

# A Generalized Layered Radiative Transfer Model in the Vegetation Canopy

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## ABSTRACT

In this paper, a generalized layered model for radiation transfer in canopy with high vertical resolution is developed. Differing from the two-stream approximate radiation transfer model commonly used in the land surface models, the generalized model takes into account the effect of complicated canopy morphology and inhomogeneous optical properties of leaves on radiation transfer within the canopy. In the model, the total leaf area index (LAI) of the canopy is divided into many layers. At a given layer, the influences of diffuse radiation angle distributions and leaf angle distributions on radiation transfer within the canopy are considered. The derivation of equations serving the model are described in detail, and these can deal with various diffuse radiation transfers in quite broad categories of canopy with quite inhomogeneous vertical structures and uneven leaves with substantially different optical properties of adaxial and abaxial faces of the leaves. The model is used to simulate the radiation transfer for canopies with horizontal leaves to validate the generalized model. Results from the model are compared with those from the two-stream scheme, and differences between these two models are discussed.

**Key words:** generalized model of radiation transfer; non-uniform canopy; distributions of leaf angle and radiation angle; optical properties of adaxial and abaxial leaves

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## 1. Introduction

The plant canopy interacts with the atmosphere above and the soil surface below, and the processes are very complex. The physical processes of water and heat exchange and photosynthetic and biochemical processes (producing CO<sub>2</sub> transport) inside the canopy require full understanding of the radiation transfer process within the canopy. In research on the land surface process and model development, it is important to study the radiation transfer within a canopy and soil system for following reasons: First, the radiation absorbed by leaves in the canopy is one of the important factors affecting photosynthesis, water transpiration and energy exchange of the canopy. Secondly, the energy reaching the ground surface plays a very important role in determining the energy and water exchange in the soil layer. Thirdly, the canopy optical properties, for instance canopy reflectance, are important parameters in remote sensing studies to determine leaf amounts, plant biomass and forest resources. So

the canopy radiation transfer plays a key role in the development of the land surface physical and biochemical models, and is a very important research field.

Study of radiation transfer in the canopy is a classical subject. Due to the complexity of the canopy structure and its optical properties, there is still no perfect way to handle it. Monsi and Saeki (1953) proposed that the light decay in a canopy is subject to an exponential law, and they also gave a mathematical model for calculating the extinction coefficient of the canopy. But, this kind of exponential model is a little bit coarse to describe the radiation transfer in a canopy with a complex structure and affect the accuracy of the calculation.

Since then, many different mathematical formulations based on a great deal of observation data in radiative transfer within the canopy were developed depending primarily on their intended purposes (e.g., De Wit, 1965; Monteith, 1973; Lemeur, 1973; Norman, 1979, 1980; Ross, 1975), and they represent great

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progress in the research of canopy radiation. A comparatively successful theory for considering radiation transfer in the canopy was based on the assumption that the plant stand was a turbid medium, and the leaves were the elements for absorbing and scattering the radiation. This turbidity theory came from astronomy. Former U.S.S.R. scientists modified it and used it to describe the radiation transfer in plants in the 1960s. Ross (1981) and Myneni et al. (1988, 1989) discussed the theory and reviewed the past work on the radiation transfer theory in the plant canopy in detail. They proposed a general scheme for dealing with the radiation transfer in the canopy but did not present effective mathematical models to quantitatively solve the very complicated problems in a real canopy. Myneni et al. (1989) systemically summarized tens of mathematical models for specific cases of radiation transfer in the canopy. But most of the models were limited to cases such as the adaxial and abaxial leaf optical properties being even, the leaves in canopy being horizontal, or distributed symmetrically and so on. There is still a lack of effective methods to deal with the cases such as the leaf layers in different heights of the canopy being heterogeneous, the adaxial and abaxial leaf optical properties being different, the distribution of incident sky radiation being anisotropic, or the leaves in the canopy not being horizontal or distributed symmetrically.

Based on geometric optical theory, De Wit (1965), Norman and Jarvis (1975), Norman (1979, 1980) and Goudriaan (1977) proposed some schemes for radiation transfer in the canopy by strictly considering the interception, absorption, scattering (scattering is reflection + transmission) of radiation by the vegetation canopy with more complicated leaf angle distributions, and they developed real numerical models suitable for some simple cases such as the adaxial and abaxial leaf optical properties being the same and the vertical structure of the canopy layers being uniform. In the schemes, the azimuth and inclination angle distributions of the leaves and light are divided into several sectors (such as three, six or nine), and then they calculate the interception, absorption and scattering of light from different directions by leaves with various orientations in the canopy. The whole canopy is vertically divided into many layers to describe more accurately the mutual interaction among the different layers. Each layer is made so thin that the effect of multi-scattering within the thin layer can be neglected. These kinds of models can be called analog-physical models or layered models. The layered model developed by Norman and Jarvis (1975) took into account the different optical properties of adaxial and abaxial leaf surfaces, but the derived numerical model was simply based on the diffuse radiation angle distribution being isotropic and the diffuse radiation transfer

being considered as two streams (one upward and one downward). Goudriaan (1977) set up a layered model that could deal with the transfer of radiation with an anisotropic angle distribution of incident and diffuse radiation in the canopy, but it could not deal with the different adaxial and abaxial leaf optical properties.

In land surface model (LSM) development, Dickinson (1983) and Sellers (1985) introduced the two-stream scheme of atmosphere radiation transfer (Meador and Weaver, 1980) to handle the radiation transfer in the plant canopy in LSMs and to calculate the hemispheric canopy albedo of the radiation. Under some conditions, an accurate solution can be obtained from the scheme, which is very simple and efficient. The two-stream approximate model is now widely used in current land surface process models (e.g., Sellers, 1985; Sellers et al., 1986, 1996a, b; Dickinson et al., 1986, 2002). However, the derivation of the governing equations for the two-stream model used in canopy radiation transfer is based on the assumption that the incident sky diffuse radiation and the scattered radiation in the canopy are all isotropic in inclination, that the vertical structure of the canopy is uniform and that the optical properties of the adaxial and abaxial leaf surfaces are the same. Therefore the conclusion is that the two-stream model is not applicable to very general cases, for example, (1) when the sky radiation and the diffuse radiation in the canopy are distributed anisotropically, (2) when the optical properties of the adaxial and abaxial surfaces of the leaves are not necessarily the same, and (3) when the vertical structure of the canopy is not uniform. Besides, a critical problem has not been solved yet, namely determining the accuracy of the two-stream model when it is applied to the radiation transfer within the canopy.

In this paper, based on the concept of layered models, we develop a more generalized layered model with effective mathematical equations. The model can not only deal with more general distributions of sky diffuse radiation and diffuse radiation in the canopy, but it can also with the different adaxial and abaxial leaf optical properties and structures in the vertical canopy layers. The model seems to be a little complicated but it can be further simplified to a certain degree. The main aim of this paper is first to derive the basic equations of the generalized model and verify its rationality, and then compare the preliminary results obtained from this model with the two-stream model in order to find some issues in the different outputs from the two models. The simplification of the model will be undertaken in a future paper.

## 2. Radiation transfer in the canopy

### 2.1 Sky radiation distribution

The radiation within the canopy is composed of

two components: (1) the direct beam radiation, that is, the part of the sunbeam not being intercepted by leaves, and (2) the diffuse radiation consisting of unintercepted sky diffuse radiation and the radiation intercepted and then scattered (reflected and transmitted) by plant branches, leaves and soil surface.

The sunbeam has only one inclination angle, equal to sun elevation angle. But for the sky diffuse radiation, its intensity is generally supposed to be symmetrically distributed in the azimuthal direction, but in the inclined direction, its intensity in the entire 90° sector is generally defined by a distribution in several sub-sectors (such as 3, 6 or 9 sub-sectors in common use), and there are two commonly assumed distributions for intensity in every sector. One is the uniform overcast sky distribution (UOC) which states that the sky has an isotropic (law of cosines) downward radiation. Its distribution table according to 9 inclinations is given as  $B_u$ :

$$B_u(1, \dots, 9) = \{0.030, 0.087, 0.133, 0.163, 0.174, 0.163, 0.133, 0.087, 0.030\}.$$

The other one is the standard overcast sky (SOC) proposed by Moon and Spenser (1942) and later verified by Grace (1971). Its distribution table according to 9 inclinations is given as  $B_s$ :

$$B_s(1, \dots, 9) = \{0.015, 0.057, 0.106, 0.150, 0.180, 0.184, 0.160, 0.110, 0.038\}.$$

Actually, the sky diffuse radiation has a great variety of distributions, and it is not limited to the above two assumptions. The two-stream model can only deal with the UOC uniform distribution. But the generalized model can deal with any distribution of the sky diffuse radiation.

### 2.2 Light-leaf geometric optical theory and $G$ function

The radiation transfer in the canopy depends on the direction of the incident radiation with different wavebands, canopy structure, leaf orientation and their optical properties. Radiation with different incident directions shows different contributions to leaves with different directions. If the radiation ray is perpendicular to the leaf, the leaf will intercept the radiation completely. If the ray is parallel to the leaf, the leaf will not intercept the radiation. Since the azimuths of both leaves and solar radiation vary over the range of 360°, radiation with different inclinations will either completely fall on the adaxial leaf surface or partly fall on the adaxial face and partly on the abaxial face.

To express the mutual interaction of the light-leaf optical geometry exactly, some basic knowledge about spherical trigonometry is required (see Fig. 1).  $R$  is a incident light,  $R_o$  is its projection on the horizontal surface.  $\theta$  is the inclination angle of light impinging on the leaf adaxial surface ( $N$  is the leaf normal and

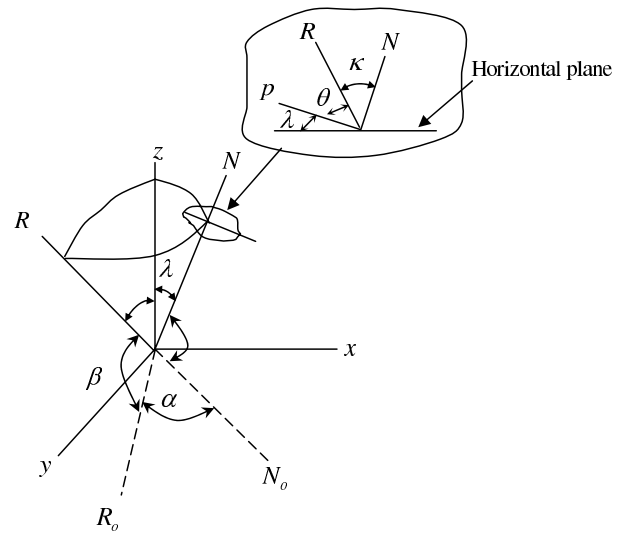


Fig. 1. Sketch map of light-leaf geometry.

$N_o$  is its projection on the horizontal surface), and  $\kappa$  is the zenith angle of the direction of incident light to the normal to the leaf adaxial surface and its complementary angle is  $\theta$ .  $\beta$  is the inclination angle of incident light to the horizontal plane.  $\lambda$  is the zenith angle of the normal of the leaf plane to the normal of the horizontal plane or the inclination angle of the leaf surface to the horizontal surface.  $\alpha$  is the difference in azimuth angles between the leaf normal and the incident ray. Since the azimuths of both the leaf normal and incident ray vary from 0° to 360° and since both the leaf and the radiation are assumed to be distributed symmetrically in azimuth, a simplification is to fix the leaf azimuth (or the incident light) angle at 0° and let the light azimuth (or leaf azimuth) angle vary from 0° to 360°. Generality will not be lost in the following derivation. Based on the cosine theorem of spherical trigonometry,  $\theta$  can be calculated by:

$$\sin \theta = \sin \beta \cos \lambda + \cos \beta \sin \lambda \cos \alpha, \quad (1)$$

where  $\sin \theta$  represents the projection of unit leaf area onto the light direction.

An important concept to describe the average projection of unit leaf area with a symmetrical distribution in azimuth onto the light direction is the  $G$  function:

$$G(\beta, \lambda) = \frac{\int_0^{2\pi} |\sin \theta| d\alpha}{\int_0^{2\pi} d\alpha}. \quad (2)$$

Here, the  $G$  function represents the effective portion of the unit leaf area to intercept the light, and it can be defined as the interception coefficient here. Since the light could either strike the adaxial leaf surface or part of the adaxial leaf surface and part of the abaxial leaf surface, the two cases of  $\lambda > \beta$  and  $\lambda \leq \beta$  must be distinguished for the calculation of  $G$ :

(1) For  $\lambda \leq \beta$ : all the light falls onto the adaxial leaf surface, thus,

$$\begin{aligned} G(\beta, \lambda) &= \frac{1}{2\pi} \int_0^{2\pi} (\sin \beta \cos \lambda + \cos \beta \sin \lambda \cos \alpha) d\alpha \\ &= \frac{1}{2\pi} (\sin \beta \cos \lambda \alpha + \cos \beta \sin \lambda \sin \alpha) \Big|_0^{2\pi} \\ &= \sin \beta \cos \lambda. \end{aligned} \quad (3)$$

(2) For  $\lambda > \beta$ : if the leaf azimuth  $\alpha$  is set to 0, there is a critical angle  $\alpha_c$  of the light azimuth. When the light azimuth is within  $0 \sim \pm\alpha_c$ , the light will fall onto the adaxial leaf surface. Otherwise, when the light is within  $\pm\alpha_c \sim \pi$ , the light will fall onto the abaxial leaf surface. The value of  $\alpha_c$  is determined by  $\sin \theta = 0$  where the incident light is parallel with the leaf plane, thus:

$$\cos \alpha_c = -\tan \beta \cot \lambda. \quad (4)$$

That is:

$$\alpha_c = \arcsin(\tan \beta \cot \lambda) + \pi/2. \quad (5)$$

Integrating Eq. (2) from  $-\alpha_c$  to  $\alpha_c$  and  $\alpha_c$  to  $2\pi - \alpha_c$ , we have:

$$\begin{aligned} G(\beta, \lambda) &= \frac{1}{2\pi} \left[ \int_{-\alpha_c}^{\alpha_c} \sin \theta d\alpha + \int_{\alpha_c}^{2\pi - \alpha_c} (-\sin \theta) d\alpha \right] \\ &= \frac{2}{\pi} \alpha_c \sin \beta \cos \lambda - \sin \beta \cos \lambda \\ &\quad + \frac{2}{\pi} \cos \beta \sin \lambda \sin \alpha_c. \end{aligned} \quad (6)$$

The leaf inclination angle in the canopy can vary within the range 0 to  $\pi/2$ . The inclination distribution of leaves can be described by the leaf angle distribution function  $g(\lambda)$  which is a normalized function and satisfies:

$$\int_0^{\pi/2} g(\lambda) \sin \lambda d\lambda \equiv 1. \quad (7)$$

Based on much field observation, De Wit (1965) pointed out that the leaf angle distributions could approximately be divided into planophile, erectophile, extremophile, plagiophile and spherical. In real applications, whole  $90^\circ$  intervals of inclination can be divided into  $n$  sub-intervals, for example,  $n = 9, 6$  or  $3$  and intervals of each subsection being  $10^\circ, 15^\circ$  or  $30^\circ$  respectively.

Thus,

$$g^*(\lambda) = \int_{10^\circ, 15^\circ \text{ or } 30^\circ} g(\lambda) \sin \lambda d\lambda$$

means the distribution density of unit leaf area within the sub-intervals of  $10^\circ, 15^\circ$  or  $30^\circ$ . Ross (1981) presented a number of distributions with 3 sub-intervals of  $0^\circ-30^\circ, 30^\circ-60^\circ$  and  $60^\circ-90^\circ$  for some plants in Figs. 25-31 and Tables I.6.1 and I.6.2 in his book.

Given the distribution function  $g(\lambda)$ , then the mean of the  $G$  function,  $\bar{G}(\beta)$ , can be obtained by integrating and averaging  $G$ :

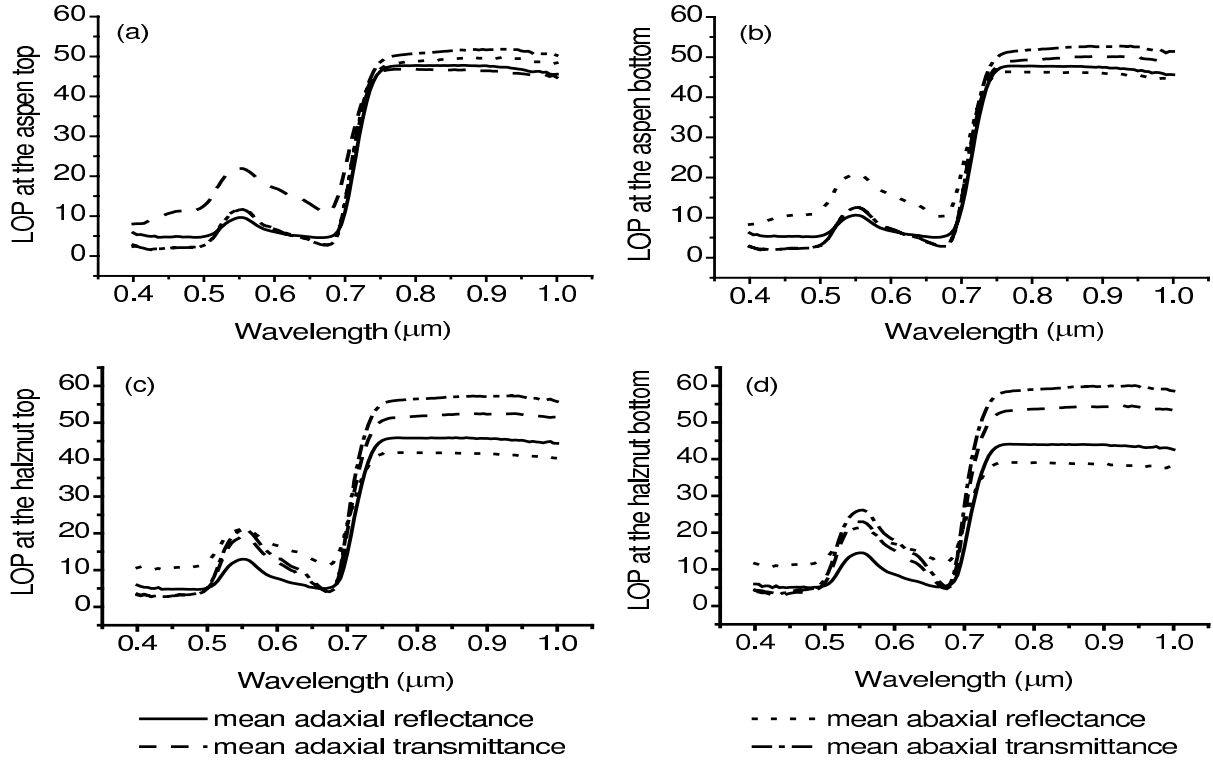
$$\bar{G}(\beta) = \int_0^{\pi/2} G(\beta, \lambda) g(\lambda) \sin \lambda d\lambda. \quad (8)$$

The function  $\bar{G}(\beta)$  indicates the averaged effective portion of the unit leaf area with a  $g(\lambda)$  distribution to intercept the light with incident angle  $\beta$ , and it can be defined as the average interception coefficient of unit leaf area. For horizontal leaves,  $\bar{G}(\beta) = \sin \beta$ ; for vertical leaves,  $\bar{G}(\beta) = 2/\pi \cos \beta$ ; and for spherical leaves with leaf angle distribution  $g(1, \dots, 9) = \{0.015, 0.045, 0.074, 0.099, 0.124, 0.143, 0.158, 0.168, 0.174\}$ ,  $\bar{G}(\beta) = 0.5$ .

### 2.3 Canopy structure

All the crowns of the plants, which are called the canopy, have a very complicated structure and physical properties. The morphology of the plant canopy in the vertical direction is generally not uniform. The orientation of its leaves at any height has various angle distributions and is not necessarily spherical in inclination or symmetrical in azimuth. The optical properties of both surfaces of the leaves may not be the same. The scattering properties of the leaves could be isotropic or anisotropic. Jiang (1996) exemplified a forest canopy in which the canopy could be divided into many species layers, such as trees, shrubs, herbs, and moss, and their optical properties were quite different. Yet, the difference in angle distribution of different layers is very obvious: the leaf angles at the lower layers are distributed mostly as planophiles, which is the result of adapting to the weak light with their horizontal and extended leaves helping them to receive more transmitted radiation; the leaf angles at the upper layers are distributed mostly as plagiophiles, and the leaf angles in the middle layers, are distributed between planophiles and plagiophiles. The measurement of leaf angles of the deciduous plants in East Tennessee of the U.S. indicates that the average leaf angle of the plants is about  $33^\circ$ , but the average leaf angles are  $38^\circ$  in the upper canopy layer (height above 17 m),  $20^\circ$  in the middle (height between 8 and 17 m), and  $10^\circ$  in the lower layers (height under 8 m) (e.g., Wang and Wang, 1999). Hence for high plants, regardless of differences in optical properties or in layer structures, it is important to divide the canopy into vertical layers for a radiation transfer study.

The adaxial and abaxial leaf optical properties can also be different. Moreover, the difference tends to be obvious sometimes from the measurements. Figure 2 gives the adaxial and abaxial aspen and hazelnut leaf reflectance and transmittance measured in the Boreas Project in the U.S. It can be found that the differences in optical properties between the adaxial and abaxial



**Fig. 2.** The adaxial and abaxial leaf reflectance and transmittance of aspen (a, b) and hazelnut (c, d) observed in the multi-layer vegetation in the Boreas experiment (Mesarch, et al., 1998). LOP: Leaf optical properties.

leaf surfaces are great. Even for adaxial and abaxial leaf surfaces with the same optical properties, the reflectance of the adaxial leaf surface will increase greatly when there is snow covering the surface, but the reflectance of the abaxial face with no snow cover will change little. Thus snow cover makes a great difference in the optical properties for adaxial and abaxial leaf surfaces. This kind of difference should be considered in the radiation transfer model.

### 3. Equations of the generalized layered model

#### 3.1 General expression for the leaf radiation transfer equation

As discussed above, a quite successful theory for radiation transfer within the vegetation canopy developed previously is based on the assumption that the canopy is a turbid continuous medium and the plant laminae and branches are the elements for absorbing and scattering the radiation. The derivation of the equation for radiation transfer in the canopy from the theory is very complicated. (Ross, 1981; Myneni et al., 1988, 1989; Huang, 1997). Here, only the final expression of the generalized radiation transfer equation presented in their works is listed:

$$-\mu \frac{\partial \phi(\beta', L)}{\partial L} + \bar{G}(\beta', L) \phi(\beta', L) =$$

$$\begin{aligned} & \frac{\omega}{4\pi} \int_{4\pi} P(\beta \rightarrow \beta') \bar{G}(\beta, L) \phi(\beta, L) d\beta + \\ & \frac{\omega}{4\pi} \bar{G}(\beta_0, L) P(\beta_0 \rightarrow \beta') I(L=0) \times \\ & \exp\left(\frac{\int_0^L -\bar{G}(\beta_0, L) dL}{\mu_0}\right), \end{aligned} \quad (9)$$

where  $\beta$  is the inclination angle of incident diffuse radiation,  $\beta_0$  is the inclination angle of the direct sunbeam,  $\beta'$  is the inclination angle of scattered diffuse radiation,  $I(L=0)$  is the beam flux at the canopy top,  $L$  is the accumulated leaf area index from the canopy top to the place in the canopy being studied,  $\phi(\beta', L)$  is the radiation flux in  $\beta'$  at  $L$ ,  $\omega$  is the leaf scattering coefficient and is the sum of leaf reflectance  $\rho$  and leaf transmittance  $\tau$ ,  $P(\beta \rightarrow \beta')$  is the normalized scattering phase function of the radiation and represents the fraction of the intercepted radiation energy which is scattered in the direction  $\beta'$ , and  $\mu = \sin \beta$  and  $\mu_0 = \sin \beta_0$ . In Eq. (9), the first term on the left side is the diffuse radiation  $\phi(\beta', L)$  change with leaf area index within the canopy, and the second term is the intercepted loss of diffuse radiation  $\phi(\beta', L)$  itself. The first term on the right side is the contribution of the diffuse light coming from all directions to  $\phi(\beta', L)$ , and the second term is the contribution of a sunbeam with direction  $\beta_0$  to  $\phi(\beta', L)$ .

Equation (9) is the general light transfer expression. It is only applied to a horizontally extended uniform leaf canopy, broad and flat leaf lamina and dense vegetation. It is an integral-differential equation and it is very difficult to find its solution for many complicated cases as discussed in section 1. Most existing solutions mainly tackle the cases of a canopy with the same adaxial and abaxial optical properties of the leaves, and/or uniform structure in the vertical direction, and/or isotropic diffuse radiation. There are few works that deal with radiation transfer in the canopy with dissimilar optical properties of adaxial and abaxial leaves, anisotropic diffuse radiation, or non-uniform vertical structure of the canopy. The aim of this paper is to develop the generalized layered model called the generalized model and to derive the equations for the model that, with computer aid, can deal with wide practical problems such as anisotropic diffuse radiation, uneven adaxial and abaxial leaf optical properties, nonuniform vertical canopy structure, or their different combinations.

**3.2 Description of the generalized model and governing equation**

The generalized model simulates the physical process of radiation transfer in a canopy (see Fig. 3). There is only one loose restriction on the model: that the canopy is required to be horizontally infinite, con-

tinuous, and uniform, and therefore there is no mutual interaction between the horizontal layers.

The transfer of the direct sunbeam radiation within the canopy will be considered first (see Fig. 3). The unintercepted part of the direct beam is transferred continuously downward by keeping its original direction, and the intercepted part turns into diffuse radiation scattering in all directions. Both the direct beam and the diffuse light falling onto the soil will scatter isotropically upward in all directions. The decay of the direct beam along with the increase of the leaf area index in the canopy obeys Beer's (exponential extinction) law as follows:

$$I(L_j) = I(L_0)e^{-\int_0^{L_j} \frac{\bar{G}(\beta_0, L)}{\sin \beta_0} dL} \tag{10a}$$

If  $\bar{G}(\beta_0, L)$  is not a function of  $L$ , then

$$I(L_j) = I(L_0)e^{-k_d \cdot L_j} \tag{10b}$$

where  $\beta_0$  is the inclination angle of the direct sunbeam to the horizontal surface,  $I(L_0)$  and  $I(L_j)$  are the beam fluxes at the top of the canopy and at layer  $j$  with accumulated leaf area  $L_j (L_j = \sum_{k=1}^{j-1} dL_k)$  respectively, and  $k_d = \frac{\bar{G}(\beta_0)}{\sin \beta_0}$  is the extinction coefficient of the sunbeam.

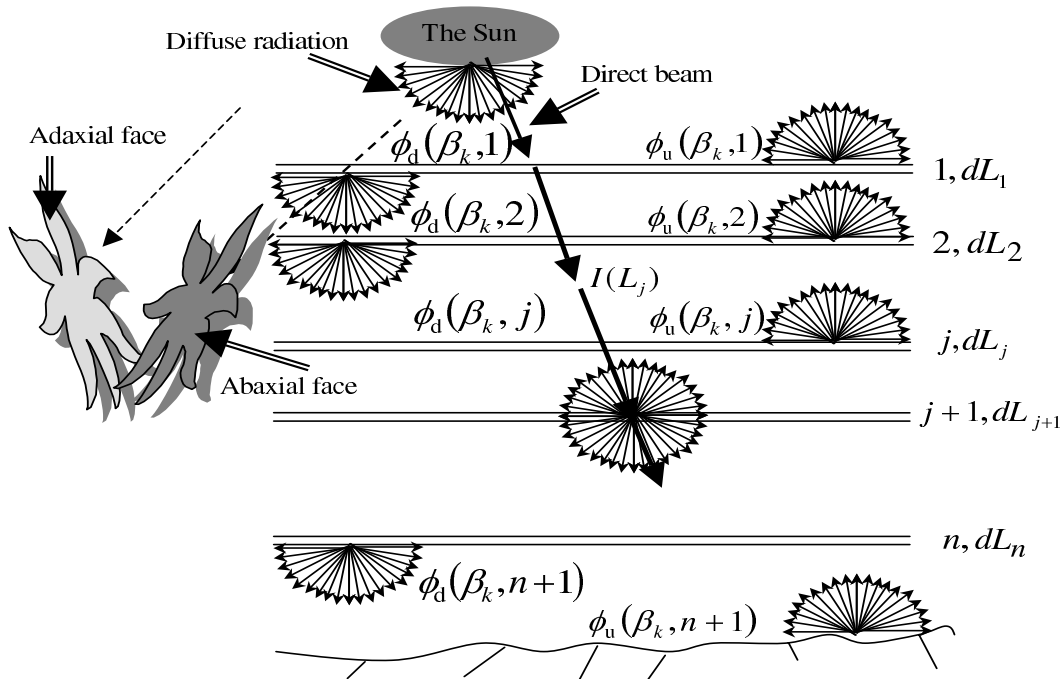


Fig. 3. Layered canopy radiation schematic diagram.

Next we consider diffuse radiation transfer within the canopy. It is a commonly accepted assumption that the radiation from the sky and that within the canopy are all symmetrically distributed in azimuth, and that the leaves are also symmetrically distributed in azimuth. As mentioned above, for simplicity but without loss of generality, the azimuth of either leaves or the diffuse radiation rays could be set to  $0^\circ$ .

In the generalized model, all diffuse radiation (including that from the sky and within the canopy) is equally divided into  $k$  (here  $k = 9$ ) sub-beams of light over the entire inclination range of  $90^\circ$ . The canopy with total leaf area index (LAI) is vertically divided into  $n$  sub-layers and the order number of each layer is from the canopy top to the bottom, for example,  $j = 1$  is for the top layer and  $j = n + 1$  is for the soil surface (refer to Fig. 3).  $dL_j$  is the leaf area index for layer  $j$ . To avoid the mutual shading interaction and multiple scattering effects within the same layer, the leaf area index in the sub-layer should be very small, and, according to the work of Goudriaan (1977),  $dL_j \leq 0.1$  is a reasonable selection.

For the convenience of derivation and depiction later on, our notation is defined as follows.  $\beta_k$  ( $k = 1, 2, 3, \dots, 9$ ) is the incident inclination angle for diffuse radiation,  $\beta'_k$  ( $k = 1, 2, 3, \dots, 9$ ) is the inclination angle of the scattered radiation from the intercepted radiation,  $\lambda_n$  ( $n = 1, 2, 3, \dots, 9$ ) is the leaf inclination angle,  $B_u(\beta'_k)$  is the scattering distribution function for the leaves as Lambertian radiators,  $B_l(\beta'_k, \lambda_n)$  is the anisotropic scattering distribution function for the canopy layer, and  $\phi_d(\beta_k, j)$  and  $\phi_u(\beta_k, j)$  are the downward and upward radiant fluxes with inclination angle  $\beta_k$  between layers  $j - 1$  and  $j$  respectively.

For diffuse radiation, the radiation intercepted by layer  $j$  with leaf area  $dL_j$  is proportional to the average projection ( $G$  function) of unit leaf area to the incident light direction, and it is inversely proportional to the sine of the inclination of the incident light. Therefore for the leaves with a specific inclination  $\lambda_n$  in layer  $j$ , the intercepted fraction is given by:

$$M_i(\beta_k, \lambda_n) = dL_j G(\beta_k, \lambda_n) / \sin \beta_k, \quad (11)$$

and the unintercepted radiation penetrating through the layer is:

$$M_t(\beta_k, \lambda_n) = 1 - M_i(\beta_k, \lambda_n). \quad (12)$$

In this generalized model, the anisotropic scattering distribution function  $B_l(\beta'_k, \lambda_n)$  adopts the expression proposed by Goudriaan (1977) which is commonly used at present (Wohlfahrt et al., 2000). It is:

$$B_l(\beta'_k, \lambda_n) = \frac{B_u(\beta'_k) M_i(\beta'_k, \lambda_n)}{\sum_{\beta_k=1}^9 B_u(\beta_k) M_i(\beta_k, \lambda_n)}. \quad (13)$$

The reason for using Eq. (13) was given by Goudriaan (1977) as follows: although the leaves in the model are Lambertian radiators, the light scattered by a layer is not isotropic, contrary to an assumption of De Wit (1965). For a certain leaf inclination  $\lambda_n$ , the projection of  $dL_j$  with the light inclination  $\beta_k$  is  $G(\beta_k, \lambda_n)$ , and the interception fraction of  $dL_j$  is given by Eq. (11). The anisotropy of the scattered light is due to the variation of the intercepted fraction  $M_i$  with  $\beta_k$ , because the scattered radiation is proportional to  $M_i$ . Only for horizontal leaves is  $M_i(\beta_k, \lambda_n)$  invariant with  $\beta_k$ . The scattered light is then isotropic so that the scattered radiant flux through a horizontal surface is distributed with  $\beta'_k$  as  $B_u(\beta'_k)$  (Goudriaan, 1977).

Defining the adaxial leaf reflectance and transmittance as  $\rho_j$  and  $\tau_j$  and the abaxial leaf reflectance and transmittance as  $\rho'_j$  and  $\tau'_j$  for layer  $j$  respectively, the equations for the downward and upward radiation leaving layer  $j$  with only a single leaf angular inclination  $\lambda_n$  are:

$$\begin{aligned} \phi_d(\beta'_k, j+1) &= \phi_d(\beta'_k, j) M_t(\beta'_k, \lambda_n) + \\ & B_l(\beta'_k, \lambda_n) \sum_{\beta_k=1}^k M_i(\beta_k, \lambda_n) \times \\ & [\phi_d(\beta_k, j)(\rho_j \zeta_f + \rho'_j \zeta_b + \tau_j \xi_f + \tau'_j \xi_b) + \\ & \phi_u(\beta_k, j+1)(\rho_j \xi_b + \rho'_j \xi_f + \tau_j \zeta_b + \tau'_j \zeta_f)] + \\ & B_l(\beta'_k, \lambda_n) I_B(z = L_j) \cdot dL_j \cdot \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} \times \\ & (\rho_j \zeta_{f,B} + \rho'_j \zeta_{b,B} + \tau_j \xi_{f,B} + \tau'_j \xi_{b,B}). \end{aligned} \quad (14a)$$

$$\begin{aligned} \phi_u(\beta'_k, j) &= \phi_u(\beta'_k, j+1) M_t(\beta'_k, \lambda_n) + \\ & B_l(\beta'_k, \lambda_n) \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) \times \\ & [\phi_d(\beta_k, j)(\rho_j \xi_f + \rho'_j \xi_b + \tau_j \zeta_f + \tau'_j \zeta_b) + \\ & \phi_u(\beta_k, j+1)(\rho_j \xi'_f + \rho'_j \xi'_b + \tau_j \zeta'_f + \tau'_j \zeta'_b)] + \\ & B_l(\beta'_k, \lambda_n) I_B(z = L_j) \cdot dL_j \cdot \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} \times \\ & (\rho_j \xi_{f,B} + \rho'_j \xi_{b,B} + \tau_j \zeta_{f,B} + \tau'_j \zeta_{b,B}). \end{aligned} \quad (14b)$$

$\xi_f$ ,  $\xi_b$ ,  $\zeta_f$  and  $\zeta_b$  are the ratios of the four parts to the total scattered contribution of  $\phi_d(\beta_k, j)$  to  $\phi_u(\beta'_k, j)$  under the condition that the intercepted radiation is scattered symmetrically. They are functions of  $\beta'_k$ ,  $\beta_k$  and  $\lambda_n$ :  $\xi_f = \xi_f(\beta_k, \beta'_k, \lambda_n)$ ,  $\xi_b = \xi_b(\beta_k, \beta'_k, \lambda_n)$ ,  $\zeta_f = \zeta_f(\beta_k, \beta'_k, \lambda_n)$ , and  $\zeta_b = \zeta_b(\beta_k, \beta'_k, \lambda_n)$ . In Eq. (14a), the first term on the right side is the radiation contributed by the unintercepted part of  $\phi_d(\beta'_k, j)$ . The

second term includes contributions from  $K$  ( $K = 9$ ) incident diffuse radiation elements at 9 respective  $\beta_k$  angles. Each element represents the radiation contributed by scattering the intercepted part of  $\phi_d(\beta_k, j)$  and  $\phi_u(\beta_k, j + 1)$ . The scattering contribution consists of four parts. Part 1 is the radiation reflected by the adaxial leaf surface with its reflectance  $\rho$ . Part 2 is the radiation reflected by the abaxial leaf surface with its reflectance  $\rho'$ . Part 3 is the radiation transmitted through the adaxial leaf surface with its transmittance  $\tau$ . And Part 4 is the radiation transmitted through the abaxial leaf surface with its transmittance  $\tau'$ . The last term is the radiation contributed by the sunbeam, which only consists of similar four parts.  $\xi_{f, B}, \xi_{b, B}, \zeta_{f, B}$  and  $\zeta_{b, B}$  are the same functions as  $\xi_f, \xi_b, \zeta_f$  and  $\zeta_b$  but they use beam inclination angle  $\beta_0$  instead of  $\beta_k$ . The subscript B refers to beam radiation. So,  $\xi_{f, B} = \xi_{f, B}(\beta_0, \beta'_k, \lambda_n)$ ,  $\xi_{b, B} = \xi_{b, B}(\beta_0, \beta'_k, \lambda_n)$ ,  $\zeta_{f, B} = \zeta_{f, B}(\beta_0, \beta'_k, \lambda_n)$  and  $\zeta_{b, B} = \zeta_{b, B}(\beta_0, \beta'_k, \lambda_n)$ . Detailed expressions of  $\xi_f, \xi_b, \zeta_f, \zeta_b, \xi_{f, B}, \xi_{b, B}, \zeta_{f, B}$  and  $\zeta_{b, B}$ , and also the verification of  $\xi'_f = \xi_b, \xi'_b = \zeta_f, \zeta'_f = \xi_b$  and  $\zeta'_b = \xi'_f$  are given in Appendix A.

Similarly to Eq. (14a), the three terms and relative coefficients on the right side of Eq. (14b) have the meanings similar to those in Eq. (14a).

For the more general case that the canopy consists of leaves with an inclination distribution function defined as  $g(\lambda_n, L_j)$ , the equations for the downward and upward radiation at  $\beta'_k$  in the layer  $j$  are given by

$$\begin{aligned} \phi_d(\beta'_k, j + 1) = & \phi_d(\beta'_k, j) \sum_{\lambda_n=1}^N g(\lambda_n, L_j) M_t(\beta'_k, \lambda_n) + \\ & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) \times \\ & \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) [\phi_d(\beta_k, j) \times \\ & (\rho_j \zeta_f + \rho'_j \zeta_b + \tau_j \xi_f + \tau'_j \xi_b) + \\ & \phi_u(\beta_k, j + 1) (\rho_j \xi_b + \rho'_j \xi_f + \tau_j \zeta_b + \tau'_j \zeta_f)] + \\ & \sum_{\lambda_n=1}^N g(\lambda_n) B_1(\beta'_k, \lambda_n) I_B(z = L_j) dL_j \times \\ & \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} (\rho_j \zeta_{f, B} + \\ & \rho'_j \zeta_{b, B} + \tau_j \xi_{f, B} + \tau'_j \xi_{b, B}), \quad (15a) \end{aligned}$$

$$\phi_u(\beta'_k, j) = \phi_u(\beta'_k, j + 1) \sum_{\lambda_n=1}^N g(\lambda_n, L_j) M_t(\beta'_k, \lambda_n) +$$

$$\begin{aligned} & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) \times \\ & \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) [\phi_d(\beta_k, j) \times \\ & (\rho_j \xi_f + \rho'_j \xi_b + \tau_j \zeta_f + \tau'_j \zeta_b) + \\ & \phi_u(\beta_k, j + 1) (\rho_j \zeta_b + \rho'_j \zeta_f + \tau_j \xi_b + \tau'_j \xi_f)] + \\ & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) I_B(z = L_j) dL_j \times \\ & \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} (\rho_j \xi_{f, B} + \\ & \rho'_j \xi_{b, B} + \tau_j \zeta_{f, B} + \tau'_j \zeta_{b, B}). \quad (15b) \end{aligned}$$

If the adaxial and abaxial leaf optical properties are even, that is,  $\rho'_j = \rho_j$  and  $\tau'_j = \tau_j$ , then Eqs. (15) become:

$$\begin{aligned} \phi_d(\beta'_k, j + 1) = & \phi_d(\beta'_k, j) \sum_{\lambda_n=1}^N g(\lambda_n, L_j) M_t(\beta'_k, \lambda_n) + \\ & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) \times \\ & \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) \{ \phi_d(\beta_k, j) \times \\ & [\rho_j(1 - \xi) + \tau_j \xi] + \phi_u(\beta_k, j + 1) \times \\ & [\rho_j \xi + \tau_j(1 - \xi)] \} + \sum_{\lambda_n=1}^N g(\lambda_n, L_j) \times \\ & B_1(\beta'_k, \lambda_n) I_B(z = L_j) dL_j \times \\ & \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} [\rho_j(1 - \xi_B) + \tau_j \xi_B], \quad (16a) \end{aligned}$$

$$\begin{aligned} \phi_u(\beta'_k, j) = & \phi_u(\beta'_k, j + 1) \sum_{\lambda_n=1}^N g(\lambda_n, L_j) M_t(\beta'_k, \lambda_n) + \\ & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) \times \\ & \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) \{ \phi_d(\beta_k, j) \times \\ & [\rho_j \xi + \tau_j(1 - \xi)] + \phi_u(\beta_k, j + 1) \\ & [\rho_j(1 - \xi) + \tau_j \xi] \} + \sum_{\lambda_n=1}^N g(\lambda_n, L_j) \times \\ & B_1(\beta'_k, \lambda_n) I_B(z = L_j) dL_j \times \\ & \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} [\rho_j \xi_B + \tau_j(1 - \xi_B)], \quad (16b) \end{aligned}$$



where  $\xi = \xi_f + \xi_b = \xi(\beta_k, \beta'_k, \lambda_n)$ ,  $\xi_B = \xi_{f,B} + \xi_{b,B} = \xi_B(\beta_k, \beta'_k, \lambda_n)$ .

If the vegetation is uniform in the vertical direction, that is,  $\rho_j = \rho$  and  $\tau_j = \tau$ , then Eqs. (16) become:

$$\begin{aligned} \phi_d(\beta'_k, j+1) = & \phi_d(\beta'_k, j) \sum_{\lambda_n=1}^N g(\lambda_n, L_j) M_t(\beta'_k, \lambda_n) + \\ & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) \times \\ & \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) \{ \phi_d(\beta_k, j) \times \\ & [\rho(1-\xi) + \tau\xi] + \phi_u(\beta_k, j+1) \times \\ & [\rho\xi + \tau(1-\xi)] \} + \sum_{\lambda_n=1}^N g(\lambda_n, L_j) \times \\ & B_1(\beta'_k, \lambda_n) I_B(z = L_j) dL_j \times \\ & \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} [\rho(1-\xi_B) + \tau\xi_B], \quad (17a) \end{aligned}$$

$$\begin{aligned} \phi_u(\beta'_k, j) = & \phi_u(\beta'_k, j+1) \sum_{\lambda_n=1}^N g(\lambda_n, L_j) M_t(\beta'_k, \lambda_n) + \\ & \sum_{\lambda_n=1}^N g(\lambda_n, L_j) B_1(\beta'_k, \lambda_n) \times \\ & \sum_{\beta_k=1}^K M_i(\beta_k, \lambda_n) \{ \phi_d(\beta_k, j) \times \\ & [\rho\xi + \tau(1-\xi)] + \phi_u(\beta_k, j+1) \times \\ & [\rho(1-\xi) + \tau\xi] \} + \sum_{\lambda_n=1}^N g(\lambda_n, L_j) \times \\ & B_1(\beta'_k, \lambda_n) I_B(z = L_j) dL_j \times \\ & \frac{G(\beta_0, \lambda_n)}{\sin \beta_0} [\rho\xi_B + \tau(1-\xi_B)]. \quad (17b) \end{aligned}$$

Using the scheme for direct and diffuse radiation above, we can model either the visible radiation (VIS) or near-infrared radiation (NIR) transfer within the canopy, provided that the incident radiation on the canopy top, the leaf area index, leaf angle distribution, and the corresponding leaf and soil optical parameters (leaf reflectance, leaf transmittance, and soil reflectance) for the specific wave band are given.

Compared with the two-stream scheme, the model with Eqs. (14) and (17) can be used in more general cases: anisotropic distribution of both incident sky radiation and diffuse radiation within the canopy, uneven optical properties of adaxial and abaxial leaf

surfaces, and differing leaf angle distributions in each layer, etc. So the model developed in this paper greatly expands the application for the research of radiation transfer within the canopy.

#### 4. Model verification

For horizontal leaves the fraction of radiation intercepted by a layer is always equal to the leaf area index of the layer, independent of the light inclination (Goudriaan, 1977). The radiative transfer in canopy with horizontal leaves will be the same under the same direct beam and diffuse radiation because of the linear superposition of the light. So, the case with horizontal leaves is a good one to verify the model's behavior. In this study, a comparison of the results from the existing two-stream model and the generalized model is conducted. Since the two-stream model can only deal with a canopy with equal adaxial and abaxial leaf optical properties and a canopy with uniform vertical morphology, our comparison will be restricted to these conditions. For the generalized model, the inclination angles of both diffuse radiation and leaves are equally divided into 9 segments. So,  $k = 1, 2, 3, \dots, 9$  in  $\beta_k$  and  $n = 1, 2, 3, \dots, 9$  in  $\lambda_n$ . Because both models follow the principle of linear superposition, the incident beam and diffuse radiation can be set to unity in the following simulations.

Several sets of simulation studies are designed for the comparison. They include incident diffuse radiation or direct beam, three leaf area indexes of 1, 5 and 8 stand for different typical vegetations, three sets of leaf optical properties (such as large, medium and small) to cover different leaves (including those with snow cover) and different radiations (such as visible or near infrared wave bands), and different values of soil reflectance including the case with snow cover. The combinations of a set of experiment with different leaf area indexes, different leaf optical properties and different soil reflectances can be classified into the 12 groups and are shown in Appendix B. Group N1 with n1-n6 sub-groups covers the combination of the leaf optical properties of the VIS waveband, soil reflectance (small or large), and leaf area index (large, medium or small). Group N2 with n7-n12 sub-groups includes the combination of the leaf optical properties in the NIR waveband, soil reflectance (large or small), and leaf area index (large, medium or small).

For the cases with only sky diffuse radiation, the results from the two models for the 12 groups are completely equal to each other. Table 1 lists the results from the 12 groups. The equality shown in Table 1 demonstrates the correctness of the generalized model developed here.

**Table 1.** Comparison of the generalized model (NEW) and the two-stream model (OLD) for the results of canopy reflectance, transmittance, absorptance, and soil absorptance, under diffuse radiation. (Leaf angles are horizontal, and the 12 experiments deal with the combinations of leaf area index, leaf reflection, leaf transmission, and soil reflection. Please find the respective parameters for the 12 cases in Appendix B.).

Case	Canopy reflectance		Canopy transmittance		Canopy absorptance		Soil absorptance		$\Delta = \text{NEW} - \text{OLD}$			
	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD	Canopy reflectance	Canopy transmittance	Canopy absorptance	Soil absorptance
n1	0.184	0.185	0.422	0.424	0.732	0.731	0.084	0.085	-0.001	-0.002	0.001	-0.001
n2	0.08	0.08	0.41	0.412	0.592	0.591	0.328	0.329	0.000	-0.002	0.001	-0.001
n3	0.056	0.056	0.012	0.012	0.942	0.942	0.002	0.002	0.000	0.000	0.000	0.000
n4	0.056	0.056	0.011	0.012	0.935	0.935	0.009	0.009	0.000	-0.001	0.000	0.000
n5	0.056	0.056	0.001	0.001	0.944	0.944	0.000	0.000	0.000	0.000	0.000	0.000
n6	0.056	0.056	0.001	0.001	0.943	0.944	0.001	0.001	0.000	0.000	-0.001	0.000
n7	0.582	0.582	0.696	0.697	0.279	0.278	0.139	0.139	0.000	-0.001	0.001	0.000
n8	0.344	0.343	0.571	0.573	0.199	0.199	0.457	0.458	0.001	-0.002	0.000	-0.001
n9	0.424	0.424	0.103	0.107	0.555	0.555	0.021	0.021	0.000	-0.004	0.000	0.000
n10	0.419	0.419	0.076	0.078	0.52	0.519	0.061	0.062	0.000	-0.002	0.001	-0.001
n11	0.42	0.42	0.021	0.025	0.575	0.575	0.004	0.005	0.000	-0.004	0.000	-0.001
n12	0.42	0.42	0.016	0.018	0.567	0.566	0.013	0.014	0.000	-0.002	0.001	-0.001

**Table 2.** Comparison of the generalized model (NEW) and the two-stream model (OLD) for the results of canopy reflectance, transmittance, absorptance, and soil absorptance, under beam radiation. (Leaf angles are horizontal, and the incident radiation is beam radiation with arbitrary angles. Please find the respective parameters of the 12 cases in Appendix B.).

Case	Canopy reflectance		Canopy transmittance		Canopy absorptance		Soil absorptance		$\Delta = \text{NEW} - \text{OLD}$			
	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD	Canopy reflectance	Canopy transmittance	Canopy absorptance	Soil absorptance
n1	0.184	0.185	0.424	0.424	0.731	0.731	0.085	0.085	-0.001	0.000	0.000	0.000
n2	0.08	0.08	0.412	0.412	0.59	0.591	0.33	0.329	0.000	0.000	-0.001	0.001
n3	0.056	0.056	0.012	0.012	0.941	0.942	0.002	0.002	0.000	0.000	-0.001	0.000
n4	0.056	0.056	0.012	0.012	0.935	0.935	0.009	0.009	0.000	0.000	0.000	0.000
n5	0.056	0.056	0.001	0.001	0.944	0.944	0.000	0.000	0.000	0.000	0.000	0.000
n6	0.056	0.056	0.001	0.001	0.943	0.944	0.001	0.001	0.000	0.000	-0.001	0.000
n7	0.584	0.564	0.698	0.746	0.276	0.287	0.14	0.149	0.02	-0.048	-0.011	-0.009
n8	0.345	0.307	0.573	0.613	0.197	0.202	0.459	0.49	0.038	-0.04	-0.005	-0.031
n9	0.425	0.385	0.104	0.123	0.554	0.613	0.021	0.006	0.04	-0.019	-0.059	0.015
n10	0.421	0.38	0.076	0.089	0.518	0.602	0.061	0.017	0.041	-0.013	-0.084	-0.044
n11	0.422	0.382	0.021	0.028	0.574	0.613	0.004	0.006	0.04	-0.007	-0.039	-0.002
n12	0.422	0.381	0.017	0.021	0.565	0.602	0.013	0.017	0.041	-0.004	-0.037	-0.004

**Table 3.** Comparison of the generalized model (NEW) and the two-stream model (OLD) for the results of canopy reflectance under diffuse radiation. (Leaf angles are vertical. Please find the respective parameters of the 6 cases in Appendix B.).

	Case					
	n7	n8	n9	n10	n11	n12
NEW	0.58	0.269	0.334	0.296	0.307	0.296
OLD	0.639	0.283	0.411	0.372	0.387	0.38
$\Delta = \text{NEW} - \text{OLD}$	-0.059	-0.014	-0.077	-0.076	-0.08	-0.084

If only beam radiation is incident on the horizontal leaves, the radiation from all incident inclinations should be the same, and they should also be the same as the result of diffuse radiation under the same condition. By the comparison of the results from Table 2 and Table 1, it is very clear that the generalized model fits the above theoretical conclusion. The results in Table 5 presented by Goudriaan (1977) also demonstrate the model correctness. However, the results of the two-stream model do not fit, that is, its diffuse result is not the same as the beam result. It seems that the results from beam and diffuse radiation are not consistent in the two-stream model. It can be found from Table 2 that, if both the leaf reflectance and transmittance are great (experiments n7–n12), the canopy reflectance differences between the two models are greater than 0.02, and the greatest difference even reaches a magnitude of 0.04.

Comparisons of the two models for the non-horizontal leaf distributions are also conducted. There are certain differences between the results. We will discuss them in detail in a future paper. Table 3 gives the reflectance differences of vertical leaves for the two models in the NIR wave band under diffuse radiation. We can see that the differences are greater than 0.05 for all the cases except case n8.

## 5. Summary

This paper discusses a generalized layered radiative transfer model in the vegetation canopy with high resolution, which is a physical analog of the radiation transfer process within the canopy. The model is quite generalized and capable. It can deal with the transfer of anisotropic sky diffuse radiation with anisotropic distributions in various canopies including those that have a nonuniform morphological structure in the vertical direction and uneven leaves with substantially different optical properties in their adaxial and abaxial faces.

The model simulates horizontal leaves in various conditions. The simulated results with horizontal leaves from this model are compared with those from the two-stream model, and these verify the correctness of the generalized model and show some differences between the two models in the NIR waveband under beam radiation. The canopy reflectance results by the generalized model are larger than those by the two-stream model with horizontal leaves under the beam radiation. The canopy reflectance differences for the cases in the NIR waveband are about 0.04 except in n7, which its leaf area index is small and the soil reflectance is large. The canopy reflectance difference in case n7 is 0.02. The comparison of the two model results for non-horizontal leaves show certain differences.

For example, the canopy reflectance results by the generalized model are less than those by the two-stream model with vertical leaves in NIR waveband under diffuse radiation. The absolute values of the canopy reflectance differences for the cases are all larger than 0.05 except in n8, in which case both of the leaf area index and the soil reflectance are large.

The canopy in the model is divided into many layers based on the principle of avoiding the effect of multi-scattering between layers. The model is more accurate for more layers in a certain degree to decrease the possible errors mathematically. But in order to save computing time and meet the requirement of the LSM models, the model must be simplified in the vertical layers.

In this generalized model, the anisotropic scattering distribution function in Eq. (13) adopts the expression by Goudriaan (1977). It was based on the Lambertian leaves, and given directly. Maybe we can derive another more reasonable expression based on the light-leaf geometry.

Future studies on the generalized model will go into greater depth to show the importance of the development of this kind of model and will simplify the model to a certain degree for its use in LSM models later on.

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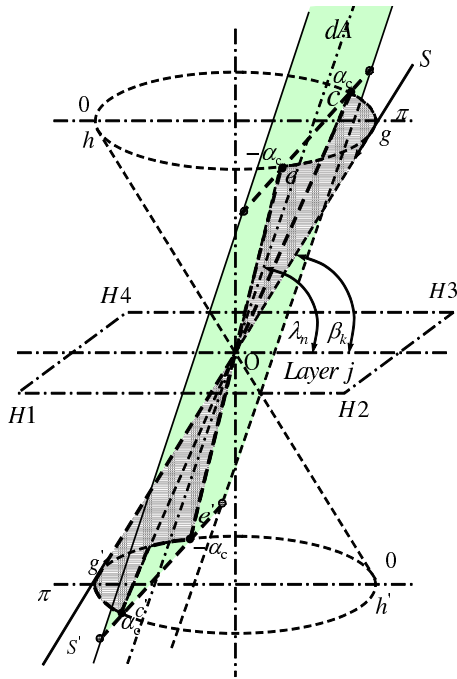
## APPENDIX A

### Partitioning of Scattered Radiation

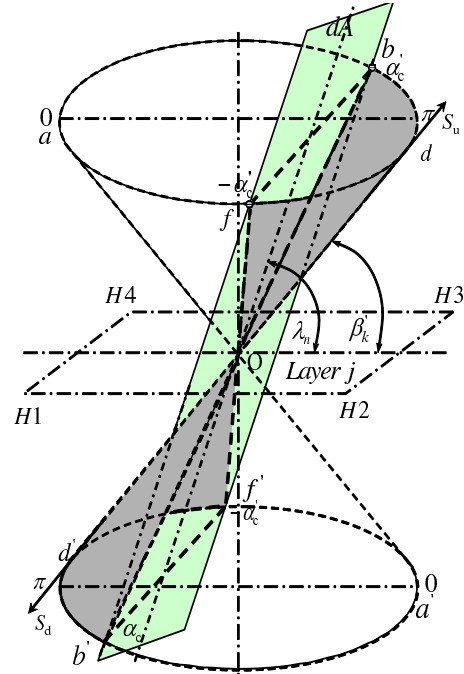
A downward radiant flux [e.g.,  $\phi_d(\beta_k, j)$ ] intercepted by layer  $j$  will be scattered (both reflected and transmitted) in all directions and contribute to  $\phi_d(\beta'_k, j+1)$  and to  $\phi_u(\beta'_k, j)$ . Similarly, an upward radiant flux  $\phi_u(\beta_k, j+1)$  intercepted by layer  $j$  will contribute to  $\phi_u(\beta'_k, j)$  and to  $\phi_d(\beta'_k, j+1)$  [Eqs. (14)]. The following derivation seeks to determine the partitioning of the contribution from the intercepted and scattered radiation of the flux  $\phi_d(\beta_k, j)$  (marked as  $S$  in Fig. A1) by leaf element  $dA$  in layer  $j$  to  $\phi_u(\beta'_k, j)$  and  $\phi_d(\beta'_k, j+1)$ .

The scattered contribution of the radiation flux  $S = \phi_d(\beta_k, j)$  to the flux  $\phi_u(\beta'_k, j)$  is defined as  $S_u$  (see Figs. A2 and A3). It consists of four parts. Part 1 is  $S_{u, \rho}$ . It is the reflected radiation at the inclination angle of  $\beta'_k$  within the azimuth angle ranging over arc  $fab$  that intercepted by the adaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at the inclination angle of  $\beta_k$  within the azimuth angle ranging

over arc  $ehc$ . Part 2 is  $S_{u,\rho}$ . It is the reflected radiation at  $\beta'_k$  within the azimuth angle ranging over  $bdf$  that intercepted by the abaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at  $\beta_k$  within the azimuth angle ranging over arc  $cge$ . Part 3 is  $S_{u,\tau}$ . It is the transmitted radiation at  $\beta'_k$  within the azimuth angle ranging over arc  $bdf$  that intercepted by the adaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at  $\beta_k$  within the azimuth angle ranging over arc  $ehc$ . Part 4 is  $S_{u,\tau'}$ . It is the transmitted radiation at  $\beta'_k$  within the azimuth angle ranging over arc  $fab$  that intercepted by the abaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at  $\beta_k$  within the azimuth angle ranging over arc  $cge$ . Thus,  $S_u = S_{u,\rho} + S_{u,\rho'} + S_{u,\tau} + S_{u,\tau'}$ . Under the condition that the intercepted radiations are scattered symmetrically by  $dA$ , the optical properties of  $dA$  are equal,



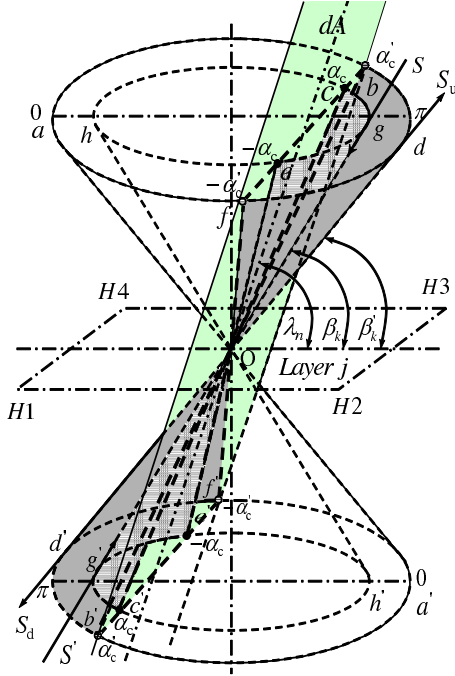
**Fig. A1.** Sketch map of the light intercepted by the adaxial and abaxial leaf surfaces. The downward incident beam of light  $S$  at the inclination angle  $\beta_k$  strikes the inclined leaf element  $dA$  with an inclination angle  $\lambda_n$ . The light within the azimuth angle ranging over arc  $ehc$  will fall onto the adaxial leaf surface, and that within the azimuth angle ranging over arc  $cge$  will fall onto the abaxial leaf surface.  $\alpha_c$  and  $-\alpha_c$  are critical angles of the light azimuth, marked as  $c$  and  $e$  respectively. It is similar for the upward incident beam of light  $S'$ , that the light over arc  $c'g'e'$  will fall onto the adaxial surface and that over arc  $e'h'e'$  will fall onto the abaxial surface.



**Fig. A2.** Sketch map of the light emitted from the adaxial and abaxial leaf surfaces.  $S_d$  and  $S_u$  are the downward and upward scattered light at  $\beta'_k$  emitted from  $dA$ .  $\alpha'_c$  and  $-\alpha'_c$  are critical angles of the light azimuth. The light over arc  $fab$  will emit from the adaxial surface, that over arc  $bdf$  will emit from the abaxial surface, over arc  $b'd'f'$  will emit from the adaxial surface, and that over arc  $f'a'b'$  will emit from the abaxial surface.

$\rho = \rho' = \tau = \tau'$ . Suppose the incident diffuse radiation is uniform and the scattered symmetrically. Then the fraction of radiation reflected at  $\beta'_k$  after the interception of the adaxial face is  $\xi_f = S_{u,\rho}/S$ , the fraction of radiation reflected at  $\beta'_k$  after the interception of the abaxial face is  $\xi_b = S_{u,\rho'}/S$ , the fraction of radiation transmitted at  $\beta'_k$  after the interception of the adaxial face is  $\zeta_f = S_{u,\tau}/S$ , and the fraction of radiation transmitted at  $\beta'_k$  after the interception of the abaxial face is  $\zeta_b = S_{u,\tau'}/S$ . So  $\xi_f$ ,  $\xi_b$ ,  $\zeta_f$  and  $\zeta_b$  represent the ratios of the radiation fluxes into each part to the total radiation flux into the conical shell at  $\beta'_k$ .

The contribution of the downward radiation flux  $S = \phi_d(\beta_k, j)$  to  $\phi_d(\beta'_k, j + 1)$  is defined as  $S_d$ , which also consists of four parts. Part 1 is  $S_{d,\rho}$ . It is the reflected radiation at  $\beta'_k$  within the azimuth angle ranging over arc  $b'd'f'$  that intercepted by the adaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at  $\beta_k$  within the azimuth angle ranging over arc  $ehc$ . Part 2 is  $S_{d,\rho'}$ . It is the reflected radiation at  $\beta'_k$  within arc  $f'a'b'$  that intercepted by the abaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at  $\beta_k$  within arc  $cge$ . Part 3 is  $S_{d,\tau}$ . It is the transmitted radiation at  $\beta'_k$  within arc  $f'a'b'$  that intercepted by the adaxial leaf surface of  $dA$ , this radiation come



**Fig. A3.** Sketch map of the light interacting with the adaxial and abaxial leaf surfaces. This is a complete figure taking on both the incident and scattered light simultaneously.

from the part of  $S$  at  $\beta_k$  within arc  $ehc$ . Part 4 is  $S_{d,\tau'}$ . It is the transmitted radiation at  $\beta'_k$  within arc  $b'd'f'$  that intercepted by the abaxial leaf surface of  $dA$ , this radiation come from the part of  $S$  at  $\beta_k$  within arc  $cge$ . Also we have  $S_d = S_{d,\rho} + S_{d,\rho'} + S_{d,\tau} + S_{d,\tau'}$ . Supposing that  $S_d$  is equal to  $S_u$  and the diffuse radiation is uniform, then based on the same reasoning as above we can see that  $S_{d,\rho}$  is equal to  $S_{u,\tau}$ ,  $S_{d,\rho'}$  is equal to  $S_{u,\tau'}$ ,  $S_{d,\tau}$  is equal to  $S_{u,\rho}$ , and  $S_{d,\tau'}$  is equal to  $S_{u,\rho'}$ . Thus the corresponding ratios of the four parts to  $S_d$  as  $S_{d,\rho}/S_d$ ,  $S_{d,\rho'}/S_d$ ,  $S_{d,\tau}/S_d$  and  $S_{d,\tau'}/S_d$  are equal to  $\zeta_f$ ,  $\zeta_b$ ,  $\xi_f$  and  $\xi_b$  respectively.

$\xi_{f,B}$ ,  $\xi_{b,B}$ ,  $\zeta_{f,B}$  and  $\zeta_{b,B}$  have the same meanings as  $\xi_f$ ,  $\xi_b$ ,  $\zeta_f$ , and  $\zeta_b$ , except the incident downward radiant flux is the sunbeam, where the subscript B stands for the beam. That is to say, if the downward radiant flux is a sunbeam, the four ratios of the four parts contributing to  $\phi_d(\beta'_k, j+1)$  downwardly are  $\xi_{f,B}$ ,  $\xi_{b,B}$ ,  $\zeta_{f,B}$  and  $\zeta_{b,B}$ , and to  $\phi_u(\beta'_k, j)$  upwardly are  $\xi_{f,B}$ ,  $\xi_{b,B}$ ,  $\zeta_{f,B}$  and  $\zeta_{b,B}$ .

Now the above reasoning deals with the case of the contribution to the flux  $\phi_u(\beta'_k, j)$  or  $\phi_d(\beta'_k, j+1)$  at  $\beta'_k$  from the flux  $\phi_d(\beta'_k, j)$  at  $\beta_k$ . The calculation of the contribution of the flux  $\phi_u(\beta_k, j)$  at  $\beta_k$  intercepted by layer  $j$  to the flux  $\phi_u(\beta'_k, j)$  or  $\phi_d(\beta'_k, j+1)$  is similar to the above. That is, for the radiation  $\phi_u(\beta'_k, j)$  scattered upward, the ratios of the four parts to the total upward scattered radiation [marked as  $\xi'_f$ ,  $\xi'_b$ ,  $\zeta'_f$ , and  $\zeta'_b$

in Eq. (14a)] are equal to  $\zeta_b$ ,  $\zeta_f$ ,  $\xi_b$  and  $\xi_f$  respectively. For the radiation  $\phi_u(\beta'_k, j)$  scattered downward, the ratios of the four parts to the total downward scattered radiation are equal to  $\xi_b$ ,  $\xi_f$ ,  $\zeta_b$  and  $\zeta_f$  respectively.

The expressions for  $\xi_f$ ,  $\xi_b$ ,  $\zeta_f$  and  $\zeta_b$  as dependent on  $\beta_k$ ,  $\beta'_k$ , and  $\lambda_n$  can be derived as follows.

The radiance  $R$  ( $\text{J m}^{-2} \text{sr}^{-1} \text{s}^{-1}$ ) of a Lambertian radiator element  $dA$  with unit surface, and receiving a downward beam of light with unit radiant flux at an incident inclination angle of  $\theta$ , is given by

$$R = \sin \theta. \quad (\text{A1})$$

The energy flow  $\psi$  ( $\text{J s}^{-1}$ ) of radiation reflected or transmitted by the element  $dA$  under an angle  $\theta'$  into a solid angle  $d\omega'$  equals

$$\psi = R \sin \theta' d\omega', \quad (\text{A2})$$

where  $d\omega'$  is given by

$$d\omega' = \cos \beta'_k d\beta'_k d\alpha'. \quad (\text{A3})$$

All angles without a prime refer to incident radiation and those with a prime to reflected or transmitted radiation.

The angles of the direction of incident and reflected or transmitted radiation inclined to the element  $dA$  are calculated according to Eq. (1)

$$\sin \theta = \sin \beta_k \cos \lambda_n + \cos \beta_k \sin \lambda_n \cos \alpha, \quad (\text{A4a})$$

$$\sin \theta' = \sin \beta'_k \cos \lambda_n + \cos \beta'_k \sin \lambda_n \cos \alpha'. \quad (\text{A4b})$$

where  $\lambda_n$  is the inclination of the leaf element  $dA$  to horizontal plane H1-H2-H3-H4 (Fig. A1),  $\beta_k$  and  $\beta'_k$  are those to H1-H2-H3-H4 (like layer  $j$ ) for downward incident radiation  $R$  and for upward reflected or transmitted radiation respectively, and  $\alpha$  and  $\alpha'$  are the azimuths of the incident and reflected or transmitted radiations respectively.

For reflected radiation,  $\sin \theta$  and  $\sin \theta'$  should be either both positive or both negative. The critical values of the azimuths  $\alpha$  and  $\alpha'$  for which  $\sin \theta$  and  $\sin \theta'$  are zero are denoted by  $\alpha_c$  and  $\alpha'_c$  (See Figs. A1, A2 and A3), and are given by

$$\alpha_c = \pi/2 + \arcsin \left( \frac{\tan \beta_k}{\tan \lambda_n} \right) \quad \beta_k < \lambda_n, \quad (\text{A5a})$$

$$\alpha_c = \pi \quad \beta_k \geq \lambda_n, \quad (\text{A5b})$$

and likewise for  $\alpha'_c$  where  $\beta_k$  is replaced by  $\beta'_k$ .

According to the reasoning above, in the case of  $S_{d,\rho}$ ,  $\sin \theta$  and  $\sin \theta'$  are both positive and  $\xi_f$  can be calculated by:

$$\begin{aligned} \xi_f &= \int_0^{\alpha'_c} \int_0^{\alpha_c} \sin \theta \sin \theta' d\alpha d\alpha' \cos \beta'_k d\beta'_k / S_t = \\ & b_1 \cos \beta'_k d\beta'_k / S_t, \\ & b_1 = a_1 \alpha_c \alpha'_c + a_2 \alpha_c \sin \alpha'_c + \\ & a_3 \sin \alpha_c \alpha'_c + a_4 \sin \alpha_c \sin \alpha'_c. \end{aligned} \quad (\text{A6a})$$

In the case of  $S_{d,\rho'}$ ,  $\sin \theta$  and  $\sin \theta'$  are both negative and  $\xi_b$  can be calculated by:

$$\begin{aligned} \xi_b &= \int_{\alpha_c}^{\pi} \int_{\alpha'_c}^{\pi} \sin \theta \sin \theta' d\alpha d\alpha' \cos \beta'_k d\beta'_k / S_t = \\ & b_2 \cos \beta'_k d\beta'_k / S_t, \\ b_2 &= [\alpha_1(\pi - \alpha_c)(\pi - \alpha'_c) + \\ & a_2(\pi - \alpha_c)(-\sin \alpha'_c) + \\ & a_3(-\sin \alpha_c)(\pi - \alpha'_c) + \\ & a_4(-\sin \alpha_c)(-\sin \alpha'_c)]. \end{aligned} \quad (\text{A6b})$$

In the case of  $S_{d,\tau}$ ,  $\sin \theta$  is positive and  $\sin \theta'$  is negative, and  $\zeta_f$  can be calculated by:

$$\begin{aligned} \zeta_f &= - \int_0^{\alpha_c} \int_{\alpha'_c}^{\pi} \sin \theta \sin \theta' \psi d\alpha d\alpha' \cos \beta'_k d\beta'_k / S_t = \\ & b_3 \cos \beta'_k d\beta'_k / S_t, \\ b_3 &= a_1 \alpha'_c (\alpha_c - \pi) + a_2 (\alpha_c - \pi) \sin \alpha'_c + \\ & a_3 \sin \alpha_c \alpha'_c + a_4 \sin \alpha'_c \sin \alpha_c. \end{aligned} \quad (\text{A6c})$$

In the case of  $S_{d,\tau'}$ ,  $\sin \theta$  is negative and  $\sin \theta'$  is positive, and  $\zeta_b$  can be calculated by:

$$\begin{aligned} \zeta_b &= - \int_0^{\alpha'_c} \int_{\alpha_c}^{\pi} \sin \theta \sin \theta' \psi d\alpha d\alpha' \cos \beta'_k d\beta'_k / S_t = \\ & b_4 \cos \beta'_k d\beta'_k / S_t, \\ b_4 &= a_1 \alpha_c (\alpha'_c - \pi) + a_2 \alpha_c \sin \alpha'_c + \\ & a_3 \sin \alpha_c (\alpha'_c - \pi) + a_4 \sin \alpha_c \sin \alpha'_c. \end{aligned} \quad (\text{A6d})$$

It should be pointed out that:

$$\xi_f + \xi_b + \zeta_f + \zeta_b = 1.0.$$

Therefore,

$$S_t = (b_1 + b_2 + b_3 + b_4) \cos \beta'_k d\beta'_k. \quad (\text{A7})$$

The auxiliary variables  $a_1, a_2, a_3, a_4$  are given by

$$a_1 = \sin \beta_k \sin \beta'_k \cos^2 \lambda_n, \quad (\text{A8a})$$

$$a_2 = \sin \beta_k \cos \beta'_k \sin \lambda_n \cos \lambda_n, \quad (\text{A8b})$$

$$a_3 = \sin \beta'_k \cos \beta_k \sin \lambda_n \cos \lambda_n, \quad (\text{A8c})$$

$$a_4 = \cos \beta_k \cos \beta'_k \sin^2 \lambda_n. \quad (\text{A8d})$$

## APPENDIX B

### Design of Experiment

The experiments n1–n12 indicate the status about various combinations of leaf area index, leaf reflection, and transmission, and soil reflection.

Group N1:

n1: LAI=1,  $\rho = 0.1$ ,  $\tau = 0.1$ ,  $\rho_s = 0.8$ ;

n2: LAI=1,  $\rho = 0.1$ ,  $\tau = 0.1$ ,  $\rho_s = 0.2$ ;

n3: LAI=5,  $\rho = 0.1$ ,  $\tau = 0.1$ ,  $\rho_s = 0.8$ ;

n4: LAI=5,  $\rho = 0.1$ ,  $\tau = 0.1$ ,  $\rho_s = 0.2$ ;

n5: LAI=8,  $\rho = 0.1$ ,  $\tau = 0.1$ ,  $\rho_s = 0.8$ ;

n6: LAI=8,  $\rho = 0.1$ ,  $\tau = 0.1$ ,  $\rho_s = 0.2$ ;

Group N2:

n7: LAI=1,  $\rho = 0.5$ ,  $\tau = 0.3$ ,  $\rho_s = 0.8$ ;

n8: LAI=1,  $\rho = 0.5$ ,  $\tau = 0.3$ ,  $\rho_s = 0.2$ ;

n9: LAI=5,  $\rho = 0.5$ ,  $\tau = 0.3$ ,  $\rho_s = 0.8$ ;

n10: LAI=5,  $\rho = 0.5$ ,  $\tau = 0.3$ ,  $\rho_s = 0.2$ ;

n11: LAI=8,  $\rho = 0.5$ ,  $\tau = 0.3$ ,  $\rho_s = 0.8$ ;

n12: LAI=8,  $\rho = 0.5$ ,  $\tau = 0.3$ ,  $\rho_s = 0.2$ .

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