

Is Model Parameter Error Related to a Significant Spring Predictability Barrier for El Niño events? Results from a Theoretical Model

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ABSTRACT

Within a theoretical ENSO model, the authors investigated whether or not the errors superimposed on model parameters could cause a significant “spring predictability barrier” (SPB) for El Niño events. First, sensitivity experiments were respectively performed to the air–sea coupling parameter, α and the thermocline effect coefficient μ . The results showed that the uncertainties superimposed on each of the two parameters did not exhibit an obvious season-dependent evolution; furthermore, the uncertainties caused a very small prediction error and consequently failed to yield a significant SPB. Subsequently, the conditional nonlinear optimal perturbation (CNOP) approach was used to study the effect of the optimal mode (CNOP-P) of the uncertainties of the two parameters on the SPB and to demonstrate that the CNOP-P errors neither presented a unified season-dependent evolution for different El Niño events nor caused a large prediction error, and therefore did not cause a significant SPB. The parameter errors played only a trivial role in yielding a significant SPB. To further validate this conclusion, the authors investigated the effect of the optimal combined mode (i.e. CNOP error) of initial and model errors on SPB. The results illustrated that the CNOP errors tended to have a significant season-dependent evolution, with the largest error growth rate in the spring, and yielded a large prediction error, inducing a significant SPB. The inference, therefore, is that initial errors, rather than model parameter errors, may be the dominant source of uncertainties that cause a significant SPB for El Niño events. These results indicate that the ability to forecast ENSO could be greatly increased by improving the initialization of the forecast model.

Key words: ENSO predictability, optimal perturbation, error growth, model parameters

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1. Introduction

The ENSO, a prominent climate phenomenon in the coupled ocean–atmosphere system of the tropical Pacific, has a significant impact on global climate. Hence, ENSO has received a great deal of attention in the scientific literature. While significant progress has been made in ENSO theory and predictability over the years, especially through the Tropical Ocean Global Atmosphere (TOGA) program (see the review by Wang and Picaut, 2004), considerable uncertain-

ties still exist in realistic ENSO predictions (Jin et al., 2008; Tang et al., 2008). In particular, for forecasts that are made before and throughout the spring, ENSO predictions tend to be much less successful. This low predictability has been related to the so-called “spring predictability barrier” (SPB) of ENSO (Webster and Yang, 1992).

Some studies have explored the SPB by studying the transient growth of an initial error. From the perspective of error growth, the SPB is referred to as the phenomenon with ENSO forecasting that has a

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large prediction error; in particular, a prominent error growth occurs during the spring when the prediction is made before spring (Mu et al., 2007a, 2007b; Duan et al., 2009; Yu et al., 2009). Chen et al. (2004) suggested that this predictability barrier could be reduced through improvements in initialization. Furthermore, Moore and Kleeman (1996) and Samelson and Tziperman (2001) demonstrated that linear singular vector (LSV) errors could cause a SPB. Mu et al. (2007a) used the conditional nonlinear optimal perturbation (CNOP; see the next section) approach to investigate the SPB for ENSO events in a theoretical model and showed that the CNOP-type initial errors cause a more significant SPB than do the LSV-type errors. Duan et al. (2009) also identified two types of initial error that cause a significant SPB and emphasized the importance of initial error patterns in the SPB. Recently, Yu et al. (2009) further illustrated that random initial errors fail to cause a SPB. Based on these previous works, it can be inferred that the occurrence of the SPB may be closely related to patterns of initial errors.

Prediction uncertainties are generally caused by initial and model errors. In realistic predictions of ENSO, the SPB phenomenon is also often illustrated under the condition that there exist both initial and model errors in the predictions. Furthermore, an increasing number of studies have indicated that model errors influence the ability to forecast ENSO (Wu and Anderson, 1993; Flügel and Chang, 1998; Latif et al., 1998; Liu, 2002; Zhang et al., 2003; Garay, 2004; Williams, 2005; Zheng et al., 2009), in which one important aspect is the effect of the uncertainties of model parameters on ENSO modeling (Zebiak and Cane, 1987; Wu and Anderson, 1993; Liu, 2002). In particular, some of these studies have emphasized that the external forcing characterized by model parameters amplify the amplitude of ENSO, while others have suggested that the parameterized external forcing lowers the ENSO amplitude. Nevertheless, these works indicate that the uncertainties superimposed on the model parameters may have an important effect on ENSO predictability.

In this study, the authors have attempted to address the following questions/issues: (1) whether or not model parameter errors cause a significant SPB; (2) the role of model parameter errors in yielding a SPB; and (3) which errors, model or initial, are the dominant source of the uncertainties that cause a significant SPB? Mu et al. (2010) showed that CNOP represents the combined mode of initial error and model parameter error that induces the largest prediction error at the time of prediction and that has the potential to yield a significant SPB. In the present study, the

CNOP approach was used to investigate the questions posed about SPB by studying the combined mode of initial and model errors that may result in a significant SPB.

The paper is organized as follows. The CNOP approach is introduced in the next section. The theoretical model adopted in this study is discussed in section 3. In section 4, the effect of the errors superimposed on a single parameter on the SPB is explored through sensitivity experiments. Using the CNOP approach, the effect of the optimal mode of the multiple parameter errors is investigated in section 5. In addition, a group of experiments is discussed in section 5 to demonstrate the importance of initial errors in yielding a SPB. Finally, a summary and discussion of the results are presented in section 6.

2. Methods

The first part of the present study was to investigate the effects of model parameter errors on the SPB. To investigate the uncertainties of a single parameter, the magnitude of the errors of a parameter was modulated and a sensitivity analysis performed. To investigate the combined modes of multi-parameter errors, or of the initial error and model errors, the CNOP approach (Mu et al., 2003, 2010) was used in the experiments. CNOP is now introduced briefly, below.

The evolution equations for the state vector w are:

$$\begin{cases} \frac{\partial w}{\partial t} + F(w, \mathbf{p}, t) = 0, \\ w|_{t=0} = w_0, \end{cases} \quad (1)$$

where $w(\mathbf{x}, \mathbf{p}, t) = (w_1(\mathbf{x}, \mathbf{p}, t), w_2(\mathbf{x}, \mathbf{p}, t), \dots, w_n(\mathbf{x}, \mathbf{p}, t))$ and w_0 is the initial state; $\mathbf{x} = (x_1, x_2, \dots, x_n)$ and t is time. Furthermore, $\mathbf{p} = (p_1, p_2, \dots, p_n)$ is the model parameter vector, and F is a nonlinear operator.

Assuming that the dynamical system equation and the initial state are known exactly, the future state can be determined by integrating Eq. (1) with the appropriate initial condition. The solution to Eq. (1) for the state vector, w , at time τ is given as:

$$w(\mathbf{x}, \tau) = M_\tau(\mathbf{p})(w_0). \quad (2)$$

where $M_\tau(\mathbf{p})$ is the propagator of Eq. (1), with the parameter vector \mathbf{p} , and propagates the initial value to time τ in the future, as described by Eq. (2).

Solutions of Eq. (2) are $U(t)$ and $U(t) + u(t)$, with initial values U_0 and $U_0 + u_0$, respectively. Thus, we have:

$$\begin{aligned} U(\tau) &= M_\tau(\mathbf{p})(U_0), \\ U(\tau) + u(\tau) &= M_\tau(\mathbf{p})(U_0 + u_0), \end{aligned} \quad (3)$$

where \mathbf{u}_0 is the initial perturbation of a time-dependent state $\mathbf{U}(t)$ (hereafter referred to as reference state), and $\mathbf{u}(\tau)$ describes the nonlinear evolution of the initial perturbation.

In addition, while assuming that the parameter perturbation vector \mathbf{p}' is superimposed on the parameter vector \mathbf{P} , we obtain:

$$\begin{aligned} \mathbf{U}(\tau) &= M_\tau(\mathbf{P})(\mathbf{U}_0), \\ \mathbf{U}(\tau) + \mathbf{u}_p(\tau) &= M_\tau(\mathbf{P} + \mathbf{p}')(\mathbf{U}_0), \end{aligned} \quad (4)$$

where $\mathbf{u}_p(\tau)$ describes the departure from the reference state $\mathbf{U}(\tau)$ caused by \mathbf{p}' .

When one considers that there exists both an initial perturbation and parameter perturbation in Eq. (2), then:

$$\begin{aligned} \mathbf{U}(\tau) &= M_\tau(\mathbf{P})(\mathbf{U}_0), \\ \mathbf{U}(\tau) + \mathbf{u}_{p,I}(\tau) &= M_\tau(\mathbf{P} + \mathbf{p}')(\mathbf{U}_0 + \mathbf{u}_0), \end{aligned} \quad (5)$$

where $\mathbf{u}_{p,I}(\tau)$ is the departure from the reference state $\mathbf{U}(\tau)$ caused by the combined mode of initial and model parameter perturbations.

The nonlinear optimization problem is defined as follows:

$$J_1(\mathbf{u}_{0\delta}, \mathbf{p}'_\delta) = \max_{\mathbf{u}_0 \in C_\eta, \mathbf{p}' \in C_\sigma} J(\mathbf{u}_0; \mathbf{p}'), \quad (6)$$

where

$$J(\mathbf{u}_0, \mathbf{p}') = \|M_\tau(\mathbf{P} + \mathbf{p}')(\mathbf{U}_0 + \mathbf{u}_0) - M_\tau(\mathbf{P})(\mathbf{U}_0)\|,$$

\mathbf{u}_0 and \mathbf{p}' respectively are a perturbation vector superimposed on the reference state $\mathbf{U}(t)$, with an initial value of \mathbf{U}_0 , and the parameter \mathbf{P} , with $\mathbf{u}_0 \in C_\eta$, $\mathbf{p}' \in C_\sigma$ as the constraint conditions. By solving Eq. (6), one can obtain the optimal combined mode of the initial perturbation and parameter perturbation, $(\mathbf{u}_{0\eta}; \mathbf{p}'_\sigma)$, for a given constraint that induces the largest departure from the reference state $U(t)$ at time τ . Mu et al. (2010) called this optimal combined mode CNOP. CNOP has two special cases. One is CNOP-I, denoted by $\mathbf{u}_{0\eta,I}$, which represents the initial perturbation that has the largest nonlinear evolution at a prediction time, and is obtained by solving the following optimization problem:

$$J_{u_0}(\mathbf{u}_{0\delta,I}) = \max_{\mathbf{u}_0 \in C_\eta} \|M_\tau(\mathbf{P})(\mathbf{U}_0 + \mathbf{u}_0) - M_\tau(\mathbf{P})(\mathbf{U}_0)\|. \quad (7)$$

The other case is CNOP-P, denoted by $\mathbf{p}_{\sigma,p}$, which describes the parameter perturbation that results in the largest departure from a given reference state, and can be obtained by evaluating the following optimization problem:

$$J_p(\mathbf{p}_{\sigma,p}) = \max_{\mathbf{p}' \in C_\sigma} \|M_\tau(\mathbf{P} + \mathbf{p}')(\mathbf{U}_0) - M_\tau(\mathbf{P})(\mathbf{U}_0)\|. \quad (8)$$

Physically, CNOP represents the optimal combined mode of initial error and the model parameter error, while CNOP-I, in perfect model experiments, acts as the optimal initial error, and CNOP-P, in perfect initial condition experiments, represents the optimal parameter error. In their respective scenarios, they cause the largest prediction error. In this paper, the authors use the physics of CNOPs to study the SPB. CNOP, CNOP-I, and CNOP-P can be solved by the optimization algorithm of SQP2 with an adjoint model.

3. Theoretical model for the coupled tropical ocean–atmosphere

With a number of simplifications, Wang and Fang (1996) (hereafter referred to as WF96) simplified Zebiak and Cane (1987) intermediate coupled ocean–atmosphere model to a theoretical one. This theoretical model consists of two time-dependent ordinary differential equations: one describing the evolution of the anomalous SST, T , in the equatorial eastern Pacific, and the other depicting the evolution of the anomalous thermocline depth, h .

$$\frac{dT}{dt} = a_1 T - a_2 h + \sqrt{\frac{2}{3}} T(T - a_3 h), \quad (9)$$

$$\frac{dh}{dt} = b(2h - T), \quad (10)$$

where

$$\begin{aligned} a_1 &= \overline{T}_z + \overline{T}_x - \alpha_s, \\ a_2 &= (\mu + \delta_1) \overline{T}_x, \\ a_3 &= \mu + \delta_1, \\ b &= \frac{2}{\delta(1 - 3\varepsilon)}. \end{aligned}$$

In this model, the linear terms in Eq. (9) describe the vertical advection by the anomalous upwelling of the ocean mean temperature $(\overline{T} - \overline{T}_z)$ and the vertical advection by the mean upwelling of the anomalous ocean temperature $\overline{T}(T - \mu h)$, as well as the linear damping $(-\alpha_s T)$ with α_s as cooling coefficient. The coefficients a_1 and a_2 involve the basic state parameters \overline{T}_x and \overline{T}_z , which characterize the mean temperature differences between the equatorial eastern and western basins and between the mixed-layer and subsurface-layer water, respectively. These basic state parameters can be time-dependent, reflecting the climatological annual cycle of the basic state. The quadratic term in Eq. (9) is derived from the nonlinear temperature advection by the anomalous upwelling of the anomalous temperature. Furthermore, this term represents the nonlinear coupling between surface layer thermodynamics and upper ocean dynamics (thermocline depth

fluctuations). The linear terms in Eq. (10) depict the effect of equatorial waves on the thermocline adjustment, $2bh$, and the effect of the wind forcing where the wind forcing is proportional to the zonal SST gradient and (in WF96) is reduced by $-bT$ by assuming that the SST anomaly vanishes at the tropical western Pacific. Two non-dimensional coupling parameters are presented in this model. One is the air-sea coupling coefficient, $\alpha = (L_0/L_y)^2$, which measures the strength of the ocean currents' feedback per unit wind speed anomaly (see WF96); here, L_0 is the oceanic Rossby radius of deformation and L_y is the characteristic meridional length scale of the coupled ENSO mode. The other coupled parameter is the thermocline effect coefficient μ , which measures the degree of coupling between the thermocline fluctuation and the SST. The meanings and typical values of other parameters are listed in Table 1 of WF96.

The steady solution $O(0,0)$ of the WF96 model represents the climatological mean equilibrium state (including an annul cycle), in which both the SST and the depth of the thermocline are normal. To numerically solve this model, a fourth-order Runge-Kutta scheme with a time step of $dt = 0.01$ (representing one day) was used.

4. Can the model parameter errors cause a significant SPB for El Niño events?

Mu et al. (2007a) demonstrated that initial errors, CNOP-I type errors (see section 2), yield the most significant SPB for El Niño events in perfect model experiments. However, prediction uncertainties, as mentioned in the introduction, are generally caused by initial errors and model errors. Thus, it is of interest to determine if model errors can also cause a SPB.

Uncertainties of the parameterization processes are an important kind of model error (Mu et al., 2002), which may yield prediction errors. In the WF96 model, there are two fundamental coupling parameters. One is the air-sea coupling coefficient, α ; the other is the thermocline effect parameter, μ . These two parameters are often determined empirically in the model. In this study, values were chosen as $\alpha=0.0212$, $\mu=1.525$, which guaranteed that the model ENSO exhibited an irregular oscillation. Obviously, there exist uncertainties superimposed on these two parameters. In this study, the authors examined if the errors of these two parameters yielded a SPB for El Niño events by performing perfect initial condition experiments.

The SPB is related to the seasonal growth rates of prediction errors. Therefore, we evaluated the prediction errors caused by the parameter errors. Here, the

norm,

$$\|\mathbf{u}(t)\| = \sqrt{T'(t) + h'(t)},$$

is used to measure the development of the prediction errors, where $T'(t)$ and $h'(t)$ are the SST anomaly (SSTA) and thermocline depth anomaly components of the prediction errors, respectively. To estimate the seasonal growth rate of the prediction errors, each year was divided into four seasons, starting with January to March (JFM), followed by April to June (AMJ) and so forth. Then, the slopes, $\kappa = \partial\gamma(t)/\partial t$, of the curve $\gamma(t) = \|\mathbf{u}(t)\|$ for different seasons were estimated, where $\mathbf{u}(t)$ represents the evolution of the prediction errors for El Niño events and the slope indicates the growth rates of prediction errors for the different seasons. In particular, if it is assumed that the prediction error at the start time of a season is $\|\mathbf{u}(t_1)\|$ and at the end of the season is $\|\mathbf{u}(t_2)\|$, the growth rate of the prediction error for the season can be roughly estimated by evaluating

$$\kappa \approx \frac{\|\mathbf{u}(t_2)\| - \|\mathbf{u}(t_1)\|}{t_2 - t_1}.$$

Since each season possesses a common time interval length, we simply use the values of $\|\mathbf{u}(t_2)\| - \|\mathbf{u}(t_1)\|$ here to indicate the tendency, κ , of the growth of the prediction errors for each season. A positive (negative) value of κ corresponds to an increase (decrease) in the errors, and the larger the absolute value of κ , the faster the increase (decrease) in the errors.

4.1 Characteristics of the seasonal evolutions of the prediction errors caused by uncertainties in the parameter α

With the initial constraint $\|\mathbf{U}_0\| = \max\{|T_0|, |h_0|\} \leq \rho_0$, Duan (2003) demonstrated that the initial anomalies, $(T_0, h_0) = (-\rho_0, \rho_0)$ (ρ_0 is a positive number), are most likely to evolve into El Niño events in the WF96 model. These initial anomalies have robust patterns of negative (positive) SST and positive (negative) thermocline depth anomalies, which agree with the observations qualitatively (Duan et al., 2004) and act as the optimal precursors to El Niño events (see Duan et al., 2004).

Furthermore, the authors of the present study investigated whether or not parameter errors can cause a significant SPB in model El Niño events induced by the above optimal precursors. We chose $\rho_0=0.05$ and 0.08 , and obtained two initial precursor anomalies $(T_0, h_0) = (-0.05, 0.05)$ and $(-0.08, 0.08)$, which are dimensionless and correspond to dimensional $(-0.1^\circ\text{C}, 2.5 \text{ m})$ and $(-0.16^\circ\text{C}, 4.0 \text{ m})$ for Niño-3 SSTA and thermocline depth anomaly. We considered these initial precursors to occur in January, April, July, and

Table 1. Seasonal growth rates of prediction errors caused by errors of α for $U_{W,Jan}$. The numbers in bold denote the largest growth rate of prediction errors.

$\Delta\alpha$	JFM	AMJ	JAS	OND
1%	0.0012	0.0021	0.0014	-0.0003
5%	0.0060	0.0108	0.0078	-0.0014
10%	0.0124	0.0228	0.0172	-0.0023
15%	0.0191	0.0360	0.0286	-0.0026
-1%	0.0012	0.0020	0.0014	-0.0003
-5%	0.0057	0.0099	0.0064	-0.0016
-10%	0.0112	0.0189	0.0117	-0.0033
-15%	0.0164	0.0271	0.0161	-0.0049

October, respectively. For each initial precursor time, the two initial anomalies induced two different intensities of El Niño events. The initial anomaly $(-0.05, 0.05)$ developed a weak El Niño event, while the $(-0.08, 0.08)$ anomaly evolved into a relatively strong El Niño event. For convenience, we denote the weak (strong) El Niño events with initial precursor times of January, April, July, and October as $U_{W,Jan}(U_{S,Jan})$, $U_{W,Apr}(U_{S,Apr})$, $U_{W,Jul}(U_{S,Jul})$, and $U_{W,Oct}(U_{S,Oct})$, respectively.

If $\Delta\alpha$ is the error in the coupling parameter α , then the seasonal growth rates of the prediction errors caused by $\Delta\alpha$ can be investigated. In the numerical experiments, different values of $\Delta\alpha$ were used. For realistic predictions using a numerical model, it is necessary to verify that the model is able to simulate the main features of the observed ENSO. According to this precondition, we determined the error bounds of the parameter α as $\Delta\alpha \in [-15\%\alpha, 15\%\alpha]$, which provided a result in which the modeled ENSO continued to persist with an irregular oscillation similar to the observed ENSO events. With this perturbed parameter, we predicted the above-predetermined eight El Niño events induced by the optimal precursors for a one-year lead time by integrating the model for 12 months. The El Niño events with an initial anomaly $(T_0, h_0) = (-0.05, 0.05)$, i.e. $U_{W,Jan}$, were investigated first. The prediction errors caused by the uncertainties of the parameter α were obtained for this El Niño event. Therefore, the seasonal growth rates of the prediction errors could be estimated. The error growth rates caused by different magnitudes of the parameter errors for the El Niño events $U_{W,Jan}$ are shown in Table 1. It can be seen that the largest growth rate in the prediction errors caused by the uncertainties in α occurred in the AMJ season, which corresponds to the time when most of the climate models yielded a prediction barrier. However, the error growth rates for each season, even in the spring time (i.e. the AMJ season in the WF96 model), were very small and hence negligible. It is conceivable that the corresponding prediction errors were also very small.

For the relatively strong El Niño event, $U_{S,Jan}$, with an initial anomaly $(-0.08, 0.08)$, similar results were obtained. The largest growth rates in the prediction errors caused by uncertainties in α tended to occur in AMJ, but the magnitudes were very small and can be neglected.

We have shown that, although the prediction errors resulting from uncertainties in the parameter α exhibit a seasonal-dependent evolution, for either a relatively strong or weak El Niño with an initial precursor time in January, they are very small and have a trivial effect on the prediction results of the El Niño event. In these cases, we cannot say that a SPB occurred for this kind of El Niño event. To further examine whether or not the uncertainties of the parameter α can cause a SPB for other kinds of El Niño events, we also investigated the cases of the El Niño events $U_{W,Apr}(U_{S,Apr})$, $U_{W,Jul}(U_{S,Jul})$, and $U_{W,Oct}(U_{S,Oct})$, whose initial precursor times occur in April, July, and October, respectively.

Similarly, we superimposed different magnitudes of errors on the parameter α and investigated the seasonal growth rates of the resultant prediction errors on the different kinds of El Niño events. The results show that, only in predictions of the El Niño event $U_{S,Jul}$, the largest growth rates in the prediction errors occurred for the AMJ season; while for predictions of the El Niño events $U_{W,Apr}$, $U_{S,Apr}$, $U_{W,Oct}$, $U_{S,Oct}$, and $U_{W,Jul}$, the most significant error growth did not occur in AMJ (Table 2). In particular, for the El Niño event $U_{S,Oct}$, the largest error growth rates in the prediction errors did not all correspond to a common season with increasing magnitudes in the parameter errors (Table 3). Furthermore, the prediction errors caused by the errors in the parameters α and μ were very small and could be neglected.

From the above results, it is demonstrated that the largest growth in the prediction errors caused by uncertainties in parameter α does not always occur in the AMJ season; furthermore, the growth in prediction errors is very small and results in a negligible prediction error for the El Niño events. Mu et al. (2007a) showed

Table 2. Seasonal growth rates of prediction errors caused by errors of α for $U_{W,Oct}$. The numbers in bold denote the largest growth rate of prediction errors.

$\Delta\alpha$	OND	JFM	AMJ	JAS
1%	0.0011	0.0015	0.0009	0.0004
5%	0.0057	0.0076	0.0047	0.0027
10%	0.0116	0.0159	0.0101	0.0072
15%	0.0178	0.0249	0.0163	0.0139
-1%	0.0011	0.0014	0.0009	0.0003
-5%	0.0054	0.0070	0.0041	0.0011
-10%	0.0105	0.0135	0.0077	0.0008
-15%	0.0155	0.0195	0.0110	-0.0006

that the SPB for El Niño events characterizes predictions as having a large uncertainty, In particular, the largest error growth tends to occur in the AMJ season when the predictions are made before the spring (see also Duan et al., 2009). It is therefore inferred that parameter errors do not cause a significant SPB for El Niño events. This finding indicates that the uncertainties in the parameter α could not be responsible for the occurrence of the SPB for El Niño events. Next, we investigate whether or not the uncertainties in the parameter μ can cause a SPB.

4.2 Characteristics of seasonal evolutions of the prediction errors caused by uncertainties in the coupling parameters μ

The parameter μ is another important coupling parameter in the WF96 model. In this section, we explore the characteristics of the seasonal evolutions of the prediction errors caused by uncertainties in μ for El Niño events.

With a predetermined error bound, $\Delta\mu \in [-15\%\mu, 15\%\mu]$, numerical experiments similar to those of the coupling parameter α were performed. The results demonstrate that for the El Niño events $U_{W,Jan}$ and $U_{S,Jan}$, the largest growth rates of the prediction errors caused by the uncertainties in the parameter μ occurred in the OND season, with increasing magnitudes of parameter errors; while for the El Niño events $U_{W,Oct}$ and $U_{S,Oct}$, the largest error

growth rates occurred in the JAS season. However, for the El Niño events $U_{W,Jul}$ and $U_{S,Jul}$, the prediction errors caused by uncertainties in μ tended to grow significantly in the AMJ season. These results demonstrate that the largest error growth associated with uncertainties in μ did not always occur in the AMJ season for El Niño events with different initial precursor times. That is to say, the prediction errors caused by uncertainties in μ did not have an obvious season-dependent evolution and thus failed to yield a significant SPB.

In sections 4.1 and 4.2, the uncertainties of the parameters α and μ , respectively, have been considered, and it has been demonstrated that they do not cause a significant SPB for El Niño events. Despite these results, we are still unable to conclude that the model errors do not yield a significant SPB for El Niño events in the WF96 model. In fact, in realistic predictions, the multiple parameters in the model are simultaneously uncertain. Therefore, we should investigate if the combined mode of uncertainties of multiple parameters can cause a SPB. However, a great deal of combined modes of uncertainties of multiple parameters exist, and in order to determine which of these causes a SPB, each combined mode of the parameter errors should be examined. Obviously, such experiments are very difficult to perform. Nevertheless, if the combined error mode that has the largest effect on prediction can be found, and this error mode does

Table 3. Seasonal growth rates of prediction errors caused by errors of α for $U_{S,Oct}$. The numbers in bold denote the largest growth rate of prediction errors.

$\Delta\alpha$	OND	JFM	AMJ	JAS
1%	0.0018	0.0024	0.0024	0.0017
5%	0.0090	0.0124	0.0125	0.0100
10%	0.0184	0.0258	0.0269	0.0246
15%	0.0284	0.0405	0.0436	0.0447
-1%	0.0017	0.0024	0.0023	0.0015
-5%	0.0086	0.0114	0.0109	0.0061
-10%	0.0168	0.0219	0.0205	0.0091
-15%	0.0246	0.0316	0.0290	0.0093

Table 4. Constraint conditions associated with CNOP-P.

Constraint conditions		Constraint conditions	
C-1	$ \Delta\alpha \leq 1\% \alpha, \Delta\mu \leq 2\% \mu;$	C-2	$ \Delta\alpha \leq 1\% \alpha, \Delta\mu \leq 4\% \mu;$
C-3	$ \Delta\alpha \leq 1\% \alpha, \Delta\mu \leq 6\% \mu;$	C-4	$ \Delta\alpha \leq 1\% \alpha, \Delta\mu \leq 8\% \mu;$
C-5	$ \Delta\alpha \leq 2\% \alpha, \Delta\mu \leq 2\% \mu;$	C-6	$ \Delta\alpha \leq 2\% \alpha, \Delta\mu \leq 4\% \mu;$
C-7	$ \Delta\alpha \leq 2\% \alpha, \Delta\mu \leq 6\% \mu;$	C-8	$ \Delta\alpha \leq 2\% \alpha, \Delta\mu \leq 8\% \mu;$
C-9	$ \Delta\alpha \leq 3\% \alpha, \Delta\mu \leq 2\% \mu;$	C-10	$ \Delta\alpha \leq 3\% \alpha, \Delta\mu \leq 4\% \mu;$
C-11	$ \Delta\alpha \leq 3\% \alpha, \Delta\mu \leq 6\% \mu;$	C-12	$ \Delta\alpha \leq 3\% \alpha, \Delta\mu \leq 8\% \mu;$

Table 5. Seasonal growth rates of prediction errors caused by the CNOP-P errors for $U_{W,Jan}$. The numbers in bold denote the largest growth rate of prediction errors.

Constraints of parameter errors	JFM	AMJ	JAS	OND
C-1	0.0028	0.0023	0.0019	0.0118
C-2	0.0049	0.0039	0.0084	0.0239
C-3	0.0071	0.0056	0.0149	0.0355
C-4	0.0092	0.0074	0.0215	0.0467
C-5	0.0038	0.0038	-0.0019	0.0100
C-6	0.0057	0.0046	0.0041	0.0241
C-7	0.0078	0.0060	0.0110	0.0363
C-8	0.0099	0.0076	0.0180	0.0480
C-9	0.0048	0.0056	-0.0029	0.0053
C-10	0.0066	0.0059	-0.0002	0.0235
C-11	0.0085	0.0068	0.0068	0.0369
C-12	0.0106	0.0081	0.0141	0.0492

not cause a SPB for El Niño events, we could conclude that any combined mode of parameter errors does not cause a SPB in the WF96 model. The CNOP approach (see section 3) can be used to find such an error mode. Indeed, the CNOP-P error yields the largest prediction error and has the most significant negative effect on prediction. Consequently, in the following section, we will explore whether the CNOP-P error causes a SPB for the predetermined El Niño events.

5. Characteristics of the seasonal evolution of the prediction errors caused by the CNOP-P errors

In numerical simulations of ENSO, the values of the parameters in the model, as mentioned in the last section, are generally chosen to be sufficient for a requirement, i.e. the determined parameters must make the modeled ENSO persist with an irregular oscillation with typical features of the observed ENSO events. Furthermore, in realistic predictions, a well-performing model is generally used to forecast ENSO. It is conceivable that when the errors are simultaneously superimposed on both α and μ in the WF96 model, their error bounds, to satisfy the above requirement, may be different from the errors that are only added to a single parameter, as shown in the last sec-

tion. In fact, their error bounds are determined as follows:

$$C_\sigma = \{(\Delta\alpha, \Delta\mu) \mid |\Delta\alpha| \leq 3\% \alpha, |\Delta\mu| \leq 8\% \mu\}. \quad (11)$$

With this constraint on the parameter errors, the El Niño events $U_{W,Jan}(U_{S,Jan})$, $U_{W,Apr}(U_{S,Apr})$, $U_{W,Jul}(U_{S,Jul})$, and $U_{W,Oct}(U_{S,Oct})$ were predicted for one year starting from their respective initial precursory times, and the seasonal growth rates of the prediction errors caused by the CNOP-P errors were estimated.

Let $p' = (\Delta\alpha, \Delta\mu)$ be the errors of the two parameters. We adopt Eq. (8) as the objective function to compute the CNOP-P errors. In this case, U_0 in Eq. (8) represents the initial state of the model El Niño events, and J determines the maximum prediction errors of the El Niño events. The error evolution, as in the last section, is measured by the norm

$$\|\mathbf{u}(t)\| = \sqrt{T'(t) + h'(t)},$$

where $T'(t)$ and $h'(t)$ are, respectively, the SSTA and thermocline depth anomaly components of the prediction errors caused by the parameter errors. In the error bound, C_σ , of the parameter errors, different magnitudes of the constraint conditions were chosen to

Table 6. Seasonal growth rates of prediction errors caused by the CNOP-P errors for $U_{W,Oct}$. The numbers in bold denote the largest growth rate of prediction errors.

Constraints of parameter errors	OND	JFM	AMJ	JAS
C-1	0.0028	0.0017	0.0044	0.0205
C-2	0.0049	0.0044	0.0104	0.0367
C-3	0.0070	0.0062	0.0164	0.0524
C-4	0.0091	0.0058	0.0223	0.0677
C-5	0.0037	0.0025	0.0032	0.0245
C-6	0.0057	0.0033	0.0092	0.0420
C-7	0.0077	0.0044	0.0154	0.0585
C-8	0.0098	0.0057	0.0216	0.0744
C-9	0.0047	0.0037	0.0027	0.0275
C-10	0.0065	0.0040	0.0080	0.0470
C-11	0.0085	0.0048	0.0143	0.0645
C-12	0.0105	0.0059	0.0208	0.0811

obtain the CNOP-P. The error bounds on the parameter α ranged from $1\%\alpha$ to $3\%\alpha$, and on parameter μ from $2\%\mu$ to $8\%\mu$. Twelve combinations of the error bounds on the parameters α and μ were then obtained (Table 4; values of α and μ are referred to as in section 3). With these error bounds, the CNOP-P errors of all the predetermined El Niño events were computed. These computations demonstrated that for each El Niño event there exists a CNOP-P error. Furthermore, these CNOP-P errors correspond to the parameter perturbations that lie on the boundary of the domain defined by the constraint and had a robust pattern of $(\Delta\alpha, \Delta\mu) = (a, b)$ (a and b are positive numbers). For example, the CNOP-P error for the El Niño event $U_{W,Jan}$ with the constraint $|\Delta\alpha| \leq 1\%\alpha$, $|\Delta\mu| \leq 2\%\mu$ is $(1\%\alpha, 2\%\mu)$. With these perturbed parameters, we integrated the WF96 model and obtained prediction errors for the El Niño events. The seasonal growth rates, κ , of the prediction errors, i.e. the slopes of the curve $\gamma(t) = \|\mathbf{u}(t)\|$ for different seasons, could be estimated. We found that for the El Niño events with different initial precursor times, the largest growth rates of the prediction errors did not always occur in the AMJ season. Furthermore, with increasing magnitudes of the parameter errors, the seasons in which the largest error growth for each El Niño event occurred were not common. In Tables 5 and 6, the cases of the El Niño events $U_{W,Jan}$ and $U_{W,Oct}$ are listed as examples. It is clear that the CNOP-P errors did not have an obvious season-dependent evolution with the largest growth in the AMJ season; furthermore, they yielded a trivial magnitude of prediction error. This indicates that the CNOP-P errors could not have caused a significant SPB. Although the CNOP-P errors are most likely to cause a SPB due to their large negative effect on predictions, the results shown here demonstrate that the CNOP-P errors do not cause a

significant SPB in the WF96 model. Therefore, it is inferred that the other combined modes of the errors of the two parameters in the WF96 model are scarcely able to cause a significant SPB. That is to say, the model errors could not yield a significant SPB. Mu et al. (2007a) demonstrated that initial errors could cause a significant SPB for El Niño events; moreover, the largest growth in prediction errors caused by initial errors in the WF96 model always occurred in the AMJ season and yielded a considerable prediction error. The question of whether or not an initial error is the dominant source of uncertainties that result in a significant SPB arises; this question is addressed in the next section.

6. Characteristics of seasonal evolutions of the prediction errors caused by the combined mode of initial and model errors

Prediction errors in realistic predictions are generally caused by initial and model errors. The SPB phenomenon described by some studies has also been illustrated under the condition that there exists both initial and model errors in predictions. Next, in the present study, the seasonal evolution of the prediction error caused by the combined mode of initial and model parameter errors is investigated, and the results compared with those related to model parameter errors (see section 5) and initial error (see Mu et al., 2007a). In doing so, an attempt is made to identify the dominant source of uncertainties that cause a significant SPB for El Niño events.

In these experiments, we used the WF96 model with perturbed initial conditions and model parameters to predict the predetermined El Niño events and to estimate the season-dependent predictability of the El Niño events. In the WF96 model, there are two variables, Niño-3 SSTA, T , and thermocline depth

Table 7. Seasonal growth rates of prediction errors caused by the CNOP errors for $U_{W,Jan}$ with the parameter error bound $|\Delta\alpha| \leq 3\%\alpha$ and $|\Delta\mu| \leq 8\%\mu$ and initial error bounds ranging from 0.01 to 0.05. The numbers in bold denote the largest growth rate of prediction errors.

Initial error bounds	JFM	AMJ	JAS	OND
0.01	0.0316	0.0430	0.0617	0.0665
0.02	0.0609	0.0741	0.0832	0.0774
0.03	0.0905	0.1054	0.1059	0.0895
0.04	0.1202	0.1368	0.1299	0.1035
0.05	0.1498	0.1681	0.1555	0.1198

Table 8. Seasonal growth rates of prediction errors caused by the CNOP errors for $U_{W,Jul}$ with the parameter error and initial error as in Table 7. The numbers in bold denote the largest growth rate of prediction errors.

Initial error bounds	JAS	OND	JFM	AMJ	JAS
0.01	0.0327	0.0361	0.0557	0.0913	-0.1064
0.02	0.0643	0.0592	0.0848	0.1463	-0.1954
0.03	0.0962	0.0825	0.1156	0.2238	-0.2369
0.04	0.1282	0.1060	0.1482	0.3317	-0.2466
0.05	0.1600	0.1296	0.1827	0.4829	-0.2128

anomaly, h , and two empirical coupling parameters, α and μ . In the numerical experiments, we considered the initial errors of the variables T and h , denoted by $\mathbf{u}_0 = (T'_0, h'_0)$; and the uncertainties in the parameters α and μ as the model errors (see section 5). In imperfect model experiments, there exist combined modes of initial error and model errors. CNOP represents the optimal combined mode of the initial error and the model parameter error that causes the largest prediction errors, and it is therefore most likely to yield a significant SPB.

For each constraint shown in Table 4, we took different magnitudes of initial errors to estimate the prediction errors of the El Niño events. The use of these different magnitudes yielded similar results. For simplicity, we will only use the case of the parameter error constraint $|\Delta\alpha| \leq 3\%\alpha; |\Delta\mu| \leq 8\%\mu$ to describe the results. In this case, the initial errors, in terms of the measurement $\|\mathbf{u}_0\| = \max\{|T'_0|, |h'_0|\}$, were chosen to range from 0.01 to 0.05. With these constraints, we obtained the CNOP errors of the predetermined El Niño events by using Eq. (6). Similarly, we estimated the seasonal growth rate of the prediction errors caused by the CNOP errors by computing the slopes of the curve

$\gamma(t) = \|\mathbf{u}(t)\|$ for the different seasons.

By investigating the prediction errors caused by the resultant CNOPs, we found that the prediction errors always exhibited an obvious season-dependent evolution for El Niño events with initial precursor times of July and October (Tables 8 and 9) and were significantly larger than those caused by the CNOP-P errors shown in section 5. Note that the El Niño events with the initial precursory time of July were predicted for 15 months and the error growth rates for the corresponding five seasons were estimated, in order to avoid understanding that the large error growth in the AMJ season was due to the accumulation of error growth. However, for the El Niño events with an initial precursor time of January, the largest error growth associated with the CNOPs did not always occur in the AMJ season (Table 7). In fact, only when the initial errors were relatively large did the most significant growth of the prediction error caused by the CNOPs arise in the AMJ season. However, Mu et al. (2007a) demonstrated that the optimal initial error in a constraint, CNOP-I, (i.e. the initial error that has the largest negative effect on predictions; see Section 2) always yields the SPB for El Niño events, regardless

Table 9. Seasonal growth rates of prediction errors caused by the CNOP errors for $U_{W,Oct}$ with the parameter error and initial error as in Table 7. The numbers in bold denote the largest growth rate of prediction errors.

Initial error bounds	OND	JFM	AMJ	JAS
0.01	0.0318	0.0368	0.0568	0.0089
0.02	0.0625	0.0631	0.0876	-0.0442
0.03	0.0935	0.0898	0.1227	-0.0982
0.04	0.1245	0.1167	0.1621	-0.1513
0.05	0.1555	0.1438	0.2063	-0.2015

of the magnitudes of the initial errors. It is obvious that the seasonality of the evolutions of the prediction errors caused by the CNOP-I errors was disordered by the model parameter errors. It can be inferred that the initial error, rather than the model parameter error, may be the dominant source of uncertainties that cause a significant SPB.

7. Summary and discussion

In this paper, the authors first examined the effect of the uncertainties of the coupling parameters α and μ on the SPB for El Niño events in the WF96 model through sensitivity experiments. The results showed that, within the allowable bounds on the parameter errors, the prediction errors caused by uncertainties of each of these two parameters did not exhibit an obvious season-dependent evolution; notably, they did not possess the largest error growth in spring. Furthermore, the prediction errors caused by these parameter errors were very small and were insufficient to have a large effect on predictions of El Niño events. The results from this study suggest that the uncertainties in the parameters α and μ cannot cause a significant SPB.

By considering that there may simultaneously exist errors from multiple parameters in the forecast model used in realistic predictions, the conditional nonlinear optimal perturbation (CNOP) approach was used to investigate the effects of the optimal mode (CNOP-P) of the errors of the two parameters in the WF96 model on the SPB for El Niño events. The CNOP-P errors of the two parameters did not cause a significant SPB. CNOP-P errors are known to cause the largest prediction errors in perfect model experiments and are most likely to cause a SPB. However, as demonstrated above, they did not cause a SPB; therefore, any other error modes of the two parameters will not yield a SPB. This implies that the model parameter errors may not be the dominant source of uncertainties that cause a significant SPB.

In order to further address this perspective, another set of numerical experiments were performed. The season-dependent predictability of the El Niño events associated with the coexistence of initial errors and model parameter errors were estimated by computing the CNOP errors of the WF96 model. From the numerical results, it was demonstrated that the CNOP patterns of initial error and model parameter error tended to have the largest growth in spring and yield a significant SPB phenomenon for El Niño events. Furthermore, the coherent season-dependent predictability caused by initial errors of El Niño events in the WF96 model did not exist, due to consideration

of the parameter errors. Therefore, the authors emphasize that initial errors may play the key role in yielding SPB for El Niño events. That is to say, initial errors could be the dominant source of the uncertainties that cause a significant SPB. Thus, it is understandable why Chen et al. (2004) pointed out that the predictability barrier for ENSO could be reduced by improving the initial conditions. Furthermore, the results of this paper can be considered as a motivation for the data assimilation of ENSO prediction.

The above results have been derived from a theoretical ENSO model and from qualitative arguments. The numerical experiments performed here were also of an exploratory nature. Future aims include exploring the above problems using a more realistic ENSO model. When the experiments are applied to a more realistic model, the effect of model parameter error on ENSO predictability may be better estimated.

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REFERENCES

- Chen, D., M. A. Cane, A. Kaplan, S. E. Zebiak, and D. J. Huang, 2004: Predictability of El Niño over the past 148 years. *Nature*, **428**, 733–736.
- Duan, W. S., 2003: Applications of nonlinear optimization method to the studies of ENSO predictability. Ph. D. dissertation, Institute of Atmospheric Physics, Chinese Academy of Sciences, 111pp. (in Chinese)
- Duan, W. S., M. Mu, and B. Wang, 2004: Conditional nonlinear optimal perturbation as the optimal precursors for ENSO events. *J. Geophys. Res.*, **109**, D23105.
- Duan, W. S., X. Liu, K. Y. Zhu, and M. Mu, 2009: Exploring initial errors that cause a significant spring predictability barrier for El Niño events. *J. Geophys. Res.*, **114**, C04022, doi: 10.1029/2008JC004925.
- Flügel, M., and P. Chang, 1998: Does the predictability of ENSO depend on the seasonal cycle? *J. Atmos. Sci.*, **55**, 3230–3243.
- Garay, J. Z., 2004: Influence of stochastic forcing on ENSO prediction. *J. Geophys. Res.*, **109**, C1107.
- Jin, E. K., and Coauthors, 2008: Current status of ENSO prediction skill in coupled ocean-atmosphere models. *Climate Dyn.*, **31**, 647–664.
- Latif, M., and Coauthors, 1998: A review of the predictability and prediction of ENSO. *J. Geophys. Res.*, **103**, 14357–14393.

- Liu, Z. Y., 2002: A simple model study of ENSO suppression by external periodic forcing. *J. Atmos. Sci.*, **15**, 1088–1098.
- Moore, A. M., and R. Kleeman, 1996: The dynamics of error growth and predictability in a coupled model of ENSO. *Quart. J. Roy. Meteor. Soc.*, **122**, 1405–1446.
- Mu, M., W. S. Duan, and J. C. Wang, 2002: Predictability problems in numerical weather and climate prediction. *Adv. Atmos. Sci.*, **19**, 191–205.
- Mu, M., W. S. Duan, and B. Wang, 2003: Conditional nonlinear optimal perturbation and its applications. *Nonlinear Processes in Geophysics*, **10**, 493–501.
- Mu, M., W. S. Duan, and B. Wang, 2007a: Season-dependent dynamics of nonlinear optimal error growth and El Niño–Southern Oscillation predictability in a theoretical model. *J. Geophys. Res.*, **112**, D10113, doi: 10.1029/2005JD006981.
- Mu, M., H. Xu, and W. S. Duan, 2007b: A kind of initial errors related to “spring predictability barrier” for El Niño events in Zebiak–Cane model. *Geophys. Res. Lett.*, **34**, L03709, doi: 10.1029/2006GL-27412.
- Mu, M., W. S. Duan, Q. Wang, and Y. M. Liu, 2010: An extension of conditional nonlinear optimal perturbation. *Nonlinear Processes in Geophysics*, **17**, 211–220.
- Samelson, R. G., and E. Tziperman, 2001: Instability of the chaotic ENSO: The growth-phase predictability barrier. *J. Atmos. Sci.*, **58**, 3613–3625.
- Tang, Y., Z. Deng, X. Zhou, Y. Cheng, and D. Chen, 2008: Interdecadal variation of ENSO predictability in multiple models. *J. Climate*, **21**, 4811–4833.
- Wang, B., and Z. Fang, 1996: Chaotic oscillation of tropical climate: A dynamic system theory for ENSO. *J. Atmos. Sci.*, **53**, 2786–2802.
- Wang, C., and J. Picaut, 2004: Understanding ENSO physics—A review. *Earth Climate: The Ocean–Atmosphere Interaction*, Geophysical Monograph Series, Vol. 147, American Geophysical Union, 21–48.
- Webster, P. J., and S. Yang, 1992: Monsoon and ENSO: Selectively interactive systems. *Quart. J. Roy. Meteor. Soc.*, **118**, 877–926.
- William, P. D., 2005: Modelling climate change: the role of unresolved processes. *Philosophical Transactions of the Royal Society*, **363**, 2931–2946.
- Wu, D. H., and D. L. Anderson, 1993: ENSO variability and external impacts. *J. Climate*, **6**, 1703–1717.
- Yu, Y. S., W. S. Duan, H. Xu, and M. Mu, 2009: Dynamics of nonlinear error growth and season-dependent predictability of El Niño events in the Zebiak–Cane model. *Quart. J. Roy. Meteor. Soc.*, doi: 10.1002/qj.526.
- Zebiak, S. E., and M. A. Cane, 1987: A model El Niño Southern Oscillation. *Mon. Wea. Rev.*, **115**, 2262–2278.
- Zhang, L., M. Flugel, and P. Chang, 2003: Testing the stochastic mechanism for low-frequency variations in ENSO predictability. *Geophys. Res. Lett.*, **12**, doi: 10.1029/2003GL017505.
- Zheng, F., H. Wang, and J. Zhu, 2009: ENSO ensemble prediction: initial condition perturbations vs. model parameter perturbations. *Chinese Science Bulletin*, **54**(14), 2516–2523.