq. (28), the curvature effect term and dissipative term are not discussed.

The advection term of the zonal vorticity $-\pi'\zeta_\lambda$ is equal to $-\left(\frac{u'}{\sin\theta}\frac{\partial\zeta_\lambda}{\partial\lambda}+v'\frac{\partial\zeta_\lambda}{\partial\theta}+\dot{\sigma}\frac{\partial\zeta_\lambda}{\partial\sigma}\right)$. Similarly to the mechanism of advection term in the vertical vorticity equation, if there is wind transport of the zonal vorticity from a high value area to a low value area, then a positive local zonal vorticity will be generated (Fig. 2a).

According to the definition of the divergence in the meridional plane (D_λ) and Eq. (2), the divergence term of the zonal vorticity $-\zeta_\lambda D_\lambda$ can be rewritten as $\zeta_\lambda\frac{1}{\sin\theta}\frac{\partial u'}{\partial\lambda}$. If there is a positive zonal vorticity $\zeta_\lambda>0$ and the flow in the meridional plane is convergent $D_\lambda<0\left(\frac{1}{\sin\theta}\frac{\partial u'}{\partial\lambda}>0\right)$, i.e., the zonal velocity u' increases in the zonal direction, then the zonal vortex tube will be elongated and narrowed, thereby causing an increase in the local zonal vorticity (Fig. 2b).

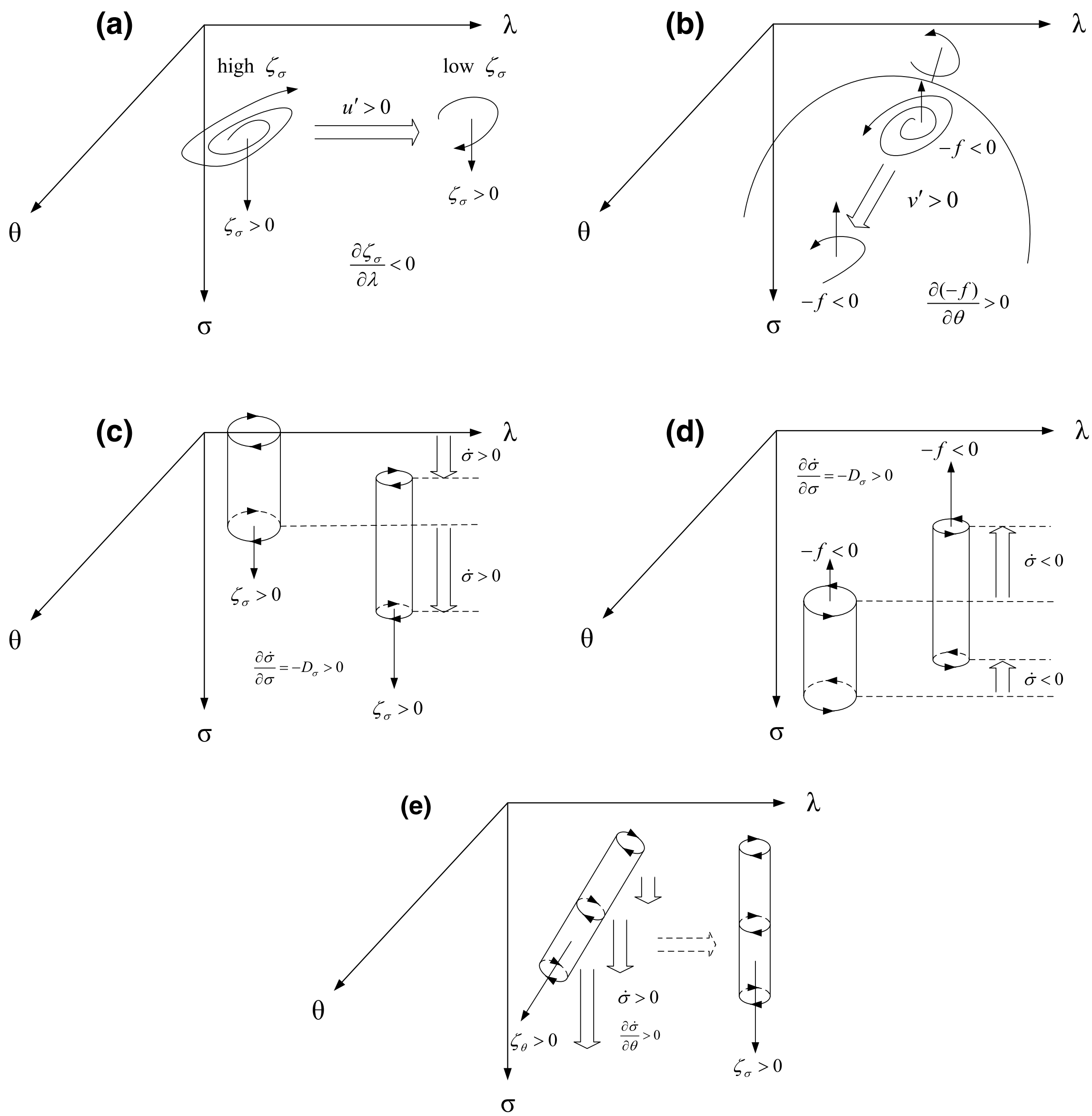

Fig. 1 Local vertical vorticity generation by the terms on the *right* side of the vertical vorticity equation: **a** advection term of the vertical vorticity, **b** advection term of the geostrophic vorticity, **c** divergence term of the vertical vorticity, **d** divergence term of the geostrophic vorticity, and **e** tilting or twisting term of the horizontal vorticity

The third term $\frac{1}{\sin\theta}\frac{\partial v'}{\partial \lambda}\frac{\partial u'}{\partial \sigma}$ on the right side of Eq. (26) represents the zonal vorticity generated by the twisting of the vertical vorticity. If there is a positive vertical vorticity $\frac{1}{\sin\theta}\frac{\partial v'}{\partial \lambda} > 0$ and the zonal velocity increases in the σ direction $\left(\frac{\partial u'}{\partial \sigma} > 0\right)$, then the vertical vorticity will be tilted (Fig. 2c), thereby resulting in the generation of positive zonal vorticity $\left(\frac{1}{\sin\theta}\frac{\partial v'}{\partial \lambda}\frac{\partial u'}{\partial \sigma} > 0\right)$.

The Coriolis force term in Eq. (26) is $-f\xi_\theta$, and the mechanism is shown in Fig. 2d. If there is a positive meridional vorticity $\zeta_\theta > 0$, then the Coriolis force $(-f)$ will deflect the meridional vorticity to the right of the direction of ζ_θ in the Northern Hemisphere, thus leading to a negative zonal vorticity $(-f\xi_\theta < 0)$. Section 7 of the supplementary material demonstrates that the effect of the Coriolis force on the horizontal vorticity is the same as that on the horizontal velocity.

The last term on the right side of Eq. (26) is the meridional ATAG term $-\frac{R_0}{a^2\sigma}\frac{\partial T}{\partial \theta}$. If there is a positive meridional ATAG $\left(\frac{\partial T}{\partial \theta} > 0\right)$ in the Northern Hemisphere, then an anomalous ascending motion and an anomalous descending motion will be generated in the low and high latitudes (Fig. 2e), respectively. As shown in Fig. 2e, the anomalous

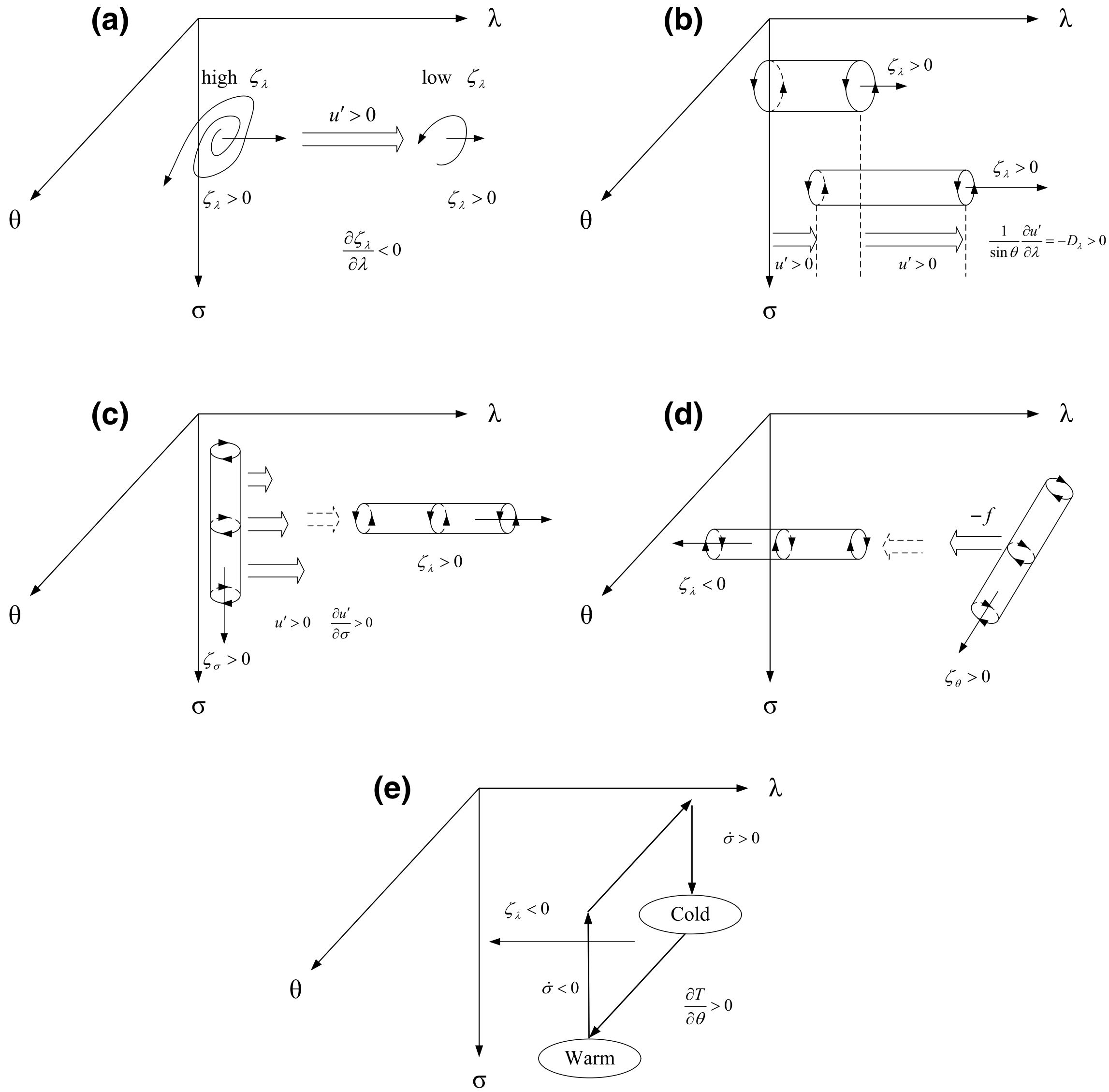

Fig. 2 Local zonal vorticity generation by the terms on the *right* side of the zonal vorticity equation: **a** advection term of the zonal vorticity, **b** divergence term of the zonal vorticity, **c** tilting or twisting term of the vertical vorticity, **d** Coriolis force term, and **e** meridional ATAG term

circulation resulting from the meridional ATAG generates a negative zonal vorticity $\left(-\frac{R_0}{a^2\sigma}\frac{\partial T}{\partial \theta} < 0\right)$.

3.2.3 Meridional vorticity equation

The physical interpretation of the meridional vorticity Eq. (27) is almost the same as that of the zonal vorticity Eq. (26) except for the direction of the vorticity. Equation (27) shows that the rate of the change of the local meridional vorticity is described by the sum of the seven terms on the right side, which are the advection term of the meridional vorticity, the divergence term of the meridional vorticity, the tilting or twisting term of the vertical vorticity, the Coriolis force term, the curvature effect term, the dissipative term and the zonal ATAG term. Similarly to the description of Eqs. (26) and (28), the curvature effect term and dissipative term in Eq. (27) are not discussed.

The advection term of the meridional vorticity $-\pi'\zeta_\theta$ can be expanded as $-\left(\frac{u'}{\sin\theta}\frac{\partial \zeta_\theta}{\partial \lambda} + v'\frac{\partial \zeta_\theta}{\partial \theta} + \dot{\sigma}\frac{\partial \zeta_\theta}{\partial \sigma}\right)$. As in the mechanisms of the advection terms in the vertical and zonal vorticity equations, if there is transport of the meridional vorticity from a high value area to a low value area by the wind, then a positive local meridional vorticity will be generated (Fig. 3a).

According to the definition of the divergence in the zonal plane (D_θ) and Eq. (2), the divergence term of the meridional vorticity $-\zeta_\theta D_\theta$ can be rewritten as $\zeta_\theta \frac{1}{\sin\theta}\frac{\partial(v'\sin\theta)}{\partial\theta}$. If there is a positive meridional vorticity $\zeta_\theta > 0$ and the flow in the zonal plane is convergent $D_\theta < 0$ $\left(\frac{1}{\sin\theta}\frac{\partial(v'\sin\theta)}{\partial\theta} > 0\right)$, i.e., the meridional velocity v' increases in the colatitude direction, then the meridional vortex tube will be elongated and narrowed, thereby causing an increase in the local meridional vorticity (Fig. 3b).

The third term $-\frac{1}{\sin\theta}\frac{\partial(u'\sin\theta)}{\partial\theta}\frac{\partial v'}{\partial\sigma}$ is the meridional vorticity generated by the twisting of the vertical vorticity. If there is a positive vertical vorticity $-\frac{1}{\sin\theta}\frac{\partial(u'\sin\theta)}{\partial\theta} > 0$ and the meridional velocity increases in the σ direction $\left(\frac{\partial v'}{\partial\sigma} > 0\right)$, then the vertical vorticity will be tilted (Fig. 3c), thereby leading to a generation of positive meridional vorticity $\left(-\frac{1}{\sin\theta}\frac{\partial(u'\sin\theta)}{\partial\theta}\frac{\partial v'}{\partial\sigma} > 0\right)$.

The Coriolis force term on the right side of Eq. (27) is $f\xi_\lambda$, and the mechanism is shown in Fig. 3d. If there is a positive zonal vorticity $\zeta_\lambda > 0$, then a positive meridional vorticity will be generated $\left(f\xi_\lambda > 0\right)$ by the deflection of Coriolis force (f).

The last term on the right side of Eq. (27) is the zonal ATAG term $\frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda}$. If there is a positive zonal ATAG $\left(\frac{\partial T}{\partial\lambda} > 0\right)$, then an anomalous ascending motion and an anomalous descending motion will be generated in the east and west (Fig. 3e), respectively. As shown in Fig. 3e, the anomalous circulation caused by the zonal ATAG generates a positive meridional vorticity $\left(\frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda} > 0\right)$.

As a summary of Sect. 3.2, we need to point out that the new dynamical equations (26)–(28) not only interpret the mechanisms of evolution of the global atmospheric circulation through the perspective of 3D vorticities but also can

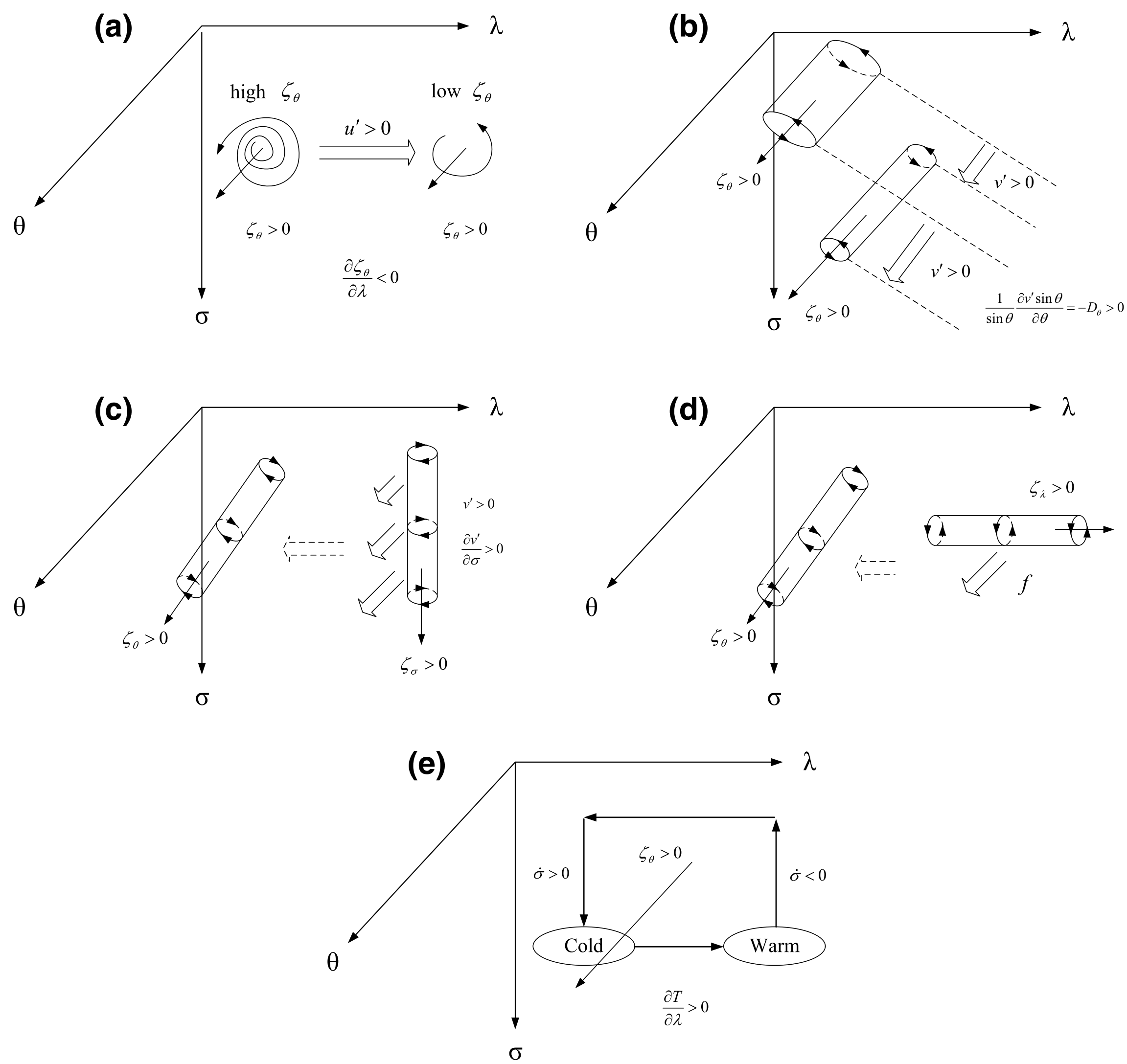

Fig. 3 Local meridional vorticity generation by the terms on the *right* side of the meridional vorticity equation: **a** advection term of the meridional vorticity, **b** divergence term of the meridional vorticity, **c** tilting or twisting term of the vertical vorticity, **d** Coriolis force term, and **e** zonal ATAG term

be used to study the source terms of the variations and nonlinear interaction processes of the horizontal, meridional and zonal circulations.

In fact, according to Eqs. (7) and (25), the 3D vorticities can be written as follows:

$$\begin{cases} \zeta_\lambda \cong \zeta_{\lambda H} + \zeta_{\lambda R} \cong \left(-\frac{\partial v'_H}{\partial \sigma}\right) + \left(-\frac{\partial v'_R}{\partial \sigma}\right), \\ \zeta_\theta \cong \zeta_{\theta W} + \zeta_{\theta R} \cong \frac{\partial u'_W}{\partial \sigma} + \frac{\partial u'_R}{\partial \sigma}, \\ \zeta_\sigma = \zeta_{\sigma R} + \zeta_{\sigma H} + \zeta_{\sigma W} = \left(\frac{1}{\sin\theta}\frac{\partial v'_R}{\partial \lambda} - \frac{1}{\sin\theta}\frac{\partial (u'_R \sin\theta)}{\partial \theta}\right) \\ \quad + \frac{1}{\sin\theta}\frac{\partial v'_H}{\partial \lambda} - \frac{1}{\sin\theta}\frac{\partial (u'_W \sin\theta)}{\partial \theta}, \end{cases} \tag{29}$$

where $\zeta_{\lambda H}$ and $\zeta_{\lambda R}$ represent the zonal vorticities of the meridional and horizontal circulations, $\zeta_{\theta W}$ and $\zeta_{\theta R}$ represent the meridional vorticities of the zonal and horizontal circulations, and $\zeta_{\sigma R}$, $\zeta_{\sigma H}$ and $\zeta_{\sigma W}$ represent the vertical vorticities of the horizontal, meridional and zonal circulations.

If we insert the component representations of ζ_λ, ζ_θ and ζ_σ into Eqs. (26)–(28), then we obtain the decomposed 3D vorticity equations of the horizontal, meridional and zonal circulations. Apparently, the nonlinear evolution processes and the source terms [especially the nonuniform global warming, i.e., the meridional and zonal ATAG terms in Eqs. (26) and (27)] of the variations of the three-pattern circulations can be investigated by using the decomposed 3D vorticity equations. That will be discussed in more detail in a future study.

3.3 Operator properties and energy conservation law

For the newly established dynamical equations of the horizontal, meridional and zonal circulations, we can demonstrate that the dissipative characteristics of the primitive equations remain preserved. In fact, for any $\psi_1 = (H_1, W_1, R_1, \varphi'_1, T_1)^T$ and $\psi_2 = (H_2, W_2, R_2, \varphi'_2, T_2)^T$, we define the inner product of ψ_1 and ψ_2 as follows:

$$(\psi_1, \psi_2) = \int_0^1 \int_0^\pi \int_0^{2\pi} (H_1 H_2 + W_1 W_2 + R_1 R_2 + \varphi'_1 \varphi'_2 + T_1 T_2) \sin\theta d\lambda d\theta d\sigma. \tag{30}$$

Thus, for Eq. (15) with the suitable boundary conditions (see Sects. 1 and 4 of the supplementary material), we have the following operator properties:

$$(C\psi_1, \psi_2) = (\psi_1, C\psi_2), (CB'C\psi_1, \psi_2) = (\psi_1, CB'C\psi_2),$$

$$(CL'C\psi_1, \psi_2) = (\psi_1, CL'C\psi_2), (CN'C\psi_1, \psi_2) = (\psi_1, -CN'^T C\psi_2),$$

which show that C, $CB'C$ and $CL'C$ are self-adjoint and $CN'C$ is anti-adjoint. In fact, the self-adjointness of C keeps the properties of operators B', N' and L' in the primitive equations (13) unchanged.

Furthermore, by multiplying $\psi = (H, W, R, \varphi', T)^T$ with both sides of Eq. (15) and taking the inner product defined by Eq. (30), we have

$$\left(CB'C\frac{\partial \psi}{\partial t}, \psi\right) + (CN'C\psi, \psi) + (CL'C\psi, \psi) = (C\xi', \psi). \tag{31}$$

By using the definition of matrix B' and the self-adjointness of $CB'C$, we have

$$\left(CB'C\frac{\partial \psi}{\partial t}, \psi\right) = \frac{1}{2}\frac{\partial}{\partial t}(CB'C\psi, \psi), \tag{32}$$

and (see Sect. 8 of the supplementary material for the specific deducing process)

$$\begin{aligned} (CB'C\psi, \psi) = & \int_0^1 \int_0^\pi \int_0^{2\pi} \left\{ a\left(-\frac{\partial^2 H}{\partial \sigma^2} + \frac{1}{\sin\theta}\frac{\partial^2 R}{\partial \lambda \partial \sigma}\right) H \right. \\ & \left. + a\left(-\frac{\partial^2 W}{\partial \sigma^2} + \frac{\partial^2 R}{\partial \theta \partial \sigma}\right) W \right\} \sin\theta d\lambda d\theta d\sigma \\ & + \int_0^1 \int_0^\pi \int_0^{2\pi} \left\{ a(-\Delta_3 R) R + \frac{R_0^2}{ac^2} T^2 \right\} \sin\theta d\lambda d\theta d\sigma \\ = & \int_0^1 \int_0^\pi \int_0^{2\pi} \left\{ a(u'_W + u'_R)^2 + a(v'_H + v'_R)^2 + \frac{R_0^2}{ac^2} T^2 \right\} \\ & \sin\theta d\lambda d\theta d\sigma. \end{aligned} \tag{33}$$

Equation (33) indicates that $(CB'C\psi, \psi)$ represents the total energy of the three-pattern circulations. Because $CN'C$ is anti-adjoint, we have $(CN'C\psi, \psi) = 0$. Thus, by using Eq. (32), we can rewrite Eq. (31) as follows:

$$\frac{1}{2}\frac{\partial}{\partial t}(CB'C\psi, \psi) + (CL'C\psi, \psi) = (C\xi', \psi), \tag{34}$$

which is the energy conservation law of the large-scale three-pattern circulations, and it shows that the energy evolution is caused by the dissipation $(CL'C\psi, \psi)$ and external forcing $(C\xi', \psi)$.

4 Basic evolution rules of the horizontal, meridional and zonal circulations

In this section, we will show the potential applications of the new dynamical equations in studying the dynamics of the horizontal, meridional and zonal circulations.

4.1 A simplified model

To highlight the basic evolution rules, we first neglect the dissipative term $CL'C\psi$ and forcing term $C\xi'$ in Eq. (15), thus resulting in the following:

$$CB'C\frac{\partial\psi}{\partial t} + CN'C\psi = 0. \tag{35}$$

In addition, because the hydrostatic equilibrium leads to large-scale horizontal motions, we are interested to know how the atmospheric circulation with only the horizontal component induces the meridional and zonal circulations. In other words, it is assumed that there is only horizontal circulation at the initial time; hence, the terms about H and W in Eq. (35) are eliminated except for the time derivative terms. Thus, the operator $CN'C$ in Eq. (35) is simplified as follows:

$$CN'C = \begin{bmatrix} 0 & 0 & m_{13} & -\frac{R_0}{a\sigma}\frac{\partial}{\partial\theta} \\ 0 & 0 & m_{23} & \frac{R_0}{a\sigma}\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} \\ 0 & 0 & m_{33} & 0 \\ -\frac{R_0}{a\sigma}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta & \frac{R_0}{a\sigma}\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda} & 0 & \frac{R_0{}^2}{ac^2}\pi' \end{bmatrix},$$

where $m_{13} = a\frac{\partial}{\partial\sigma}A'\frac{\partial}{\partial\theta} + a\frac{\partial}{\partial\sigma}\pi'\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}$, $m_{23} = a\frac{\partial}{\partial\sigma}\pi'\frac{\partial}{\partial\theta} - a\frac{\partial}{\partial\sigma}A'\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}$ and $m_{33} = a\frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}A'\frac{\partial}{\partial\lambda} - \frac{\partial}{\partial\theta}\sin\theta\pi'\frac{\partial}{\partial\theta}\right) - a\frac{1}{\sin\theta}\left(\frac{\partial}{\partial\lambda}A'\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\lambda}\pi'\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}\right)$.

Moreover, to prevent false forces from changing the energy conservation law, the anti-adjointness of $CN'C$ should be preserved after the simplification (Chou 1974, 1983), i.e., the components m_{13} and m_{23} must be zero. Then, after rearranging the terms, we arrive at a simplified model of Eq. (15) as follows:

$$\frac{\partial}{\partial t}\left(-\frac{\partial^2 H}{\partial\sigma^2} + \frac{1}{\sin\theta}\frac{\partial^2 R}{\partial\lambda\partial\sigma}\right) = \frac{R_0}{a^2\sigma}\frac{\partial T}{\partial\theta}, \tag{36}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 W}{\partial\sigma^2} - \frac{\partial^2 R}{\partial\theta\partial\sigma}\right) = \frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda}, \tag{37}$$

$$\frac{\partial}{\partial t}(\Delta_3 R) = -\pi'_R(\Delta_2 R - f), \tag{38}$$

$$\frac{\partial}{\partial t}\left(\frac{R_0{}^2}{a^2c^2}T\right) = \frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta H - \frac{\partial W}{\partial\lambda}\right) - \frac{R_0{}^2}{a^2c^2}\pi'_R T, \tag{39}$$

where $\pi'_R = \frac{u'_R}{\sin\theta}\frac{\partial}{\partial\lambda} + v'_R\frac{\partial}{\partial\theta}$ and $\Delta_2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$.

4.2 Physical interpretation of the simplified model

To illustrate the physical concepts more clearly, we insert Eq. (5) into Eqs. (36)–(39) and rewrite them as follows:

$$\frac{\partial}{\partial t}\left(-\frac{\partial v'_H}{\partial\sigma}\right) + \frac{\partial}{\partial t}\left(-\frac{\partial v'_R}{\partial\sigma}\right) = -\frac{R_0}{a^2\sigma}\frac{\partial T}{\partial\theta}, \tag{40}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial u'_W}{\partial\sigma}\right) + \frac{\partial}{\partial t}\left(\frac{\partial u'_R}{\partial\sigma}\right) = \frac{R_0}{a^2\sigma}\frac{1}{\sin\theta}\frac{\partial T}{\partial\lambda}, \tag{41}$$

$$\frac{d_h}{dt}(\Delta_2 R - f) = -\frac{\partial}{\partial t}\left(\frac{\partial^2 R}{\partial\sigma^2}\right), \tag{42}$$

$$\frac{\partial}{\partial t}\left(\frac{R_0{}^2}{a^2c^2}T\right) = \frac{R_0}{a^2\sigma}(\dot\sigma_H + \dot\sigma_W) - \frac{R_0{}^2}{a^2c^2}\pi'_R T, \tag{43}$$

where $\frac{d_h}{dt} = \frac{\partial}{\partial t} + \pi'_R$. Equations (40)–(43) show the basic evolution rules between the horizontal ATAG and the horizontal, meridional and zonal circulations.

4.2.1 Components of 3D vorticities in the simplified model

We first introduce the components of Eqs. (40)–(43). According to Eq. (29), we know that $-\frac{\partial v'_H}{\partial\sigma}$ represents the zonal vorticity of $\vec{V}'_H$ and $\frac{\partial u'_W}{\partial\sigma}$ is the meridional vorticity of $\vec{V}'_W$. In addition, $-\frac{\partial v'_R}{\partial\sigma}$ and $\frac{\partial u'_R}{\partial\sigma}$ denote the zonal and meridional vorticities of $\vec{V}'_R$, respectively. We usually use $-\frac{\partial v'_H}{\partial\sigma}$ and $\frac{\partial u'_W}{\partial\sigma}$ to represent the intensities of the meridional and zonal circulations (i.e., $\vec{V}'_H$ and $\vec{V}'_W$), respectively. We also use $\frac{\partial u'_R}{\partial\sigma}$ and $\frac{\partial v'_R}{\partial\sigma}$ to represent the baroclinicity of the horizontal circulation $\vec{V}'_R$.

Furthermore, by using Eqs. (5) and (8), we generate the following:

$$\Delta_2 R = \frac{1}{\sin\theta}\frac{\partial v'_R}{\partial\lambda} - \frac{1}{\sin\theta}\frac{\partial u'_R\sin\theta}{\partial\theta} = \zeta_{\sigma R}, \tag{44}$$

$$\frac{\partial^2 R}{\partial\sigma^2} = \frac{1}{\sin\theta}\frac{\partial v'_H}{\partial\lambda} - \frac{1}{\sin\theta}\frac{\partial u'_W\sin\theta}{\partial\theta} = \zeta_{\sigma H} + \zeta_{\sigma W}, \tag{45}$$

and

$$\Delta_2 R + \frac{\partial^2 R}{\partial\sigma^2} = \Delta_3 R = \zeta_{\sigma R} + \zeta_{\sigma H} + \zeta_{\sigma W} = \zeta_\sigma. \tag{46}$$

Then, according to Eq. (29), we know that $\Delta_2 R$ is the vertical vorticity of the horizontal circulation $\vec{V}'_R$, and $(\Delta_2 R - f)$ is the absolute vorticity of $\vec{V}'_R$. $\frac{\partial^2 R}{\partial\sigma^2}$ denotes the vertical vorticities of the meridional circulation $\vec{V}'_H$ (represented by $\zeta_{\sigma H}$) and the zonal circulation $\vec{V}'_W$ (represented by $\zeta_{\sigma W}$). In addition, by using Eqs. (4) and (7), we can easily prove

$$\frac{1}{\sin\theta}\frac{\partial u'_W}{\partial\lambda}+\frac{1}{\sin\theta}\frac{\partial(v'_H\sin\theta)}{\partial\theta}=\frac{1}{\sin\theta}\frac{\partial u'}{\partial\lambda}+\frac{1}{\sin\theta}\frac{\partial(v'\sin\theta)}{\partial\theta}=D_\sigma, \tag{47}$$

where D_σ represents the horizontal divergence of the actual atmospheric circulation. Equation (47) suggests that the vertical vorticities of $\vec{V}'_H$ and $\vec{V}'_W$ are caused by the horizontally divergent motions. Thus, Eqs. (45) and (46) show that $\Delta_2 R$, $\frac{1}{\sin\theta}\frac{\partial v'_H}{\partial\lambda}$ and $-\frac{1}{\sin\theta}\frac{\partial u'_W\sin\theta}{\partial\theta}$ are actually a decomposition of the vertical vorticity of atmosphere (Hu et al. 2017).

4.2.2 *Dynamics between the horizontal ATAG and three-pattern circulations*

Similarly to Sect. 3.2, Fig. 4 shows the physical meaning of Eqs. (40)–(42). First, if we assume that the initial horizontal circulation $\vec{V}'_R$ in the simplified model is barotropic, i.e., $\frac{\partial u'_R}{\partial\sigma}=-\frac{\partial v'_R}{\partial\sigma}=0$, then the horizontal ATAG in the isobaric surface is zero, that is, $\frac{1}{a\sin\theta}\frac{\partial T}{\partial\lambda}=-\frac{1}{a}\frac{\partial T}{\partial\theta}=0$. According to initial conditions $H=W=0$ (there is only horizontal circulation at the initial time), we then have $-\frac{\partial v'_H}{\partial\sigma}=\frac{\partial u'_W}{\partial\sigma}=0$ and $\frac{\partial^2 R}{\partial\sigma^2}=0$ by using Eqs. (40)–(42). Thus, the meridional and zonal circulations will not be induced, and Eq. (42) shows that the evolution of the horizontal circulation $\vec{V}'_R$ is controlled by the conservation law of the absolute vorticity, i.e., $\frac{d_h}{dt}(\Delta_2 R-f)=0$.

If the initial horizontal circulation $\vec{V}'_R$ is baroclinic, that is, $\frac{\partial u'_R}{\partial\sigma}\neq 0$ and $-\frac{\partial v'_R}{\partial\sigma}\neq 0$, then the horizontal ATAG in the isobaric surface is not zero. Equations (40) and (41) indicate that $-\frac{1}{a}\frac{\partial T}{\partial\theta}$ and $\frac{1}{a\sin\theta}\frac{\partial T}{\partial\lambda}$ will induce the evolution of $\vec{V}'_H$ and $\vec{V}'_W$. In addition, the horizontal ATAG will also cause the evolution of the baroclinicity of $\vec{V}'_R$ [denoted by $\frac{\partial}{\partial t}(-\frac{\partial v'_R}{\partial\sigma})$ and $\frac{\partial}{\partial t}(\frac{\partial u'_R}{\partial\sigma})$ in Eqs. (40) and (41)] that further has effects on the evolution of $\vec{V}'_H$ and $\vec{V}'_W$.

For the constant horizontal ATAG, Eqs. (40) and (41) show that the meridional circulation $\vec{V}'_H$ will be strengthened when $-\frac{\partial v'_R}{\partial\sigma}$ is decreasing. In contrast, $\vec{V}'_H$ will be weakened when $-\frac{\partial v'_R}{\partial\sigma}$ is increasing. We reached similar conclusions for the zonal circulation $\vec{V}'_W$. The relationships between the baroclinicity of $\vec{V}'_R$ and the intensity of $\vec{V}'_H$ (or $\vec{V}'_W$) are essentially the transformation between the zonal vorticities (or meridional vorticities) of $\vec{V}'_R$ and $\vec{V}'_H$ (or $\vec{V}'_W$).

Furthermore, according to Eq. (45), the meridional and zonal circulations induced by the horizontal ATAG and the baroclinicity of the horizontal circulation will cause $\frac{\partial^2 R}{\partial\sigma^2}\neq 0$. Then, the vertical vorticity ζ_σ will be partitioned into three parts, i.e., $\zeta_\sigma=\zeta_{\sigma R}+\zeta_{\sigma H}+\zeta_{\sigma W}$. Thus, Eqs. (42) and (46) show that the absolute vorticity $(\Delta_2 R-f)$ of circulation $\vec{V}'_R$ is not conserved, because of the transformation of the vertical vorticities between the horizontal circulation and vertical circulations (meridional and zonal circulations). The change in $(\Delta_2 R-f)$ is generally determined by the evolution of the vertical vorticities of $\vec{V}'_H$ and $\vec{V}'_W$. Specifically, according to Eqs. (42) and (45), $(\Delta_2 R-f)$ will increase when the meridional and zonal circulations have a negative change in vertical vorticity, i.e., $\left(\frac{\partial\zeta_{\sigma H}}{\partial t}+\frac{\partial\zeta_{\sigma W}}{\partial t}\right)<0$, and $(\Delta_2 R-f)$ will decrease if $\left(\frac{\partial\zeta_{\sigma H}}{\partial t}+\frac{\partial\zeta_{\sigma W}}{\partial t}\right)>0$. Furthermore, the thermodynamic Eq. (43) shows that the vertical circulations $\vec{V}'_H$ and $\vec{V}'_W$ and the advection transport by the horizontal circulation will conversely cause the redistribution of the air temperature field.

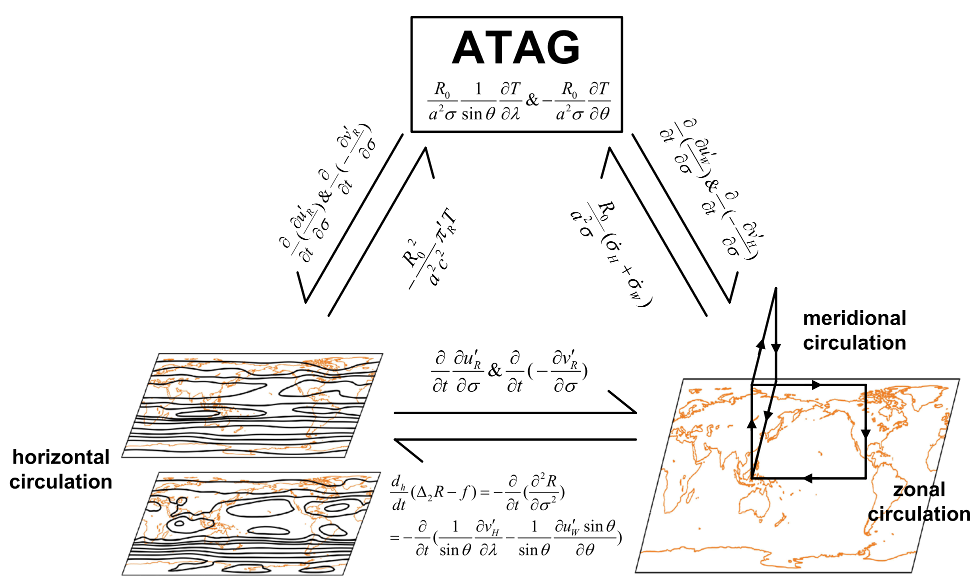

Fig. 4 Schematic diagram of the basic evolution rules of the horizontal ATAG, the horizontal circulation and the meridional and zonal circulations

5 Summary

In this study, a novel three-pattern decomposition model of global atmospheric circulation (TPDGAC) was used to establish the dynamical equations of the horizontal, meridional and zonal circulations. Furthermore, the physical meaning of the newly established equations was investigated and a simplified model was presented to demonstrate the potential applications of the new dynamical equations in studying the dynamics of the Rossby, Hadley and Walker circulations with the nonuniform global warming.

Because the TPDGAC can equivalently represent the global atmospheric circulation by the horizontal, meridional and zonal circulations, we can incorporate the three-pattern decomposition model (TPDM) into primitive equations of atmospheric dynamics to obtain new dynamical equations of the three-pattern circulations. The new equations completely describe the evolution mechanisms of the horizontal, meridional and zonal circulations from the perspective of the evolutions of 3D vorticities.

Similarly to the physical meaning of the vertical vorticity equation, the zonal and meridional vorticity equations reveal the evolution dynamics of the meridional and zonal circulations, respectively. The rate of change of the local zonal (or meridional) vorticity is given by the sum of the advection term of the zonal (or meridional) vorticity, the divergence term of the zonal (or meridional) vorticity, the tilting or twisting term of the vertical vorticity, the Coriolis force term, the curvature effect term, the dissipative term, and the meridional (or zonal) air temperature anomaly gradient (ATAG) term. The meridional (or zonal) ATAG is an important factor in the evolution of the zonal (or meridional) vorticity.

A simplified model of the newly established equations reveals the basic evolution rules between the ATAG and the horizontal, meridional and zonal circulations. It indicates that the horizontal ATAG not only induces the meridional and zonal circulations but also causes the evolution of the baroclinicity of the horizontal circulation. In addition, the baroclinicity of the horizontal circulation in turn has effects on the evolution of the meridional and zonal circulations. For the constant horizontal ATAG, the meridional and zonal circulations are strengthened when the baroclinicity of the horizontal circulation decreases. In contrast, the meridional and zonal circulations are weakened when the baroclinicity of the horizontal circulation increases.

Furthermore, the simplified model shows that the horizontal circulation and the induced meridional and zonal circulations cause a decomposition of the vertical vorticity. The transformation of the three decomposed vertical vorticities prevents the conservation of the absolute vorticity of the horizontal circulation, and the change is approximately described by the evolution of the vertical vorticities of the meridional and zonal circulations. The absolute vorticity of the horizontal circulation increases or decreases when the meridional and zonal circulations have negative or positive changes in vertical vorticity.

This study and its prequel of the TPDGAC (Hu et al. 2015) present a new dynamical theory of the global horizontal, meridional and zonal circulations. Because the horizontal circulation is the global generalization of the Rossby wave at middle–high latitudes and the meridional and zonal circulations are the global generalizations of the Hadley and Walker circulations at low latitudes, appropriately simplifying the newly established dynamical equations provides potential opportunities for studying the dynamical mechanisms of the variations in and interactions among the Rossby, Hadley and Walker circulations.

As a next step, we will numerically simulate the simplified model to quantitatively analyze the relationships between the ATAG and the horizontal, meridional and zonal circulations. We will also use the new dynamical equations to determine the source terms that cause obvious variations in the three-pattern circulations and will investigate the relationships between changes in the Rossby, Hadley and Walker circulations and the nonuniform global warming (ATAG) in recent decades.

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