An Economical Consistent Dissipation Operator and Its Applications to the Improvement of AGCM①

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ABSTRACT

This paper introduces a new consistent dissipation operator. It is based on the explicit square conservation scheme and the theory of consistent dissipation. The operator makes full use of the advantages of the Leap-frog scheme, i.e., its second order time precision and its explicit solution manner. Meanwhile, it overcomes the fatal disadvantage, the absolute instability in computations, of the scheme. When it is applied to the explicit square conservation scheme, the time precision of the scheme reaches to third order. Especially, the computational stability of this scheme is as good as the third order explicit Runge-Kutta scheme. The CPU time required in computations by the scheme is less than that required by the explicit square conservation scheme with the consistent dissipation operator constructed from the Runge-Kutta method. Therefore, the new operator is an economical one. The application of the operator to the improvement of the dynamical model of the L_2 IAP AGCM shows its time-saving property and its good effects.

Key words: Consistent dissipation, Explicit square conservation, Time-saving

I. INTRODUCTION

The theory of consistent dissipation operator is put forward by Wang et al. (1994). It improves the precision and the computational stability of the explicit square conservation scheme (Wang et al., 1990) and makes the scheme practicable. A class of high-precision consistent dissipation operator is constructed by using the classic explicit Runge-Kutta method (Wang et al., 1994), which plays important roles both in improving the simulation effects and saving computational time, of some real numerical simulation problems. Can an economical consistent dissipation operator be constructed based on the aforesaid theory? After a careful study on this problem, we can say that the answer is affirmative. By using the Leap-frog scheme, a new second-order consistent dissipation operator is established, which needs only one iteration while an old second-order one in Wang and Ji (1994) requires two iterations. Therefore, it saves more time than the old one does. The detailed discussion of the new operator will be given in the coming sections.

II. THE CONSTRUCTION OF THE OPERATOR

Consider the evolution equation in operator form:

$$\frac{\partial F}{\partial t} = LF \ , \tag{1}$$

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where L is a zero operator:

$$(LF,F)=0 . \tag{2}$$

The equation is of the square conservation:

$$\frac{\partial}{\partial t}\frac{1}{2}\|F\|^2=0 . \tag{3}$$

The Leap–frog scheme to solve Eq.(1) can be written as

$$\frac{F^{n+1}-F^{n-1}}{2\Delta t}=LF^n \tag{4}$$

or

$$F^{n+1}=F^{n-1}+2\Delta tLF^n . \tag{4$'$}$$

This scheme has two advantages. First, it is an explicit scheme and can be solved directly. Next, it is of second–order precision. However, this scheme is absolutely unstable, and usually needs a proper smoothing or filtering, such as the Shuman smoothing. The smoothing might lessen the precision of the scheme and even destroy the physical relations of energy transformation and exchange. For example, it will break down the energy conservation property. Thus, this scheme is not a practicable scheme for numerical simulations.

Anyway, we can use the advantages of the Leap–frog scheme to construct a time–saving consistent dissipation operator. The solution from the Leap–frog is not regarded as the discrete solution of the equation at $(n+1)$–th step (F^{n+1}), but only as an approximation of F^{n+1}. This approximation solution is marked as $\tilde{F}^{n+1}$:

$$\tilde{F}^{n+1}=F^{n-1}+2\Delta tLF^n . \tag{4$''$}$$

By using $\tilde{F}^{n+1}$, a consistent dissipation operator B can be constructed:

$$BF^n=\frac{\alpha L\tilde{F}^{n+1}+\beta LF^n+\gamma LF^{n-1}}{\Delta t} , \tag{5}$$

where α, β, γ are all undetermined parameters. According to the definition of second–order consistent dissipation operator (Wang et al., 1994):

$$B(F)^n=\left(\frac{\partial^2 F}{\partial t^2}\right)^n+\frac{\Delta t}{3}\left(\frac{\partial^3 F}{\partial t^3}\right)^n+O(\Delta t^2) , \tag{6}$$

the parameters can be determined as:

$$\alpha=\frac{5}{6} ,\quad \beta=-\frac{2}{3} ,\quad \gamma=-\frac{1}{6} , \tag{7}$$

and the new consistent dissipation operator is established:

$$BF^n = \frac{1}{6\Delta t}[5L\tilde{F}^{n+1} - 4LF^n - LF^{n-1}] \ . \tag{5$'$}$$

The new operator is, in form, very similar to the second-order implicit consistent dissipation operator:

$$BF^n = \frac{1}{6\Delta t}[5LF^{n+1} - 4LF^n - LF^{n-1}] \ , \tag{8}$$

that is constructed in Wang Bin's Doctor Dissertation (1992). When F^{n+1} in (8) is replaced by the solution of Leap-frog $\tilde{F}^{n+1}$, the implicit operator B defined by (8) becomes the new explicit operator, with which the explicit square conservation scheme (Wang et al., 1990):

$$\frac{F^{n+1} - F^n}{\Delta t} = LF^n + \varepsilon_n \Delta t BF^n \tag{9}$$

will be more economical.

III. A NEW EXPLICIT SCHEME IN GENERAL USE

According to Wang (1995), the explicit square conservation scheme with a second-order consistent dissipation operator is of third-order precision, and $\varepsilon_n = 0.5 + O(\Delta t^2)$. If set, $\varepsilon_n = 0.5$, it is easy to prove that Scheme (9) is still of third-order precision. By replacing Δt in the left side of (9) by an adjustable interval τ_n and substituting (5)$'$ into (9), a new explicit scheme is formed:

$$\frac{F^{n+1} - F^n}{\tau_n} = \frac{1}{12}\left[5L\tilde{F}^{n+1} + 8LF^n - LF^{n-1}\right] , \tag{10}$$

where $\tilde{F}^{n+1}$ is determined by (4)$''$ and the time interval in (4)$''$ is still Δt, not τ_n. Similar to Wang et al., (1996), there are two theorems on Scheme (10):

Theorem 1: If L is a zero operator, i.e., L satisfies (2), and τ_n is determined by the following formula:

$$\tau_n = \frac{\Delta t}{6\|AF^n\|^2}\left[5(L\tilde{F},\tilde{R}_1) + (LF^{n-1},\tilde{R}_2)\right] , \tag{11}$$

where

$$\begin{cases} \tilde{R}_1 = \dfrac{\tilde{F}^{n+1} - F^n}{\Delta t} \ , \quad \tilde{R}_2 \dfrac{F^n - F^{n-1}}{\Delta t} \\ AF^n = \dfrac{1}{12}\left[5L\tilde{F}^{n+1} + 8LF^n - LF^{n-1}\right] , \end{cases} \tag{12}$$

then Scheme (10) keeps the quadratic conservation law exactly and has third-order precision.

Theorem 2: If the operator L does not satisfy the condition (2), but τ_n still obeys (11)–(12), then Scheme (10) is of third-order precision.

Also, it is not difficult to prove the third theorem according to the method given by Li et al., (1984):

Theorem 3: Scheme (10) has the same absolute stability interval as the third order explicit Runge–Kutta scheme: $\bar{\tau} = \mu\tau \in (-2.51, 0)$ where $\mu = \inf\left\{\frac{\delta LF}{\delta F}\right\}$

These theorems denote that Scheme (10) determined by (11)–(12) can be used not only for square conservation systems but also for other general systems. Especially, the scheme has third-order precision and good computational stability. Therefore, it is suitable for the simulations with long-term numerical integrations.

IV. APPLICATION

The establisment of the new explicit scheme with exact quadratic conservation law is for applications to real simulation problems, such as the atmospheric and oceanic problems. One application of the new scheme introduced here, is to improve the IAP atmospheric general circulation model (AGCM). The original AGCM uses the Leap–frog scheme to solve the dynamic equations. The scheme cannot keep the conservation of the effective energy. Even the mass conservation property is also hard to be maintained, without any correction. Hereby, the computing effects may be influenced. For example, a small system near the Antarctic region can be easily found from the observed 500 hPa height field in July (see Fig.1), but the original dynamical model is unable to simulate this system (see Fig.2). Therefore, the computing scheme used in the model should be modified.

We use the new scheme to improve the IAP AGCM, i.e., use it to replace the Leap–frog scheme in the model. It is found that good computing effects and time benefit can be obtained from the improved AGCM. Without FFT filering near the poles, the improved model shows being of good physical conservation properties and good computational stability. It keeps the conservations of effective energy and mass all the time and the adjustable time interval τ_n is always near to, but greater than $\Delta t (= 100$ s) (see Table 1). In the same situation, the time interval used by the original model can only be about 50 s. In real simulations, FFT filtering is usually adopted near the poles, in order to lengthen the time interval and to save the computing time. In this case, the improved model can set $\Delta t = 800$ s, and needs about 1200 s for a monthly numerical integration. While, the original model can only use 360 s as the time interval and requires about 1324 s for the same integration. This denotes that the improved model is more economical than the original one. It saves only about 10% of CPU time because the new scheme needs one iteration at each step while the Leap–frog scheme does not. Furthermore, the computing effects from the improved model are also better than those from the original one. Fig. 3 shows that the improved model can simulate the small system near the antarctic pole, found in Fig. 1. Thus, the new scheme, constructed from the aforesaid economical consistent dissipation operator, is very effective and worthy of popularization and application.

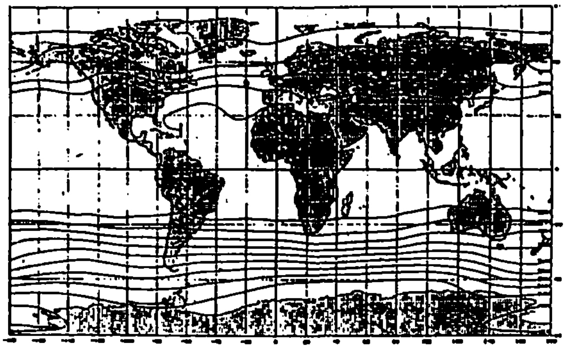

Fig. 1. The observed 500 hPa height field in July.

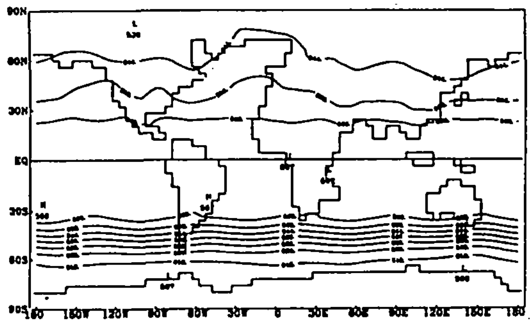

Fig. 2. The simulated 500 hPa height field in July from the original model.

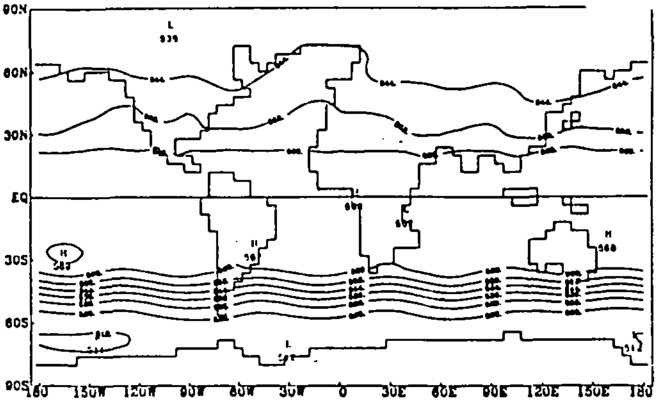

Fig. 3. The simulated 500 hPa height field in July from the improved model.

Table 1. Evolutions of Effective Energy, Mass and Time Interval of the New Scheme under the Standard Stratification Approximation

Step	Effective enewrgy	Mass	Time interval
00100	3.3860237e9	3250544	100.0734
00200	3.3890237e9	3250544	100.0668
00400	3.3890237e9	3250544	100.0567
00800	3.3890237e9	3250544	100.0450
01600	3.3890234e9	3250544	100.0240
03200	3.3890237e9	3250545	100.0107
06400	3.3890234e9	3250544	100.0260
12800	3.3890237e9	3250544	100.2663
25600	3.3890237e9	3250544	100.3467

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