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Retrieval of Atmospheric and Oceanic Parameters and the Relevant Numerical Calculation

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ABSTRACT

It is well known that retrieval of parameters is usually ill-posed and highly nonlinear, so parameter retrieval problems are very difficult. There are still many important theoretical issues under research, although great success has been achieved in data assimilation in meteorology and oceanography. This paper reviews the recent research on parameter retrieval, especially that of the authors. First, some concepts and issues of parameter retrieval are introduced and the state-of-the-art parameter retrieval technology in meteorology and oceanography is reviewed briefly, and then atmospheric and oceanic parameters are retrieved using the variational data assimilation method combined with the regularization techniques in four examples: retrieval of the vertical eddy diffusion coefficient; of the turbulivity of the atmospheric boundary layer; of wind from Doppler radar data, and of the physical process parameters. Model parameter retrieval with global and local observations is also introduced.

Key words: variational data assimilation, parameter retrieval, ill-posed problem, regularization

1. Introduction

It is well known that numerical prediction of atmospheric and oceanic motions is reduced to solving a set of nonlinear partial differential equations with initial and boundary conditions, which is often called a direct problem. Direct problems are essentially, given the models (equations and initial and boundary conditions), the process of seeking solutions and making predictions. In recent years, a variety of methods such as variational data assimilation (Le Dimet and Talagrand, 1986, Talagrand and Courtier, 1987; Lorenc, 1988; Gao and Cou, 1995; Navon, 1997; Wang et al., 2000, Huang et al., 2005b), the Kalman filter (Courtier et al., 1993), and the ensemble Kalman filter (Evensen, 2003) have been proposed to boost the accuracy of numerical weather prediction and to extend prediction range. Data assimilation is an analysis technique in which the observed information is accumulated into the model state by taking advantage of consistency constraints with the law of time evolution and physical properties. This approach belongs to the category of inverse problems.

Parameter retrieval is another kind of inverse problem, which has been successfully applied to the geosciences, material sciences, biological sciences, and atmospheric and oceanic sciences. Since the pioneering work of Gelfand et al. (1951), parameter retrieval problems in the hyperbolic equations have been widely studied and great success has been achieved, while the variational method is usually used to solve the parameter retrieval problem in the elliptic equations (Tosaka et al., 1999), which is similar to the adjoint method of variational data assimilation in meteorology and oceanography to some extent. For more detail about the regularization theory, readers may refer to Groetsch (1993).

In meterology, an early detailed survey also addressed issues of adjoint parameter identification as provided by Le Dimet and Navon (1988). Zou et al. (1992) estimated the magnitude of the nudging coefficient in the National Meteorological Center (NMC)

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adiabatic version of the spectral Medium Range Forecast (MRF) model, while Wang (1993) and Wang et al. (1995) estimated the same coefficient using the FSU (Florida State University) adiabatic spectral model. Stauffer and Bao (1993) performed a parameter estimation of nudging coefficients in a 1-D linearized shallow water equation model. Louis and Zivkovid (1994) carried out physical parameter estimation in a simplified single column model.

In oceanography, studies on adjoint parameter estimation were first carried out by Panchang and O'Brien (1989) for determining the bottom drag coefficient in a tidal channel. Phase speeds were estimated in a review paper by Smedstad and O'Brien (1991) using the adjoint method in a reduced gravity model for the tropica1 Pacific Ocean. Richardson and Panchang (1992) noted the difficulties associated with parameter estimation (eddy viscosity) by adjoint methods due to their being beset by instabilities and nonuniqueness (in general, ill-posedness) when identifying parameters distributed in the space and time domain, especially when the data is noisy. To overcome these difficulties, they proposed the insertion of an additional constraint in the cost functional J, meaning that the parameter profiles should be smoothly varying, that is, they proposed a compromise between the data-model misfit minimization and the solution smoothness. For detailed references of the parameter estimation problems in meteorology and oceanography, readers can refer to the reviews by Navon (1997), Zhu et al. (1998), and Le Dimit and Yang (2002), etc.

In this paper, research on parameter retrieval in recent years, especially that from the authors, are reviewed, and attention is mainly focused on the variation method combined with the regularization techniques and their application to some particular examples by the authors. Theoretical analyses and numerical tests indicate that the variation method combined with the regularization techniques and proper choice of regularization parameters can improve the convexity of the cost functional, constrain ill-posedness, and hence speed up the convergence of numerical solutions and boost the accuracy of solutions.

2. Some remarks on parameter retrieval and regularization theory

As mentioned above, parameter retrieval belongs to the class of inverse problems. Therefore parameter retrieval possesses some basic characteristics of the inverse problems. As noted by Yeh (1986) in the context of ground water flow parameter estimation, the inverse or parameter estimation problem is often ill-posed and beset by instability and non-uniqueness, particularly if one seeks parameters distributed in the space-time domain.

2.1 *An example of an ill-posed model parameter retrieval*

The ill-posedness concept goes back to Hadamard. He claimed (Kirsch, 1996) that a model for a physical problem had to be well-posed in the sense that the following three properties are satisfied:

(1) There exist solutions of the problem (existence)

(2) There is at most one solution of the problem (uniqueness)

(3) The solution depends continuously on the initial and boundary data (stability)

A model for which at least one of the above three properties does not hold is called ill-posed or improperly posed. The following example is given by Kirsch (1996)

$$\begin{cases} -\dfrac{d}{dx}\left[a(x)\dfrac{du}{dx}\right] = f(x)\ , \quad x \in (0,1) \\ a(0)\dfrac{du}{dx}(0) = b_0\ , \quad a(1)\dfrac{du}{dx}(1) = b_1\ , \end{cases} \tag{2.1}$$

where $a(x)$ is a diffusion coefficient, and $f(x)$ represents a source term. If $b_0, b_1, f(x)$ and the observation data $u_{\rm obs}(x)$ are given, some cases emerge as follows in the retrieval of $a(x)$.

(1) The solutions may not be unique.

(2) The solutions are not stable. When $\| u^{\varepsilon} - u_{\rm obs} \| \leqslant \varepsilon$ for a very small $\varepsilon > 0$, then $\| a_{\varepsilon}(x) - a_{\rm T}(x) \|$ becomes very big, where $a_{\rm T}(x)$ stands for the true solution.

(3) Numerical instability. For a given $u_{|textobs}$, the solutions can be obtained by the least squares approach. However, the numerical calculation of the derivatives of $u_{\rm obs}$ will lead to numerical instability (Huang and Wu, 2001).

2.2 *An example of a highly non-linear model parameter retrieval*

Even if the direct problems are linear, the inverse problems are usually nonlinear. The example given by Guo (1998) is presented to demonstrate the high-nonlinearity of the parameter retrieval.

$$\begin{cases} \dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}\ , \quad (t,x) \in (0,T) \times (0,1) \\ u|_{t=0} = 0, u|_{x=0} = f(t)\ , \quad u|_{x=1} = 0\ , \end{cases} \tag{2.2}$$

where k is a constant. Given k and $f(t)$, then there exists a unique solution of system (2.2), and the solution can be expressed as

$$u(t,x) = -k\int_0^t \frac{\partial M[k(t-\tau),x]}{\partial x} f(\tau)d\tau\ , \tag{2.3}$$

where $M(t,x) = \dfrac{1}{\sqrt{\pi t}} \displaystyle\sum_{n=-\infty}^{n=\infty} e^{-\frac{(x+2n)^2}{4t}} (t>0)$.

If $f(t)$ is given and k is an unknown to be determined, then additional observation information is required. For instance, the observational data h satisfies:

$$-\rho ck\frac{\partial u}{\partial x}(t_0, 0) = h \ , \tag{2.4}$$

where ρ, c, h are constants and $0 < t_0 < T$. By the solution (2.3), from Eq. (2.4) the nonlinear equation of k can be obtained as

$$\rho ck\int_0^{t_0} M[k(t-\tau), 0]f'(\tau)d\tau = h \ , \tag{2.5}$$

which is an algebraic equation of high-nonlinearity.

2.3 *Impacts of observational data on parameter retrievals*

Below is an example given by Romanov (1987) showing the impacts of observational data on parameter retrievals. The model is as follows

$$\begin{cases} L_q u = \dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial^2 u}{\partial x^2} + q(x)u = 0 \\ u|_{t=0} = 0, \ u_t|_{t=0} = \delta(x) \ , \end{cases} \tag{2.6}$$

where $\delta(x)$ is the Dirac delta function, and the parameter $q(x)$ is to be determined by additional observational data. However, the properties of the observational data are vitally important to the well-posedness of the problem, which is related closely to the physical background and mathematical structure of the problem. If the observational data at the point $x = 0$ are $u|_{x=0} = f_1(t)$ and $u_x|_{x=0} = f_2(t)$, then under some conditions, there exists a local solution $q(x)$. If the observation data at the point $x = x_0$ ($x_0 > 0$) are $u|_{x=x_0} = f_1(t)$ and $u_x|_{x=x_0} = f_2(t)$, then $q(x)$ can be uniquely determined in the region $x \geqslant x_0$. In the region $x < x_0$, however, $q(x)$ cannot be uniquely determined.

2.4 *The outline of the regularization principle*

We now briefly introduce the concept of regularization or the regularization principle. Inverse problems are usually ill-posed, i.e., solutions probably do not exist; or even if solutions exist, the solutions are probably not-unique; or, solutions exist and are unique, but the only solution is unstable at the initial and boundary conditions, etc. Hence, for wider applications to practical problems, classical solutions must be generalized to ones which are called "solution-like" (Groetsch, 1993).These solutions are required to satisfy two conditions: (1) the solution-like solutions must be unique, and (2) if the solutions to a model exist, then the solution-like solutions must be among them. Specifically, for

$$\mathrm{K}x = g \tag{2.7}$$

where K is a linear bounded operator, we want to seek a solution, called solution-like, such that the minimum norm least square solution x^+ satisfies

$$\begin{cases} \parallel \mathrm{K}x^+ - g \parallel = \min \\ \parallel x^+ \parallel = \min \ . \end{cases}$$

It can be proved that x^+ exists and is unique. If K is a nonlinear operator, then under some conditions, there exist similar results, which are omitted here.

Generally speaking, g is obtained by measurement. Denote such a measurement with an error δ ($\parallel g_\delta - g \parallel \leqslant \delta$) by g_δ. We want to find a stable approximate solution x_δ such that

$$\parallel \mathrm{K}x_\delta - g_\delta \parallel \leqslant \delta, \ \parallel x_\delta \parallel = \min, \ \text{and} \ x_\delta \to x^+ \ .$$

This solution is called the minimum norm solution with discrepancy δ. This variational problem is provably equivalent to (Kress, 1989)

$$J[x] = \parallel \mathrm{K}x - g_\delta \parallel^2 + \gamma \parallel x \parallel^2 = \min \ ,$$

where $J[x]$ is referred to as the Tikhonov functional, $\parallel x \parallel^2$ the stable functional, and γ the regularization parameter. The regularization parameter γ is uniquely determined according to the equation

$$\parallel \mathrm{K}x_{\delta,\gamma(\delta)} - g_\delta \parallel = \delta \ ,$$

which is called the discrepancy principle. The stable functional $\parallel x \parallel^2$ varies according to the specific problem, and may be $\int_\Omega |x|^2 d\Omega$, $\int_\Omega (|x|^2 + |\nabla x|^2) d\Omega$, or $\parallel \mathrm{L}x \parallel^2$ where L is a closed linear operator. The stable functional is different from penalty terms with an imposed smoothness constraint.

3. Retrieval of the vertical eddy diffusion parameter of a 1-D sea temperature model

For illustration, we will use a one-dimensional heat-diffusion model formulated for describing the vertical distribution of sea temperature in time (Huang et al., 2004a; Huang et al., 2003). The governing equation is:

$$\begin{cases} \dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial z}\left(K\dfrac{\partial T}{\partial z}\right) + f(t, z) \ , \\ T|_{t=0} = U(z), \ \ K\dfrac{\partial T}{\partial z}\bigg|_{z=0} = \varphi(t), \ \ K\dfrac{\partial T}{\partial z}\bigg|_{z=H} = 0 \ , \end{cases} \tag{3.1}$$

where $T = T(t, z)$ is sea temperature, $K = K(t, z)$ is the vertical eddy diffusion coefficient,

$$f(t, z) = -\frac{\nu I_0}{\rho_0 c_p}\exp(-\nu z) \ ,$$

ρ_0 is the sea water density, ν is the sea water specific capacity, ν is the light diffusion coefficient, H is the depth of the ocean upper layer, I_0 is the transmission component of solar radiation at the sea surface, and

$$\varphi(t) = \frac{Q(t)}{\rho_0 c_p} - \frac{I_0}{\rho_0 c_p}\bigg|_{z=0} \ ,$$

where $Q(t)$ is the net heat flux at the sea surface.

Assume γ, c_p, ρ_0, I_0 and $Q(t)$ are given, and that $U(z)$ and $K(t,z)$ are unknown and need to be determined by the data assimilation method. If $T_{\rm obs}$ is given on the whole domain, the estimations of the initial condition $U(z)$ and the model parameter $K(t,z)$ are performed simultaneously in the framework of variational data assimilation with the regularization technique (Huang et al., 2004a). If $T_{\rm obs}(t,0)$ is given, i.e., the observations are taken only at the sea surface, then the cost functional is defined as

$$J[U,K] = \frac{1}{2}\int_0^\tau [T(t,0)-T_{\rm obs}(t,0)]^2 dt + \frac{\gamma^2}{2}\int_0^\tau\int_0^H K(t,z)\left(\frac{\partial T}{\partial z}\right)^2 dtdz\,. \quad (3.2)$$

Through a series of variational calculations (Huang and Han, 2003), the following adjoint equations and adjoint boundary conditions are obtained

$$\begin{cases} -\dfrac{\partial P}{\partial t} = \dfrac{\partial}{\partial z}\left(K\dfrac{\partial P}{\partial z}\right) - \gamma^2\dfrac{\partial}{\partial z}\left(K\dfrac{\partial T}{\partial z}\right)\,, \\ P|_{t=\tau}=0,\ \ K\dfrac{\partial P}{\partial z}\Big|_{z=0} = -[T(t,0)-T_{\rm obs}(t)] \\ \qquad + \gamma^2 K\dfrac{\partial T}{\partial z}\Big|_{z=0}\,,\ \ K\dfrac{\partial P}{\partial z}\Big|_{z=H}=0\,. \end{cases} \quad (3.3)$$

And the gradients of the cost functional (3.2) with respect to U and K are

$$\nabla_U J = p(0,z),\ \ \nabla_K J = -\frac{\partial T}{\partial z}\frac{\partial p}{\partial z} + \frac{1}{2}\gamma^2\left(\frac{\partial T}{\partial z}\right)^2\,.$$

Then, numerical experiments are performed.

Take $U(z)=\sin z, K=1$. A perturbation is added to the eddy diffusion coefficient, $K=1+0.05(z-H)$. One experiment is calculated with $\gamma=0$, and the other with $\gamma=0.001$. The results of the numerical experiments are shown in Fig. 1. It can be seen from the figure that, if $\gamma=0$, i.e., without the regularization term, then E_k (E_k is the norm of eddy diffusion coefficient error) increases with the iteration number; if $\gamma\neq 0$, then J decreases quikly with the iteration number and the accuracy is improved.

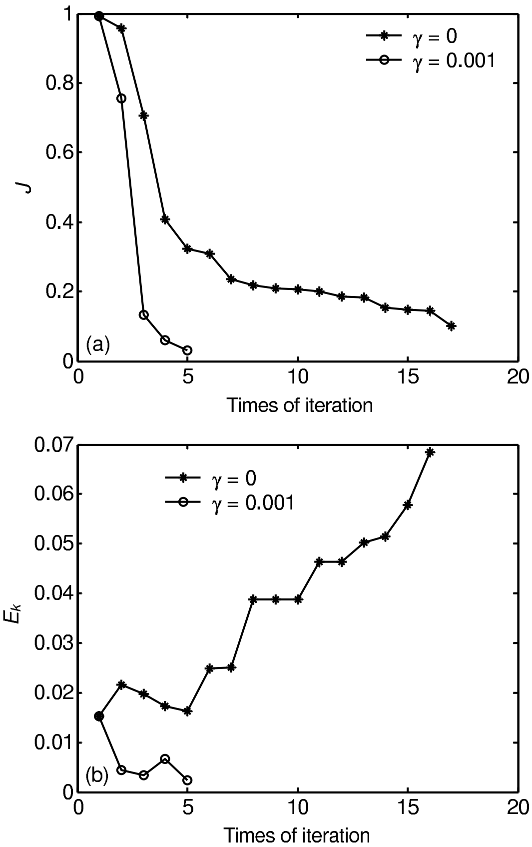

Fig. 1. (a) The iteration process of J with and without regularization; (b) The variation of error E_k with and without regularization .

4. Retrieval of the turbulivity of the atmospheric boundary layer

According to classic Ekman theory, the wind vector is viewed as an Ekman spiral along the vertical direction for constant turbulivity and geostrophic wind. However, the complexity of the structures of wind in the atmospheric boundary layer hints that this assumption is problematic. For instance, the geostrophic wind, though a good approximation to the real atmospheric wind, is still different from it. Therefore, the temporal and spatial distribution of the turbulivity should be determined according to the structures of the true wind (Wang et al., 2005).

The stationary atmospheric boundary layer is governed by the following model

$$\begin{cases} \dfrac{\partial}{\partial z}\left(\kappa\dfrac{\partial u}{\partial z}\right) + f(v-v_g)=0\,, \\ \dfrac{\partial}{\partial z}\left(\kappa\dfrac{\partial v}{\partial z}\right) - f(u-u_g)=0\,, \end{cases} \quad (4.1)$$

with the boundary conditions

$$(u,v)|_{z=0}=(0,9),\ \ (u,v)|_{z=H}=(u_{{\rm g},H}, v_{{\rm g},H})\,, \quad (4.2)$$

where κ is the turbulivity, u and v are the zonal and meridional components of velocity respectively, $u_{\rm g}$ and $v_{\rm g}$ the zonal and meridional components of geostrophic wind respectively, f the Coriolis parameter, and $u_{{\rm g},H}$ and $v_{{\rm g},H}$ the values of $u_{\rm g}$ and $v_{\rm g}$ at the upper boundary.

If the observations $u_{\rm obs}$ and $v_{\rm obs}$ are known, the turbulivity κ can be retrieved. If only the wind field of a single station is known and the distribution of the pressure field is unknown, then the geostrophic wind $u_{\rm g}$ and $v_{\rm g}$ cannot be determined and must be regarded as parameters to be retrieved. So the present problem is to determine $\kappa, u_{\rm g}$ and $v_{\rm g}$, in which the following functional is minimized,

$$\begin{aligned} J[\kappa, u_{\rm g}, v_{\rm g}] =& \frac{1}{2}\int_0^H [(u-u_{\rm obs})^2+(v-v_{\rm obs})^2]dz \\ &+\frac{\gamma}{2}\int_0^H \kappa\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]dz\,. \end{aligned} \tag{4.3}$$

The retrieval of multiple parameters is usually ill-posed, which leads to the application of the regularization principle. The second term in J is a stable functional and γ is the regularization parameter. As the first order norm of (u, v), the stable functional can overcome the ill-posedness and diminish oscillation in the retrieval.

Through a series of variational calculations, the following adjoint equations and adjoint boundary conditions are obtained

$$\begin{cases} \dfrac{\partial}{\partial z}\left(\kappa\dfrac{\partial P}{\partial z}\right)-fQ=u-u_{\rm obs}+\gamma f(v-v_{\rm g})\,, \\ \dfrac{\partial}{\partial z}\left(\kappa\dfrac{\partial Q}{\partial z}\right)+fP=v-v_{\rm obs}-\gamma f(u-u_{\rm g})\,, \\ (P,Q)|_{z=H}=(0,0)\,. \end{cases} \tag{4.4}$$

The gradients of the functional J are

$$\begin{cases} \nabla_\kappa J=\gamma\left[\left(\dfrac{\partial u}{\partial z}\right)^2+\left(\dfrac{\partial v}{\partial z}\right)^2\right]+\left(\dfrac{\partial u}{\partial z}\dfrac{\partial P}{\partial z}+\dfrac{\partial v}{\partial z}\dfrac{\partial Q}{\partial z}\right)\,, \\ \nabla_{u_{\rm g}}J=-fQ\,,\quad \nabla_{v_{\rm g}}J=fP\,. \end{cases} \tag{4.5}$$

As the first step in the retrieval of structures of the atmospheric boundary layer by real observations, an ideal numerical test is performed to verify the validation of the adjoint method combined with the regularization principle.

Let the thickness of the atmospheric boundary layer be H=2000 m and the Coriolis parameter at 40°N be f=0.0000937442 s^{-1}.

Similar to the literature (Tan, 2000), the turbulivity is taken according to the following expression

$$\kappa(z)=\begin{cases} 2\times10^{-8}z^3-1.23\times10^{-4}z^2+0.0685z+4\,, \\ \qquad z<500\ {\rm m^2\,s^{-1}} \\ \dfrac{2500}{z-250}\,,\quad z\geqslant 500\ {\rm m^2\,s^{-1}}\,. \end{cases} \tag{4.6}$$

Obviously this profile is smooth enough. Let the geostrophic wind profile be

$$u_{\rm g}(z)=4+\frac{z}{2000}(10-4),$$
$$v_{\rm g}(z)=1.5+\frac{z}{2000}(6-1.5)\,.\ \ ({\rm in\ m\,s^{-1}})$$

Under these conditions, the true wind field (or "wind observations") u and v can be calculated from the Ekman equations.

The initial guess values of the turbulivity and geostrophic wind are $k(z)$=8 m^2 s^{-1}, $u_{\rm g}(z)=8$ m s^{-1} and $v_{\rm g}(z)=3$ m s^{-1}.

Because of the limitation of space in this paper, the retrieval of real sounding wind data in Luohe, Henan Province, China on 22 April 1992 is omitted here (see Wang et al., 2006).

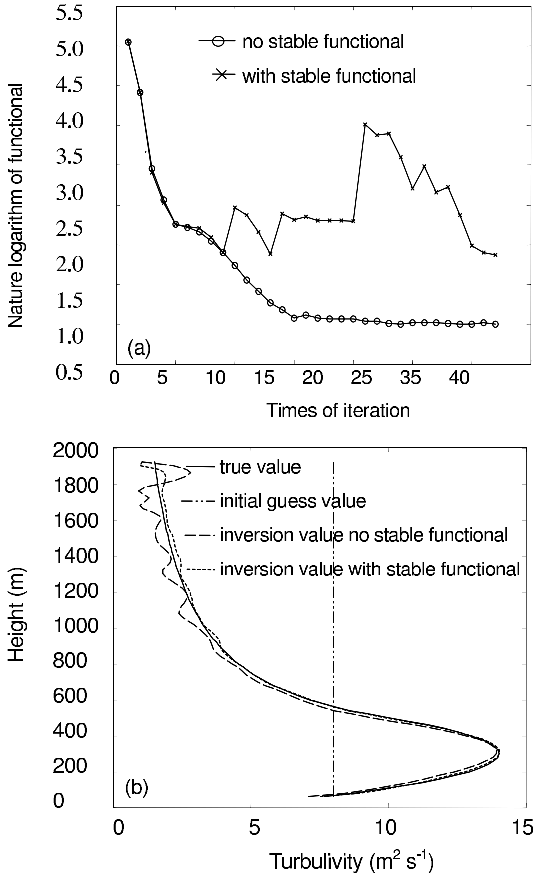

Fig. 2. (a) two kinds of functionals varying with iteration time; (b) comparison of the true value, initial guess value and retrieval of turbulivity with and without a stabilized functional.

5. 2-D wind retrieval from single-Doppler data

Modern weather Doppler radars are important tools to investigate microscale and mesoscale weather systems, but their direct observations are limited to the reflectivity and the radial-velocity component. So, it is necessary to obtain 2-D or 3-D wind fields by retrieval methods. Compared to other retrieval methods, the variational adjoint method has obvious merits. Furthermore, it is easy to incorporate other observational data into the analysis. Because the cost of Doppler radar is high, the New Generation Radar Network in China (CINRAD) can only provide us with single-Doppler radar data in most areas. So it will be the main research direction to use the variational adjoint method to retrieve the wind fields from single-Doppler data in the future.

With the development of variational theory, many retrieval techniques based on the variational method have been proposed. Wu et al.(1997) retrieved the horizontal wind fields with the adjoint equations after simplifying the vertical physical variables in the atmospheric motion equations. The results of the method are encouraging, but it is still difficult to specify boundary conditions. Qiu and Xu (1992), Xu and Qiu (1994), and Xu et al. (1994) developed simple adjoint methods to retrieve horizontal wind fields using the reflectivity advection equation or the radial-wind momentum equation. Their results show that using data over multiple time levels can not only increase the accuracy of the retrieval but also make the method less sensitive to the errors in the observation; neglecting the influence of weak vorticity and divergence can suppress false and small structures in the retrieved wind fields because of noise in the observation data; reasonably setting the weights in the cost function can improve the retrieval results greatly. Gal-Chen and Zhang (1993) proposed an advection retrieval scheme that retrieves three-dimensional wind fields from multiple time levels of reflectivity and radial-wind data by making a Lagrangian conservation assumption for the reflectivities and a steady-state assumption for the eddies in the wind field. The experiment results show that the steady-state assumption is satisfied easily and the retrieval results are improved obviously when the retrievals are performed in a moving frame of reference; proper filtering can reduce errors in the retrieved wind fields. Shapiro et al's (1996) two-scalar algorithm assumes incompressible and frozen-turbulence flow, then recovers the three-dimensional wind fields from the conservation equation of reflectivity. But the assumptions are too strict to be satisfied. And how to retrieve wind fields in deep, moisture convective systems is still a difficult question.

In China, much work on single-Doppler velocity retrievals has been done. Wei et al. (1998) advanced the objective method of the selection of functional weighting factors. Experiments show that the optimal selection of weights is good for improving the retrieval of model parameters. Qiu et al. (2000) presented a variational method for retrieving wind vectors from single-Doppler wind measurements, in which initial forecast fields are subsequently created from the retrieved winds. A numerical experiment is performed by using the Advanced Regional Prediction System (ARPS) developed by Center for Analysis and Prediction of Storms (CAPS) in the United States. The experiment shows that the Doppler radar data can be used to improve the analysis of the initial fields and short-term numerical prediction of mesoscale weather.

As we will see in this paper, based on the variational adjoint method and regularization methods, theoretical analyses and numerical experiments are performed for wind and related parameter retrievals from single-Doppler radar data (Cao et al., 2005). The method uses the mass continuity equation as a weak constraint, in addition to the strong constraint of the reflectivity conservation equation. The wind retrievals are classified into two cases: wind fields without and with vortexes, the latter of which is studied in this paper.

The 2-D reflectivity conservation equation in the polar coordinate system is:

$$\begin{cases} \dfrac{\partial \eta}{\partial t}+v_r\dfrac{\partial \eta}{\partial r}+\dfrac{v_\alpha}{r}\dfrac{\partial \eta}{\partial \alpha}=k\left(\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial \alpha^2}\right)\eta\ , \\ \qquad (t,r,\alpha)\in(0,T)\times(r_1,r_2)\times(\alpha_1,\alpha_2)\ , \\ \eta|_{t=0}=\phi(r,\alpha)\ ,\ \ \eta|_{r=r_1}=\varphi_1(t,\alpha)\ , \\ \eta|_{r=r_2}=\varphi_2(t,\alpha)\ ,\ \ \eta|_{\alpha=\alpha_1}=\psi_1(t,r)\ , \\ \eta|_{\alpha=\alpha_2}=\psi_2(t,r)\ . \end{cases} \tag{5.1}$$

Here $\eta(t,r,\alpha)$ denotes the radar reflectivity, and (v_r,v_α,k) are the radial and tangent velocities and the coefficient of eddy viscosity respectively, and the other quantities have their usual meanings.

The unknown parameters (v_r,v_α,k) will be retrieved or estimated from the Doppler radar data $\eta_{\rm obs},v_{\rm obs}$, with the model (5.1) as the physical constraint. Because three unknown variables are to be retrieved from just one equation, the solution of (5.1) must be ill-posed. To overcome the ill-posedness of the solutions, the cost functional can be defined as:

$$J[v_r(r,\alpha),v_\alpha(r,\alpha),k]=J_1+J_2+J_3+J_4\ , \tag{5.2}$$

here:

$$J_1=\frac{1}{2}\int_0^T\int_{r_1}^{r_2}\int_{\alpha_1}^{\alpha_2}[(\eta-\eta_{\rm obs})^2+\gamma_1(v_r-v_{r,\rm obs})^2]d\Omega\ ,$$

(constraint of the observation field),

$$J_2=\frac{\gamma_2}{2}\int_0^T\int_{r_1}^{r_2}\int_{\alpha_1}^{\alpha_2}\left(\frac{\partial(rv_r)}{r\partial r}+\frac{\partial v_\alpha}{r\partial\alpha}\right)^2d\Omega\ ,$$

(weak constraint),

$$J_3=\frac{\gamma_3}{2}\int_0^T\int_{r_1}^{r_2}\int_{\alpha_1}^{\alpha_2}(|\nabla v_r|^2+|\nabla v_\alpha|^2)d\Omega\ ,$$

(regularization term),

$$J_4 = \frac{\gamma_4}{2}\int_0^T \int_{r_1}^{r_2} \int_{\alpha_1}^{\alpha_2} (v_\alpha - \bar{v}_\alpha)^2 d\Omega \,,$$

(constraint of the background field).

η is the reflectivity field predicted from model (5.1). (v_r, v_α, k) are the retrieved variables. $\eta_{\rm obs}, v_{r,{\rm obs}}$ are observations from the radar. γ_1, γ_2 and γ_4 are weighting coefficients, and γ_3 is the regularization parameter. The ultimate goal for the retrieval is to optimally estimate the three unknowns (v_r, v_α, k) through the minimization of the cost functional.

The gradient of J with respect to the unknowns (v_r, v_α, k) can be calculated as:

$$\left\{\begin{array}{l} \nabla_{v_r} J = \int_0^T \Big\{ \gamma_1 (v_r - v_{r,{\rm obs}}) - p\dfrac{\partial \eta}{\partial r} \\ \qquad -\gamma_2 \dfrac{\partial}{\partial r}\left[\dfrac{\partial (r v_r)}{r\partial r} + \dfrac{\partial v_\alpha}{r\partial \alpha}\right] \\ \qquad -\gamma_3 \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}\left(r\dfrac{\partial v_r}{\partial r}\right) + \dfrac{\partial}{\partial \alpha}\left(\dfrac{\partial v_r}{r\partial \alpha}\right)\right]\Big\} dt \,, \\ \nabla_{v_\alpha} J = \int_0^T \Big\{ -p\dfrac{\partial \eta}{r\partial \alpha} - \gamma_2 \dfrac{\partial}{r\partial \alpha}\left[\dfrac{\partial (r v_r)}{r\partial r} + \dfrac{\partial v_\alpha}{r\partial \alpha}\right] \\ \qquad -\gamma_3 \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}\left(r\dfrac{\partial v_\alpha}{\partial r}\right) + \dfrac{\partial}{\partial \alpha}\left(\dfrac{\partial v_\alpha}{r\partial \alpha}\right)\right]\Big\} dt \,, \\ \nabla_k J = \left\langle p \,, \; \left(\dfrac{\partial^2}{\partial r^2} + \dfrac{\partial}{r\partial r} + \dfrac{\partial^2}{r\partial \alpha^2}\right)\eta \right\rangle \,. \end{array}\right. \tag{5.3}$$

Below is the numerical test. For convenience, the observationals are chosen as the solution to (5.1) superimposed onto a disturbance, and the wind field contains a vortex structure, viz., a Rankine vortex. The minimization algorithm used in this paper is the Quasi-Newton algorithm. The domain parameters are as follows: r_1=500 m, r_2=2500 m, α_1=0, and $\alpha_2 = 2\pi$.

Case 1 The 2-D vortex wind field and coefficient of eddy viscosity are retrieved from the observational data with the mass conservation as a weak constraint. The optimal weights are: $\gamma_1 = 4.0, \gamma_2 = 0.05, \gamma_3$=0.0, and $\gamma_4 = 0.0$. The true wind field, retrieved wind field and vector errors are shown in Figs. 3–4. The retrieved value for the coefficient of eddy viscosity is k=124.47 (the true value $k_t = 125.0$).

Case 2 The background field is incorporated into the cost functional and each weight is specified as follows: $\gamma_1 = 4.0, \gamma_2 = 0.05, \gamma_3 = 0.0$, and $\gamma_4 = 2.0$. The retrieved wind field and vector errors are shown in Figs. 7 and 8. The retrieved value for the coefficient of eddy viscosity diffusion is $k = 124.43$. As indicated in the figures, the accuracy of retrieval for wind is improved although that of the eddy diffusion coefficient declines a little.

Case 3 The regularization is incorporated into the cost functional. The results of the numerical experiments reveal that the regularization term helps to improve the accuracy of wind to a certainty. And the

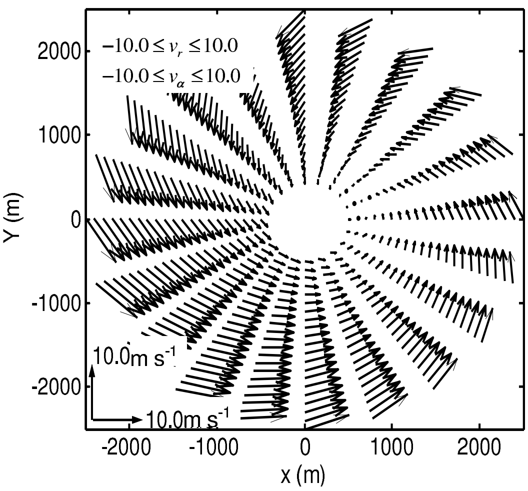

Fig. 3. True vortex wind field at $t = T/4$.

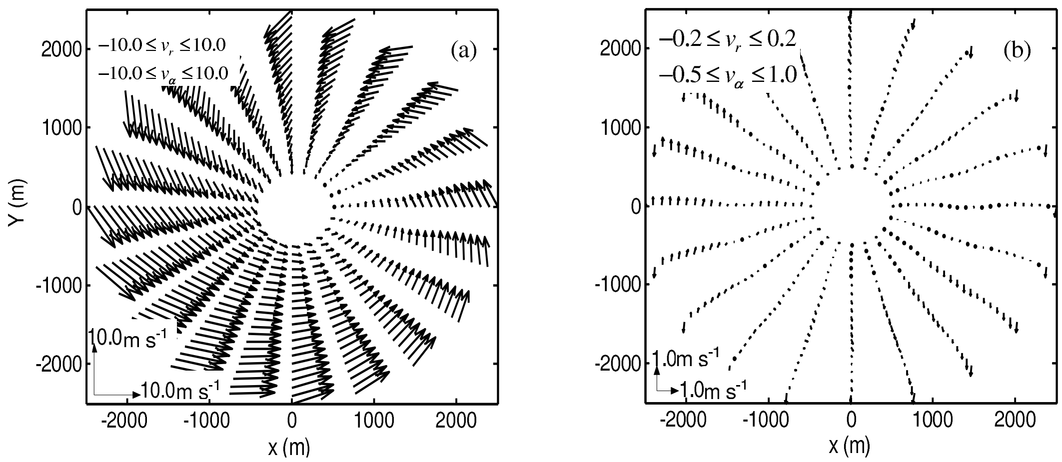

Fig. 4. (a) Retrieved vortex wind without the background field; (b) error vectors without the background field.

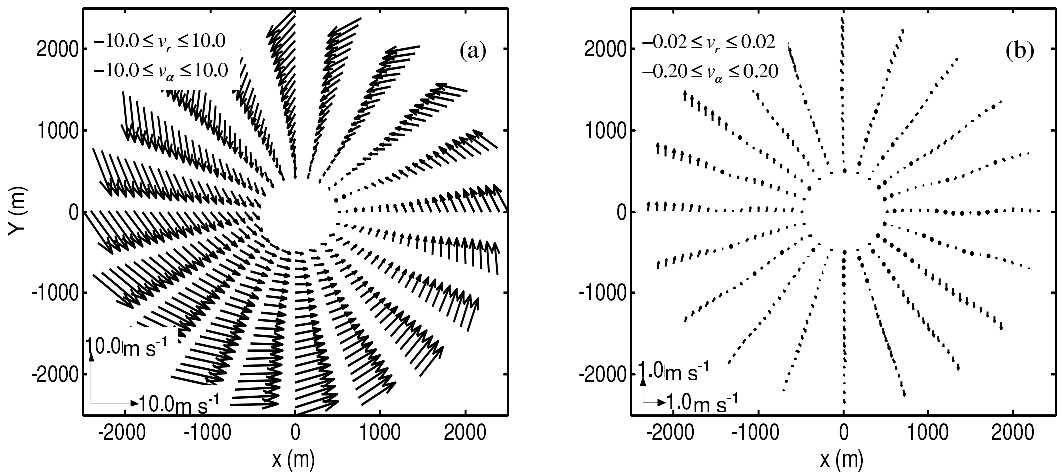

Fig. 5. (a) Retrieved vortex wind with background; (b) error vectors with background.

result is sensitive to the value of its weight. Each weight is specified as follows: $\gamma_1 = 4.0, \gamma_2 = 0.05, \gamma_3 = 0.35$, and $\gamma_4 = 2.0$. The retrieved wind field is similar to that in Fig. 7 and is not given here. The retrieved coefficient of eddy viscosity is $k = 124.79$.

In the above experiments, the two-dimensional vortex wind field and coefficient of eddy viscosity are retrieved simultaneously from the reflectivity data and radial-wind data with the mass continuity as a weak constraint, in addition to the strong constraint of the reflectivity conservation equation. The numerical experiments indicate that the wind field is well retrieved and its accuracy is improved after adding the background term to the cost functional. At the same time, the regularization term helps to improve the precision of retrieval to a certainty. But the result is sensitive to the value of the weights due to the fact that the vortex wind changes significantly in space itself.

Based on the results for the artificial data, natural 2-D wind fields are retrieved with low altitude radar data from the Nanjing Doppler radar (omitted here due to the limitation of space), and the results are promising.

6. Retrieval of physical process parameters

The adjoint method holds only for differentiable systems. While adopting the parameterization of physical processes in prediction models, the models usually contain discontinuous physical processes called "on-off", processes (Xu, 1996a; Zou, 1997; Mu and Wang, 2003). For example, Zou (1997) studied the assimilation of moist physical processes with "on-off" switches by theoretical analysis and numerical modeling; Xu (1998), however, pointed out that, although some of the formulations in Zou (1997) and Xu (1998) are similar, there are substantial discrepancies in the derivations of the adjoint. The authors can also refer to Xu (1997), and Xu and Gao (1999). Also for this same problem, Wang et al. (2002) proved the existence of the classical functional gradient concerning the initial condition; Wang et al. (2005) also proved the existence of the classical functional gradient concerning the parameter. Here, we apply a different way, i.e., we introduce the weak form of the original model by which the adjoint model is derived directly; and the functional gradients with respect to both the initial condition and the parameter can also be obtained in the classical sense. (Huang et al., 2004b).

The following simple non-differentiable ordinary differential system (Xu, 1996a) is taken as an example to demonstrate the weak form method for generalized variational data assimilation.

$$\begin{cases} \dfrac{da}{dt} = F(t) + \beta H_+(q - q_c) , \\ q_{t=0} = q_0 \end{cases} \tag{6.1}$$

where $q(t)$ is the specific humidity, β and q_0 are constants, q_c is the critical point, $q_c > q_0, F(t) > -\beta, F(t) \geqslant \delta > 0, F(t)$ is a differentiable function, and $H_+(\cdot)$ is the Heaviside function. It is obvious that Eq. (6.1) is well-posed on $[0, T]$ and that $q(t)$ is continuous. Take T such that there exists a unique τ_c on $[0, T]$, and $q(\tau_c) = q_c$. The following space is introduced:

$$\mathbb{S}^1(0,T) = \{p(t)|p(t) \in H^1(0,T) \text{ and } p|_{t=T} = 0\} .$$

So the weak form of (6.1) can be written as:

$$\begin{aligned} \int_0^T \frac{\partial q}{\partial t} p(t)dt &= \int_0^T F(t)p(t)dt \\ &\quad + \int_0^T \beta H_+(q - q_c)p(t)dt \\ &= \int_0^T F(t)p(t)dt + \int_0^T \beta p(t)dt . \end{aligned} \tag{6.2}$$

Table 1. The comparison of assimilation results of three tests.

	The initial values of the retrieval			The number of iterations	The results of the retrieval		
	q_0	q_c	J		q_0	q_c	J
Test 1	0.05	0.25	9.1573	28	0.2439	0.4707	0.0012
Test 2	0.05	0.55	1.3018	6	0.25	0.4676	0.0
Test 3	0.42	0.55	2.7382	17	0.25	0.4622	0.0

Denoting the observation of q on $[0, T]$ by $q_{\rm obs}$, the optimization problem is to find q_0 and q_c such that

$$J[q_0, q_c] = \frac{1}{2}\int_0^T [q(t) - q_{\rm obs}(t)]^2 dt = \min\ . \qquad (6.3)$$

Set perturbations to q_0 and q_c such that $q_0 \to \tilde{q}_0 = q_0 + \alpha U, q_c \to \tilde{q}_c = q_c + \alpha\xi$, and $\tau_c \to \tilde{\tau}_c, q(t) \to \tilde{q}(t)$, and denote $\hat{q} = \lim\limits_{\alpha\to 0}(\tilde{q} - q)/\alpha, \hat{\tau}_c = \lim\limits_{\alpha\to 0}(\tilde{\tau} - \tau)/\alpha$, which are in the sense of $L^2(0, T)$. From Eq. (6.2),

$$-q(0)p(0) - \int_0^T q(t)\frac{dp(t)}{dt}dt = \int_0^T F(t)p(t)dt + \int_{\tau_c}^T \beta p(t)dt\ , \qquad (6.4)$$

so

$$-Up(0) - \int_0^T \frac{\tilde{q}(t) - q(t)}{\alpha}\frac{dp(t)}{dt}dt = \int_{\tilde{\tau}_c}^{\tau_c} \frac{\beta}{\alpha}p(t)dt\ . \qquad (6.5)$$

When $\alpha \to 0$,

$$-Up(0) - \int_0^T \hat{q}(t)\frac{dp(t)}{dt}dt = -\beta p(\tau_c)\hat{\tau}_c\ . \qquad (6.6)$$

Because

$$q_c = q(\tau_c) = q_0 + \int_0^{\tau_c} F(\tau)d\tau\ ,$$

$$\tilde{q}_c = q_c + \alpha\xi = q_0 + \alpha U + \int_0^{\tilde{\tau}_c} F(\tau)d\tau\ ,$$

then

$$\xi = U + F(\tau_c)\hat{\tau}_c\ , \qquad (6.7)$$

and with Eq. (6.6), Eq. (6.7) can be written as

$$-Up(0) - \int_0^T \hat{q}(t)\frac{dp(t)}{dt}dt = -\beta p(\tau_c)\frac{\xi - U}{F(\tau_c)}\ . \qquad (6.8)$$

Introduce the generalized adjoint model:

$$\begin{cases} -\dfrac{dp(t)}{dt} = q - q_{\rm obs}\ , \\ p|_{t=T} = 0\ , \end{cases} \qquad (6.9)$$

so

$$-Up(0) + \int_0^T \hat{q}(t)(q - q_{\rm obs})dt = -\beta p(\tau_c)/F(\tau_c)\cdot U + \beta p(\tau_c)/F(\tau_c)\cdot\xi\ .$$

Through

$$J'[q_0, q_c; U, \xi] = \int_0^T \hat{q}(q - q_{\rm obs})dt = \nabla q_0 J\cdot U + \nabla q_c J\cdot\xi\ ,$$

the gradients of the cost functional (6.3) with respect to q_0 and q_c are obtained respectively:

$$\begin{cases} \nabla q_0 J = p(0) + \beta p(\tau_c)/F(\tau_c)\ , \\ \nabla q_c J = -\beta p(\tau_c)/F(\tau_c)\ . \end{cases} \qquad (6.10)$$

Suppose that the true values of the parameters in model (6.1) are $F(t) = 2.0, \beta = -1.5, q_c = 0.46, q_0 = 0.25$ and $T = 1.0$. The time step in the numerical experiments is $dt = 0.005$.

Integrate (6.1) with the true values of q_0 and q_c mentioned above, and save the results of q as the observation $q_{\rm obs}$. Then use the perturbed values of q_c and q_0 as the initial guesses. The descent algorithm used here is the Quasi-Newton method. In order to examine the effectiveness of the retrieval, three tests with different perturbed values of q_c and q_0 are implemented.

The perturbed values and the results of the retrieval are illustrated in Table 1. From the table it can be seen that all the results of the three tests are ideal. Figure 6 shows the track of the cost functional descending in the process of iteration. From the figure it can be seen that the domain of the contour is divided by the line between the points (2.1, 2.1) and (5.0, 5.0). On the bottom right hand side the contours

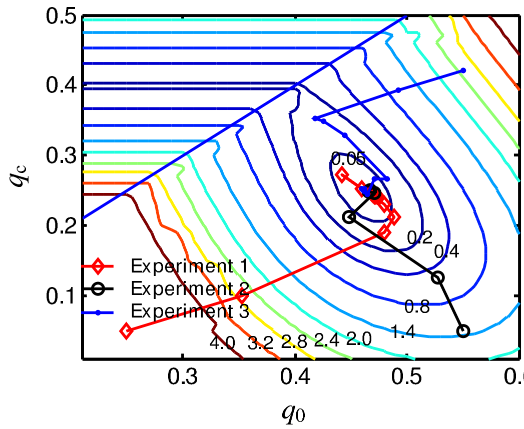

Fig. 6. The track of the cost functional descending in the process of iteration.

are distributed elliptically while on the top left hand side, as $q_0 > q_c$ here, the contours is vary only with q_0. From the result of the experiment we can conclude that the method effective in resolving the problems of this type.

7. Other applications of VAR and the regularization method

Other applications of VAR and the regularization method include the retrieval of GPS dropsonde's motion (Fang and Huang, 2004; Fang et al., 2004), adjustment of the wind field by the variational analysis method (Lan et al., 2004; Huang et al., 2005a), assimilation of typhoon tracks and retrieval of model parameters (Xiang et al., 2004), retrieval of parameters in the ENSO cycle using the observational data Tropical Atmosphere and Oceans (TAO) (Du, 2004), retrieval of parameters in the 1-D shallow water model (Pan and Huang, 2004; Du et al., 2004), and application of the improved discrepancy principle in retrievals of atmosphere infrared remote sensing (Li and Huang, 2001). It is suggested that readers refer to these papers for the details.

8. Summary

Because of its ill-posedness and high nonlinearity, the parameter retrieval problem is very difficult to solve. In this paper, the variational data assimilation method combined with the regularization techniques was employed to retrieve the following atmospheric and oceanic parameters: the vertical eddy diffusion coefficient, the turbulivity of the atmospheric boundary layer, the wind with Doppler radar data, and the physical process parameters. Model parameter retrieval with global and local observations was also studied. The results in this paper indicated that the application of the regularization techniques helped to reduce numerical oscillations in the iterations, to speed up the convergence of solutions, and to boost the accuracy of solutions.

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