ADVANCES IN ATMOSPHERIC SCIENCES, VOL. 24, NO. 6, 2007, 1077–1085

Nonlinear Atmospheric and Climate Dynamics in China (2003–2006): A Review

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(Received 30 January 2007; revised 16 May 2007)

ABSTRACT

Recent advances in the study of nonlinear atmospheric and climate dynamics in China (2003–2006) are briefly reviewed. Major achievements in the following eight areas are covered: nonlinear error dynamics and predictability; nonlinear analysis of observational data; eddy-forced envelope Rossby soliton theory; sensitivity and stability of the ocean's thermohaline circulation; nonlinear wave dynamics; nonlinear analysis on fluctuations in the atmospheric boundary layer; the basic structures of atmospheric motions; some applications of variational methods.

Key words: nonlinear dynamics, predictability, blocking, stability, nonlinear waves, variational methods

DOI: 10.1007/s00376-007-1077-7

1. Introduction

The atmosphere is a complex, nonlinear system, which involves complex, nonlinear interactions among all components of the atmosphere. So, the study of nonlinear atmospheric dynamics is very important. Nonlinear atmospheric dynamics is an interdisciplinary branch of atmospheric sciences, geophysical fluid dynamics and nonlinear sciences, whose complexity and nonlinearity present many physical and mathematical challenges for scientists in many fields (Li and Wang, 2006). Continuous development of nonlinear atmospheric dynamics plays an important role in understanding the basic processes in the atmosphere, as well as providing the theory and method for the prediction of atmospheric motions. The research activities of Chinese scientists in this field have been vigorous and productive during the last three decades, and some important advances in nonlinear adjustment processes, nonlinear stability and instability, predictability study, multiple equilibria dynamics, meso- and microscale nonlinear dynamics, and nonlinear blocking dynamics etc. have been made (Li and Chou, 2003; Diao et al., 2004). Since the last International Union of Geodesy and Geophysics (IUGG) General Assembly (2003), four years have passed. In these four years, some new studies in China have been carried out in many areas of nonlinear atmospheric and climate dynamics. This paper summarizes the major results.

2. Nonlinear error dynamics and its applications to the predictability of weather and climate

Because the atmosphere is a complex nonlinear system and there exist a lot of limitations using the linearized error growth equation to study its predictability (Lacarra and Talagrand, 1988; Mu, 2000), it is necessary to propose a new approach based on nonlinear error growth dynamics for quantifying atmospheric

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predictability. Ding and Li (2007) and Li et al. (2006) applied nonlinear error growth equations of nonlinear dynamical systems instead of linear approximation equations to discuss the evolution of initial perturbations, and introduced a novel concept of nonlinear local Lyapunov exponent (NLLE). The NLLE depends generally on the initial state in the phase space, the initial error and evolution time, which is quite different from the global Lyapunov exponent, and the local Lyapunov exponents based on linear error dynamics (Eckmann and Ruelle, 1985; Nese, 1989; Kazantsev, 1999; Ziehmann et al., 2000).

In order to study the average predictability of the whole system, the whole ensemble mean of the NLLE should be defined. According to chaotic dynamical system theory and probability theory, Ding and Li (2007) proved a saturation theorem of mean relative growth of initial error (RGIE) derived by the mean NLLE. That is, for a chaotic dynamic system, the mean RGIE will necessarily reach a saturation value in a finite time interval. Once the mean RGIE reaches the saturation, at that moment almost all predictability of chaotic dynamic systems is lost. Therefore, the predictability limit can be defined as the time at which the mean RGIE reaches its saturation level.

On the other hand, in order to measure predictability of a specified state with certain initial uncertainties and to investigate the distribution of the predictability limit, Ding and Li (2006) introduced the local ensemble mean of the NLLE. Using this, the local average predictability limit of a chaotic system could be quantitatively determined by examining the evolution of the local mean relative growth of initial error (LRGIE). The local ensemble mean of the NLLE, different from the whole ensemble mean of the NLLE, could show local error growth dynamics of subspace on an attractor in the phase space. Moreover, in practice the local average predictability limit itself might be regarded as a predictand to provide an estimation of accuracy of prediction results.

The NLLE defined in Ding and Li (2007) and Li et al. (2006) actually only characterizes the growth rate of initial perturbation along the most rapidly growing direction. Li and Ding (2006) extended the concept of the NLLE from one dimension to multi-dimension. Based on the NLLE along the most rapidly growing direction, for an n-dimensional chaotic dynamic system, they obtained the first m NLLE spectra along other directions by the growth rate of the volume of m-dimensional subspace spanned by the first m error vectors (m=2, 3, $\cdots$, n). In a chaotic system, each error vector tends to fall along the local direction of most rapid growth. Due to the finite precision of computer calculations, the collapse toward a common direction causes the orientation of all error vectors to become indistinguishable. Li and Ding (2006) further pointed out that this problem could be overcome by the repeated use of the Gram-Schmidt reorthogonization (GSR) procedure on the vector frame.

Nonlinear error dynamics may be applied to predictability studies of finite-size initial error, and demonstrates superiority in determining the limit of predictability of chaotic systems in comparison with the linear approach. Recently, Chen et al. (2006) applied nonlinear error dynamics to the predictability analysis of atmospheric observation data, and obtained the spatiotemporal characteristics of the predictability limit of the 500-hPa geopotential height field. Other aspects, including the spatiotemporal characteristics of the predictability limit of other different variables, such as temperature, precipitation etc., the decadal change of the predictability limit, the relationships among the predictability limits of motion on various time and space scales, and the prediction of predictability etc., will be subjects of future research by employing nonlinear error dynamics.

3. Nonlinear analysis of observational data

Global change science is a relatively new research field. As one of the most important branches of global change science, climate change is paid great attention by all governments throughout the world. Although many studies have been performed to investigate past climate change, the causes of some significant climate events (such as the Medieval Warm Period and the Little Ice Age) are still not clear. For the Little Ice Age in particular, the timing of its onset and passing is highly controversial (Bryson, 1974; Thompson et al., 1989; Briffa, 2000). For further studying past climate change, especially to reveal the rules of global climate change in the past 2000 years and to predict future climate change, Wan et al. (2005a) introduced a new method making use of the dynamical lag correlation exponent (named Q index), a dynamics exponent based on phase-space reconstruction, which can effectively discern the similarities or differences between the dynamics of the two series. With the Q index, the authors analyzed the dynamics structure of some typical climatic proxies. Their results showed that the dynamics of climatic proxies are almost similar, and that regional climate change reflects that of global climate change. In other words, regional climate is controlled by the global climate change. Furthermore, they also found that there are two dynamics jump periods (namely 700–900 years and 1300–1700 years) in the past 2000 years of the climate system, which may correspond to the Medieval Warm Period and the Little Ice Age, respectively.

The climate system is nonlinear, non-stationary and hierarchical, which makes it even harder to detect and analyze abrupt climate changes. Based on the Student's t-test, Bernaola-Galván et al. (2001) recently proposed a heuristic segmentation algorithm to segment the time series into several subsets with different scales, which is more effective in detecting the abrupt changes of nonlinear time series. Feng et al. (2005) tried to verify the effectiveness of the heuristic segmentation algorithm in dealing with nonlinear time series by an ideal time series. Through detecting and analyzing the information of abrupt climate changes contained in the dendrochronology of the recent 2000 years, the authors succeeded in distinguishing abrupt changes with different scales. The research based on the newly defined parameter of abrupt change density showed that human activity might have led to the unbalanced distribution of serial and spares segments of abrupt climate changes in the recent 1000 years, which may be one of the manifestations of global temperature change.

Most of the present statistical climate prediction methods (which mainly include empirical, mathematical and physical statistical methods) are based on the hypothesis that the system is stationary. However, observations, in particular for the climate data, are often nonlinear/non-stationary and multi-hierarchical, which makes prediction very difficult. Aiming to address this problem, Wan et al. (2005b) introduced a new prediction model in which, firstly, using the empirical mode decomposition, the observation sequence is made stationary and a variety of intrinsic mode functions (IMF) are obtained; then, the IMFs are separately predicted by the mean generating function model; and finally, with the optimal subset regression model, part of the predictions are used as new samples to fit the original series directly, or step-by-step, and a system of prediction equations is set up. Climate sequences prediction research showed that the individual IMF, especially the eigen-IMF, has more stable predictability than that of its sources. The trend of development in climate prediction lies in researching the mechanism and hierarchy of the climate system, constructing the corresponding climate prediction model. An attempt has been accomplished in their work. It is expected that the model proposed can open up a new and effective method for climate prediction or evaluation.

4. Eddy-forced envelope Rossby soliton theory

Although several theoretical models have been proposed to explain the role of high-frequency eddies in exciting a blocking circulation (Haines and Marshall, 1987; Malanotte-Rizzoli and Malguzzi, 1987; Malguzzi, 1993; Luo, 2000), some questions for block-eddy interaction remain unresolved. To further describe the interaction between an incipient block (planetary scale) and short synoptic-scale eddies, Luo (2005a) proposed a new forced envelope Rossby soliton model in an equivalent barotropic beta-plane channel. It is shown that the planetary-scale projection of the nonlinear interaction between synoptic-scale eddies is the most important contributor to the amplification and decay of the planetary-scale blocking dipole or anticyclone, while the synoptic-planetary-scale interaction contributes significantly to the downstream development of pre-existing synoptic-scale eddies. Large-scale topography plays a secondary role compared to synoptic-scale eddies in exciting the block. However, it plays a role in inducing a standing planetary-scale ridge prior to block onset, which fixes the geographical location of the block and induces meridional asymmetry in the flow. In particular, the topographically induced planetary-scale ridge that is almost in phase with a dipole component of blocking flow is found to be a controlling factor for the northward deflection of storm tracks associated with blocking anticyclones.

Using the same block-eddy interaction model mentioned above but ignoring the effect of topography, Luo (2005b) first examined the role of westward traveling planetary waves in blocking circulations associated with synoptic-scale eddies. He found that a typical retrograde blocking anticyclone can arise through the interaction of an incipient block with synoptic-scale perturbations when a planetary-scale ridge with zonal wavenumber 1 shifts westward from the east of the dipole component in a pre-existing planetary-scale flow prior to block onset. However, a noticeable northward deflection of synoptic eddies cannot be observed in this case. Luo (2005c) also examined the relationship between blocked flow and deformed eddies using numerical and analytical methods. It was found that the blocking flow and synoptic-scale eddies are symbiotically dependent upon one another during their interaction. The low-frequency variability in an isolated block flow and associated synoptic eddy activity is caused by low-frequency eddy forcing by pre-existing synoptic eddies prior to block onset. The interaction of pre-existing eddies with an isolated blocked flow induces two types of eddies: Z-type eddies with a meridional monopole structure and M-type eddies having a meridional tripole structure, which play different roles in the deformation of eddies. These eddies tend to disperse their energy toward the downstream with the same group velocity as that of the block. At the same time, the zonal wavenumbers of blocked flow and deformed (Z-type and M-type) eddies can be shown to satisfy two conserved quantities.

In addition, Luo (2005d) further extended the eddy-forced envelope Rossby soliton theory proposed in Luo (2005a) to include zonal mean flow with a shear and discussed the impact of the horizontal shear of basic flow on isolated vortex pair blocks in association with synoptic-scale eddies. To simplify the problem, an assumption of a weak shear is used to allow analytical solutions. The type of the shear of the mean flow is found to have a significant impact on the strength and persistence of isolated vortex pair blocks and the deformation of synoptic-scale eddies.

5. Sensitivity and stability of the ocean's thermohaline circulation

To study nonlinear mechanisms of amplification, Berloff and Meacham (1996) modified the linear singular vector (LSV) technique and Mu (2000) proposed the concept of nonlinear singular vectors (NSVs) and nonlinear singular values (NSVAs). These concepts were successfully applied by Mu and Wang (2001) and Durbiano (2001) to study finite-amplitude stability of flows in two-dimensional quasigeostrophic and shallow water models, respectively. In Mu et al. (2003), the concept of conditional nonlinear optimal perturbations (CNOPs) was introduced and applied to study the nonlinear effects in predictability studies, sensitivity analysis and stability research. Mu et al. (2004) considered the sensitivity and stability of the thermohaline circulation (THC) in finite amplitude perturbations within a simple model context by using the approach of CNOP. It was shown that linearly stable thermohaline circulation states can become nonlinearly unstable and the properties of the perturbations with optimal nonlinear growth are determined. An asymmetric nonlinear response to perturbations exists with respect to the sign of finite amplitude freshwater perturbations, on both thermally-dominated and salinity-dominated thermohaline flows. This asymmetry is due to the nonlinear interaction of the perturbations through advective processes.

Sun et al. (2005) also utilized CNOP to investigate the decadal variation of THC in a simple coupled ocean-atmospheric two-box model. Specifically, the passive variabilities found in this model are due to non-normal and nonlinear growth of initial perturbations. These variabilities, measured as the recovering time of perturbations, can cause decadal variability of THC. These results are of potential application to the interpretation of past climate change linked to THC variations in a nonlinear regime.

6. Nonlinear waves and their applications in studies of the atmosphere and ocean

It is important to be able to find the exact solutions of nonlinear wave equations in nonlinear problems. Recently, a number of methods have been proposed, such as the homogeneous balance method (Wang, 1995), the hyperbolic tangent function expansion method (Parkes and Duffy, 1997; Yang et al., 2001), the trial function method (Kudryashov, 1990), and the sine-cosine method (Yan, 1996). These methods are widely applied to solve nonlinear wave equations exactly and many solutions are obtained. However, much more complicated nonlinear equations or sets of equations still exist that cannot be very easily and directly solved; one has to seek their approximate or asymptotic solutions. Liu et al. (2003a) and Fu et al. (2003a,b,c, 2004a,b) proposed several new methods to solve nonlinear evolution equations, including the Jacobi elliptic function expansion method, new transformation method, a method related to elliptic equations, the power series expansion method, and so on. Using these methods, the authors obtained the classical solitary wave solutions (Fu et al., 2004b, 2005a; Zhao et al., 2005). In addition, the generalized periodic wave solutions were also derived (Liu et al., 2003a; Fu et al., 2005a). These solutions are of benefit to our understanding of the irregularities in period and multiple structures of atmospheric motions (Chen et al., 2005; Fu et al., 2004c,d, 2005b). In fact, there are indeed ample patterns in motions of geophysical fluid from our studies of motions of the atmosphere and ocean.

7. Nonlinear analysis of fluctuations in the atmospheric boundary layer and in fields related to weather and climate variables

It has been found that there exists universal statistical law in the atmospheric motions over broad temporal or spatial ranges (Dmowska and Saltzman, 1999). Starting from time series analysis, Liu et al. (2003b), Fu et al. (2003d), Wang et al. (2005a), Shi et al. (2005), and Liu et al. (2005a) applied new methods put forward in fluid mechanics, such as hierarchical similarity law and extended self-similarity, to analyze the statistical symmetry in fluctuations in the atmospheric boundary layer and in meteorological variables. It was shown that there exists hierarchical similarity in atmospheric motions over broad ranges, and there are different hierarchical similarity laws for different conditions, which originate from the internal factors among atmospheric systems. For example, stratification is an important factor for velocity fluctuations in the atmospheric boundary layer, and it can influence the hierarchical similarity of velocity fluctuations. Finally, the difference of the hierarchical similarity is determined by the gravity waves or solitary waves found in the atmospheric boundary layer. Due to the existence of these gravity waves or solitary waves, the statistical symmetry and energy cascade of turbulence will be changed in the atmospheric boundary layer.

8. The basic structures of atmospheric motions and the relation between solitary waves and wavelets

Liu et al. (2003c, 2004a), Liu et al. (2003d, 2006), and Chi et al. (2004) found that the steady states of atmospheric motions were spiral patterns and these kinds of structure originate from the balance between several forces, such as dissipation force, in the controlled equations of atmospheric motions. These kinds of spiral structure are related either to cone-shaped spirals or to column-shaped spirals, which are formed either from updraft due to surface convergence or downdraft due to divergence at high levels, or from downdraft due to convergence at high levels or updraft due to surface divergence.

Liu et al. (2003e, 2004b), Liu et al. (2003f, 2004c, 2005b), and Fu et al. (2005c) also found that the common Mexican hat wavelet satisfied linear ordinary differentiation equations with variable coefficients and its shape was also a kind of solitary wave. This therefore showed that the wavelet was a kind of solitary wave.

9. Some applications of variational methods

9.1 *Inverse problems in atmospheric science and their applications*

In recent years, a variety of methods have been proposed to boost accuracy of numerical weather prediction, such as variational data assimilation (VDA). The aim of VDA is to obtain initial and boundary conditions and parameters of models, statistically or dynamically from observational data. Generally speaking, the problem with VDA is characterized by ill-posedness, and belongs to the category of inverse problems (Huang et al., 2005a, 2006).

Initial conditions and model parameters retrieval belong to the class of inverse problems. It is ill-posed to determine the initial conditions and model parameters, which are distributed in space and time with global and local observations by adjoint method. In numerical experiments, the solution is very sensitive to the first guess and iteration steps, and the calculation is unstable to some extent without regularization. Huang and Wu (2001) and Zhang and Huang (2003) utilized the regularization method and overcame the ill-posedness of the problem to some extent. In the case of local observations, it improved the accuracy and stability of the solution, especially for the determination of model parameters in which the descent speed of the cost functional and the accuracy are both improved. However, the ill-posedness of the problem is very complicated; more efforts should be made in future (Pan and Huang, 2004; Du et al., 2004). In addition, Huang et al. (2004a), Huang and Han (2003), Cao et al. (2005), Fang et al. (2004), Fang and Huang (2004), and Wang et al. (2006)) applied the VDA method combined with regularization techniques and the optimal control idea to retrieve atmospheric and oceanic parameters, including the vertical eddy diffusion coefficient, the turbulence of the atmospheric boundary layer, the wind with Doppler radar data, and GPS dropsonde' s motion, obtaining good results.

Xiang et al. (2004, 2006) employed the adjoint method to a statistical dynamical tropical cyclone prediction model (SD-90). The results indicated that by using the adjoint method, five tropical cyclone (TC) tracks all fitted well, and forces acting on TCs are well retrieved. They also applied the same method to a portion of the track data of TC 9804, which indicated that when the amount of data of a TC track is sufficient, the algorithm is stable. If a TC track data is obtained every three hours, their results further prove the applicability of the algorithm to TCs with complicated mesoscale structures.

9.2 *Variational adjustment and decomposition of the wind field*

The variational optimization analysis method (VOAM) for a 2-D flow field suggested by Sasaki (1970) can be used efficiently in most cases. However, in cases where there are high frequency noises in the 2-D or 3-D flow field, it appears to be inefficient. Based on Sasaki's VOAM, Huang et al. (2005b,c) and Lan et al. (2004) proposed a generalized variational optimization analysis method (GVOAM) with regularization ideas, which could deal well with flow fields containing high frequency noises. A numerical test shows that observational data can be both variationally-optimized and filtered, and therefore the GVOAM is an efficient method.

By using variational analysis, Huang et al. (2007) used a linear and nonviscous 2-D Boussinesq equation on an f-plane to extract basic flows from the observational wind field. Their results indicated that, on the one hand, extracted basic flow may satisfy the controlling equations, but on the other hand, the value obtained from the average functional difference between the basic flow and observations along the vertical direction is minimal. Subtracting the basic flow from the observational wind field gives disturbance wind fields, which can be used to study instability problems and temporal evolvement of the disturbance field (Huang et al., 2004b).

9.3 *Studies of variational data assimilation for a coupled air-sea model*

For the prediction of ENSO, the accuracy of the model, including the parameters and initial value of the model is important. This can, in recent years, be retrieved by variational data assimilation methods. However, when the nonlinearity of the model is strong enough, the effect of the improvement made by 4-D variational data assimilation (4-DVAR) may be poor due to bad approximation of the tangent linear model to the original model. So, Du et al. (2006) introduced the idea of optimal control to improve the effect of 4-DVAR in the inversion of the parameters of a nonlinear dynamic ENSO model. Their results indicated that when the terminal controlling term is added to the cost functional of 4-DVAR, which originates from the optimal control, the effect of the inversion may be largely improved compared to traditional 4-DVAR, as can be particularly obvious from the phase orbit of the model variables. In addition, their results also suggested that the method of 4-DVAR in combination with optimal control can not only reduce the error resulting from the inaccuracy of the model parameters, but can also correct the parameters themselves.

9.4 *New approach to variational data assimilation with "on-off" processes*

Moist physical processes characterized by "on-off" switches are strongly nonlinear. The impact of "on-off" switches on VDA has been intensively investigated from idealized simple models to sophisticated forecast models in recent years.

Mu and Wang (2003) presented a new method based on the nonlinear perturbation equation (NPE) for an idealized model to calculate the gradient of the CF in the presence of "on-off" switches. This method provides an accurate gradient of CF in time-continuous cases. In discrete cases, it is useful to obtain the global descent direction of the CF in optimization, and helpful to find the global minimum (Wang et al., 2002). In addition, the robustness of the NPE method in using the VDA method to estimate both the initial conditions (IC) and parameters is also verified (Wang et al., 2005b).

Using an idealized PDE model, Mu and Zheng (2005) demonstrated that the traditional time discretization dealing with the nonlinearity caused by switches of the governing equation could induce awful zigzagging in both the CF and the numerical solution of the model. The authors proposed a method, which is a generalization of Xu's (1997) intermediate interpolation method, to eliminate the zigzag phenomenon. The potential merits of this treatment are that the convergence in the minimization processes of the VDA is improved and satisfactory optimization retrievals are obtained. Furthermore, they also demonstrate theoretically and numerically that if the "on-off" switches in the forward model are not properly numerically treated, the discrete CF gradients (even the one-sided gradient of CF) with respect to some ICs do not exist, and the solution of the corresponding tangent linear model (TLM) obtained by the conventional approach would not be a good first-order linear approximation to the nonlinear perturbation solution of the governing equation. Consequently, the validity of the adjoint approach in VDA with parameterized physical processes could not be guaranteed (Zheng and Mu, 2006).

10. Summary

Progress in the study of nonlinear atmospheric and climate dynamics achieved by Chinese scientists in the period 2003–2006 has been reviewed. The progress includes eight aspects: (1) nonlinear error dynamics and its applications to the predictability of weather and climate; (2) applications of nonlinear methods to observational data analysis; (3) the introduction of a new eddy-forced envelope Rossby soliton theory; (4) studies of the sensitivity and stability of the ocean's thermohaline circulation by using a new approach of conditional nonlinear optimal perturbation (CNOP); (5) nonlinear wave dynamics and its application to studies of the atmosphere and ocean; (6) nonlinear analysis of fluctuations in the atmospheric boundary layer; (7) studies of the basic structures of atmospheric motions and the relation between solitary waves and wavelets; and (8) some applications of variational methods, including variational data assimilation (VDA), the variational optimization analysis method (VOAM), and so on. These achievements shed some light on some atmospheric nonlinear phenomena and help to deepen our understanding of them. However, we are still far away from exposing the nature or cause of complex nonlinear phenomena. There are still some important problems left for future study and improvement, especially in basic theory and methods. Many more achievements in this research area are expected to be realized in the future.

Acknowledgements. This work was supported by the National Natural Science Foundation of China (Grant Nos. 40325015 and 40675046).

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