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Paired-Domination in Claw-Free Cubic Graphs

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Abstract. A set S of vertices in a graph G is a paired-dominating set of G if every vertex of G is adjacent to some vertex in S and if the subgraph induced by S contains a perfect matching. The minimum cardinality of a paired-dominating set of G is the paired-domination number of G, denoted by $\gamma_{\rm pr}(G)$. If G does not contain a graph F as an induced subgraph, then G is said to be F-free. In particular if $F = K_{1,3}$ or $K_4 - e$, then we say that G is claw-free or diamond-free, respectively. Let G be a connected cubic graph of order n. We show that (i) if G is $(K_{1,3}, K_4 - e, C_4)$-free, then $\gamma_{\rm pr}(G) \le 3n/8$; (ii) if G is claw-free and diamond-free, then $\gamma_{\rm pr}(G) \le 2n/5$; (iii) if G is claw-free, then $\gamma_{\rm pr}(G) \le n/2$. In all three cases, the extremal graphs are characterized.

Key words. Bounds, Claw-Free cubic graphs, Paired-domination

1. Introduction

Domination and its variations in graphs are now well studied. The literature on this subject has been surveyed and detailed in the two books by Haynes, Hedetniemi, and Slater [7, 8]. In this paper we investigate paired-domination in cubic claw-free graphs.

A *matching* in a graph G is a set of independent edges in G. The cardinality of a maximum matching in G is denoted by $\beta'(G)$. A *perfect matching M* in G is a matching in G such that every vertex of G is incident to a vertex of M.

Paired-domination was introduced by Haynes and Slater [9]. A *paired-dominating set*, denoted PDS, of a graph G is a set S of vertices of G such that every vertex is adjacent to some vertex in S and the subgraph induced by S contains a

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perfect matching. Every graph without isolated vertices has a PDS since the end-vertices of any maximal matching form such a set. The *paired-domination number* of G, denoted by $\gamma_{pr}(G)$, is the minimum cardinality of a TDS.

A *total dominating set*, denoted TDS, of a graph G with no isolated vertex is a set S of vertices of G such that every vertex is adjacent to a vertex in S (other than itself). Every graph without isolated vertices has a TDS, since $S = V(G)$ is such a set. The *total domination number* of G, denoted by $\gamma_t(G)$, is the minimum cardinality of a TDS. Clearly, $\gamma_t(G) \leq \gamma_{pr}(G)$ for every connected graph of order $n \geq 2$. Total domination in graphs was introduced by Cockayne, Dawes, and Hedetniemi [2].

For notation and graph theory terminology we in general follow [7]. Specifically, let $G = (V, E)$ be a graph with vertex set V of order n and edge set E. For a set $S \subseteq V$, the subgraph induced by S is denoted by $G[S]$. A *cycle* on n vertices is denoted by C_n and a *path* on n vertices by P_n. The minimum degree (resp., maximum degree) among the vertices of G is denoted by $\delta(G)$ (resp., $\Delta(G)$).

We call $K_{1,3}$ a claw and $K_4 - e$ a diamond. If G does not contain a graph F as an induced subgraph, then we say that G is F-free. In particular, we say a graph is *claw-free* if it is $K_{1,3}$-free and *diamond-free* if it is $(K_4 - e)$-free. An excellent survey of claw-free graphs has been written by Faudree, Flandrin, and Ryjáček [4].

In this paper we show that if G is a connected $(K_{1,3}, K_4 - e, C_4)$-free cubic graph of order $n \geq 6$, then $\gamma_{pr}(G) \leq 3n/8$, while if G is a connected claw-free and diamond-free cubic graph of order $n \geq 6$, then $\gamma_{pr}(G) \leq 2n/5$. We show that if G is a connected claw-free cubic graph of order $n \geq 6$ that contains $k \geq 1$ diamonds, then $\gamma_{pr}(G) \leq 2(n + 2k)/5$. Finally, we show that a connected claw-free cubic graph has paired-domination number at most one-half its order. In all cases, the extremal graphs attaining the upper bounds are characterized.

2. $(K_{1,3}, K_4 - e, C_4)$-free Cubic Graphs

To obtain sharp upper bounds on the paired-domination number of $(K_{1,3}, K_4 - e, C_4)$-free cubic graphs, we shall need a result due to Hobbs and Schmeichel [11] who established a lower bound on the maximum number $\beta'(G)$ of independent edges in a cubic graph having so-called super-hereditary properties. As a consequence of this result, we have the following lower bound on $\beta'(G)$ when G is a cubic graph.

Theorem 1 [11]. *If G is a connected cubic graph of order n, then $\beta'(G) \geq 7n/16$ with equality if and only if G is the graph shown in Fig. 1.*

Using Theorem 1, we show that the paired-domination number of a $(K_{1,3}, K_4 - e, C_4)$-free cubic graph is at most three-eights its order.

Theorem 2. *If G is a connected $(K_{1,3}, K_4 - e, C_4)$-free cubic graph of order $n \geq 6$, then there exists a PDS of G of cardinality at most $3n/8$ that contains at least one vertex from each triangle of G. Furthermore, $\gamma_{pr}(G) = 3n/8$ if and only if G is the graph shown in Fig. 2.*

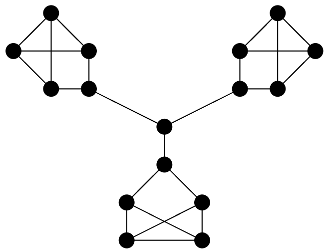


Fig. 1. The unique connected cubic graph G with $\beta'(G) = 7n/16$

Proof. Since G is $(K_{1,3}, K_4 - e)$-free and cubic, every vertex of G belongs to a unique triangle of G, and so $n \equiv 0(\text{mod}3)$. Let G' be the graph of order $n' = n/3$ whose vertices correspond to the triangles in G and where two vertices of G' are adjacent if and only if the corresponding triangles in G are joined by at least one edge. Then, since G is connected and C_4-free, G' is a connected cubic graph. Thus, by Theorem 2, $\beta'(G') \geq 7n'/16$ with equality if and only if G' is the graph shown in Fig. 1. Let M' be a maximum matching in G' (of cardinality $\beta'(G')$).

We now construct a PDS S of G as follows: For each edge $u'v' \in M'$, we select an edge uv of G that joins a vertex u in the triangle corresponding to u' and a vertex v in the triangle corresponding to v', and we add the vertices u and v to S, while for each vertex of G' that is not incident with any edge of M', we add two vertices from the corresponding triangle in G. Then S is a PDS of G that contains at least one vertex from each triangle of G. Thus, since $|S| = 2|M'| + 2(n' - 2|M'|) = 2(n' - |M'|)$,

$$\gamma_{\text{pr}}(G) \leq 2(n' - \beta'(G')) \leq 2\left(n' - \frac{7n'}{16}\right) = \frac{9n'}{8} = \frac{3n}{8}.$$

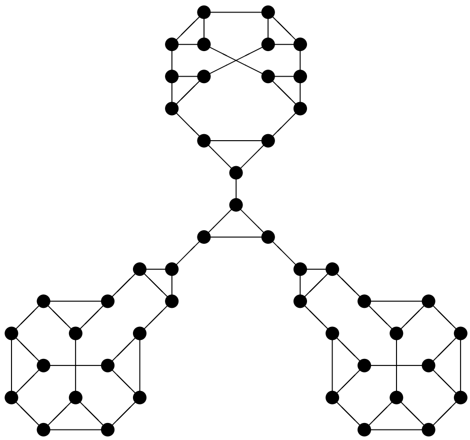


Fig. 2. The unique connected cubic $(K_{1,3}, K_4 - e, C_4)$-free graph G with $\gamma_{\text{pr}}(G) = 3n/8$

Furthermore, if we have equality throughout this inequality chain, then $\beta'(G') = 7n'/16$ and G' is the graph shown in Fig. 1. But then G must be the graph shown in Fig. 2. Conversely, it can be checked that the graph G of Fig. 2 satisfies $n = 48$ and $\gamma_{\mathrm{pr}}(G) = 18$. □

3. Claw-free Cubic Graphs

If we remove the restriction that G is C_4-free in Theorem 2, then we show in this subsection that the upper bound on the paired-domination number of G increases from three-eights its order to two-fifths its order. For this purpose we first prove the following result, our proof of which is along similar lines to the proof of Hobbs and Schmeichel in [11].

Theorem 3. *If G is a connected graph of order n with* $\delta(G) = 2$ *and* $\Delta(G) = 3$ *such that every vertex of degree* 2 *belongs to a path with an even number of internal vertices of degree* 2 *between two not necessarily distinct end-vertices of degree* 3, *then* $\beta'(G) \geq 2n/5$ *with equality if and only if G is the graph shown in Fig. 3.*

Proof. By a theorem of Berge [1], for any graph G

$$\beta'(G) = \frac{1}{2}\left(n - \max_{S \subseteq V(G)} \{\mathrm{o(G-S)} - |\mathrm{S}|\}\right),$$

where $\mathrm{o(G-S)}$ denotes the number of odd components of $G - S$. Thus it suffices to show that for the graph G satisfying the conditions of our theorem,

$$\max_{S \subseteq V(G)} \{\mathrm{o(G-S)} - |\mathrm{S}|\} \leq \frac{\mathrm{n}}{5}. \tag{1}$$

Let S be a smallest subset of $V(G)$ on which the maximum in (1) is attained. If $S = \emptyset$, then (1) is satisfied. Hence we may assume $|S| \geq 1$. Let $v \in S$ and let $S' = S - \{v\}$. Then, by our choice of S, $\mathrm{o(G-S')} \leq \mathrm{o(G-S)} - 2$, implying that v must be adjacent to three distinct odd components of $G - S$. Thus every vertex of S is adjacent to three distinct odd components of $G - S$. Furthermore, since G is connected and $\Delta(G) = 3$, every component of $G - S$ is odd. In particular, we note that no vertex of degree 2 is in S, and so each (odd) component of $G - S$ contains an odd number of vertices of degree 3 in G, plus possibly an even number of

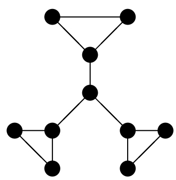


Fig. 3. A graph G with $\beta'(G) = 2n/5$

vertices of degree 2 in G. It follows that there are an odd number of edges joining S and any component of $G - S$.

For $k \geq 0$, let c_{2k+1} denote the number of components H of $G - S$ that are joined to S by exactly $2k + 1$ edges. If $k = 0$, then since $\delta(G) = 2$, H has order at least 3. Furthermore, $|V(H)| = 3$ if and only if H is a triangle consisting of two adjacent vertices of degree 2 and their common neighbor of degree 3 in G. If $k \geq 1$, then the sum of the degrees in H of the vertices of H is at least $2(|V(H)| - 1)$ since H is connected. On the other hand, this sum is equal to $3|V(H)| - d_2 - (2k + 1)$ where $d_2 \geq 0$ denotes the number of vertices of H of degree 2 in G. Consequently, $|V(H)| \geq 2k + d_2 - 1 \geq 2k - 1$. Hence,

$$|V(H)| \geq \begin{cases} 3 & \text{if } k = 0 \\ 2k - 1 & \text{if } k \geq 1. \end{cases}$$

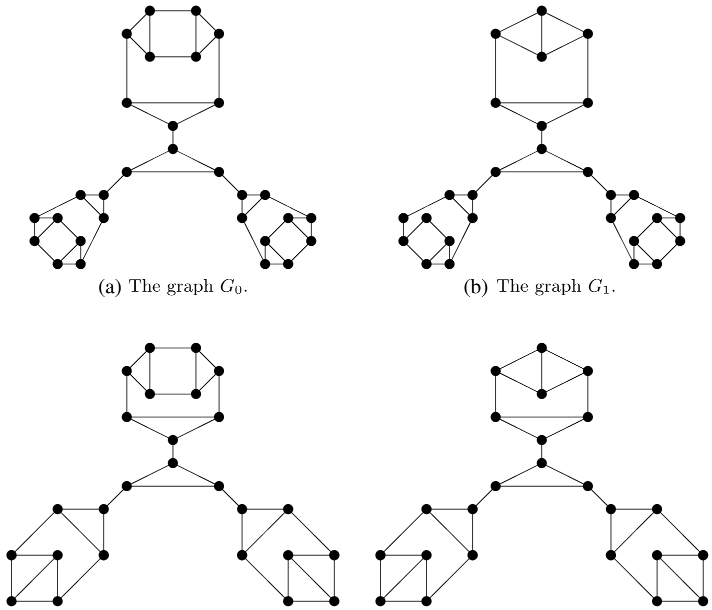
Proceeding now exactly as in the proof of Hobbs and Schmeichel in [11] we obtain (1). Furthermore, their proof shows that if we have equality in (1), then each component of $G - S$ that is joined to S by exactly one edge has order exactly 3 (and is therefore a triangle consisting of two adjacent vertices of degree 2 and their common neighbor of degree 3 in G) while $c_{2k+1} = 0$ for $k \geq 1$. Since G is connected, G is therefore the graph shown in Fig. 3. □

Using Theorem 3, we present a sharp upper bound on the paired-domination number of a claw-free cubic graph.

Theorem 4. *If G is a connected claw-free cubic graph of order $n \geq 6$ that contains $k \geq 0$ diamonds, then there exists a PDS of G of cardinality at most $2(n + 2k)/5$ that contains at least one vertex from each triangle of G. Furthermore, $\gamma_{\text{pr}}(G) = 2(n + 2k)/5$ if and only if $G \in \{G_0, G_1, G_2, G_3\}$ where G_0, G_1, G_2, and G_3 are the four graphs shown in Fig. 4.*

Proof. If $n = 6$, then G is the prism $K_3 \times K_2$, $k = 0$, and there exists a PDS of G of cardinality $2 < 12/5$ that contains one vertex from each triangle of G. Hence we may assume that $n \geq 8$.

Since G is a claw-free and cubic, every vertex of G belongs to a unique triangle or to a unique diamond of G. Let G' be the graph of order $n' = (n + 2k)/3$ whose vertices correspond to the triangles in G and where two vertices of G' are adjacent if and only if the corresponding triangles in G share a common edge or are joined by at least one edge. Each triangle of G that belongs to no diamond is joined to three other triangles by one edge each or to a triangle by one edge and to another one by two edges. Therefore the triangles of G in no diamond that are joined to only two other triangles can be gathered by pairs forming a subgraph shown in Fig. 5(a) (where u and v are distinct but possibly adjacent). Each diamond in G corresponds to two adjacent vertices of degree two in G'. Thus, G' is either an even cycle or satisfies the conditions of Theorem 3 (two vertices of degree 2 in G' belong to a triangle of G' if they correspond in G either to a subgraph shown in Fig. 5(a) with $uv \in E(G)$ or to a subgraph shown in Fig. 5(b) with $xy \in E(G)$).



(a) The graph G_0. (b) The graph G_1.

(c) The graph G_2. (d) The graph G_3.

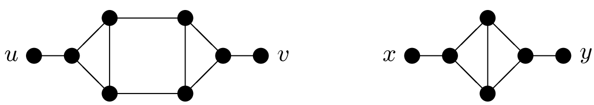
Fig. 4. The four connected cubic claw-free graph G_k, $0 \le k \le 3$, with k copies of $K_4 - e$ and with $\gamma_{\rm pr}(G_k) = 2(n + 2k)/5$

In both cases, $\beta'(G') \ge 2n'/5$ with equality if and only if G' is the graph shown in Fig. 3. Let M' be a maximum matching in G' (of cardinality $\beta'(G')$) and let S be a PDS of G as constructed in the proof of Theorem 2. Then S is a PDS of G that contains at least one vertex from each triangle of G. Thus, since $|S| = 2(n' - |M'|)$,

$$\gamma_{\rm pr}(G) \le 2(n' - \beta'(G')) \le 2\left(n' - \frac{2n'}{5}\right) = \frac{6n'}{5} = \frac{2(n+2k)}{5}.$$

Furthermore, if we have equality throughout this inequality chain, then $\beta'(G') = 2n'/5$ and G' is the graph shown in Fig. 3. But then $k \le 3$ and G must be one of the four graphs G_k shown in Fig. 4. Conversely, it can be checked that for $k \in \{0, 1, 2, 3\}$ the graph G_k of Fig. 4 contains k diamonds and satisfies $\gamma_{\rm pr}(G_k) = 2(n + 2k)/5$.

As an immediate consequence of Theorem 3, we have the following result.



(a) (b)

Fig. 5. Two subgraphs of G

Theorem 5. *If G is a connected claw-free and diamond-free cubic graph of order $n \geq 6$, then there exists a PDS of G of cardinality at most $2n/5$ that contains at least one vertex from each triangle of G. Furthermore, $\gamma_{pr}(G) = 2n/5$ if and only if $G = G_0$ where G_0 is the graph shown in Fig. 4(a).*

Haynes and Slater [9] showed that the paired-dominating set problem is NP-complete. We remark that since the constructions of the graph G' from G and of a maximum matching M' of G' in the proof of Theorems 2 and 4 are polynomial, the proof of Theorems 2 and 3 provides a polynomial algorithm to construct a PDS (and therefore a TDS) of G of order at most $3n/8$ or $2n/5$ or $2(n+2k)/5$ in the considered classes.

As a further consequence of Theorem 4, we show that the paired-domination number of a claw-free cubic graph is at most one-half its order and we characterize the extremal graphs. For this purpose, we say that a diamond in a claw-free cubic graph is of **type-1** if the two vertices not in the diamond that are neighbors of the degree two vertices of the diamond are not adjacent, and of **type-2** otherwise. Hence the diamond shown in Fig. 5 is of type-1 if $xy \notin E(G)$ and of type-2 if $xy \in E(G)$.

Let F_1, F_2 and F_3 be the three cubic claw-free graphs shown in Fig. 6.

Theorem 6. *If G is a connected claw-free cubic graph of order n, then $\gamma_{pr}(G) \leq n/2$ with equality if and only if $G \in \{K_4, F_1, F_2, F_3, G_3\}$ where F_1, F_2 and F_3 are the graphs shown in Fig. 6 and G_3 is the graph shown in Fig. 4(d).*

Proof. We proceed by induction on the order n of a connected claw-free cubic graph. If $n = 4$, then $G = K_4$ and $\gamma_{pr}(G) = 2 = n/2$, while if $n = 6$, then $G = K_3 \times K_2$ and $\gamma_{pr}(G) = 2 < n/2$. This establishes the bases cases. Suppose then that $n \geq 8$ is even and that for every connected claw-free cubic graph G' of order $n' < n$, $\gamma_{pr}(G') \leq n'/2$ with equality if and only if $G' \in \{K_4, F_1, F_2, F_3, G_3\}$. Let G be a connected claw-free cubic graph of order n.

If G is diamond-free, then by Theorem 4, $\gamma_{pr}(G) \leq 2n/5$. Hence we may assume that G contains at least one diamond. Let F be the subgraph of G shown in Fig. 7 where x and y are distinct but possibly adjacent.

Claim 1. *If G has a diamond of type-1, then $\gamma_{pr}(G) \leq n/2$ with equality if and only if $G \in \{F_1, F_2, F_3\}$.*

Proof. We may assume that the diamond $G[\{u, v, w, z\}]$ is of type-1, and so $xy \notin E(G)$. Let G' be the connected claw-free cubic graph of order $n' = n - 4$ obtained from G by deleting the vertices u, v, w, z (and their incident edges) and adding the edge xy. By the inductive hypothesis, $\gamma_{pr}(G') \leq n'/2$. Let S' be a minimum PDS of G'. If $\{x, y\} \subseteq S'$, let $S = S' \cup \{u, w\}$ if the edge xy belongs to a perfect matching in $G'[S']$, and let $S = S' \cup \{u, v\}$ otherwise. If $x \notin S'$, let $S = S' \cup \{u, v\}$. If $x \in S'$ and $y \notin S'$, let $S = S' \cup \{v, w\}$. In all cases, S is a PDS of G, and so $\gamma_{pr}(G) \leq |S| \leq n/2$. Furthermore, if $\gamma_{pr}(G) = n/2$, then $\gamma_{pr}(G') = n'/2$ and so, by the inductive hypothesis, $G' \in \{K_4, F_1, F_2, F_3, G_3\}$. Unless $G' = K_4$, the

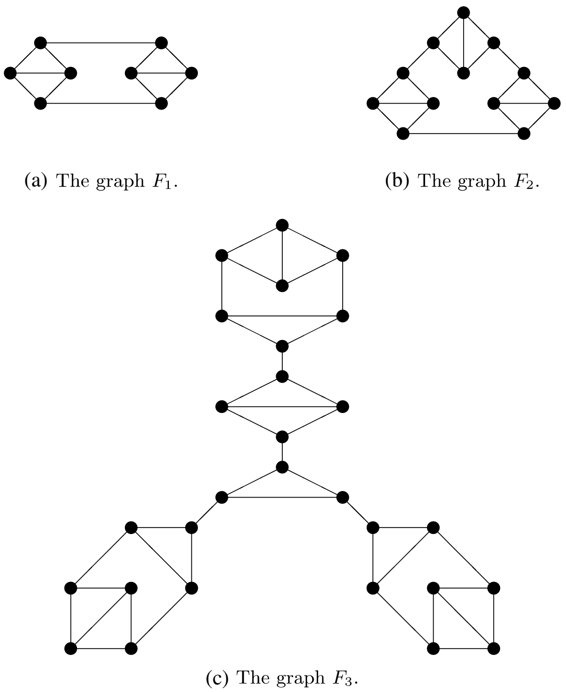


Fig. 6. Three connected cubic claw-free graphs

edge xy does not belong to a triangle of G' for otherwise G would contain a claw. If $G' \in \{F_2, F_3\}$, then $\gamma_{\mathrm{pr}}(G) < n/2$ (irrespective of the choice of the edge xy), a contradiction. Hence either $G' = K_4$, in which case $G = F_1$, or $G' = F_1$ in which case $G = F_2$, or $G' = G_3$, in which case $G = F_3$. □

Claim 2. *If every diamond of G is of type-2, then $\gamma_{\mathrm{pr}}(G) \le n/2$ with equality if and only if $G = G_3$.*

Proof. Note that $xy \in E(G)$. Let a be the common neighbor of x and y, and let b be the remaining neighbor of a. Let $N(b) = \{a, c, d\}$. Since G is claw-free, $G[\{b, c, d\}] = K_3$. Let c' and d' be the neighbors of c and d, respectively, that do not belong to the triangle $G[\{b, c, d\}]$. If $c' = d'$, then G contains a diamond of type-1, contrary to assumption. Hence, $c' \neq d'$. If c' and d' belong to a common diamond, then $n = 14$ and $\gamma_{\mathrm{pr}}(G) = 6$. Hence we may assume that

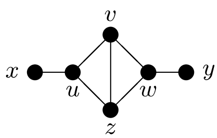


Fig. 7. A subgraph F

$N(c') \cap N(d') = \emptyset$. Thus the triangle containing c' is vertex-disjoint from that containing d'. Furthermore, these two triangles are not contained in a diamond (for otherwise such a diamond would be of type-1). It follows that the only vertices within distance 3 from b that belong to a diamond are u and w. Hence we can uniquely associate the eight vertices of the set $V(F) \cup \{a, b\}$ with the diamond induced by $\{u, v, w, z\}$. Therefore if G has k diamonds, $k \leq n/8$. Thus, by Theorem 4, $\gamma_{\text{pr}}(G) \leq 2(n+2k)/5 \leq n/2$. Furthermore, it follows that in this case $\gamma_{\text{pr}}(G) = n/2$ if and only if $G = G_3$. □

The desired result of Theorem 6 now follows from Claims 1 and 2. □

We show next that the upper bound on the paired-domination number of a claw-free cubic graph presented in Theorem 4 can be improved if we add the restriction that the graph is 2-connected.

Theorem 7. *If G is a* 2*-connected claw-free cubic graph of order* $n \geq 6$ *that contains* $k \geq 0$ *diamonds, then* $\gamma_{\text{pr}}(G) \leq (n+2k)/3$.

Proof. If $n = 6$, then $G = K_3 \times K_2$, $k = 0$, and so $\gamma_{\text{pr}}(G) = 2 = (n+2k)/3$. Hence we may assume that $n \geq 8$. Let G' be the graph of order $n' = (n+2k)/3$ constructed in the proof of Theorem 4. Then, G' is either an even cycle or satisfies the conditions of Theorem 3. Since G is 2-connected, so too is G'.

We show that G' has a perfect matching M'. If G' is an even cycle, this is immediate. Assume then that $\Delta(G') = 3$ and that every vertex of degree 2 belongs to a path with an even number of internal vertices of degree 2 between two not necessarily distinct end-vertices of degree 3 in G'. Hence the subgraph of G' induced by its vertices of degree two contains a perfect matching M^*. We now transform G' into a 2-connected cubic graph G'' by replacing each edge $xy \in M^*$ in G' with a $K_4 - e$ (and so x and y are not adjacent in the resulting $K_4 - e$). Let x' and y' denote the two new vertices of the resulting $K_4 - e$. Since every 2-connected cubic graph has a perfect matching, G'' has a perfect matching M''. We now construct a perfect matching M' of G' from the matching M'' as follows. For each edge $xy \in M^*$, if $x'y' \in M''$, then we remove $x'y'$ from the matching, while if $\{xx', yy'\} \subset M'$ (resp., $\{xy', x'y\} \subset M'$), then we replace the edges xx' and yy' (resp., xy' and $x'y$) with the edge xy. Hence, $\beta'(G') = n'/2$.

Let S be a PDS of G as constructed from M' as in the proof of Theorem 2. Then, $\gamma_{\text{pr}}(G) \leq |S| = 2|M'| = n' = (n+2k)/3$. □

As an immediate consequence of Theorem 7, we have the following result.

Theorem 8. *If G is a* 2*-connected claw-free and diamond-free cubic graph of order* $n \geq 6$, *then* $\gamma_{\text{pr}}(G) \leq n/3$.

4. Total Domination

Since $\gamma_t(G) \leq \gamma_{\text{pr}}(G)$ for all graphs G, and since $\gamma_t(G) = \gamma_{\text{pr}}(G)$ for the graph G of Fig. 2 and for the graph $G = G_0$ of Fig. 4(a), we remark that the results of both

Theorem 2 and Theorem 5 are still valid for total domination (i.e., in the statement of these theorems we can replace "PDS" by "TDS" and "$\gamma_{pr}(G)$" by "$\gamma_t(G)$"). However if $G \in \{F_2, F_3, G_3\}$ where F_2 and F_3 are the graphs shown in Fig. 6 and G_3 is the graph shown in Fig. 4(d), then $\gamma_t(G) < \gamma_{pr}(G)$. Hence we have the following immediate consequence of Theorem 6.

Theorem 9. *If G is a connected claw-free cubic graph of order n, then $\gamma_t(G) \leq n/2$ with equality if and only if $G = K_4$ or $G = F_1$ where F_1 is the graph shown in Fig. 6.*

The inequality of Theorem 9 was established in [3] but the graphs achieving equality were not characterized. We also remark that the conjecture in [6] that every connected graph with minimum degree at least three has total domination number at most one-half its order is completely proved in several manuscripts. We show in [5] that if G is a connected claw-free cubic graph of order at least ten, then the upper bound of Theorem 9 can be improved.

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