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Evolutionary Optimisation of Mechanical Structures: Towards an Integrated Optimisation

J. L. Marcelin

Laboratoire Sols Solides Structures, Domaine Universitaire, Grenoble, France

Abstract. *This work results from the synthesis of author's works on the applications of stochastic techniques (genetic algorithms with neural networks) for the optimisation of mechanical structures. The emphasis of this work is on the practical aspects and the feasibility of the aforementioned techniques. The research strategy consists in substituting, for finite element calculations in the optimisation process, an approximate response of a neural network. More precisely, the paper describes the use of backpropagation neural networks in creating function approximations for use in computationally intensive design optimisation based on genetic algorithms. An example of application for space frame optimisation of a helicopter tail boom is given in this paper, for which we can talk of integrated optimisation. This example (including displacement and frequency constraints) show the use of neural networks as a function approximation strategy to limit the computational costs associated with stochastic search methods.*

Keywords. Genetic algorithms; Neural networks; Structural optimisation

1. Introduction: The Need for an Integrated Optimal Design Process

The search for the best compromise between economic, mechanical and technological imperatives has always been the primary objective of the mechanical engineer. The methods used to achieve these objectives have evolved considerably over the last few years. The author's experience in optimisation began in 1983. At this time, design would come first, then calculation and finally, optimisation. In practice, and during experience of the optimisation of the shape of mechanical structures between 1985 and 1990, many extreme cases were encountered. In these cases, the question of optimisation was not dealt with until damage had occurred in-service: the author's industrial partners realised (often too late) that their designing left quite a bit to be desired. They would then call for the author's help in using optimisation programs to supply them with an improved 'shape'. These shapes were reached despite the technological limitations being very severe at this stage; so severe, in fact, that engineers were powerless to resolve the problem. Innumerable problems such as this were dealt with.

Figure 1 exemplifies this well. Some results in

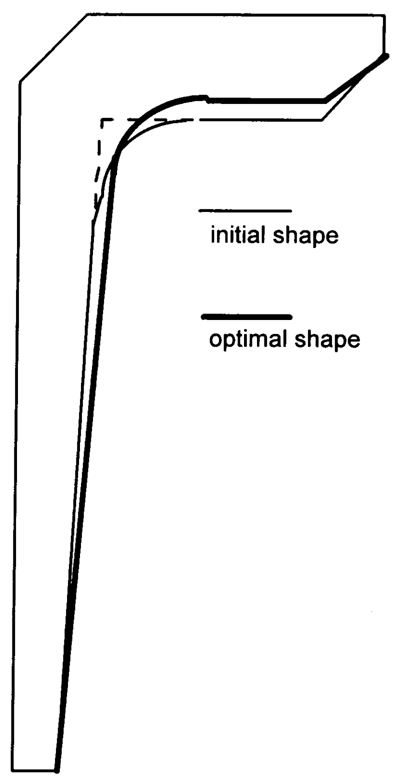

Fig. 1. Optimal shape design of a rotor blade support.

Correspondence and offprint requests to: Dr J. L. Marcelin, Laboratoire Sols Solides Structures, 3S UMR CNRS C5521, Domaine Universitaire, BP No 53, 38041 Grenoble, cedex 9, France. Email: jean-luc.marcelin@ujf-grenoble.fr

investigating the optimal shape of a helicopter gear box are presented here. The part is located just below the rotor blades. This axisymmetrical structure is long enough to be considered as being clamped at its base. The design variables and constraints were given by the manufacturer. The objective is the minimisation of the maximal tangential stress along the exterior boundary. Figures 1 and 2 show the initial and final boundaries and the tangential stress distribution along the exterior boundary.

Such an approach to design has become unthinkable nowadays. The economic competitive climate has increased, design and manufacture delays have been reduced, thus the numerous overlaps that this approach involves have become prohibitive. In short, optimisation can no longer be separated from the act of design. It is now accepted that in an integrated design approach, optimisation has to begin from the design stage, taking into consideration the constraints of both specification and those induced by different materials. Optimisation is therefore made easier because constraints or limitations can more easily be varied in accordance with all those involved with the project. This was not the case in the example above.

In this work, it will be shown that the integration of optimisation from the design phase is, according to the author, possible thanks to new optimisation techniques. A number of optimisation methods are popular at the moment, known as *probabilistic* or *stochastic* optimisation. For example, the simulated annealing method or genetic algorithms, whose principle advantages are an assured convergence without using derivatives, and eventual functions with discrete and non-derivable variables, even though deterministic methods of optimisation (called *gradient methods*) necessitate a calculation that is resistant to these sensitivities. Genetic algorithms rely on the natural laws of selection, which allow a living organism to adapt to a given environment. From these principles, it seems sensible to apply genetic algorithms to the optimisation of structures of mechanical structures. As will be shown in the example of a helicopter's tail boom, from the beginning of the design process, genetic algorithms will allow adaptation of the mechanical object to its environment, and to the specifications.

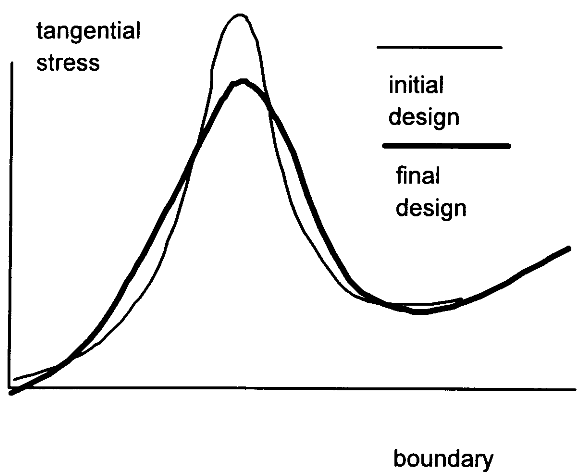

Fig. 2. Tangential stress variations along the exterior boundary.

After a presentation of the methods and tools used, this paper focuses on an application concerned with the mechanics and calculation of structures (Section 3). It will be seen later on, through the example of Section 3, that the integrated optimisation of mechanical structures has almost become a reality. On the other hand, it will be seen in the conclusion that the difficulties are more important in an integrated, optimal design process for mechanical systems, because of the complexity of the problems encountered. Nevertheless, it will be seen that integrated optimisation, and even alternatives to artificial intellegence techniques, can effectively be considered for precise problems of mechanical technology.

2. Methods Used: Optimisation Tools Adapted to Mechanical Technology

The author's experience began with the optimisation of the shape of mechanical structures (2D and axisymmetrical), although this was in the context of traditional design [1–4]. Mathematical optimisation programs were quite difficult to use, and not sufficiently versatile to be adapted quickly to new cases. An optimal integrated design could not be achieved easily with normal mathematical programming techniques, which require a formulation heavily adapted to each particular problem. It will be shown in this article that stochastic techniques are ideally suited to integrated optimisation, and to mechanical technology problems in particular. Note that the essential characteristics of the problems are as follows:

- the design variables are often a mixture of discrete and continuous values;
- they are often highly constrained by strict technological constraints.

2.1. Genetic Algorithms

The problem is to maximize a function of n variables. The principle of Genetic Algorithms (GA) is to make a population of individuals evolve according to a replica of Darwinian theories. The starting point is a population of individuals chosen randomly and coded by binary numbers (as an example, because we can have codings other than binary codings), called *chromosomes*. From this point, the algorithm generates, more or less randomly, new populations formed from individuals, increasingly more adapted to a given, well-defined environment. Selections and reproductions are made from the best performing parents of the population from which they come: they are stochastic or deterministic. The creation of these offspring is done by the application of genetic operators (mutation, crossing). It is always stochastic. The new replacement population is created by selecting the best performing individuals from among either the offspring or the parents of the offspring. The replacement is either stochastic or deterministic [5].

The essential advantage of these methods is that they operate simultaneously on a test space of the solutions. In all cases, the convergence is assured towards an extreme. This extreme is not necessarily the absolute extreme, but it has a greater chance of being so than if a traditional gradient method is used [5]. In effect, a stochastic method explores a larger solution space. In addition, another essential advantage of these methods lies in the small number of assumptions required for the objective function.

2.2. Neural Networks

The operation of Artificial Neural Networks (ANNs), as their name suggests, takes inspiration from biological neural networks. A large part of their vocabulary has thus been 'borrowed' to describe them. Details of the theory can be found in Jodouin [6]. Discussion of artificial neural networks will be condensed here, as this paradigm is well documented in the literature. The use of neural networks for simulation or modelisation will be done in two stages: one phase, which is called *apprenticeship*, using finite element calculations; followed by a *calculation* or *generalisation* phase. In the present case, neural networks should be able to estimate an objective function or a cost function of entry or design variables. In the case of the optimisation procedure and binary coding for the chromosomes of the GA (for example, because we can have codings other than binary codings), the exterior entries will be 0s or 1s, which will correspond to the chromosome's digits.

The application of neural networks to modelisation, especially for simulation of the calculations for mechanical structures, is relatively recent, and seems promising from the results obtained [7–9]. Parallels do exist, however, in adaptative reponse surface methods, where polynomial response surfaces are used in lieu of the ANNs. The continuation to modelisation seems natural, as the action of modeling a process or behaviour necessitates knowledge of the principle characteristics of the process or behaviour. The network knows how to extract these characteristics, and can therefore be memorised easily. On the other hand, this ability to model exploits the adaptation qualities of networks, allowing them to improve as they are exploited. In this work, effective neural networks were used, at the current level of knowledge, and for which apprenticeship and generalisation/calculation algorithms are described by Jodouin [4]. This neural network, quite easily programmed, is a three-layer network with a sigmoid neural function called a MLP (Multi-Layer Perceptron). In this work, the neural network has been used in a 'train as you get the training exemplars' mode during optimisation. This is different from previous approaches, where the training exemplars are generated first, the neural network is then trained to provide function approximations, and the trained network is ultimately linked to the optimiser. Figure 3 summarises this global strategy. After the first n generations (calculated using an exact FEM-based analysis, n being left with the choice of the user within sight of the errors made on the approximations of the objective function), the neural network learning is carried out in parallel with the optimisation process. For the neural network, we must recall that the entry variables are the digits of the chromosomes when using the GA, and the exit variables are the objective function and constraints of the optimisation problem. The neural network is used in a 'train as you get the training exemplars' mode during optimisation. Training is carried out with the results of the first n generations. When a new design is generated and analysed through the neural network approximator, some fractions of new designs reanalysed using exact FEM-based analysis can be used in training an enhanced network, as can be seen in Fig. 3 at generations j and j+1. More precisely, updating of the neural network can be carried out when the GA has already trimmed the problem. In our examples, we have enhanced the base of training using exact

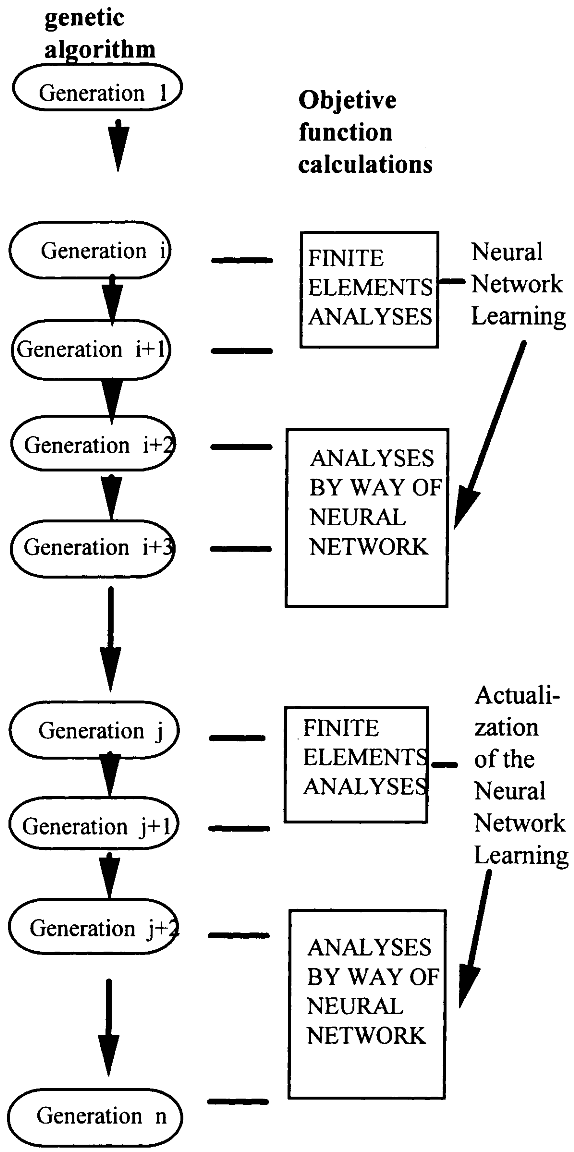

Fig. 3. The global strategy used.

FEM-based analysis for all of the individuals in several successive generations (in general, two or three successive generations). There is no general rule with regard to the moment when a reactualisation of the training base must be carried out. This moment is left to the user within sight of the evolution of convergence of the GA, and the evolution of the errors made on the approximations of the objective function.

3. Integrated Optimisation of Mechanical Structures

Here we give an example of a space frame optimisation subject to frequency constraints: a helicopter tail boom. This example was used in the past [10,11] and was dealt with by way of conventional optimisation methods. It is an example of space frame optimisation subject to frequency and displacement constraints. We are going to apply our new global strategy to the example.

A space frame idealisation of a helicopter tail boom is shown in Fig. 4 along, with node numbers and dimensions, given in centimeters. Element connectivities can be found in Table 1. The frame supports eight concentrated masses located at nodes 13, 14, 15, 16, 25, 26, 27 and 28. Each mass has a constant value of 22.68 kg. Some forces are applied at nodes 25, 26, 27 and 28, and are given in Table 4. All members are made of thin-wall tubes. Four possible cross-sectional shapes for the members are shown in Fig. 5. The sizes and properties of cross-sectional shapes are given in Table 3.

All members are made of the same material ($E = 7.2E7$ kN/m^2, Poisson ratio = 0.3, mass per volume = 0.00277 kg/cm^3). The frame is designed for minimum weight subject to constraints placed on the fundamental frequency and the maximum nodal displacement which occurs at nodes 25, 26, 27 and 28 (in the y direction). The lower limit on the fundamental frequency is 0.25 Hz, and the upper limit on the maximum displacement is 2 cm.

The optimisation problems previously solved [10,11] were different because continuous variables were used. The methods used were also different. In Lust and Schmit [10], the method is based on the application of a full gamut of approximation concepts. In Woo [11], a class of a generalised hybrid constaint approximation that requires only the first-order constraint function derivatives has been developed to overcome the inherent nonlinearity of the frequency constraint.

In this paper, the optimisation problem is solved using discrete variables. The 48 structural elements are arranged in 12 groups, as shown in Table 2. Each group can take one of the four cross-sectional shapes given in Table 3. The coding for the chromosomes is made of 12 digits (one for each group of elements). Each digit can take the values 1, 2, 3 or 4. For example, the chromosome 431111111111 signifies that the first group of elements takes the cross-sectional shape number 4, the second group takes the cross-sectional shape 3, and the other ten groups take the cross-sectional shape 1.

The main objective of this problem is the minimisation of total mass M with frequency and displacement constraints. These constraints are taken into account in a penalisation function. The constraints are, $g1 = (f\text{min}/f) - 1 < 0$ (f frequency); $g2$

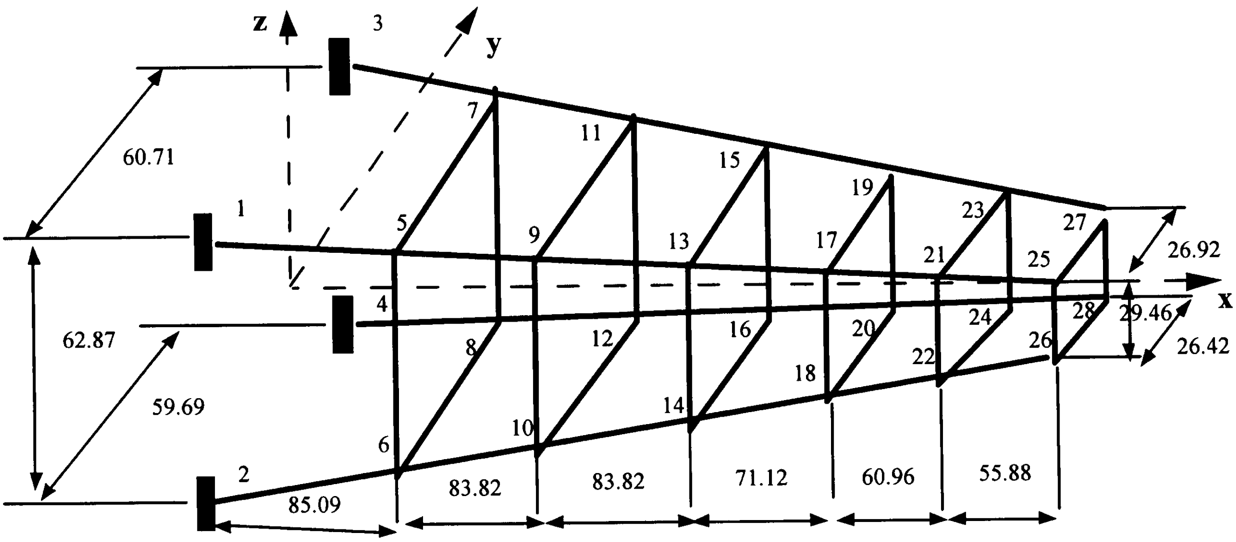

Fig. 4. Helicopter tail boom.

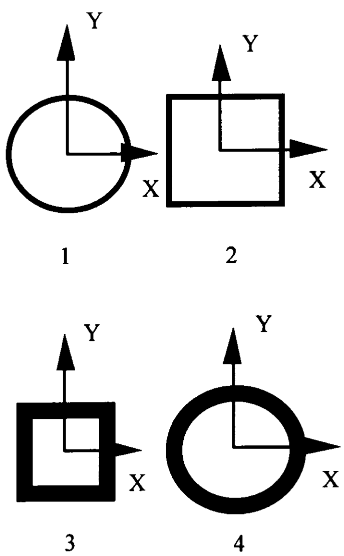

Fig. 5. Cross-sectional shapes.

$= u/u_{\max}-1 < 0$ (u maximum displacement). The penalised objective function is, $F = M\ (1 + k1C1 + k2C2)$, with $Ci = 0$ if $gi < 0$, and $Ci = gi$ if $gi > 0$; $k1$ and $k2$ are weighting coefficients (we have taken $k1{=}k2{=}k$).

The genetic population is made of 100 chromosomes. The global strategy given in Fig. 3 is applied. After the five first generations (frequency and displacement being calculated by the finite elements method), neural network learning is carried out for the calculation of frequency and displacement constraints. For the neural network, we must recall that the entry variables are the digits of the chromosomes when using the GA, and the exit variables are the frequency and the maximum displacement. The neural network is used in a 'train as you get the training exemplars' mode during optimisation. The training is carried out with the results of the five first generations, so the training uses 500 vectors. To test the accuracy of the neural network approximations, 1000 other vectors have been chosen randomly to compare finite element calculations with neural network approximations. Table 5 gives the relative errors between the two calculations. For the displacement by example, it must be noted that 93.5% (of the 1000 vectors) have a relative error less than 1%, and none of the 1000 vectors has an error greater than 2%. 400 generations are then made, and only neural network approximations are used for these 400 further generations. The savings in computational resources is very important, because the 100 analyses necessary for each generation are made 60 times faster than when using finite element analyses for each of the 100 elements of the population.

The best final designs are given in Table 6 for four different coefficients k. For final design, in each case the frequency is at the lower bound of 0.25 Hz and the maximum displacement is at the upper bound of 2 cm. The final design material distributions are almost the same for all cases. The final structural weight obtained in the different cases is approximately 53 Hz. Finally, it is interesting to note the intuitively satisfying result whereby the lighter designs contain slightly larger members at the base (fixed) end of the structure.

Table 1. Connectivities

Element	Node 1	Node 2	Element	Node 1	Node 2	Element	Node 1	Node 2	Element	Node 1	Node 2
1	1	5	13	10	9	25	13	17	37	22	21
2	2	6	14	12	11	26	14	18	38	24	23
3	3	7	15	9	11	27	15	19	39	21	23
4	4	8	16	10	12	28	16	20	40	22	24
5	6	5	17	9	13	29	18	17	41	21	25
6	8	7	18	10	14	30	20	19	42	22	26
7	5	7	19	11	15	31	17	19	43	23	27
8	6	8	20	12	16	32	18	20	44	24	28
9	5	9	21	14	13	33	17	21	45	26	25
10	6	10	22	16	15	34	18	22	46	28	27
11	7	11	23	13	15	35	19	23	47	25	27
12	8	12	24	14	16	36	20	24	48	26	28

Table 2. Groups of elements

Group	Elements of the group				Group	Elements of the group			
1	1	2	3	4	7	25	26	27	28
2	5	6	7	8	8	29	30	31	32
3	9	10	11	12	9	33	34	35	36
4	13	14	15	16	10	37	38	39	40
5	17	18	19	20	11	41	42	43	44
6	21	22	23	24	12	45	46	47	48

Table 3. Sizes and properties of sections

Cross-section	1	2	3	4
Interior (cm)	4	9	6	4.5
Exterior (cm)	4.15	9.2	6.8	5
Surface (cm^2)	3.77	7.2	10.24	14.92
Inertias I_{xx}=I_{yy} (cm^4)	30.16	48.6	70.18	168.81
Torsional inertia J (cm^4)	60.32	145.8	111.8	337.62

Table 4. Forces applied to the structure

Node	25	26	27	28
F_X (kN)	6.76	6.76	−6.76	−6.76
F_Y (kN)	7.67	−6.20	7.67	−6.20

4. General Conclusions and Synthesis

4.1. Conclusions on Neural Networks

This paper has presented an application of GAs in problems of structural optimisation, where savings in computational resources are achieved by using a feedforward (backpropagation) neural network as a universal function approximator. In fact, there is substantial literature where neural network-based function approximations have been coupled with GA-based search techniques. The originality of this work is that the neural network has been used in a 'train as you get the training exemplars' mode during the optimisation. This is different from previous approaches, where the training exemplars are first generated, the neural network is then trained to provide function approximations, and the trained network is ultimately linked to the optimiser.

In the course of our numeric experimentation on neural networks, they have seemed to present some limitations. These limitations are not to do with data processing: currently, thanks to improvements in computers, it is possible to use neural networks of a significant size. It is more the absence of an established theoretical knowledge of the functioning of the networks that renders their use as being delicate. Various problems have needed to be dealt with, such as the necessity to study the feasibility of every application before the numerable numeric experiments, the uncertain sizing of a network, or the absence of theory for the anticipatory calculation

Table 5. Relative errors

Error (%)	−<1	1<−<2	2<−<3	3<−<4	4<−<5	5<−
Frequency (%)	53.8	29.3	10.2	4.0	2.0	0.7
Displacement (%)	93.5	6.5	0	0	0	0

Table 6. Solutions

Case	Penality factor k	Solution chromosome	Mass (kg)	Frequency neural network (Hz)	Frequency finite element (Hz)	Displacement neural network (cm)	Displacement finite element (cm)
1	0.9	41 21 21 21 21 11	50.538	0.2449	0.2450	2.027	2.040
2	1	41 22 21 21 21 11	52.389	0.2516	0.2532	1.989	1.999
3	5	41 22 21 21 21 11	52.389	0.2516	0.2532	1.989	1.999
4	10	41 31 21 21 21 11	53.365	0.2534	0.2537	1.955	1.958

of errors. Fortunately, various new techniques are now available to predict the bounds on neural network-based function approximations. While such bounds tend to be conservative, they nevertheless exist, and can be used as guidelines with which to generate a network architecture, as well as for the distribution of training data for the network.

For the modelling of mechanical structures, it is reasonable to wonder if the use of a simple method derived from the Rayleigh–Ritz method (well known in the field of vibrations) would not be more suitable for the problems considered here. This idea can be illustrated using a well known example in statistics. Suppose that one wanted to optimise the number and situation of stiffeners on a given plate. As with the neural network, one can begin to evaluate a number of representative solutions by finite element methods. In a neuromimetic strategy, these solutions act as the ‘learning’. In a Rayleigh–Ritz type strategy, the solutions are used to find the stiffness corresponding to a new configuration of stiffeners, without having to redo the finite element calculations. The new solution is searched in the form of a weighted sum of test solutions calculated previously. To find the weighting coefficients, or weights (as with neural networks), mechanics offers a reliable theory: the weights are obtained by minimising the total potential energy of the plate in question. It is also possible to take the minimisation of the error, or of the residue on the equilibrium equation, as the criterion.

The method then appears to be more a Galerkin weighted residue method. One notes a certain similarity between these strategies and the neural networks, the difference being that mechanics offers a rigorous error criterion. In their favour, neural networks have the advantage of having a better capability to adapt. Moreover, there is nothing to stop us operating a neural network using the error criterion of mechanics to optimise and control the weights. It is proposed to test these strategies in the near future.

Section 3 of this work has been given over to neural networks, which present a number of intrinsic qualities. These qualities may eventually make the networks superior to the conventional mechanical methods discussed earlier. The first quality is parallelism: the networks are made up of elementary units which can calculate simultaneously (one of the reasons for the superiority of the brain). They are also very capable of adapting. Finally, they can resolve the imprecise, recognise the vague, and so prove to be highly robust.

4.2. Towards an Optimal Integrated Design for Mechanical Systems

As was shown in Section 3, it is possible to tend towards an optimal integrated design for mechanical structures. The GA technique proves itself to be extremely effective, successful and easy to use for constrained optimisation problems, when the cost of calculation is reasonable. It is possible to talk of integrated optimisation due to the versatility, ease of use and adaptability of genetic algorithms. In contrast, as soon as it is desirable to work at a higher level, such as the optimisation of complex mechanical structures, the calculations quickly become prohibitive and render genetic algorithms absolutely inoperable. The idea of this paper is to model these structures by neural networks which may, as previous numerical experience has shown, allow the process to be given the power to carry out the calculations, once learning and various adjustments have been undertaken.

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