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ORIGINAL PAPER

Hamiltonian Cycles in Normal Cayley Graphs

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Abstract

It has been conjecture that *every finite connected Cayley graph contains a hamiltonian cycle*. Given a finite group G and a connection set S, the Cayley graph $Cay(G, S)$ will be called *normal* if for every $g \in G$ we have that $g^{-1}Sg = S$. In this paper we present some conditions on the order of the elements of the connexion set which imply the existence of a hamiltonian cycle in the graph and we construct it in an explicit way.

Keywords Cayley graph · Hamiltonian cycle · Normal connection set

Mathematics Subject Classification 05C45 · 05C99

1 Introduction

The problem of finding hamiltonian cycles in graphs is a difficult problem, and since 1969 has received great attention by the Lovász Conjecture which states that every vertex-transitive graph has a hamiltonian path. A variant of the Lovász Conjecture on hamiltonian paths states that every finite connected Cayley graph contains a hamiltonian cycle (see, for instance [1,5,9,12]). In particular, there are several works on the existence of hamiltonian cycles in Cayley graphs of different types (see, for instance [2,3,6,7,14]).

Let G be a finite group. A subset $S \subseteq G$ will be called symmetric if $S = S^{-1}$. Given a symmetric subset $S \subseteq G \backslash \{e\}$ (with e the identity of G), the Cayley graph $Cay(G, S)$ is the graph with vertex set G and a pair $\{\alpha, \beta\}$ is an edge of $Cay(G, S)$ if and only if there is $s \in S$ such that $\alpha = \beta s$ (since S is symmetric, observe that $s^{-1} \in S$ and $\beta = \alpha s^{-1}$). Because S is symmetric and does not contain the identity

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we only work with simple Cayley graphs. A Cayley graph $Cay(G, S)$ will be called *normal* if for every $\alpha \in G$, $\alpha^{-1}S\alpha = S$. In the literature there is another definition of normal Cayley graph, which is different from the one used in this paper, that said that a Cayley graph on a group G is normal if the right regular representation of the group G is normal in the full automorphism group of the graph (see, for instance [10,13]).

In [8] the authors proved the following.

Theorem 1 *Let $G = \langle \delta_1, \delta_2 \rangle$ be a group. If $Cay(G, S)$ is a normal Cayley graph such that $\{\delta_1, \delta_2\} \subseteq S$ then $Cay(G, S)$ contains a hamiltonian cycle.*

In this paper we present the following result, that concludes a great amount of normal Cayley graphs are hamiltonian.

Theorem 2 *Let $G = \langle \delta_1, \ldots, \delta_m \rangle$ be a group, and suppose that $|\langle \delta_1 \rangle||\langle \delta_2 \rangle| > m+1$. If $Cay(G, S)$ is a normal Cayley graph with $\{\delta_1, \ldots \delta_m\} \subseteq S$, then $Cay(G, S)$ contains a hamiltonian cycle.*

In the last theorem we do not know any counterexamples if the condition on the order of elements in S is removed. We think that this hypothesis is not needed but the removal of this would require a new and different proof. The condition guarantees that there are enough vertices in each coset to construct the proposed Hamiltonian cycle.

For general concepts we may refer the reader to [4,11].

2 Notation and Previous Results

In order to prove the main theorem, we need some definitions and previous results.

Let G be a group, let G_0 be a subgroup of G, and let

$$\mathcal{P} = \{a_0G_0, a_1G_0, \ldots, a_nG_0\}$$

be the partition of G in cosets induced by the subgroup G_0 (with a_0 the identity element of G). For each $0 \le i \le n$, $C(a_iG_0)$ will denote the subdigraph of $Cay(G, S)$ induced by the set of vertices a_iG_0. Given two isomorphic vertex disjoint subgraphs H and H' of $Cay(G, S)$, if there is an isomorphism Ψ between H and H' such that for every $x \in V(H)$, $\{x, \Psi(x)\}$ is an edge of $Cay(G, S)$, then we will say that H and H' are *attached* (*by* $\Psi : H \to H'$).

Lemma 3 *For every $0 \le i, j \le n$, $C(a_iG_0) \cong C(a_jG_0)$. Moreover, for every $0 \le i \le n$ and $\delta \in S$, $C(a_iG_0)$ and $C(\delta a_iG_0)$ are attached (by the map $a \to \delta a$).*

Proof Given $a_iG_0, a_jG_0 \in \mathcal{P}$ let $\Phi : a_iG_0 \to a_jG_0$ defined, for each $g \in G_0$, as $\Phi(a_ig) = a_jg$. If $\Phi(a_ig) = \Phi(a_ig_1)$ then $a_jg = a_jg_1$, so $g = g_1$. Therefore Φ is injective and since all cosets have the same cardinality, Φ is bijective. If a_ig_1 and a_ig_2 are adjacent in $C(a_iG_0)$ then $g_1^{-1}a_i^{-1}a_ig_2 = g_1^{-1}g_2 \in S$. Therefore

$$\Phi(a_ig_1)^{-1}\Phi(a_ig_2) = g_1^{-1}a_j^{-1}a_jg_2 = g_1^{-1}g_2 \in S$$

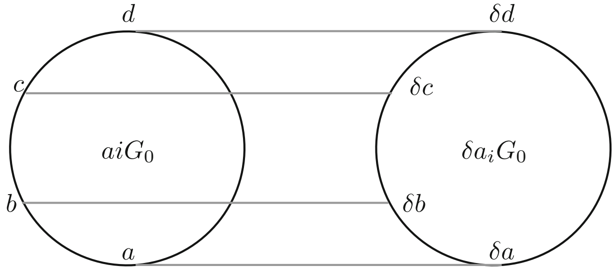

Fig. 1 The cosets $a_i G_0$ and $\delta a_i G_0$ are attached

and then $\Phi(a_i g_1)$ and $\Phi(a_i g_2)$ are adjacent in $C(a_j G_0)$, and the first part of the lemma follows. For the second part, let $a \in a_i G_0$ and $\delta a \in \delta a_i G_0$. Clearly the map $a \to \delta a$ define an isomorphism between $C(a_i G_0)$ and $C(\delta a_i G_0)$ and since S is normal, $a^{-1}\delta a \in S$, therefore $\{a, \delta a\}$ is an edge in $Cay(G, S)$ (see Fig. 1) and the lemma follows. □

Let $G = \langle \delta_1, \delta_2, \ldots, \delta_m \rangle$ be a group generated by $m \geq 3$ elements. Let $Cay(G, S)$ be a normal Cayley graph with connection set S such that $\{\delta_1, \ldots, \delta_m\} \subseteq S$. Let $G_0 = \langle \delta_1, \delta_2 \rangle$ and let

$$\mathcal{P} = \{a_0 G_0, a_1 G_0, \ldots, a_{n-1} G_0, a_n G_0\}.$$

be the partition of G in cosets induced by the subgroup G_0 (with a_0 the identity element of G).

We denote as $D(\mathcal{P}, S)$ the digraph with vertex set $\mathcal{P}$ and there is an arc $(a_i G_0, a_j G_0)$ in $D(\mathcal{P}, S)$ if and only if $a_j G_0 = \delta a_i G_0$ for some $\delta \in \{\delta_1, \ldots, \delta_m\}$.

Lemma 4 *For every $1 \leq j \leq n$, there is a $(G_0, a_j G_0)$- directed path in $D(\mathcal{P}, S)$.*

Proof Let $a_j G_0 \in \mathcal{P}$ and let $a_j = \delta_{l_k}^{j_k} \ldots \delta_{l_1}^{j_1}$, with $\delta_{l_i} \in \{\delta_1, \ldots, \delta_m\}$ and $i \in \{1, \ldots, k\}$. A directed walk W with initial vertex G_0 and final vertex $a_j G_0$ is

$$W = (G_0, \delta_{l_1} G_0, \ldots, \delta_{l_1}^{j_1} G_0, \delta_{l_2} \delta_{l_1}^{j_1} G_0, \ldots, \delta_{l_2}^{j_2} \delta_{l_1}^{j_1} G_0, \ldots, \delta_{l_k}^{j_k} \ldots \delta_{l_1}^{j_1} G_0 = a_j G_0).$$

On the other hand, if there is a directed walk, then there is a directed path, which is subgraph of the walk, with the same initial and final vertices. We can conclude that the $(G_0, a_j G_0)$-directed path in $D(\mathcal{P}, S)$ exists. □

Let $D^*(\mathcal{P}, S)$ be a spanning subdigraph of $D(\mathcal{P}, S)$ with a minimal set of arcs such that for every $a_j G_0 \in V(D^*(\mathcal{P}, S)) \backslash \{G_0\}$ there is a $(G_0, a_j G_0)$-directed path in $D^*(\mathcal{P}, S)$. Notice that G_0 has indegree 0 in $D^*(\mathcal{P}, S)$.

Lemma 5 *$D^*(\mathcal{P}, S)$ is a rooted tree, with root in G_0.*

Proof Consider the following.
Claim 1. Except for G_0, all the other vertices in $D^*(\mathcal{P}, S)$ has indegree 1.
Since for every $a_jG_0 \in V(D^*(\mathcal{P}, S))\backslash\{G_0\}$ there is a (G_0, a_jG_0)-directed path it follows that, except for G_0, every vertex has indegree at least 1. On the other hand, if there is a vertex a_kG_0 and two different arcs (a_iG_0, a_kG_0) and (a_jG_0, a_kG_0) in $D^*(\mathcal{P}, S)$, by the minimality of $D^*(\mathcal{P}, S)$ it follows that every (G_0, a_kG_0)-directed path in $D^*(\mathcal{P}, S)$ uses both arcs (a_iG_0, a_kG_0) and (a_jG_0, a_kG_0) which is a contradiction.
Claim 2. There is no cycle in $D^*(\mathcal{P}, S)$.
Suppose there is a cycle C in $D^*(\mathcal{P}, S)$. By claim 1 it follows that C has to be a directed cycle and therefore, since G_0 has indegree 0, G_0 is not a vertex of C. Thus, there is a directed path from G_0, which is not a vertex of C, to some vertex x of C which is impossible since, by claim 1, x has indegree 1.

From Lemma 4, claims 1 and 2 and by definition of $D^*(\mathcal{P}, S)$, $D^*(\mathcal{P}, S)$ is a tree with root G_0. □

Notice that $D^*(\mathcal{P}, S)$ is an underlying structure in the normal Cayley graph $Cay(G, S)$. The vertices of $D^*(\mathcal{P}, S)$ are cosets and all its arcs represent multiple edges in $Cay(G, S)$ which joints two cosets, indeed, two vertices joint by an arc in $D^*(\mathcal{P}, S)$ represents two cosets attached in $Cay(G, S)$ and these cosets as induced subgraphs are isomorphic (see Lemma 3).

Given a coset $aG_0 \in \mathcal{P}$, let $N^+_{D^*(\mathcal{P},S)}(aG_0)$ denotes its out-neighborhood in $D^*(\mathcal{P}, S)$. Let

$$T[aG_0] = \{aG_0\} \cup N^+_{D^*(\mathcal{P},S)}(aG_0)$$

and let $M[aG_0]$ be the subgraph of $C(G, S)$ induced by the set of vertices

$$\bigcup_{hG_0 \in T[aG_0]} V(C(hG_0)).$$

Lemma 6 *Let $aG_0 \in \mathcal{P}$ be a coset and suppose there is a hamiltonian cycle $C = (b_1, b_2, \ldots, b_{m_0})$ in $C(aG_0)$, with $m_0 > m + 1$.*

- (i) *For every $\delta aG_0 \in T[aG_0]\backslash\{aG_0\}$, $(\delta b_1, \delta b_2, \ldots, \delta b_{m_0})$ is a hamiltonian cycle in $C(\delta aG_0)$.*
- (ii) *For every pair of vertices $b_t, b_{t-1} \in V(C)$ (with the index of b mod m_0) there is a (b_t, b_{t-1})-hamiltonian path P in $M[aG_0]$ such that for every $\delta aG_0 \in T[aG_0]\backslash\{aG_0\}$ there is s_δ such that the hamiltonian path*

$$P_{\delta a} = (\delta b_{s_\delta}, \delta b_{s_\delta - 1}, \ldots, \delta b_1, \delta b_{m_0}, \ldots, \delta b_{s_\delta + 1})$$

of $C(\delta aG_0)$ is a subpath of P.

Proof Let $aG_0 \in \mathcal{P}$ be a coset and suppose there is a hamiltonian cycle $(b_1, b_2, \ldots, b_{m_0})$ in $C(aG_0)$, with $m_0 > m + 1$.

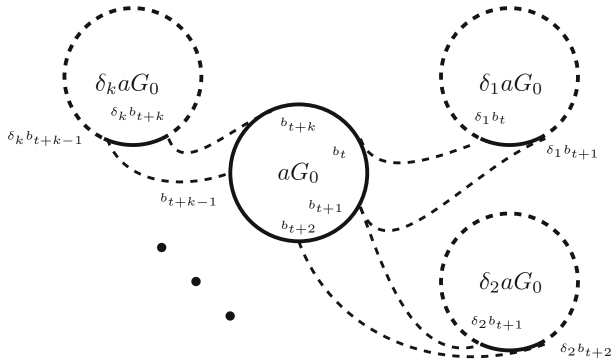

Fig. 2 Hamiltonian cycle in $T[aG_0]$

From Lemma 3 we see that for every $\delta aG_0 \in T[aG_0]\backslash\{aG_0\}$, $C(aG_0)$ and $C(\delta aG_0)$ are attached (by the map $a \to \delta a$). From here it follows that for every $\delta aG_0 \in T[aG_0]\backslash\{aG_0\}$, $(\delta b_1, \delta b_2, \ldots, \delta b_{m_0})$ is a hamiltonian cycle in $C(\delta aG_0)$.

Without loss of generality, let $T[aG_0] = \{aG_0\} \cup \{\delta_i aG_0 : i = 1, \ldots, k\}$. Observe that $k \leq m$ (see Fig. 2).

Let $b_t \in V(C)$ and let

$$\begin{gathered}
P = (b_t, \delta_1 b_t, \delta_1 b_{t-1}, \delta_1 b_{t-2}, \ldots, \delta_1 b_1, \delta_1 b_{m_0}, \ldots, \delta_1 b_{t+1}, \\
b_{t+1}, \delta_2 b_{t+1}, \delta_2 b_t, \delta_2 b_{t-1}, \ldots, \delta_2 b_1, \delta_2 b_{m_0}, \ldots, \delta_2 b_{t+2}, \\
b_{t+2}, \delta_3 b_{t+2}, \delta_3 b_{t+1}, \delta_3 b_t, \ldots, \delta_3 b_1, \delta_3 b_{m_0}, \ldots, \delta_3 b_{t+3}, \\
\ldots \\
b_{t+k-1}, \delta_k b_{t+k-1}, \delta_k b_{t+k-2}, \delta_k b_{t+k-3}, \ldots, \delta_k b_1, \delta_k b_{m_0}, \ldots, \delta_k b_{t+k}, \\
b_{t+k}, b_{t+k+1}, \ldots, b_{m_0}, b_1, \ldots, b_{t-1}).
\end{gathered}$$

From here the result follows. □

3 Proof of the Theorem 2

Let $G = \langle \delta_1, \ldots, \delta_m \rangle$ be a group, and suppose that $|\langle \delta_1 \rangle||\langle \delta_2 \rangle| > m+1$. Let $Cay(G, S)$ be a normal Cayley graph with $\{\delta_1, \ldots \delta_m\} \subseteq S$, let $G_0 = \langle \delta_1, \delta_2 \rangle$, and let

$$\mathcal{P} = \{a_0 G_0, a_1 G_0, \ldots, a_n G_0\}$$

be the partition of G in cosets induced by the subgroup G_0 (with a_0 the identity element of G).

Claim 1. The Cayley graph $Cay(G_0, S \mid_{G_0})$ is a normal Cayley graph.

Since $Cay(G, S)$ is normal, for every $g \in G$, $g^{-1}Sg = S$. Therefore for every $g \in G_0$ we see that $g^{-1}(S \mid_{G_0})g = g^{-1}(S \cap G_0)g = S \cap G_0 = S \mid_{G_0}$, and the claim follows.

Since $G_0 = \langle \delta_1, \delta_2 \rangle$ and $\{\delta_1, \delta_2\} \subseteq S \mid_{G_0}$, from Claim 1 and Theorem 1 it follows that $Cay(G_0, S \mid_{G_0})$ is hamiltonian. Observe that $Cay(G_0, S \mid_{G_0})$ is a spanning subdigraph of $C(G_0)$ and therefore $C(G_0)$ is hamiltonian. Let $C = (b_1, b_2, \ldots, b_{m_0})$ be a hamiltonian cycle in $C(G_0)$.

Let $D^*(\mathcal{P}, S)$ be a spanning subdigraph of $D(\mathcal{P}, S)$ defined as in the previous section, and for each $k \geq 1$ let $\mathcal{T}_k$ be the subdigraph of $D^*(\mathcal{P}, S)$ induced by the set of vertices (cosets) $aG_0 \in \mathcal{P}$ such that the distance in $D^*(\mathcal{P}, S)$ from G_0 to aG_0 is at most k. For each $k \geq 1$, let $\mathcal{L}_k$ be the set of leaves of $\mathcal{T}_k$.

We will prove the result by showing, by induction on k, that for every $k \geq 1$ the subgraph of $Cay(G, S)$ induced by the set of vertices

$$\bigcup_{aG_0 \in V(\mathcal{T}_k)} V(C(aG_0))$$

contains a hamiltonian cycle C_k such that for each $aG_0 \in \mathcal{L}_k$ there is a hamiltonian cycle $(d_1, \ldots, d_{m_0})$ of $C(aG_0)$ and an integer s_a such that the hamiltonian path

$$(d_{s_a}, d_{s_a - 1}, \ldots, d_1, d_{m_0}, \ldots, d_{s_a + 1})$$

of $C(aG_0)$ is a subpath of C_k.

(i) $k = 1$

Observe that $\mathcal{T}_1 = \mathcal{T}[G_0]$ and $\mathcal{L}_1 = \mathcal{T}[G_0] \backslash \{G_0\}$. Since $C = (b_1, b_2, \ldots, b_{m_0})$ is a hamiltonian cycle in $C(G_0)$, by Lemma 6 we see that for every $\delta G_0 \in T[G_0] \backslash \{G_0\} = \mathcal{L}_1$, $(\delta b_1, \delta b_2, \ldots, \delta b_{m_0})$ is a hamiltonian cycle in $C(\delta G_0)$; and there is a (b_2, b_1)-hamiltonian path P in $M[G_0]$ such that for every $\delta G_0 \in T[G_0] \backslash \{G_0\}$ there is s_δ such that the hamiltonian path

$$(\delta b_{s_\delta}, \delta b_{s_\delta - 1}, \ldots, \delta b_1, \delta b_{m_0}, \ldots, \delta b_{s_\delta + 1})$$

of $C(\delta G_0)$ is a subpath of P. Therefore,

$$C_1 = P + \{b_1, b_2\}$$

is a hamiltonian cycle with all the properties we need.

(ii) Suppose that the statement is true for $1 \leq l \leq k$.

(iii) $l = k + 1$

By induction hypothesis, the subgraph of $Cay(G, S)$ induced by the set of vertices

$$\bigcup_{aG_0 \in V(\mathcal{T}_k)} V(C(aG_0))$$

contains a hamiltonian cycle C_k such that for each $aG_0 \in \mathcal{L}_k$ there is a hamiltonian cycle $(d_1, \ldots, d_{m_0})$ of $C(aG_0)$ and an integer s_a such that the hamiltonian path

$$(d_{s_a}, d_{s_a - 1}, \ldots, d_1, d_{m_0}, \ldots, d_{s_a + 1})$$

of $C(aG_0)$ is a subpath of C_k.
Observe that $V(\mathcal{T}_{k+1})\backslash V(\mathcal{T}_k) = \mathcal{L}_{k+1}$ and that $\{T[aG_0]\backslash\{aG_0\}\}_{aG_0\in\mathcal{L}_k}$ is a partition of $\mathcal{L}_{k+1}$. Let $\mathcal{L}_k = \{q_1G_0, \ldots q_rG_0\}$.
Let $(d_1, \ldots, d_{m_0})$ be the hamiltonian cycle of $C(q_1G_0)$ and
$(d_{s_{q_1}}, d_{s_{q_1}-1}, \ldots, d_1, d_{m_0}, \ldots, d_{s_{q_1}+1})$ be the hamiltonian path of $C(q_1G_0)$ which is contained as a subpath in C_k. Let Q_k^1 be the $(d_{s_{q_1}}, d_{s_{q_1}-1})$-path obtained from C_k by deleting the edge $\{d_{s_{q_1}}, d_{s_{q_1}-1}\}$.
From Lemma 6, we see that for every $\delta q_1G_0 \in T[q_1G_0]\backslash\{q_1G_0\}$,
$(\delta d_1, \delta d_2, \ldots, \delta d_{m_0})$ is a hamiltonian cycle in $C(\delta q_1G_0)$; and there is a $(d_{s_{q_1}}, d_{s_{q_1}-1})$-hamiltonian path P_{q_1} in $M[q_1G_0]$ such that for every $\delta q_1G_0 \in T[q_1G_0]\backslash\{q_1G_0\}$ there is s_δ such that the hamiltonian path

$$(\delta d_{s_\delta}, \delta d_{s_\delta-1}, \ldots, \delta d_1, \delta d_{m_0}, \ldots, \delta d_{s_\delta+1})$$

of $C(\delta q_1G_0)$ is a subpath of P_{q_1}. Thus, $C_k^1 = Q_k^1 \circ P_{q_1}$ is a hamiltonian cycle of the subgraph of $C(G, S)$ induced by the set of vertices

$$\bigcup_{hG_0\in V(\mathcal{T}_k)} V(C(hG_0)) \cup \bigcup_{hG_0\in T[q_1G_0]\backslash\{q_1G_0\}} V(C(hG_0))$$

such that for every $\delta q_1G_0 \in T[q_1G_0]\backslash\{q_1G_0\}$ there is s_δ such that the hamiltonian path

$$(\delta d_{s_\delta}, \delta d_{s_\delta-1}, \ldots, \delta d_1, \delta d_{m_0}, \ldots, \delta d_{s_\delta+1})$$

of $C(\delta q_1G_0)$ is a subpath of C_k^1.
For the step j, with $1 < j \leq r$, let Q_k^j be the $(d_{s_{q_j}}, d_{s_{q_j}-1})$-path obtained from C_k^{j-1} by deleting the edge $\{d_{s_{q_j}}, d_{s_{q_j}-1}\}$ (which belongs to $C(q_jG_0)$) and attached to it the $(d_{s_{q_j}}, d_{s_{q_j}-1})$-path P_{q_j} (which exists by Lemma 6). Following this procedure we obtain a hamiltonian cycle $C_k^r = C_{k+1}$ of the subgraph of $C(G, S)$ induced by

$$\bigcup_{hG_0\in V(\mathcal{T}_{k+1})} V(C(hG_0))$$

such that for each $aG_0 \in \mathcal{L}_{k+1}$ there is a hamiltonian cycle $(d_1, \ldots, d_{m_0})$ of $C(aG_0)$ and an integer s_a such that the hamiltonian path

$$(d_{s_a}, d_{s_a-1}, \ldots, d_1, d_{m_0}, \ldots, d_{s_a+1})$$

of $C(aG_0)$ is a subpath of C_{k+1}. From here, the result follows. □

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