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Marking Games and the Oriented Game Chromatic Number of Partial k-Trees

H.A. Kierstead[1] and Zs. Tuza[2]

[1] Department of Mathematics, Arizona State University, Tempe, AZ 85287-1804, USA
[2] Computer and Automation Institute, Hungarian Academy of Sciences; and Department of Computer Science, University of Veszprém, Hungary

Abstract. Nešetřil and Sopena introduced the concept of oriented game chromatic number. They asked whether the oriented game chromatic number of partial k-trees was bounded. Here we answer their question positively.

1. Introduction

Let $G = (V, E)$ be a graph and let C be a set of t colors. A *proper C-coloring* of G is a function $c : V \to C$ such that $c(u) \neq c(v)$ for every edge $uv \in E$. The *coloring game* is played by two players, Alice and Bob. The players take turns playing with Alice playing first. Alice's goal is to provide a proper C-coloring of G and Bob's goal is to prevent her from doing so. A play by either player consists of choosing a vertex $v \in V$ that has not yet been colored, and then coloring v properly with a color from C, i.e., with a color that is different from any color already assigned to a neighbor of v. If at some time all t colors in C have been assigned to the neighbors of some uncolored vertex, then Bob wins; otherwise Alice wins when all the vertices are properly C-colored. The *game chromatic number* of G, denoted $\chi_g(G)$, is the least t such that Alice has a winning strategy when the coloring game is played on G with a set of colors C of size t. The game chromatic number of a class of graphs is the maximum game chromatic number of any graph in the class.

Faigle et al. [6] showed that the game chromatic number of the class of forests is 4 and the game chromatic number of the class of interval graphs with clique number ω is between $2\omega - 2$ and $3\omega - 2$, where ω denotes clique number. Kierstead and Trotter [13] showed that the game chromatic number of the class of planar graphs is between 7 and 33. This upper bound was improved by Dinski and Zhu in [5] to 30. We will mention further improvements based on marking games in the next section. Game chromatic number is closely related to the concept of precoloring extension, which was studied by Biró, Hujter, and Tuza in [1] and

Hujter and Tuza in [10] and [11]. A similar game was considered by Harary and Tuza in [9].

In this article we study games played on partial k-trees. Recall that a chordal graph is a graph G such that every cycle C of length greater than 3 has a chord in G. A subgraph H of a chordal graph G is a partial k-tree if $\omega(G) \leq k+1$, where $\omega(G)$ denotes the clique number of G. For example, interval graphs are chordal graphs and outerplanar graphs are partial 2-trees. A graph is said to have tree width k if it is a partial k-tree, but not a partial $(k-1)$-tree.

Very recently Nešetřil and Sopena [16] introduced the oriented version of the coloring game. Let $\vec{G} = (V, A)$ be an oriented graph. By this we mean that A is a set of directed edges and if $\overrightarrow{uv} \in A$, then $\overrightarrow{vu} \notin A$. A function c from $V(\vec{G})$ to a set of colors C is an oriented C-coloring of $\vec{G}$ if (i) $c(u) \neq c(v)$ for every arc $\overrightarrow{uv}$ and (ii) $c(u) = c(x)$ implies that $c(v) \neq c(w)$ for all arcs $\overrightarrow{uv}$ and $\overrightarrow{wx}$ (v and w not necessarily distinct). It follows easily that c is an oriented C-coloring of $\vec{G}$ if and only if there exists a tournament $\vec{T} = (C, D)$ on C such that if $\overrightarrow{uv} \in A$, then $\overrightarrow{c(u)c(v)} \in D$, i.e. c is a homomorphism from $\vec{G}$ to $\vec{T}$. The oriented chromatic number, $\vec{\chi}(\vec{G})$, of $\vec{G}$ is the least t such that $\vec{G}$ has an oriented coloring with t colors.

The oriented version of the coloring game is played on a fixed oriented graph $\vec{G}$ with a fixed tournament $\vec{T} = (C, D)$. Again the two players Alice and Bob take turns playing with Alice going first. Now Alice's goal is to provide a homomorphism from $\vec{G}$ to $\vec{T}$ and Bob's goal is to prevent her from doing so. A play by either player consists of choosing a vertex $v \in V$ that has not yet been colored, and then coloring v properly (with respect to the orientation D) with a color from C. This means that if v is to be colored α and w has already been colored β, then $\overrightarrow{vw} \in A$ implies $\overrightarrow{\alpha\beta} \in D$ and $\overrightarrow{wv} \in A$ implies $\overrightarrow{\beta\alpha} \in D$. If this were the only requirement, then Bob would have an easy winning strategy. He would play so that the two endpoints of a directed path P of length two are colored with the same color before the middle vertex u of P is colored. It is easy to see that then there would be no way to properly color u. To prevent this triviality, we also require that v be colored with a color that is different from the color of any other vertex connected to v by a directed path (in either direction) of length two. Bob wins if at some point before all the vertices of V are colored one of the players does not have a legal move; otherwise Alice wins after all the vertices have been properly colored. The oriented game chromatic number, $\mathrm{ogcn}(\vec{G})$, is the least t for which there exists a tournament $\vec{T}$ on t vertices such that Alice can win the oriented coloring game on $\vec{G}$ with $\vec{T}$.

Nešetřil and Sopena [16] showed that the oriented game chromatic number of a graph G is at most $\Delta^2(G)$. They also showed that there exists a constant upper bound on the oriented game chromatic number of outerplanar graphs. They asked (i) whether there exists a constant upper bound on the oriented game chromatic number of planar graphs, and (ii) whether for fixed k there exists a constant upper bound on the oriented game chromatic number of partial k-trees. The first question was answered positively by Kierstead and Trotter [14]. The main result of this article provides a positive answer to the second question.

2. Marking Games

Marking games are simplified versions of coloring games that have proved useful in bounding game and oriented game chromatic numbers. A marking game is played by two players Alice and Bob with Alice playing first. At the start of the game all vertices are unmarked. A play by either player consists of marking an unmarked vertex. The game ends when all the vertices have been marked. Different versions of the game differ in the way the score of the game is computed. For any $t \in \{1, \dots, |V|\}$, let M^t denote the set of marked vertices after t plays and $U^t = V - M^t$ denote the set of unmarked vertices after t plays. So $|M^t| = t$. For i a positive integer and any vertices u and v, we say that u and v are *i-close* after t plays, denoted $u \equiv_i^t v$, if there exists a $u - v$ path of length at most i all of whose internal vertices are unmarked after t plays. For an unmarked vertex u let

$$S_i^t(u) = \{v \in M^t : u \equiv_i^t v\}.$$

The *score* of the *i-marking game* is

$$\max\{|S_i^t(u)| : 1 \leq t \leq |V| \wedge u \in U^t\}.$$

Extending the notation and terminology of [16] we define the *i-Go number*, $\mathrm{Go}_i(G)$, of G to be the least s such that Alice has a strategy that results in a score of at most s in the i-marking game. When $i = 1$ we refer to the Go number of G and when $i \geq |V|$ we refer to the *complete Go number*, $\mathrm{cGo}(G)$. In this case $S_i^t(u)$ is the number of marked vertices connected to u by paths of any length whose internal vertices are unmarked. We set $S^t(u) = S_{|V|}^t(u)$. If t is clear from the context we may simply write M, U, and $S(u)$. Zhu [18] defined the *game coloring number*, $\mathrm{col}_g(G)$, of G to be one more than what we are calling the Go number of G.

A play of a marking game determines a linear ordering on the vertices of G by $x < y$ if x is marked before y. The importance of the Go number of G is that if Alice uses the strategy for the 1-marking game on G that guarantees a score of $\mathrm{Go}(G)$ to choose vertices to color, then she can win the coloring game using a set of $\mathrm{Go}(G) + 1$ colors just by coloring with First-Fit. It follows easily that

$$\chi_g(G) \leq \mathrm{col}_g(G) = \mathrm{Go}(G) + 1 \leq \mathrm{Go}_i(G) + 1 \leq \mathrm{cGo}(G) + 1.$$

Faigle et al. [6] actually bounded the game chromatic number of forests by showing that the Go number of a forest is at most 3. The present authors [15] extended this result to $\chi_g(G) \leq 6k - 2$ for all partial k-trees G. Guan and Zhu [8] proved that $\mathrm{Go}(G) \leq 6$, for all outerplanar graphs G. Zhu [18] showed that $\mathrm{Go}(G) \leq 18$ for all planar graphs G and [19] that $\mathrm{Go}(G) \leq 3k + 1$ for all partial k-trees G. Kierstead [12] showed that $\mathrm{Go}(G) \leq 17$ and thus $\chi_g(G) \leq 18$, for all planar graphs G. The fundamental result on oriented game chromatic number of Nešetřil and Sopena [16] depends on another parameter, the eGo number. We will not define this parameter here. Rather, it suffices to note that $\mathrm{Go}(G) \leq \mathrm{eGo}(\vec{G}) \leq \mathrm{Go}_2(G) \leq \mathrm{cGo}(G)$, where $\vec{G}$ is any orientation of the simple graph G. Nešetřil and Sopena proved that there is an exponential upper bound on $\mathrm{ogcn}(\vec{G})$ in terms of $\mathrm{eGo}(\vec{G})$. Kierstead and Trotter [14] showed that $\mathrm{Go}_2(G)$ is bounded for any planar graph, thus establishing an upper bound for the oriented game chromatic number of any planar graph. The following theorem is the main result of this paper.

Theorem 1. *For every partial k-tree G,* $\mathrm{cGo}(G) \leq 6k - 3$.

Corollary 2. *For a fixed positive integer k there exists a constant upper bound on the oriented game chromatic number of any partial k-tree.*

Corollary 3. *For every partial k-tree G,* $\chi_g(G) \leq 6k - 2$.

As noted above Zhu proved the stronger bound of $\chi_g(G) \leq \mathrm{Go}(G) + 1 \leq 3k + 2$ for partial k-trees. In the proof of Theorem 1 we expend considerable effort to get the tightest upper bound that we can. If we were only interested in a bound of the form $6k + O(1)$ we would not need to deal with secondary separators in the proof.

For a fixed graph $G = (V, E)$ and $S \subset V$, let $N(S) = \{v \in V : vs \in E$, for some $s \in S\}$.

3. Special Properties of Chordal Graphs

In this section we review some important properties of chordal graphs. An *intersection representation* of a graph $G = (V, E)$ is a collection $F = \{S_v : v \in V\}$ of sets such that $xy \in E$ if and only if $S_x \cap S_y \neq \emptyset$. The collection F is a *subtree representation* of G if each of the sets in F is the vertex set of a subtree of some fixed tree $T = (U, D)$. The following characterization of interval graphs was discovered independently by Buneman [3], Gavril [7], and Walter [17]. (Actually W. T. Trotter discovered this result before any of the published versions, but a referee decided that it was trivial and useless.)

Proposition 4. *A graph is chordal if and only if it has a subtree representation.* □

Suppose that $w : V \to \mathbb{N}$ is a weight function from the vertices of G to the non-negative integers. If $S \subset V$, then the weight $w(S)$ of S is defined by $w(S) = \sum_{v \in S} w(v)$. The following proposition is an easy exercise.

Proposition 5. *Let* $w : U \to \mathbb{N}$ *be a weight function on a tree* $T = (U, D)$. *Then there exists a vertex* $u \in U$ *such that each component of* $T - u$ *has weight at most* $\frac{1}{2} w(U)$. □

Using Propositions 4 and 5 it is easy to obtain the following extension of Proposition 5 to chordal graphs.

Proposition 6. *Let $w : V \to \mathbb{N}$ be a weight function on a chordal graph $G = (V, E)$ with $\omega(G) = k + 1$. Then G has a separating set $W \subset V$ such that $|W| \leq k + 1$ and for every component C of $G - W$ both:*

1. $w(C) \leq \frac{1}{2} w(V)$ *and*
2. *there exists $W_C \subset W$ such that $|W_C| \leq k$ and C is a component of $G - W_C$.*

Proof. Let $\{S_v : v \in V\}$ be a subtree representation of G, where each S_v is a subtree of the tree $T = (U, D)$. Moreover assume that T is the smallest tree in which G has a subtree representation. For each subtree S_v arbitrarily pick a root r_v. Define a weight function $w' : U \to \mathbb{N}$ by $w'(u) = \sum_{r_v = u} w(v)$. Then $w'(U) = w(V)$. By Proposition 5 there exists a vertex $u \in U$ such that each component of $T - u$ has weight at most $\frac{1}{2} w'(U) = \frac{1}{2} w(V)$. Let $W = \{v \in V : u \in S_v\}$. Clearly W is a clique, and so $|W| \leq k + 1$. For each component $C \subset G - W$, there exists a component $C' \subset T - u$ such that for every $v \in C$, $S_v \subset C'$. Thus each component of $G - W$ has weight at most $\frac{1}{2} w(V)$. This proves (1). For (2) we use the minimality of T. Let $u' \in C'$ such that $uu' \in D$ and $W' = \{v \in V : u' \in S_v\}$. By the minimality of T, $W \neq W'$, and so $|W \cap W'| \leq k$. It is easy to check that C is a component of $G - W_C$ for $W_C = W \cap W'$. □

We shall refer to the separator W of Proposition 6 as a *primary* separator. The separators W_C are called *secondary* separators. Finally we will need:

Proposition 7. *If $G = (V, E)$ is a chordal graph with a minimal cutset $S \subset V$, then S is a clique.* □

4. Marking Games on Partial k-Trees

In this section we prove Theorem 1. Let H be a partial k-tree. Then there exists a chordal graph G such that H is a subgraph of G and $\omega(G) = k + 1$. Since $\mathrm{cGo}(H) \leq \mathrm{cGo}(G)$, it suffices to show that $\mathrm{cGo}(G) \leq 6k - 3$. We may assume that $k > 1$, since otherwise G is a forest and $\mathrm{cGo}(G) \leq 3$. At a given stage in the marking game, the components of the subgraph $G[U]$ of G induced by the set U of unmarked vertices are called *unmarked components*. Let $C^t = \{D : D \text{ is an unmarked component at the end of play } t\}$ be the partition of U^t into unmarked components. For $S \subset V$, let $M^t(S) = M^t \cap (S \cup N(S))$ be the *closed marked neighborhood* of S. We must provide Alice with a strategy such that after any play t of the marking game, for any unmarked component $D \in C^t$, $|M^t(D)| \leq 6k - 3$.

Suppose that on play t one of the players marks a vertex $v \in D$, where D is an unmarked component in C^{t-1}. Then $U^t = U^{t-1} - \{v\}$. Let $N = \{D_1, \ldots, D_s\}$ be the set of components of $G[D - \{v\}]$. Then

$$C^t = (C^{t-1} - \{D\}) \cup N.$$

For each *old* unmarked component $D^* \in C^{t-1} - \{D\}$, $|M^t(D^*)| = |M^{t-1}(D^*)|$. For each *new* unmarked component $D' \in N$, $|M^t(D')| \le |M^{t-1}(D)| + 1$. Moreover if D', $D'' \in N$ are distinct, then

$$|M^t(D')| + |M^t(D'')| \le |M^{t-1}(D)| + k + 2.$$

To see this, note that $M^t(D^*) \subset M^{t-1}(D) \cup \{v\}$, for all $D^* \in N$. Also $M^t(D') \cap M^t(D'')$ is contained in a minimal cut set S separating D' from D''. Since G is chordal, S is a clique, and so

$$|M^t(D') \cap M^t(D'')| \le |S| \le \omega(G) \le k + 1.$$

It now follows, using the assumption that $k > 1$, that if $|M^{t-1}(D)| \le 6k - 3$, then there is at most one new unmarked component $D' \in N$ such that $|M^t(D')| \ge 4k - 1$. If there is such a component D', we say that D' is the *heir* of D; otherwise we say that D is *heirless*. Further, we interpret the notion of heir transitively. So if D' is an heir of D and D'' is an heir of D', then we say that D'' is an heir of D.

We shall say that an unmarked component $D \in C^t$ is *dangerous* if $|M^t(D)| \ge 4k - 1$ or $|M^t(D)| = 4k - 2$ and Alice chooses D (decides to mark one of the vertices of D) on play $t + 1$. We say that D_0 is a *progenitor* if D_0 is dangerous, but D_0 is not the heir of a dangerous unmarked component. In this case, if $D' \in C^s$ is an heir of D_0, then D_0 is the progenitor of D'. If in addition $i = |M^s \cap D_0|$, i.e., i vertices of D_0 have been marked after play s, then D' is an i-th *generation* heir of D_0. Finally, D_0 is heirless after i generations if D_0 does not have an i-th generation heir.

Alice will use the following general strategy. Whenever Bob marks a vertex in a component $D \in C^{t-1}$, which has an heir $D' \in C^t$, Alice will respond by marking a vertex in D'. Her plan is to leave the progenitor D_0 of D', (or D' if D' is a progenitor) heirless after $2k + 1$ generations by marking the vertices of an appropriate separator of D_0. In the mean time she will make sure that $M(D^*) \le 6k - 3$ for any heir D^* of D_0.

We are now prepared to state Alice's strategy. At the start of the game she partitions G into connected components C^0 and marks any vertex. Now suppose that it is Alice's turn to play after Bob has just made play t by marking the vertex v in the unmarked component $D \in C^{t-1}$. If D is heirless then Alice chooses any unmarked component $D^* \in C^t$. Otherwise Alice chooses the heir D^* of D in C^t.

Case 1. D^* is not dangerous. Alice marks any vertex in $U^t \cap D^*$.

Case 2. D^* is a progenitor. Alice sets $L = L(D^*) = D^* \cup M^t(D^*)$ and assigns a weight of 1 to every marked vertex of L and a weight of 0 to every unmarked vertex of L. Thus the weight of L is $|M^t(D^*)|$. Alice then chooses a $(k+1)$-separator $W \subset L(D^*)$ as in Proposition 6. So, setting $I = I(D^*) = \{F : F$ is a component of $G[L - W]\}$, for every $F \in I$, $|M^t \cap F| \le 2k - 1$. Moreover $U^t \cap F$

can be separated from the rest of D^* by a subset $W_F \subset U^t \cap W$ of size at most k. If $|U^t \cap W| \leq k$, then we let $W_F = U^t \cap W$; otherwise we let W_F be an appropriate k-subset of $U^t \cap W$ separating F from $G[L - F - N(F)]$. We call W the primary separator of D^*.

Subcase 2a. $|U^t \cap W| \leq k$. Alice assigns D^* the secondary separator $S = U^t \cap W$. In this case $S = W_F$ for each $F \in I$. Alice marks any vertex in S.

Subcase 2b. Not Subcase 2a and $|M^t \cap F| \leq 2k - 2$, for every $F \in I$. Since not Subcase 2a, $M^t \cap W = \emptyset$. Thus

$$4k - 2 \leq |M^t(D^*)| = \sum_{F \in I} |M^t \cap F| = \sum_{S \subset W} \left(\sum_{W_F = S} |M^t \cap F| \right),$$

and so there exists a k-set $S \subset W$ such that $\sum_{W_F=S} |M^t \cap F| \geq \left\lceil \frac{4k-2}{k+1} \right\rceil \geq 2$. Alice assigns D^* the secondary separator S and marks a vertex $v \in U^t \cap S$.

Subcase 2c. Not Subcase 2a and there exists $F \in I$ such that $|M^t \cap F| = 2k - 1$. Then $|W_F| = k$ and so for any $F, F' \in I$, $W_F \cap W_{F'} \neq \emptyset$. Since $|M^t(D^*)| \leq 4k - 1$, there is at most one other set $F' \in I$ such that $|M^t \cap F'| = 2k - 1$. Thus $\bigcap \{W_F : |M^t \cap F| = 2k - 1\} \neq \emptyset$. Alice marks a vertex in this intersection, but puts off the assignment of a secondary separator.

Case 3. D^* is an heir of a progenitor $D_0 \in C^s$. Then D^* inherits a primary separator W from D_0. If D_0 or an heir has been assigned a secondary separator S, then D^* inherits S. Otherwise D_0 was treated in Subcase 2c after play s by Bob, Alice has marked exactly one vertex in D_0 (more specifically, in W), and Bob has marked at most one vertex in D_0. Choose $F \in I(D_0)$ such that $(i)|M^s \cap F| = 2k - 1$ and, if possible, $(ii)|M^t \cap F| = 2k$. Note that if (ii) holds then Bob has just marked a vertex in F and so there is at most one such F. Let $S = W_F$ be the secondary separator of D^*. In either case, Alice marks a vertex in D^*, preferring one in $U^t \cap W$, and preferring still one that is also in $U^t \cap S$.

This completes our description of Alice's strategy. We must show that this is a winning strategy. Let $D_0 \in C^{t_0}$ be any progenitor. Suppose that D_0 was assigned the primary separator W and D_0 or some heir of D_0 was assigned the secondary separator S. Let $D_i \in C^{t_i}$ be the heir, if any, of D_0 in the i-th generation and let v_i be the vertex in D_{i-1} that is marked to form D_i. Note that

$$|M^{t_i}(D_i)| \leq |M^{t_{i-1}}(D_{i-1})| + 1 \leq 4k - 1 + i. \qquad (*)$$

It suffices to prove the following lemma.

Lemma 8. *If Alice follows the above strategy then:*

1. $v_1 \in S$.
2. $|M^{t_i}(D_i)| \leq 4k - 1 + i \leq 6k - 3$, *if* D_i *exists and* $i \leq 2k - 2$.

3. $|M^{t_{2k-1}}(D_{2k-1})| \leq 6k - 4$, *if* D_{2k-1} *exists.*
4. $|M^{t_{2k}}(D_{2k})| \leq 6k - 3$, *if* D_{2k} *exists.*
5. D_0 *is heirless after* $2k + 1$ *generations.*

Proof. (1) In all cases, except Subcase 2c, the secondary separator S is picked before v_1 is chosen and then v_1 is chosen from S. In Subcase 2c there are at most two possibilities for S and v_1 is chosen from the intersection of the possibilities for S.

(2) This is immediate from $(*)$.

(3) Assume that Alice has a $(2k-1)$-st generation heir D_{2k-1}. Then after play t_{2k-1} Alice has marked all k vertices in the secondary separator $S \subset D_0$, since she marked a vertex of S on play $t_1 = t_0 + 1$ and after any play t_i that Bob marked a vertex in D_0, Alice made play t_{i+1} by marking another vertex of S. First consider F such that $W_F = S$.

$$|M(U^{t_{2k-1}} \cap F)| \leq 2k - 1 + 2k - 1 \leq 4k - 2.$$

So $U^{t_{2k-1}} \cap F$ contains uncolored components that are not heirs. Also, regardless of the case in which D_0 was treated, $\sum_{W_F = S} |M^{t_0} \cap F| \geq 2$. It follows that

$$|M^{t_{2k-1}}(D_{2k-1})| \leq \sum_{W_F \neq S} |M^{t_0} \cap F| + 2k - 1 \leq (4k-1) - 2 + (2k-1) \leq 6k - 4.$$

(4) This is immediate from (3) and $(*)$.

(5) Recall that $D_0 \subset W \cup \bigcup_{F \in I} F$. After $2k + 1$ vertices of D_0 have been marked, all the vertices of the separator W are marked. Thus any heir of D_0 is contained in some $F \in I$. We have already seen that if $W_F = S$ then F does not contain an heir of D_0 even after play t_{2k-1}. So if D_0 was treated in Subcase 2a we are done. So suppose that $W_F \neq S$. Then $|S| = k = |W_F|$. So there is a vertex $v \in S - W_F$. It follows that v is not adjacent to any vertex in F. If D_0 was treated in Subcase 2b, then

$$|M(U^{t_{2k+1}} \cap F)| \leq (2k-2) + 2k = 4k - 2.$$

If D_0 was treated in Subcase 2c, then $|M^{t_2} \cap F| \leq 2k - 1$. So

$$|M(U^{t_{2k+1}} \cap F)| \leq (2k-1) + 2k - 1 = 4k - 2. \qquad \square$$

This completes the proof of Theorem 1.

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