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ORIGINAL ARTICLE

An improved Chaotic Harris Hawks Optimizer for solving numerical and engineering optimization problems

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Abstract
Harris Hawk's Optimizer (HHO) is a recently developed meta-heuristics search algorithm with inherent capability to explore global minima and maxima. However, the local search of the basic HHO algorithm is sluggish and has slow convergence rate due to its poor exploitation capability. In the present work, exploration and exploitation phase of HHO have been improved using a chaotic variant of the present optimizer. The proposed chaotic variant has been simulated and tested for 23 standard test functions and 10 different engineering design optimization problems of real life. To check the efficacy of the proposed algorithm, the test results of the proposed CHHO algorithm have been compared with others recently developed and well-known classical optimizers, such as PSO, DE, SSA, MVO, GWO, DE, MFO, SCA, CS, TSA, PSO-DE, GA, HS, Ray and Sain, MBA, ACO, MMA, etc. The experimental results reveal that the suggested method outperforms on most of the test functions and engineering design challenges with superior convergence.

Keywords Meta-heuristic · Chaotic · Exploitation · Harris Hawks Optimizer · Convergence

1 Introduction

Machine learning and computational intelligence is advancing rapidly for solving complex optimization problems. Optimization is the process of selecting of best possible solution amongst the given set of alternatives. Generally, gradient approaches are applied to solve local optimization with only one minimum or maximum point. It has been noticed that discontinuous methods are difficult to differentiate the global solution of multimodal functions in a single run. These types of non-linear un-constrained convex optimization could be precisely solved by meta-heuristics algorithms. Complex engineering problems are effectively tackled by selecting a heuristic path which provides an appropriately worthy solution to an optimization problem [1]. These days various meta-heuristics algorithms have been anticipated by taking inspiration from natural phenomena and social activities. These optimization techniques are very flexible and may be applied to engineering and design problems. These meta-heuristic algorithms may be broadly categorized as swarm-intelligence-based, evolutionary algorithms, physics-based algorithms, and human-based. The first category of population-based meta-heuristics mimics collective or social behaviors, such as moving in swarms, flocks, and herds. Some of the swarm Intelligence approaches are: Ant Colony Optimization (ACO) [2], Particle Swarm Optimization (PSO) [3], Artificial Bee Colony (ABC) [4], Bat-inspired Algorithm (BA) [5], Grey Wolf Optimization (GWO) [1], Moth Flame Optimization (MFO) [6], Krill Herd algorithm(KH) [7], Water Cycle algorithm(WCA) [8], Animal migration optimization(AMO) [9], Imperialist Competitive Algorithm (ICA) [10], Branch and Bound (BB) [11], and Harris Hawk Optimizer(HHO) [12]. The second category is Evolutionary Algorithms (EAs), such as Genetic algorithm (GA) [13], differential evolution (DE) [14], Evolution strategy (ES) [15] and Genetic programming (GP), and mimic behaviors, such

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as selection, recombination, and mutation. The third class utilizes some physical laws includes Gravitational search algorithm (GSA) [16], Big-Bang Big-Crunch (BBBC) [17], Multi-Verse Optimizer(MVO) [18], and Sine–Cosine Algorithm(SCA) [19]. The last category mimics certain human behaviors which include some of the well-known algorithms, such as Tabu search (TS) [20], Teaching–learning-Based Optimization (TLBO) [21], Socio Evolution, and Learning Optimization (SELO) [22], Biogeography-Based algorithm [23], etc. A momentary assessment of these metaheuristics and hybrid search algorithms has been portrayed in Table 1.

In last few decades, we witnessed numerous new optimization methodologies which have been introduced to improve system performance with different objectives. It has been seen that a common feature of these metaheuristics algorithms is searching process which involves diversification (exploration phase) and intensification (exploitation phase). However, local minima stagnation is the major drawback of these heuristics algorithms resulting in premature convergence. In the proposed work, we intend to improve the performance of Harris Hawks Optimizer (HHO) by chaotic strategy. HHO is a stochastic metaheuristic algorithm developed by Heidari et al. [12].

The main feature of HHO is to mimic the collective hunting by adopting four strategies, which includes encircling, surprise pouch, soft besiege and hard besiege. HHO is a simple, fast and efficient method to solve complex optimization including discrete, continuous, constraint and unconstraint problems. The major advantages of HHO are simplicity in methodology, ability to escape safely from local minima stagnation, flexibility in operation, improved performance and ease of adoption. However, besides all of these advantages, there are some flaws in HHO. The major limitations are possibility of being trapped in local minima while solving large multimodal and composition optimization problems, inability to maintain proper balance between global and local search, undesirable performance in case of multi-dimensional problems [60]. This is consistent with the observations presented by Heidari et al. [12], who notified that HHO gives poor performance in some circumstances of Unimodal and multimodal test functions. Nevertheless, No free lunch theorem allows for further modification and improvement, as no single method is efficient to tackle all kinds of optimization problems. In a very short span of time, several HHO variants have been developed by researchers and find wide applications in solving optimization problems in different domains, such as engineering design, manufacturing problems, power quality, image segmentation, drug design, networking and pattern recognition problems. From the vast HHO research variants, we have selected some of the specific work for precise comparison and interpretation. Yildiz et al. [61] presented an effective solution to manufacturing optimization problems by solving grinding optimization problem using HHO, GOA and MVO. The comparison results show that the proposed method gives improved results in handling optimum optimization variables. Abbasi et al. [62] adopted HHO to explore more prominent solution in lowering entropy generation of the microchannel considering velocity and temperature constraints. Moayedi et al. [63] incorporated HHO with ANN to find stability of soil slopes which is one of the major issues concerned with civil engineering design problem. It was noted that HHO-ANN method efficiently provides better fitted structure. Chen et al. [64] developed a hybrid algorithm to improve local search capability of basic HHO by combining chaotic maps, multi-population approach and differential strategy. In the proposed work, logistic mapping has been used to enhance exploitation phase. The comparative assessment reveals that proposed method is capable ensuring a balance between exploration and exploitation phase. Firouzi et al. [65] have solved the complications associated with cracks in cantilever beam design by applying impact testing and explored location and depth of crack for Euler–Bernoulli beam using basic and hybridized algorithms. In article [66], authors have incorporated Gaussian bare bone (GB) strategy with HHO to maintain balance between global and local search capabilities. It has been analyzed from this study that for better results, the program has to run for a large number of iterations. Chiwen et al. [67] presented an improved version of HHO by information exchange between search agents. A set of nine benchmark problems and seven engineering design problems were addressed effectively to check potentiality of the method. Elkadeem et al. [68] demonstrated performance analysis of three standard IEEE system considering renewables and distributed generator by applying hybrid HHO–PSO method. All of these studies attempted to optimize different objectives by modifying classical HHO. A common flaw noted from these variants is lack of diversity in their search process and possibility of being struck in local minima. This premature convergence may result in inferior performance for some of Unimodal and multimodal benchmark functions. Authors have used different topologies, such as hybrid, binary and chaotic strategies, for improving performance of various systems. This chaotic approach finds wide scope for developing new algorithms based on chaotic maps. The researchers are continuously working on different chaotic variants to solve optimization problems. Such Chaotic behavior has been implemented by researchers in algorithms like genetic algorithms [69], chaotic Krill Herd search [54], SCA [70], BA [71], GWO [72], PSO [73], WOA [74]. Moreover, a large number of studies revealed that basic HHO algorithm cannot escape from the local optimal solution efficiently and thus results in poor convergence efficiency. Therefore, in this work, we intend to propose a new chaotic HHO algorithm (CHHO). The chaotic

Table 1 Survey of Some existing meta-heuristic and hybrid algorithms

Algorithm	Year	Ref. no.	Number of benchmark	Problem type
Arithmetic optimization algorithm	2021	[24]	29	Engineering design problem
Archimedes optimization algorithm	2021	[25]	30	Engineering design optimization
Modified butterfly optimization algorithm	2021	[26]	14	Engineering design problem
hSMA-PS	2021	[27]	23	Standard benchmark and engineering design problem
Aquila optimizer	2021	[28]	23	Standard benchmark and engineering design problem
Spiral motion mode embedded grasshopper optimization algorithm	2021	[29]	30	Standard benchmark and engineering design problem
Hybrid variational mode decomposition (HVMD)	2021	[30]	NA	Wind turbine power output prediction
Modified krill herd	2021	[31]	NA	Economic load dispatch problem
A meliorated Harris Hawks optimizer	2021	[32]	23	Combinatorial unit commitment
Hunger game search algorithm	2021	[33]	23	Standard benchmark and engineering design problem
Soccer-inspired metaheuristics	2021	[34]	23	Optimization problems
Hybrid multi-population algorithm (HMPA)	2020	[35]	89	Standard benchmark and engineering design problem
Slime mould algorithm	2020	[36]	33	Standard benchmark and engineering design problem
Marine predators algorithm	2020	[37]	29	Engineering design optimization
Harris Hawks Optimizer	2019	[38]	29	Standard benchmark
Self-adaptive differential artificial bee colony algorithm	2019	[39]	28	Optimization
The Sailfish optimizer	2019	[40]	20	Standard benchmark
Coyote optimization algorithm	2018	[41]	40	Engineering design optimization
barnacles mating optimizer	2018	[42]	23	Design optimization
Salp Swarm algorithm	2017	[43]	19	Engineering design optimization
Electro-search algorithm	2017	[44]	10	Engineering design optimization
Grasshopper optimization algorithm	2017	[45]	19	Global optimization
GWO-SCA	2017	[46]	22	Bio-medical optimization
Lion optimization algorithm	2017	[47]	NA	Engineering design optimization
Whale optimization algorithm	2016	[48]	29	Engineering design optimization
Sine Co-Sine Algorithm	2016	[19]	19	Universal
Binary Gray Wolf optimization	2015	[49]	18	Design formulation
Fuzzy optimization Technique	2015	[50]	29	Engineering optimization
Moth-flame optimization algorithm	2015	[6]	29	Engineering design
Multi-verse optimizer	2015	[18]	19	Engineering optimization
Cuckoo search optimization algorithms	2015	[51]	28	Engineering optimization
Chaotic invasive weed optimization algorithms	2014	[52]	NA	Engineering design optimization
Symbiotic organism search	2014	[53]	26	Engineering design optimization
Chaotic Krill Herd algorithm	2014	[54]	14	NA
Interior search algorithm	2014	[55]	14	Engineering design optimization
Competition over resources	2014	[56]	8	NA
Forest optimization algorithm	2014	[57]	4	Feature weighting
Stochastic fractal search	2014	[58]	23	Engineering design optimization
Animal migration optimization	2013	[9]	23	NA
Cultural evolution algorithm	2013	[59]	7	Reliability engineering
Krill Herd algorithm	2012	[7]	20	NA
Water Cycle algorithm	2012	[8]	19	Engineering design optimization

local search (CLS) is combined with basic HHO algorithm to improve the exploitation phase of HHO.

The rest of the paper is arranged as follows: Sect. 2 comprises a brief over review related to chaotic algorithms. Section 3 describes about the back ground of proposed scheme. Section 4 first gave a description of standard benchmark functions. Section 5 includes test results of proposed algorithm and comparative analysis with well-known algorithms. Testing of 10 real-world problems is presented in Sect. 6 and finally, paper is concluded in Sect. 7.

2 Literature survey of some recent HHO and Chaotic variants

In this section, a specific associated work has been presented to explore information regarding recent innovations related to HHO variants and different chaotic strategies are explored in Table 2.

From the prescribed literature studies, it has been noticed that a wide variety of meta-heuristic and hybrid variants of HHO have been developed by the research author to fix different kinds of stochastic complexities. Some real-world problems, such as data mining, environmental issues, medicine and drugs, materials, engineering design, image segmentation, power flow, solar PV modules, feature selection, etc., were analyzed by various researchers using a heuristic approach. The solution accuracy of any algorithm depends on its capability to have a proper balance between intensification and diversification. Studies revealed that slow convergence is the common problem of most heuristic algorithms. This ultimately gives rise to reduced computational efficiency. Thus, to improve the solution efficiency, a trend of developing hybrid algorithms is escalating vastly. Also, diverse chaotic strategies have been effectively incorporated by many researchers to optimize specific objective function. The ultimate aim of these techniques is to provide an optimal solution for a pre-defined problem. Recently, a chaotic variant of HHO using the “logistic function” was presented by Chen et al. [95]. In this work, HHO was integrated with opposition-based learning along with chaotic local search to estimate PV parameters. Current–Voltage characteristics of PV modules are upgraded utilizing chaos periodicity. It is observed that this method generates optimal solution by sensing temperature difference and irradiation variance. This research is pragmatic to deal with only parameter tuning and other crucial variables skipped. Ewes et al. [75] applied HHO for enhancing local search capability of MVO while chaotic maps were employed for determining optimal parameter tuning of MVO. Total 15 benchmark functions were tested for 10 chaotic functions. It was concluded that “circle function” gave best optimal solution. Also, four real-world engineering problems were analyzed to show the propensity of present method. In this work, for recording optimal solution, a large number of parameters are altered, which may result adverse effect on HHO performance. Gao et al. [96] used Tent Chaotic function to improve the exploitation phase of HHO. It was noticed that the solutions of benchmark functions were not exploited to an appreciable level. It was noticed that authors have demonstrated only classical HHO and CHHO results. Also, comparative analysis with competitive method was not performed. In most cases, results are subjected to premature convergence with poor efficiency.

2.1 Novelty of proposed Chaotic HHO method

(i) In the proposed research, the local search capability of classical HHO has been enhanced using “Tent chaotic function”.
(ii) The eminence of the initial population has been enhanced by chaos theory.
(iii) To retain original characteristics of HHO, parameters of HHO are not altered.
(iv) The CHHO method has been effectively employed to evaluate performance of 23 standard benchmarks, and 10 real-world design problems.
(v) The effectiveness of the proposed algorithms has been inspected by Wilcoxon signed-rank test and statistical test.
(vi) The comparative analysis demonstrated in the result section revealed that the suggested method gives outstanding performance in terms of fitness evaluation and solution accuracy.

3 Background of proposed work

Harris Hawks are the intelligent raptors that reside in the United States and Mexico. For their survival, Hawks used to perform hunting in groups. The hunting process involves their inherent ability to communicate within group members to encircle, attack by making a large number of soft and hard besieges. During this process, if the target succeeds in escaping, hawks again coordinate among themselves for another attack. Meanwhile, each hawk may exchange the positions. Finally, the exhausted prey loses its energy rapidly and get struck by the hawks. The captured prey is shared equally among the group members. Remaining food if any is carried by the hawks to their nest for young hawks [97]. In this process, there is a probability for each matching strategy depending on the sites of the family associates and the prey which is mostly a rabbit. In spite of having decent convergence rate, HHO still lacks in finding best optimum solution. So, to diminish this effect and increase its proficiency, chaotic strategy is developed. Basically from the gathered literature given in Table 2, ten most significant chaotic maps extensively used in the field of optimization discovering the search space more enthusiastically and comprehensively. Out of all these available chaotic maps, Tent map is applied to HHO in the proposed work. Conventional Harris Hawks Optimizer has poor exploitation capability and lacks to discover local search space. In an endeavor to pace up Harris Hawks streamlining agent and to kill it nearby pursuit space, the suggested calculation plans to improve the local

Table 2 Literature Survey related to hybrid and chaotic variants

Sr. No.	Algorithm	Ref. no.	Year	Main findings related to proposed work
1	Chaotic Multi-Verse Harris Hawks Optimization	[75]	2020	In this work, local search capability of basic MVO is enhanced by applying chaos theory. The chaotic algorithm was tested for fifteen bench mark problems and four engineering design problems
2	Atom search optimization and Tree-seed algorithm	[76]	2020	In this research, atom search algorithm is hybridized with tree-seed algorithm to solve eighty nine benchmark problems along with seven engineering design problems. Levy flight and chaos theory was employed to develop balance between exploration and exploitation phase
3	Hybrid Multi-Population Algorithm(HMPA)	[35]	2020	This articles presents multi-population-based hybrid algorithm. Beside fifty benchmark problems, seven real-world engineering problems were found to outperform compared to other competitive methods in terms of exploration and exploitation. Levy flight and chaos theory was employed to develop balance between exploration and exploitation phase
4	HHO-IGWO	[77]	2020	Twenty-three benchmark problems along with 10 multidisciplinary engineering design problems were simulated in matlab using hybrid HHO-IGWO algorithm
5	SCA-HHO Algorithm	[78]	2020	Faults in rolling bearings were analyzed using SCA-HHO method. Results prove the superiority of SCA-HHO over other methods scaled on the basis of four validity indices
6	HHO-SCA	[79]	2020	Sixty five benchmark functions were tested using SCA-HHO algorithm. It was noticed that the proposed hybrid method gives superior results compared to other algorithms
7	Boosted Harris Hawk's Optimization (BHHO)	[80]	2020	This paper presents Boosted HHO technique for accelerating convergence rate by combining mutation parameters of DE and random exploratory steps basic HHO. Statistical results revealed proposed method provides proper balance between exploration and exploitation phases and gave superior convergence curves compared to other methods
8	Improved Harris's Hawks Optimization(IHHO)	[81]	2020	In this work, artificial tree algorithm was applied to enhance the exploitation phase of HHO for optimizing parameters of space vector machine and extreme learning machine.Twenty-three benchmark function were test to check the effectiveness of the proposed algorithm in determining accurate stock market prediction
9	IHHO and MOIHHO	[82]	2020	IHHO and MOIHHO methods were applied for determining the optimal size and location of DG at different operating power factors. The proposed method was tested on IEEE 33-bus and IEEE 69-bus system to minimize total active power loss and improve voltage profile
10	Enhanced Harris Hawks Optimization (EHHO)	[83]	2020	In this work, parameter identification of photo-voltaic model was enhanced by incorporating orthogonal-based learning. This method was to provide better solution compared to other methods
11	HHO-FORM	[84]	2020	This article presents solution for High-dimensional problems involving three numerical problems and two engineering design problems were enhanced by improving parameters of first order reliability method (FORM) by incorporating HHO
12	Quasi-reflected Harris hawks optimization algorithm	[85]	2020	In proposed work, QHHOA algorithm was tested for twenty- three benchmark functions with two variants HHO. It was observed that the hybrid method effective in solving multi-dimensional problems with superior convergence

Table 2 (continued)

Sr. No.	Algorithm	Ref. no.	Year	Main findings related to proposed work
13	HHO-ANN	[86]	2020	In this work, HHO method was applied to improve search capability of ANN to improve efficiency of prediction for two test models incorporating three different distillation systems. The analytical results prove that HHO algorithm effectively enhanced the prediction ability of the system
14	CHHO	[87]	2020	In this research, ten chaotic maps were integrated with basic HHO for tuning parameters of for marketable proton exchange membrane fuel cell
15	NOL-HHO	[88]	2020	Nonlinear opposition-based learning strategy was applied to basic HHO to improve a set of constrained codes in DNA storage. The hybrid algorithm was tested for 23 test functions.. The simulation result revealed that NOL-HHO obtained the optimal solution much faster comparing to other algorithms
16	CHHO—QWSC	[89]	2020	In this work, concrete service datasets was employed with logistic strategy for enhancing local search capability of basic HHO. The proposed chaotic HHO was effective solving QWSC problem
17	ANFIS-HHO	[90]	2019	Friction Stir Welding associated with welding problems was modified by integrating ANFIS with HHO to maximize the quality and strength of joints
18	HHODE	[91]	2019	In this work, a IEEE 30-bus test system was tested without incorporating valve-point effect and prohibited zones by implementing proposed method for optimizing power flow problems
19	DA-HHO algorithm	[92]	2019	In this work, DA-HHO algorithm was applied to a multi-layer predictive tool to modify connecting weights and biases for improving bearing capacity of foundations
20	Chaotic harmony search algorithm	[93]	2019	Combined economic emission dispatch problems were analyzed for Six test systems having 6, 10, 13, 14, 40, and 140 units with uniform distribution by applying virtual harmony and dynamic tuning algorithm parameters
21	Chaotic Grasshopper Optimization Algorithms	[94]	2018	Ten shifted and biased functions were tested with 30-dimensional and 50-dimensional benchmark problems. Further three truss bar designs were analyzed. The test results revealed that the proposed method outperforms in terms of convergence compared to other algorithms
22	Chaotic Grey wolf algorithm	[72]	2018	In this work, performance of 13 test functions were tested by applying chaotic GWO method. To check the effectiveness of the proposed method, five engineering design problems were tested by the same method

search capability of Harris Hawks Optimizer using a chaotic local Search and noted as Chaotic Harris Hawks Optimizer (CHHO). The strategy is shown briefly in the Fig. 1.

3.1 Different Chaotic functions

The concepts of probability distribution are captured by lot of meta-heuristics algorithms to gain randomness. Chaotic maps could be beneficial if randomness due to ergodicity, idleness and randomness properties is properly utilized. These chaotic criteria are fulfilled by Eq. (1).

$$o_{k+1} = f(o_k) \tag{1}$$

In Eq. (1), o_{k+1} & $f(o_k)$ are the $(k+1)^{th}$ & k^{th} chaotic number, respectively. The action of chaotic function is dependent on initial value o_0. The particular type of chaotic function will generate a solution within the standardized equations as shown in Table 3. In the proposed research, from these 10 chaotic strategies, Tent chaotic map is intuitively clubbed with basic HHO to enhance the exploitation phase of the search space.

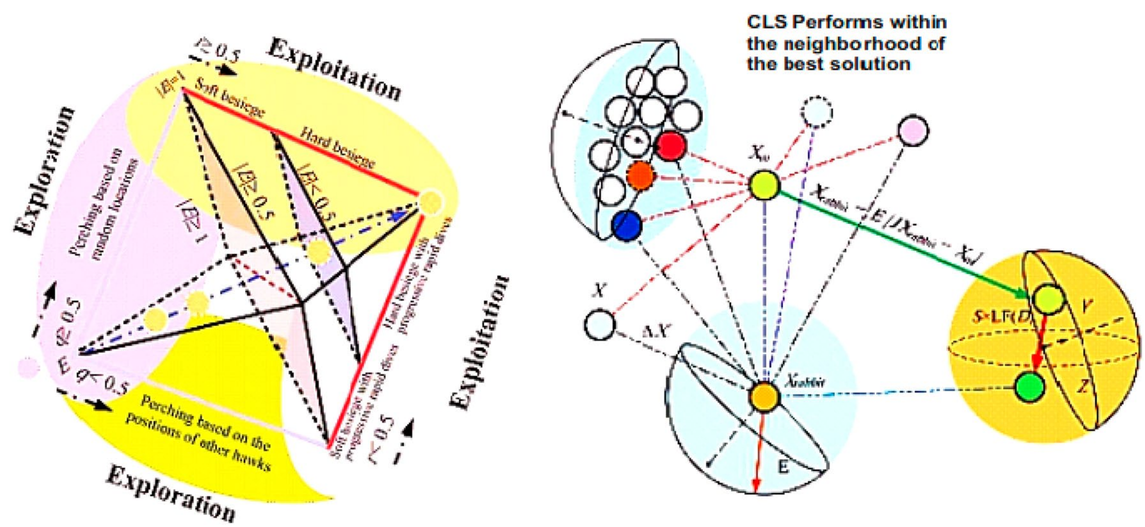


Fig. 1 Improved exploitation phase of HHO with Chaotic local search strategy (CLS)

Table 3 Chaotic functions

Sr. no.	Chaotic name	Mathematical description
1	Chebyshev	$y_{i+1} = \cos\left(\cos^{-1}(y_i)\right)$
2	Iterative	$y_{i+1} = \mathrm{Sin}\left(^{a\pi}/_{y_i}\right),\ a = 0.7$
3	Sinusoidal	$y_{i+1} = ax_i\,\mathrm{Sin}(\pi xi)\,;a = 2.3$
4	Sine	$y_{i+1} = \frac{a}{4}\,\mathrm{Sin}(\pi yi), a = 4$
5	Circle	$y_{i+1} = \mathrm{mod}\,(y_i + b - (^a/_{2\pi})\,\mathrm{Sin}(2\pi y_i),\ 1)\,; a = 0.5,\ b = 0.2$
6	Piecewise	$\begin{cases} ^{y_i}/_p & 0 \le y_i \langle p \\ ^{(y_i - p)}/_{(0.5 - p)} & p \le y_i \langle 0.5 \\ ^{(1 - p - y_i)}/_{(0.5 - p)} & 0.5 \le y_i \langle 1 - p \\ ^{(1 - y_i)}/_p & 1 - p \le y_i \langle 1 \end{cases}, p = 0.4$
7	Gauss/Mouse	$\begin{cases} 1, & y_i = 0 \\ \frac{1}{\mathrm{mod}\,(y_i, 1)} & \text{otherwise} \end{cases}$
8	Singer	$y_{i+1} = \mu\left(7.86\,y_i - 23.3\,y_i^2 + 28.75\,y_i^3 - 13.301875\,y_i^4\right),\ \mu = 1.07$
9	Logistic	$y_{i+1} = a\,y_i\,(1 - y_i),\ a = 4$
10	Tent	$y_{i+1} = \begin{cases} (y_i/0.7), & y_i < 0.7 \\ (10/3)(1 - y), & y_i \ge 0.7 \end{cases}$

3.2 Mathematical modeling of proposed CHHO

The mathematical equations are implemented based on the behavior of Harris hawks and chaotic strategy. This section includes the methodology of capturing the prey. The typical chasing technique of Harris birds is where they recognize the food and chase it using their sharp judgments while the prey not knowing hunters' plan. Let q be the probability for each equalizing attempt which depends on the position of the other family members close enough to them, which is modeled in Eq. (2), when $q < 0.5$ or perch on randomly on tall trees and modeled as in Eq. (2a) for $q \ge 0.5$.

$$X(itn+1) = \{X_{\mathrm{rand}}(itn) - r_1 \times abs(X_{\mathrm{rand}}(itn) - 2 \times r_2 \times X(itn))\,;\ q \ge 0.5 \tag{2}$$

$$X(itn+1) = \{(X_{\mathrm{prey}}(itn) - X_m(itn)) - r_3 \times (Lb + r_4 \times (Ub - Lb))\,;\ \ q < 0.5 \tag{2a}$$

where, $X(itn+1)$ represents the hawks position in next iteration (it^n), $X_{\mathrm{rand}}\,(itn)$ represents randomly selected hawks, $X(itn)$ is the current $r_1, r_2, r_3, r_4,$ and q are random values in between (0, 1) and these are modified in each iteration between upper bound (Ub) and lower bound $(Lb).X_{\mathrm{prey}}(itn)$

represents the position of prey. $X_m(itn)$ represents the mean position of Hawks which is determined by Eq. (3).

$$X_m(itn) = \frac{1}{N}\left(\sum_{i=1}^{N} X_i(itn)\right) \tag{3}$$

where $X(itn)$ is the hawk location in each iteration and N denotes total number of hawks.

Transition from Exploration to exploitation phase depends upon the escaping energy of the prey and is evaluated using Eq. (4)

$$E_A = 2 \times E_0 \times \left(1 - \frac{itn}{itn_{\max}}\right) \tag{4}$$

where E_A is avoidance energy of the prey, E_0 is the initial energy of the prey changing randomly between (− 1, 1) and $itn_{\max}$ is maximum iterations. Equation (5) is used to determine the upgraded position of hawks. The successful capture relies on chasing strategies of Hawks and escaping nature of prey depending upon Escaping energy (E_A) and Change of escape (r).Transition from exploration to exploitation phase depends upon escaping energy of prey. The prey has enough escaping energy. Hawks will first encircle and then surprise pounce is performed. Modeled in Eq. (5) and Eq. (6), Hawks perform a soft besiege for $r \geq 0.5 \& |E| \geq 0.5$.

$$X(itn+1) = \Delta X(itn) - E_A \times abs(J\,Xprey(itn) - X(itn)) \tag{5}$$

$$\Delta X(itn) = \left(X_{prey}(itn) - X(itn)\right) \tag{6}$$

where $\Delta X(itn)$ is the difference between current position of prey and location of Hawks at iteration (itn) $J = 2(1 - r_5)$ is the Jump energy which alters randomly in each iteration. r_5 is the random number within the range (0, 1). The exhausted prey fails to escape and Hawks perform hard besiege. Modeled in Eq. (7), Hawks perform a hard besiege for $r \geq 0.5$ & $|E| < 0.5$.

$$X(itn+1) = X\text{prey}(itn) - E_A \times abs(\Delta X(itn)) \tag{7}$$

$$Y = Xprey(itn) - E \times abs(JX\text{prey}(itn) - X(itn)) \tag{8}$$

$$Z = Y + S \times L_F(D) \tag{9}$$

where D = Problem's dimension, S = Range of fractal flight path by size $(1 \times D)$.

The L_F (D)-based patterns which follow the given rule in Eq. (10) and Eq. (11)

$$L_F(x) = 0.01\left(\frac{\mu \times \sigma}{|v|^{\frac{1}{\beta}}}\right) \tag{10}$$

$$\sigma = \left(\frac{\Gamma(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2\left(\frac{\beta-1}{2}\right)}\right)^{\frac{1}{\beta}} \tag{11}$$

where μ, σ are denoted as such kinds of values randomly in between (0, 1) and β is default constant set to 1.5.

At this stage, the prey has enough energy and besiege during this phase depends on Levy flight (LF) concept as modeled in Eq. (12). Hawks perform a soft besiege with rapid dives for $|E| \geq 0.5$ $\& r < 0.5$.

$$X(itn+1) = \begin{cases} Y\,;\ if\ \ F(Y) < F(X(itn)) \\ Z\,;\ if\ \ F(Z) < F(X(itn)) \end{cases} \tag{12}$$

$$Y' = X\text{prey}(itn) - E \times abs(JXprey(itn) - X_m(itn)) \tag{13}$$

$$Z' = Y' + S \times L_F(D) \tag{14}$$

The Hawks are very close to prey and perform hard besiege. Modeled in Eq. (15), Hawks perform hard Besiege with rapid dives for $|E| < 0.5$ $\& r < 0.5$.

$$X(itn+1) = \begin{cases} Y'\,;\ if\ \ F(Y') < F(X(itn)) \\ Z'\,;\ if\ \ F(Z') < F(X(itn)) \end{cases} \tag{15}$$

3.3 Algorithm of proposed work

The basic HHO is upgraded by combining the chaotic approach to further enhance the search accuracy. The pseudo-code for method proposed is as shown in Fig. 2. The flow chart showing process of algorithm is illustrated in Fig. 3.

4 Standard benchmark test functions

In the proposed CHHO algorithm, the Tent Chaotic map has been used. The mathematics of the chaotic tent map has been explained in Table 3 [75]. These standard benchmark functions are characterized by their objective fitness in parameter space within a particular dimension (Dim), range, and optimal value ($f_{\min}$).The parameter setting for the proposed algorithm is shown in Table 4. Table 5 illustrates uni-modal benchmark function from F1 to F7. Convergence curve for HHO and CHHO for the respective uni-modal (UM) benchmark is in Fig. 4. Similarly, Tables 6, 7 show test results for Multi-modal (UM) (F8 to F13) and fixed dimension (FD) (F14 to F23) functions. Figure 5 and Fig. 6 presents convergence curve for multi-modal and fixed dimension functions. The convergence curves of HHO and CHHO are indicated

PSEUDO code of the Proposed CHHO

INPUTS-: The population range is taken as N and maximum iteration number is taken as it^n
OUTPUTS- : The position of (prey) and its value of fitness
Initialization of stochastic population $X_i (i = 1, 2, 3......., N)$
While (iteration $itn_{\max}$)
For calculating the optimum robustness of Harris birds
Setting the parameter $X_{(prey)}$ as the best position of the prey
for (each Harris birds X_i
Do Update energy at primary condition E_O
$r^{0,1}$ =rand;
if $r^{0,1} < 0.7$
$r^{0,1}$ (t+1)= $r^{0,1}$ / 0.7 ;
end
if $r^{0,1} \geq 0.7$
$r^{0,1}$ (t+1)=(10/3)*(1- $r^{0,1}$);
end
$Uptate\ \ q = r^{0,1}(t+1);$
$Uptate\ \ r = r^{0,1}(t+1);$

if $|E| \geq 1$ **then** $\rightarrow$ **Phase of Exploration**
else if $q \geq 0.5$ **then**
$X(itn+1) = \{X_{randm}(itn) - r_1 \times abs(X_{randm}(itn) - 2 \times r_2 \times X(itn))$

else if $q < 0.5$ **then**
$X(itn+1) = \{(X_{prey}(itn) - X_m(itn)) - r_3 \times (Lb + r_4 \times (Ub - Lb))$
Position vector updated using $E = 2 \times E_0 \times \left(1 - \frac{itn}{itn_{\max}}\right)$

if $|E| < 1$ **then** $\rightarrow$ **Phase of Exploitation**
$X(itn+1) = \Delta X(itn) - E \times abs(JX_{prey}(itn) - X(itn))$
if $(r \geq 0.5)$ and $|E| \geq 0.5$ **then** $\rightarrow$ **placid bound**
Position vector updated using $\Delta X(itn) = \left(X_{prey}(itn) - X(itn)\right)$
else if $(r \geq 0.5)$ and $|E| < 0.5$ **then** $\rightarrow$ **Hard bound**
Position vector updated using $X(itn+1) = X_{prey}(itn) - E \times abs(\Delta X(itn))$
else if $(r < 0.5)$ and $|E| \geq 0.5$ **then** $\rightarrow$ **placid bound with advanced fast dives**
Position vector updated using $Y = X_{prey}(itn) - E \times abs(JX_{prey}(itn) - X(itn))$
else if $(r < 0.5)$ and $|E| < 0.5$ **then** $\rightarrow$ **Hard bound with advanced fast dives**
Position vector updated using $X(itn+1) = \Delta X(itn) - E \times abs(JX_{prey}(itn) - X(itn))$
end
end
end
end
Return X_{prey}

Fig. 2 Pseudo-code of Chaotic HHO algorithm

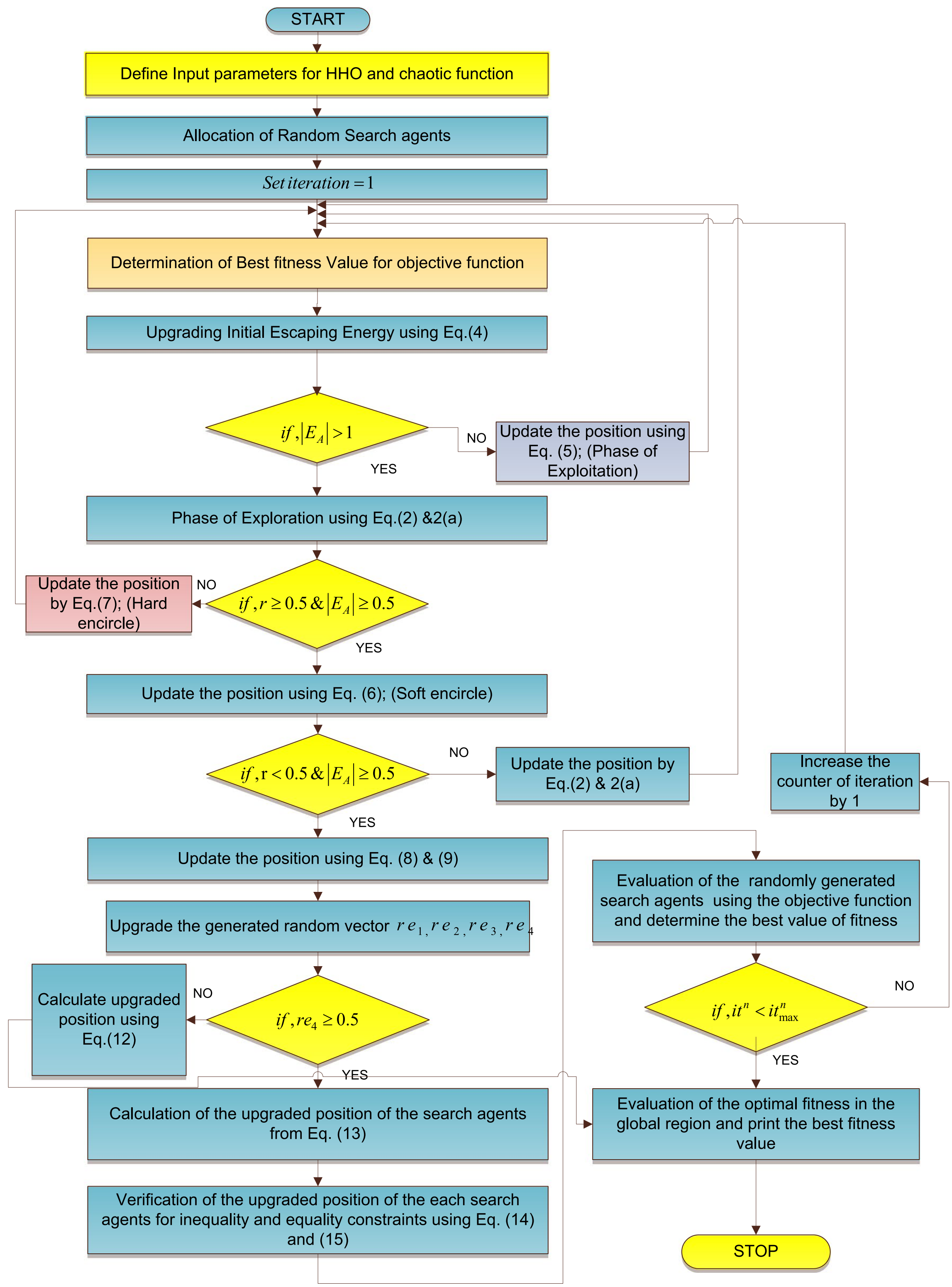


Fig. 3 Flow chart of Chaotic HHO algorithm

by green and red curve, respectively. It can be easily noticed from corresponding figures that search capability has been improved by the resultant chaotic approach with superior convergence.

Table 4 Parameter setting for the proposed method

Parameter setting	Tent_CHHO
Number of search agents	30
Number of iterations for U-Modal, M-modal, and F-Modal	500
Number of iterations for Engineering optimization design problems	500
Number of trial runs	30
Initial Parameter of Tent map (r)	0.7
Range of Tent map	(0,1)

Table 5 Unimodal test function

Unimodal function	Dim	Range	$f_{\min}$
$f_1(y) = \sum_{i=1}^{n} y_i^2$	30	[− 100, 100]	0
$f_2(y) = \sum_{i=1}^{n} \lvert y_i \rvert + \prod_{i=1}^{n} \lvert y_i \rvert$	30	[− 10,10]	0
$f_3(y) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} y_j \right)^2$	30	[− 100, 100]	0
$f_4(y) = \max_i \left\{ \lvert y_i \rvert, 1 \le i \le n \right\}$	30	[− 100, 100]	0
$f_5(y) = \sum_{i=1}^{n-1} \left[100(y_{i+1} - y_i^2)^2 + (y_i - 1)^2 \right]$	30	[− 30, 30]	0
$f_6(y) = \sum_{i=1}^{n} \left(\left[y_i + 0.5 \right] \right)^2$	30	[− 100, 100]	0
$f_7(y) = \sum_{i=1}^{n} i y_i^4 + \text{random}[0, 1]$	30	[− 1.28, 1.28]	0

5 Simulation results and discussion

In this section, results of 23 standard Benchmark functions are presented. The test systems are simulated in MATLAB 2018a Windows 10, CPU@2.10Ghz-4 GB RAM Core i5. Test results for benchmark functions are discussed with their average, worst, best, median, standard deviation. To analyze feasibility of solution, Wilcoxon sum test and statistical T test have been taken into account. The stochastic complexity of the proposed algorithm is justified and analyzed by running the algorithm for 30-trial checks and 500 re-iterations. On similar grounds, results are compared with other universally validated methods.

5.1 Testing of uni-modal functions

The search process for best position depends upon the capability of the algorithm to reach closer to origin. During the search process by various agents, there may be possibility of being entrapped far or nearby and accordingly defined in terms of exploration and exploitation. Exploration comes under global search process and exploitation falls under local search category. The outcomes for uni-modal (UM) test functions have been illustrated in Table 8. Table 9 illustrates Statistical Analysis for Uni-modal benchmark functions. Further to check the feasibility of proposed method, Dunn's test has been performed and test results for sum-of-squares (SS), degree of freedom (df), Mean square values (MS), Chi-sq Prob > Chi-sq and corresponding errors are tabulated in Table 10. Simulation time for UM Benchmark Problems utilizing CHHO is shown in Table 11. Table 12 shows compared results with other meta-heuristics search algorithms like PSO[98], GWO[1], GSA[16], BA[99], FA[100], GA[101], MFO[6], MVO[18], SMS[102], FPA[103], DE[104], ALO[105], WOA [48], etc. in terms of standard and mean deviation. In Fig. 7, a correlation between Chaotic HHO and different methods appears intermingling bend for UM (F1 to F7) shows a few ideal foci convergences nearer to optimal value. The algorithm is tested for 30-trial runs and 500 iterations as presented in Fig. 7. The test outcomes of UM (F1 to F7) have some raised points with increased convergence using CHHO revealing the effectiveness of algorithm. Box-Plot of Trial runs of U-Modal Benchmark Function CHHO compared with competitive algorithms is shown in Fig. 8.

5.2 Testing of multi-modal test function

The proposed chaotic algorithm is tested for multi-modal test function with 30-trial runs and 500 iterations as presented in Fig. 9. The outcomes for Multi-modal (MM) test functions have been illustrated in Table 13. Table 14 illustrates Statistical Analysis for Multi-modal benchmark functions. Further to check the feasibility of proposed method, Dunn's test has been performed and test results for sum-of-squares (SS), degree of freedom (df), Mean square values (MS), Chi-sq Prob > Chi-sq and corresponding errors are tabulated in Table 15. Simulation time for multi-modal (MM) Benchmark Problems with best, mean and worst utilizing CHHO is shown in Table 16. Table 17 summarizes compared results with other meta-heuristics search algorithms like PSO [98], GWO [1], GSA [16], BA [99], FA [100], GA [101], BDA [107], BPSO [108], MFO [6], MVO [18], SMS [102], DE [104], ALO [105], etc., in terms of mean and standard deviation. The test outcomes for MM (F8 to F13) have few pick points with increased convergence using CHHO revealing the effectiveness of algorithm. Box-Plot of Trial runs of M-Modal benchmark function compared with other methods is shown in Fig. 10.

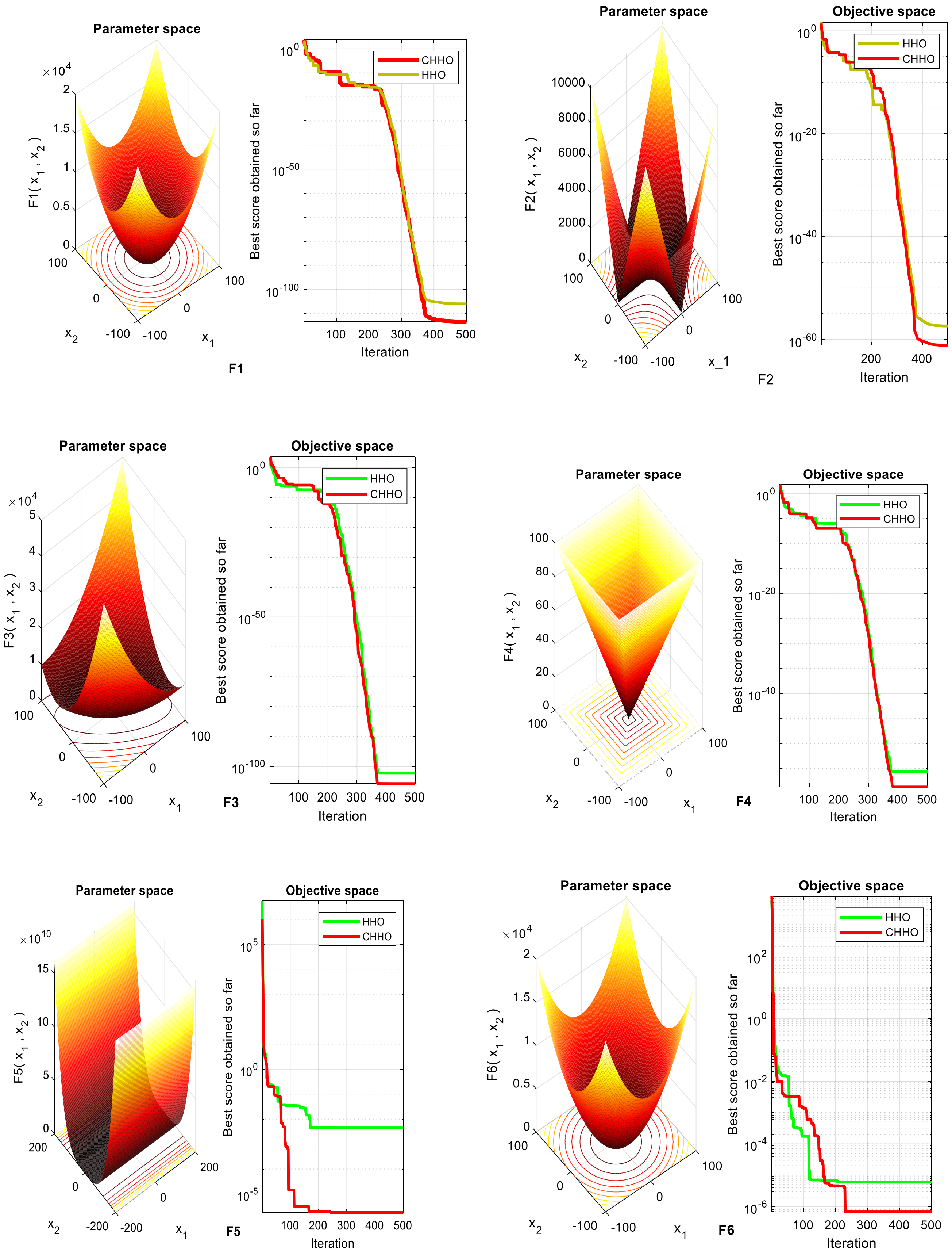


Fig. 4 Three-dimensional view of F1 to F7 along with convergence curve for HHO and CHHO

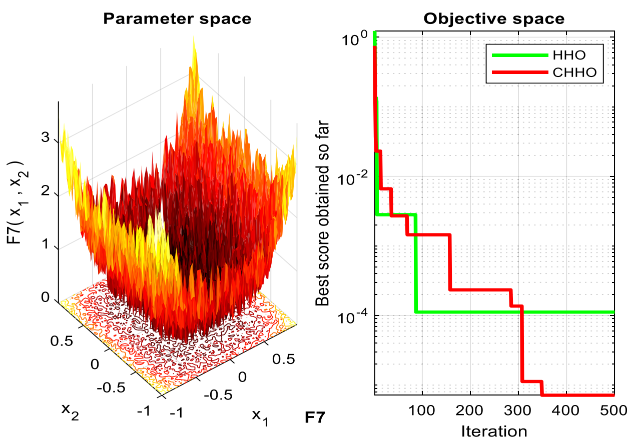


Fig. 4 (continued)

functions. Further to check the feasibility of proposed method, Dunn's test has been performed and test results for sum-of-squares (SS), degree of freedom (df), Mean square values (MS), Chi-sq Prob > Chi-sq and corresponding errors are tabulated in Table 20. Simulation time for FD Benchmark Problems utilizing CHHO is shown in Table 21. The results illustrated in Table 22 are compared with others variants, such as GWO [1], PSO [3], GSA [109], DE [14], ALO [105], BA [111], GA [110], SSA [43], DE [112], etc. in terms of mean and standard deviation. From the compared convergence curves shown in Fig. 11, it is observed that proposed tent chaotic HHO gives more superior results in terms of convergence. Box-Plot of Trial runs of FD-Modal benchmark function compared with other methods is shown in Fig. 12.

5.3 Testing of fixed dimension benchmark functions

The proposed Chaotic HHO algorithm is tested for Fixed Modal Benchmark functions (F14 to F23) for 30-trial runs and 500 iterations. The outcomes for fixed dimension (FD) test functions have been illustrated in Table 18. Table 19 illustrates Statistical Analysis for fixed-modal benchmark

6 Multi-disciplinary engineering design problems

In this section, ten different design problems are discussed which includes 3-bar truss problem, speed reducer problem, pressure-vessel design, cantilever beam design, compression design, rolling element problem welded beam, Belleville spring problem, gear train design problem, and multidisc

Table 6 Multimodal test function

Multimodal function	Dim	Limit	$f_{\min}$
$f_8(y) = \sum_{i=1}^{n} -y_i \sin\left(\sqrt{\lvert y_i \rvert}\right)$	30	[[– 500, 500]	– 418.98295
$f_9(y) = \sum_{i=1}^{n} [y_i^2 - 10\cos(2\pi y_i) + 10]$	30	[– 5.12, 5.12]	0
$f_{10}(y) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2}\right) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi y_i) + 20 + c$	30	[– 32, 32]	0
$f_{11}(y) = 1 + \sum_{i=1}^{n} \frac{y_i^2}{4000} - \prod_{i=1}^{n} \cos\frac{(y_i)}{\sqrt{i}}$	30	[– 600, 600]	0
$f_{12}(y) = \frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi z_{i+1})] + (z_n - 1)^2\right\} + \sum_{i=1}^{n} u(y_i, 10, 100, 4)$ $z_1 = 1 + \frac{y_i+1}{4}$ $u(y_i, a, k, m) = \begin{cases} k(y_i - a)^n & y_i > a \\ 0 & -a < y_i < a \\ k(-y_i - a)^n & y_i < -a \end{cases}$	30	[– 50, 50]	0
$F_{13}(x) = 0.1\left\{\begin{array}{c}\sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2\left[1 + \sin^2(3\pi x_i + 1)\right] + \\ (x_n - 1)^2\left[1 + \sin^2(2\pi x_n)\right]\end{array}\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	[– 50,50]	0

Table 7 Fixed dimension function

Fixed dimension Function	Dim	Range	$f_{\min}$
$f_{14}(y) = \left[\frac{1}{500} + \sum_{j=1}^{2} 5\frac{1}{j+\sum_{i=1}^{n}(y_i - a_{ij})^6}\right]^{-1}$	2	[− 65.536, 65.536]	1
$f_{15}(y) = \sum_{i=1}^{11}\left[a_i - \frac{y_1(b_i^2 + b_i y_2)}{b_i^2 + b_i y_3 + y_4}\right]^2$	4	[− 5,5]	0.00030
$f_{16}(y) = 4y_1^2 - 2.1y_1^4 + \frac{1}{3}y_1^6 + y_1 y_2 - 4y_2^2 + 4y_2^4$	2	[− 5, 5]	− 1.0316
$f_{17}(y) = \left(y_2 - \frac{5.1}{4\pi^2}y_1^2 + \frac{5}{\pi}y_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos y_1 + 10$	2	[− 5, 5]	0.398
$f_{18}(y) = \begin{bmatrix} 1 + (y_1 + y_2 + 1)^2 \\ (19 - 14y_1 + 3y_1^2 - 14y_2 + 6y_1 y_2 + 3y_2^2) \end{bmatrix} \times \begin{bmatrix} 30 + (2y_1 - 3y_2)^2 \times \\ \begin{pmatrix} 18 - 32y_1 + 12y_1^2 + 48y_2 \\ -36y_1 y_2 + 27y_2^2 \end{pmatrix} \end{bmatrix}$	2	[− 2, 2]	3
$f_{19}(y) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}(y_j - q_{ij})^2\right)$	3	[1, 3]	− 3.32
$f_{20}(y) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}(y_j - q_{ij})^2\right)$	6	[0, 1]	− 3.32
$f_{21}(y) = -\sum_{i=1}^{5}\begin{bmatrix} (y - a_i) \\ (y - a_i)^T + d_i \end{bmatrix}^{-1}$	4	[0, 10]	− 10.1532
$f_{22}(y) = -\sum_{i=1}^{7}\begin{bmatrix} (y - a_i) \\ (y - a_i)^T + d_i \end{bmatrix}^{-1}$	4	[0, 10]	− 10.4028
$f_{23}(y) = -\sum_{i=1}^{10}\begin{bmatrix} (y - a_i) \\ (y - a_i)^T + d_i \end{bmatrix}^{-1}$	4	[0, 10]	− 10.5363

clutch brake problem [79]. These engineering problems are abbreviated as Engineering Function (EF). Table 23 summarizes details of ten real-world design problems. The comparison of the various engineering design problem with their best mean, worst standard deviation, and p value is illustrated in Table 24. The simulation time is shown in Table 25. Relative convergence curvatures of the proposed CHHO with standard HHO are shown in Fig. 13. All design problems from EF1 to EF2 are executed for 30-trial runs with 500 iterations.

To check the effectiveness of Chaotic HHO, algorithm is tested for 30-trial runs and 500 iterations. The algorithm is tested with respect for best value, standard deviation worst value, and p value. Furthermore, a comparative analysis with recent optimization methods is provided for justifying the validity of tested results for each design problems. Figure 14 illustrates 30-trial runs iterations for ten multidisciplinary engineering problems to check the optimality of the algorithm.

6.1 EF1—three-bar truss design problem

The proposed Chaotic HHO algorithm is applied for solving problem of Truss design as shown in Fig. 15 [24]. It has two variables and three parameters. The main focus of truss design problem is to minimize weight by optimizing two parameters. In truss bar design problem, three types of constraints warping, deflection and stress are optimized to achieve the desired objective. The mathematical modeling of 3-Bar Truss is illustrated through Eq. (16.1, 16.1a, 16.1b, 16.1c, 16.1d) subject to various constraints. The results of CHHO are compared with other optimization algorithms. The results are illustrated in Table 26. It is seen that the suggested method appreciably improves the objective of cost minimization. The design problem is modeled as given below:

$$\vec{y} = [y_1, y_2] = [A_1, A_2] \tag{16.1}$$

Minimize

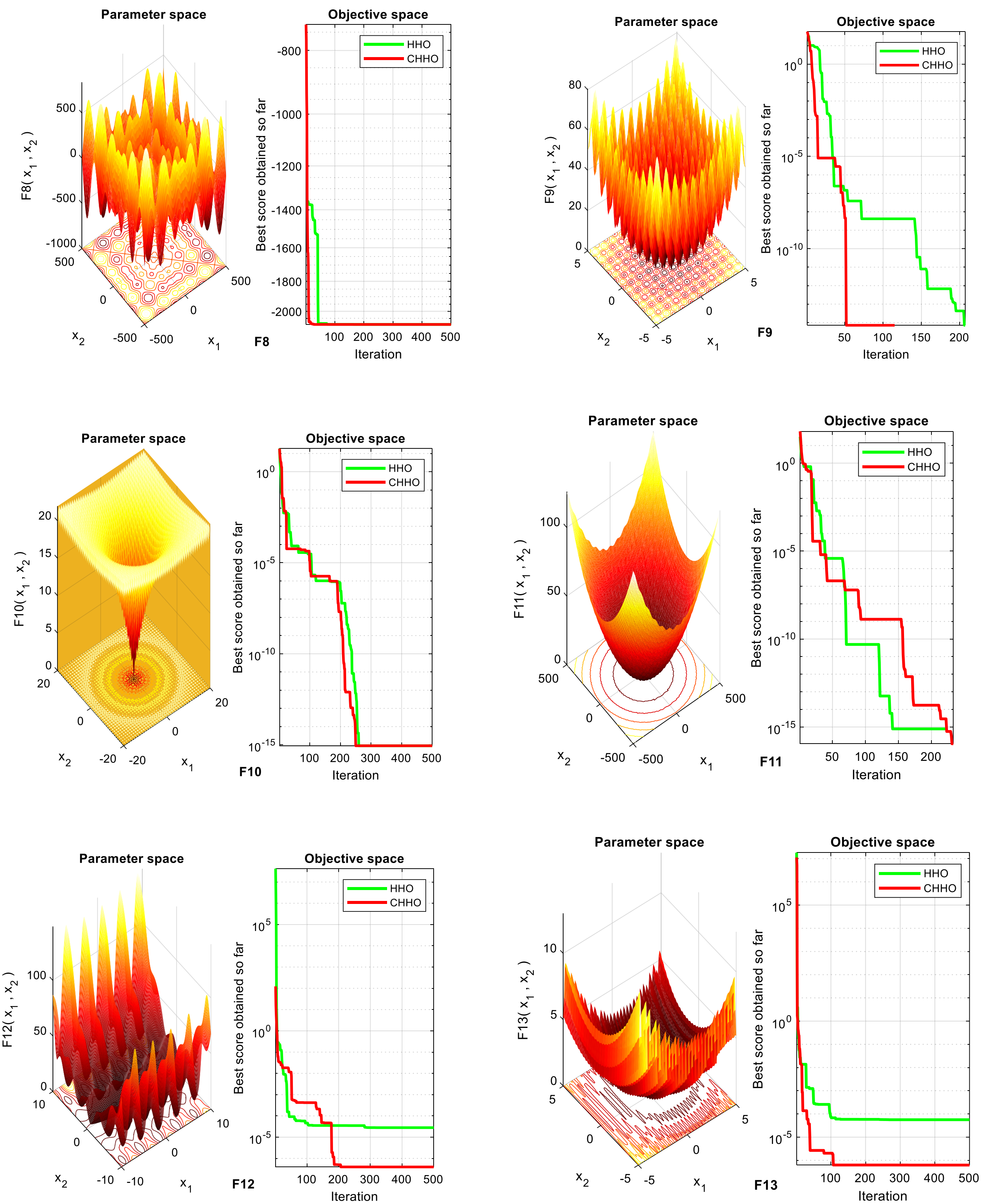


Fig. 5 Three-dimensional view of F8 to F13 along with convergence curve for HHO and CHHO

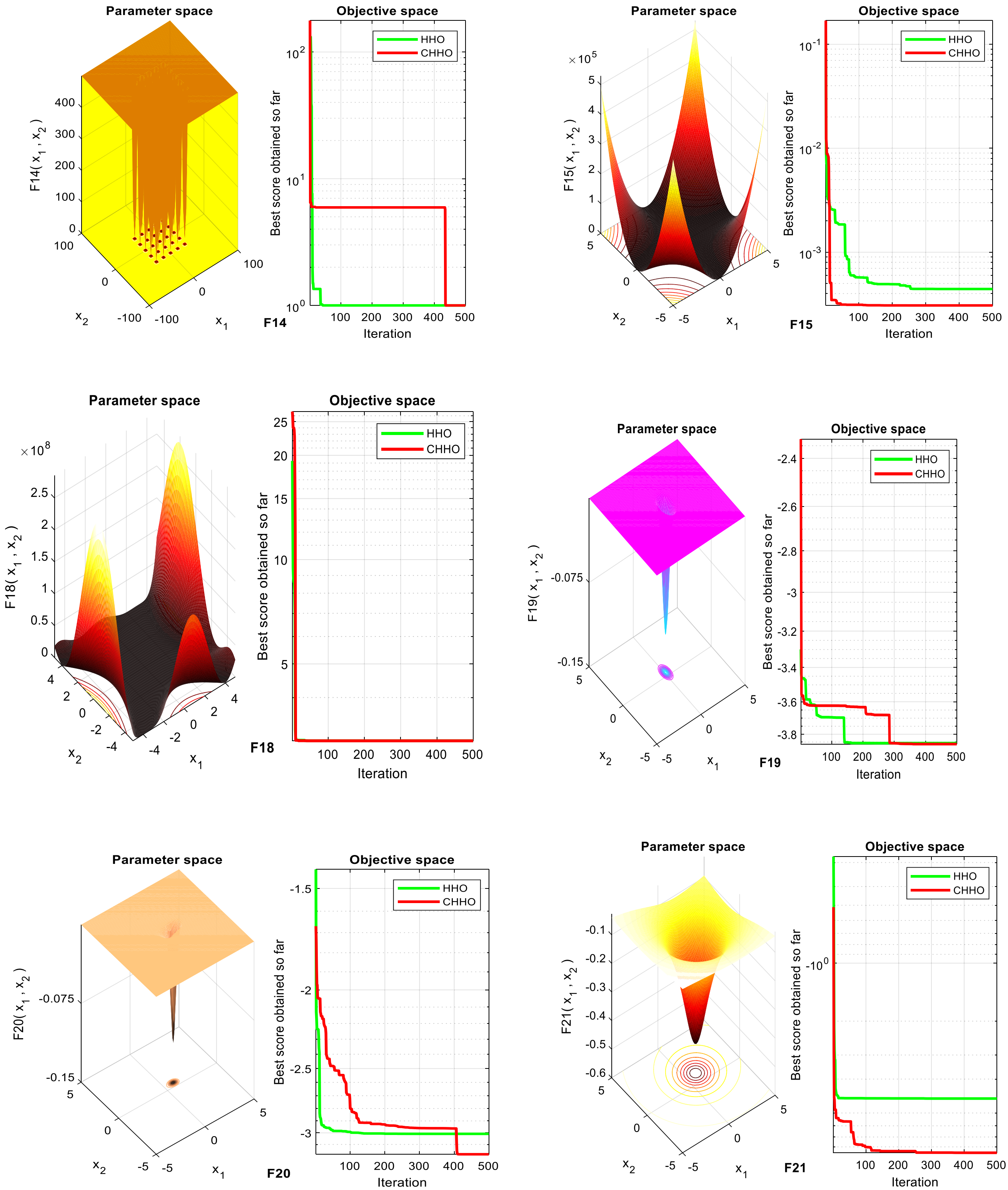


Fig. 6 Three-dimensional view of F14 to F23 along with convergence curve for HHO and CHHO

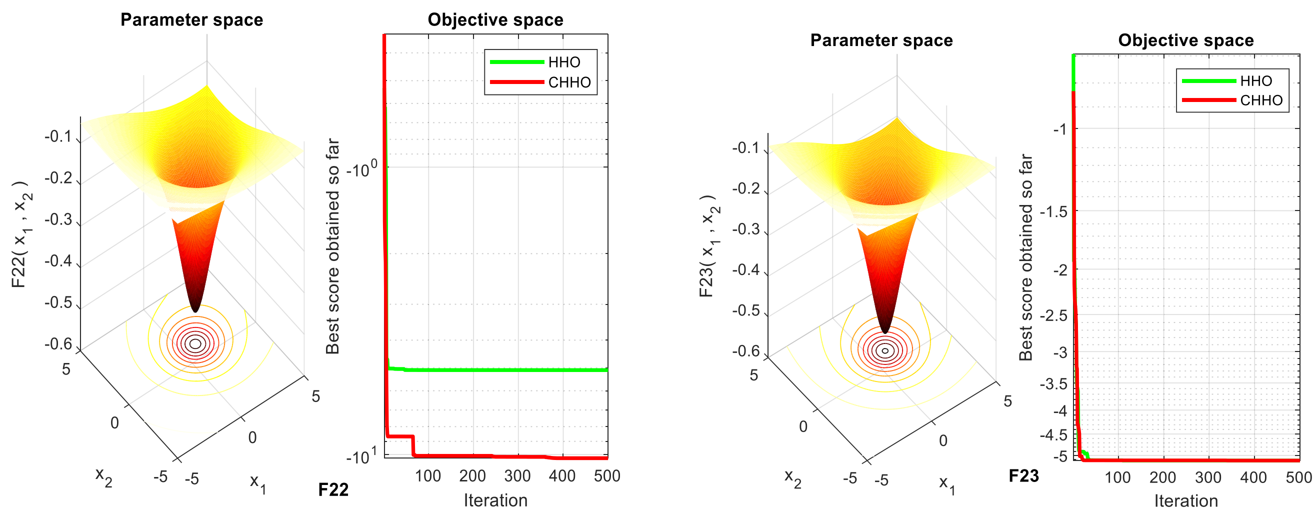


Fig. 6 (continued)

Table 8 Test results of Uni-modal benchmark functions

Function	Objective function fitness					Wilcoxon rank Sum test	*T* test	
	Mean	STD	Best	Worst	Median	*p* value	*t* value	*h* value
F1	2.29E-96	1.18E-95	1.5E-119	6.49E-95	2E-104	1.7344E-06	0.299955	0
F2	1.08E-48	5.77E-48	2.12E-60	3.16E-47	4.43E-54	1.7344E-06	0.157795	0
F3	1.54E-69	7.74E-69	7.32E-98	4.24E-68	1.56E-84	1.7344E-06	0.318537	0
F4	1.4E-48	6.39E-48	2.94E-55	3.48E-47	2.68E-52	1.7344E-06	0.237638	0
F5	0.013086	0.01999	1.59E-06	0.08431	0.003289	1.7344E-06	0.000232	1
F6	0.00016	0.000299	1.99E-08	0.001119	5.94E-05	1.7344E-06	0.000957	1
F7	0.00015	0.000159	2.54E-06	0.00069	9.11E-05	1.7344E-06	3.58E-06	1

$$f(\vec{y}) = \left(2\sqrt{2}y_1 + y_2\right) * l \tag{16.1a}$$

Subject to:

$$g_1(\vec{y}) = \frac{\sqrt{2}y_1 + y_2}{\sqrt{2}y_1^2 + 2y_1y_2}P - \sigma \le 0 \tag{16.1b}$$

$$g_2(\vec{y}) = \frac{y_2}{\sqrt{2}y_1^2 + 2y_1y_2}P - \sigma \le 0 \tag{16.1c}$$

$$g_3(\vec{y}) = \frac{1}{\sqrt{2}y_2 + y_1}P - \sigma \le 0 \tag{16.1d}$$

6.2 EF2—speed reducer design problem

The speed reducer design **is** associated with seven design parameters. The foremost objective is to minimize weight of speed reducer. This type of design problem consists of 11 constraints and 6 continuous variables as shown in Fig. 16 [116]. The seven variables are face width (z_1), teeth module (z_2), pinion teeth (z_3), first shaft length (z_4), second shaft length (z_5), the first shaft diameter z_6 and second shaft diameter (z_7). All parameters except (z_3) are continuous since it is having an integer value. The mathematical modeling for the optimal design of speed reducer is illustrated through Eq. (16.2a, 16.2b, 16.2c, 16.2d, 16.2e, 16.2f, 16.2g, 16.2h, 16.2k). The comparative analyses of CHHO with other metaheuristics method are listed in Table 27. It can be observed from the analysis that CHHO is more effective in cost minimization as compared to other methods.

Minimizing;

$$f(\vec{z}) = 0.7854z_1z_2(3.3333z_3^2 + 14.9334z_3 - 43.0934) - 1.508z_1 (z_6^2 + z_7^2) + 7.4777(z_6^3 + z_7^3) + 0.7854(z_4z_6^2 + z_5z_7^2)$$

Subject to;

$$g_1(\vec{z}) = \frac{27}{z_1z_2^2z_3} - 1 \le 0 \tag{16.2a}$$

Table 9 Statistical analysis for Uni-modal benchmark functions

Functions	No. of trial run	Minimum fitness	Maximum fitness	Mean fitness	Median fitness	First quartile (25th Percentile)	Second quartile (50th Percentile)	Third quartile (75th Percentile)	Semi Interquartile Deviation	Number of outliers	Standard Deviation
F1	30	1.4734E-119	6.4895E-95	2.29407E-96	1.9775E-104	6.4671E-108	1.977E-104	3.0748E-100	1.5374E-100	7	1.18E-95
F2	30	2.12196E-60	3.16392E-47	1.08358E-48	4.42927E-54	1.70056E-56	4.4293E-54	2.60563E-51	1.30281E-51	4	5.77E-48
F3	30	7.32349E-98	4.2382E-68	1.5387E-69	1.55924E-84	7.03988E-89	1.5592E-84	1.22112E-81	6.10559E-82	7	7.74E-69
F4	30	2.94293E-55	3.48491E-47	1.39542E-48	2.68096E-52	1.30926E-53	2.681E-52	2.65665E-50	1.32767E-50	7	6.39E-48
F5	30	1.59415E-06	0.084309909	0.013086197	0.003289218	0.00154622	0.00328922	0.016354734	0.007404257	2	0.01999
F6	30	1.99142E-08	0.00111892	0.00016028	5.94121E-05	1.7972E-05	5.9412E-05	0.00012796	5.49941E-05	3	0.000299
F7	30	2.5434E-06	0.000689678	0.000149719	9.10616E-05	4.26378E-05	9.1062E-05	0.000181665	6.95136E-05	1	0.000159

$$g_2(\vec{z}) = \frac{397.5}{z_1 z_2^2 z_3^2} - 1 \le 0 \tag{16.2b}$$

$$g_3(\vec{z}) = \frac{1.93 z_4^3}{z_2 z_3 z_6^4} - 1 \le 0 \tag{16.2c}$$

$$g_4(\vec{z}) = \frac{1.93 z_5^3}{z_2 z_3 z_7^4} - 1 \le 0 \tag{16.2d}$$

$$g_5(\vec{z}) = \frac{1}{110 z_6^3}\sqrt{\left(\frac{745.0 z_4}{z_2 z_3}\right)^2 + 16.9 \times 10^6} - 1 \le 0 \tag{16.2e}$$

$$g_6(\vec{z}) = \frac{1}{85 z_7^3}\sqrt{\left(\frac{745.0 z_5}{z_2 z_3}\right)^2 + 157.5 \times 10^6} - 1 \le 0 \tag{16.2f}$$

$$g_7(\vec{z}) = \frac{z_2 z_3}{40} - 1 \le 0 \tag{16.2g}$$

$$g_8(\vec{z}) = \frac{5 z_2}{z_1} - 1 \le 0 \tag{16.2h}$$

$$g_9(\vec{z}) = \frac{z_1}{12 z_2} - 1 \le 0 \tag{16.2i}$$

$$g_{10}(\vec{z}) = \frac{1.5 z_6 + 1.9}{12 z_2} - 1 \le 0 \tag{16.2j}$$

$$g_{11}(\vec{z}) = \frac{1.1 z_7 + 1.9}{z_5} - 1 \le 0 \tag{16.2k}$$

where $2.6 \le z_1 \le 3.6, 0.7 \le z_2 \le 0.8, 17 \le z_3 \le 28, 7.3 \le z_4 \le 8.3, 7.8 \le z_5 \le 8.3, 2.9 \le z_6 \le 3.9 and 5 \le z_7 \le 5.5$.

6.3 EF3—Pressure Vessel Engineering Problem

The design specification for this type aims to minimize cost of cylindrical pressure vessel illustrated in Fig. 17 [116]. The chaotic HHO is applied to diminish the expense which includes the material cost and welding cost to form the vessel in cylindrical form. The design variables include the thickness of the shell (Ts), the inner radius (R), the thickness of the head (Th) and the length of the cylindrical section of the vessel (L). These four variables are modeled as S1 to S4. The numerical formulations of this kind of problem are shown through Eq. (16.3, 16.3a, 16.3b, 16.3c, 16.3d, 16.3e). Table 28 shows the result analysis of proposed CHHO method with HHO, GWO, GSA, PSO, GA, DE, ACO and

Table 10 Dunn's test for unimodal benchmark function

Function	*SS*	*df*	MS	Chi-sq	Prob > Chi-sq	Error			Total	
						SS	*df*	*MS*	*SS*	*df*
F1	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F2	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F3	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F4	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F5	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F6	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F7	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29

Table 11 Simulation time for unimodal test function using Chaotic HHO algorithm

Functions	Mean time	Best time	Worst time
F1	0.264583333	0.21875	0.71875
F2	0.277604167	0.234375	0.828125
F3	0.466145833	0.390625	1.125
F4	0.315104167	0.25	0.828125
F5	0.469791667	0.390625	1.15625
F6	0.358854167	0.3125	0.859375
F7	0.43125	0.375	1.0625

BB. From the summarized results in Table 28, it has been observed that CHHO gives marginal results and gives cost-effective solution for pressure vessel design problem.

Consider:

$$\vec{s} = [s_1 s_2 s_3 s_4] = [T_s T_h R L_h] \tag{16.3}$$

Minimize;

$$f(\vec{s}) = 0.6224 s_1 s_3 s_4 + 1.7781 s_2 s_3^2 + 3.1661 s_1^2 s_4 + 19.84 s_1^2 s_3 \tag{16.3a}$$

Subject to:

$$g_1(\vec{s}) = -s_1 + 0.0193 s_3 \leq 0 \tag{16.3b}$$

$$g_2(\vec{s}) = s_3 + 0.00954 s_3 \leq 0 \tag{16.3c}$$

$$g_3(\vec{s}) = -\pi s_3^2 s_4 - \frac{4}{3}\pi s_3^3 + 1296000 \leq 0 \tag{16.3d}$$

$$g_4(\vec{s}) = s_4 - 240 \leq 0 \tag{16.3e}$$

Table 12 Comparison of Uni-Modal test function

Algorithms	Parameters	UM test function						
		F1	*F2*	*F3*	*F4*	*F5*	*F6*	*F7*
GWO [1]	STD	6.3400E-07	0.02901	7.9.1495E+01	1.31508	69.9049	0.00012	0.10028
	Mean	6.590E-29	7.180E-18	3.20E-07	5.610E-08	26.8125	0.81657	0.00221
PSO [3]	STD	0.0002.0E-04	0.04542	2.1192E+01	3.1703E+01	6.01155E+01	8.28E-05	0.04495
	Mean	1.3E-04	0.04214	7.01256E+01	1.08648	96.7183	0.00010	0.12285
MFO [6]	STD	0.00015	0.00087	188.527	5.27505	120.2607	9.87E-05	0.04642
	Mean	0.00011	0.00063	696.730	70.6864	139.1487	0.000113	0.091155
SCA [19]	STD	0.000	0.0001	0.1372	0.5823	0.0017	0.0001	0.0014
	Mean	0.000	0.000	0.0371	0.0965	0.0005	0.0002	0.000
MVO [18]	STD	0.64865	44.7459	177.0973	1.58291	1479.47	0.63081	0.02961
	Mean	2.08583	15.9247	453.200	3.12301	1272.13	2.29495	0.05199
SSA [43]	STD	0.000	1.000	0.000	0.6556	0.000	0.000	0.007
	Mean	0.000	0.2272	0.000	0.000	0.000	0.000	0.0028
CSMA[106]	STD	0	1.7E-155	0	2.8E-133	9.27916	0.003059	0.00021
	Mean	1.2E-280	3.4E-156	0	5.1E-134	5.035453	0.004431	0.0003
TENT_CHHO	STD	**1.18E-95**	**5.77E-48**	**7.74E-69**	**6.39E-48**	**0.01999**	**0.000299**	**0.000159**
	Mean	**2.29E-96**	**1.08E-48**	**1.54E-69**	**1.4E-48**	**0.013086**	**0.00016**	**0.00015**

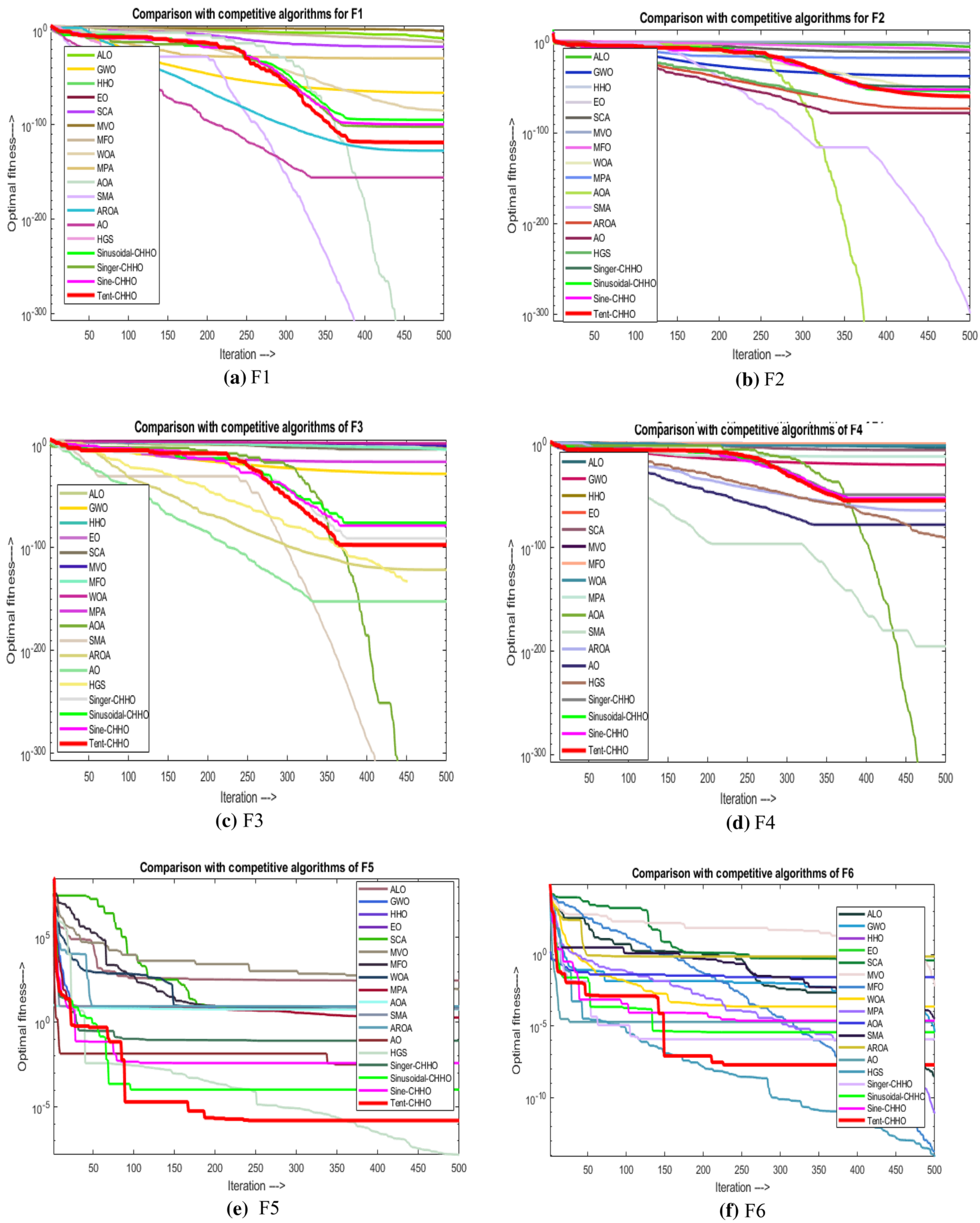
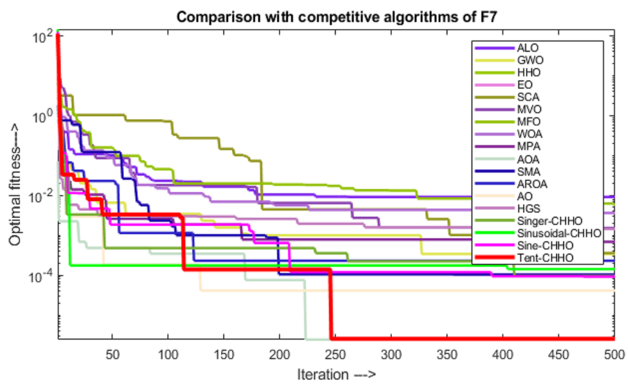


Fig. 7 Comparative convergence curve of F1 to F7 (from **a**–**g**)



(g) F7

Fig. 7 (continued)

Variable range $0 \leq s_1 \leq 99$, $0 \leq s_2 \leq 99$, $10 \leq s_3 \leq 200$, $10 \leq s_4 \leq 200$.

6.4 EF4-cantilever beam design

This is concrete engineering problem in which main focus is minimization of beam weight as shown in Fig. 18 [27]. In beam design, there are five elements l_1, l_2, l_3, l_4 and l_5. The main goal is minimization of the weight of the beam. The design problem is mathematically modeled through Eq. (16.4, 16.4a, 16.4b). Table 29 elucidates that proposed method efficiently reduces the weight of the beam compared to HHO, CS, ALO, SOS, MMA and GCA_1. The mathematical equations are as shown below:

$$\text{Consider, } \vec{l} = \left[l_1 l_2 l_3 l_4 l_5\right] \tag{16.4}$$

Minimize

$$f(\vec{l}) = 0.6224(l_1 + l_2 + l_3 + l_4 + l_5), \tag{16.4a}$$

Subject to

$$g(\vec{l}) = \frac{61}{l} + \frac{37}{l_2^3} + \frac{19}{l_3^3} + \frac{7}{l_4^3} + \frac{1}{l_5^3} \leq 1 \tag{16.4b}$$

6.5 EF5—compression spring design

Figure 19 illustrates spring design problem concerned with mechanical engineering [37]. Weight minimization is the main consideration to tackle this type of problem. There are three types of design variables: (i) wire diameter (dr) (ii) active coils (Nc) and (iii) coil diameter (Dm). The design problem is mathematically formulated through Eq. (16.5, 16.5a, 16.5b, 16.5c, 16.5d, 16.5e, 16.5f). The proposed method is applied to solve compression design problem and results are illustrated in Table 30. It is clearly seen from the analysis that CHHO method is efficient for reducing spring weight marginally.

$$\text{Consider } \vec{y} = \left[y_1 y_2 y_3\right] = [drDmNc], \tag{16.5}$$

Minimize

$$f(\vec{y}) = \left(y_3 + 2\right) y_2 y_1^2, \tag{16.5a}$$

Subject to:

$$g_1\left(\vec{y}\right) = 1 - \frac{y_2^3 y_3}{71785 y_1^4} \leq 0, \tag{16.5b}$$

$$g_2\left(\vec{y}\right) = \frac{4y_2^2 - y_1 y_2}{12566\left(y_2 y_1^3 - y_1^4\right)} + \frac{1}{5108 y_1^2} \leq 0, \tag{16.5c}$$

$$g_2\left(\vec{y}\right) = \frac{4y_2^2 - y_1 y_2}{12566\left(y_2 y_1^3 - y_1^4\right)} + \frac{1}{5108 y_1^2} \leq 0, \tag{16.5d}$$

$$g_3\left(\vec{y}\right) = 1 - \frac{140.45 y_1}{y_2^2 y_3} \leq 0, \tag{16.5e}$$

$$g_4\left(\vec{y}\right) = \frac{y_1 + y_2}{1.5} - 1 \leq 0, \tag{16.5f}$$

Variable range $0.005 \leq y_1 \leq 2.00$, $0.25 \leq y_2 \leq 1.30$, $2.00 \leq y_3 \leq 15.0$.

6.6 EF6—rolling element bearing design

The major aspect of this kind of design is to improve the dynamic load carrying capacity of rolling bearing element as illustrated in Fig. 20 [116]. There are ten parameters which decide the optimum design of bearing for improving the load-bearing power. Out of these ten variables, only five variables are of much consideration. These major variables are (i) diameter of the ball (DIMB), (ii) diameter pitch (DIMP), (iii) ball numbers (Nb), (iv) outer curvature coefficient, and (v) inner curvature coefficient. Rest of five variables only affect indirectly to the internal portion of the geometry. The design problem is mathematically formulated through Eq. (16.6, 16.6a, 16.6b, 16.6c, 16.6d, 16.6e, 16.6f, 16.6g, 16.6h, 16.6i, 16.6j, 16.6k, 16.6l). From the comparative results shown in Table 31, it can be seen that the proposed method gives superior results compared to HHO and other methods.

Maximizing;

$$C_D = f_c N^{2/3} \mathrm{DIM}_B^{1.8} \tag{16.6}$$

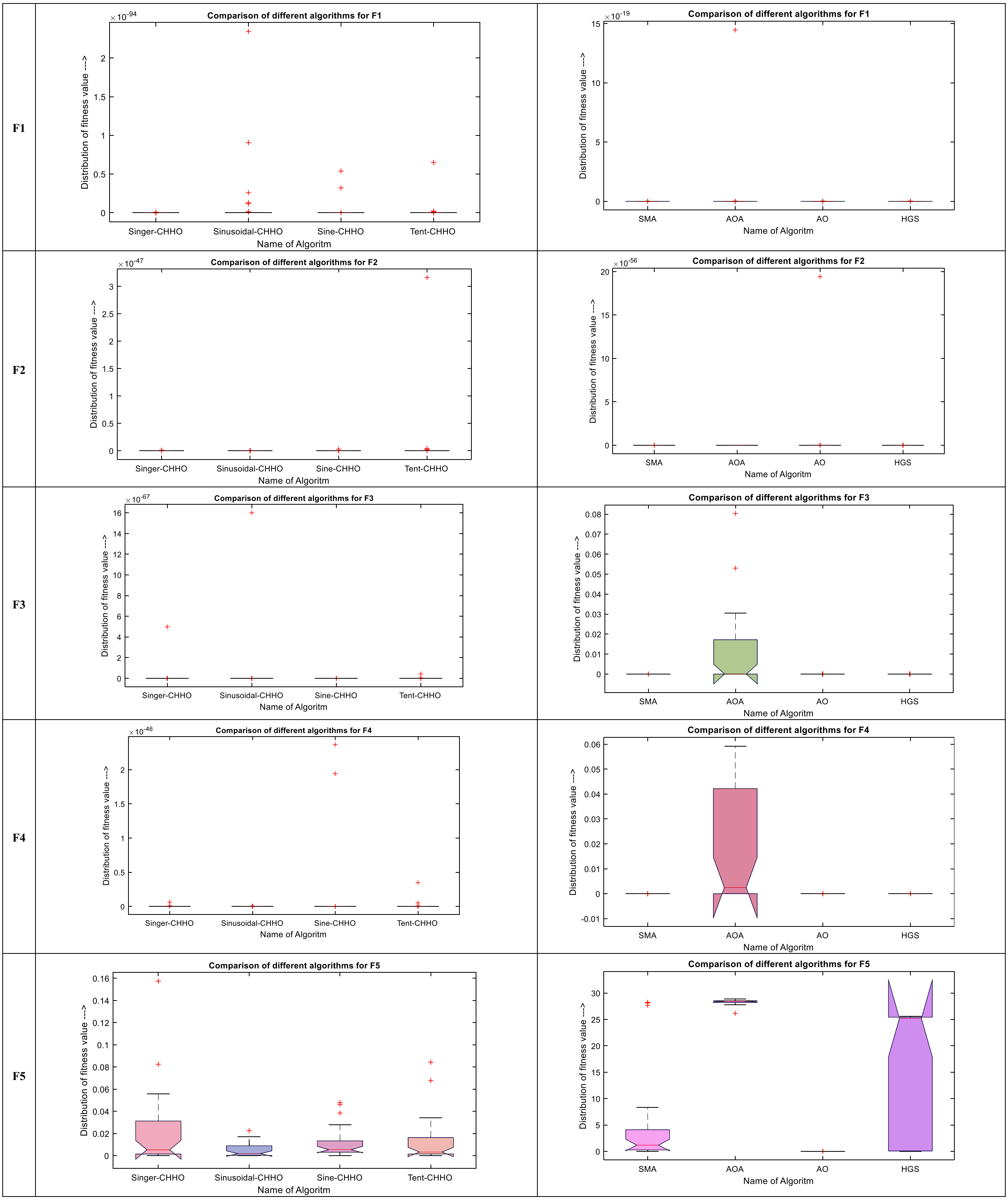


Fig. 8 Box plot of trial runs of U-modal benchmark function compared with other competitive algorithms

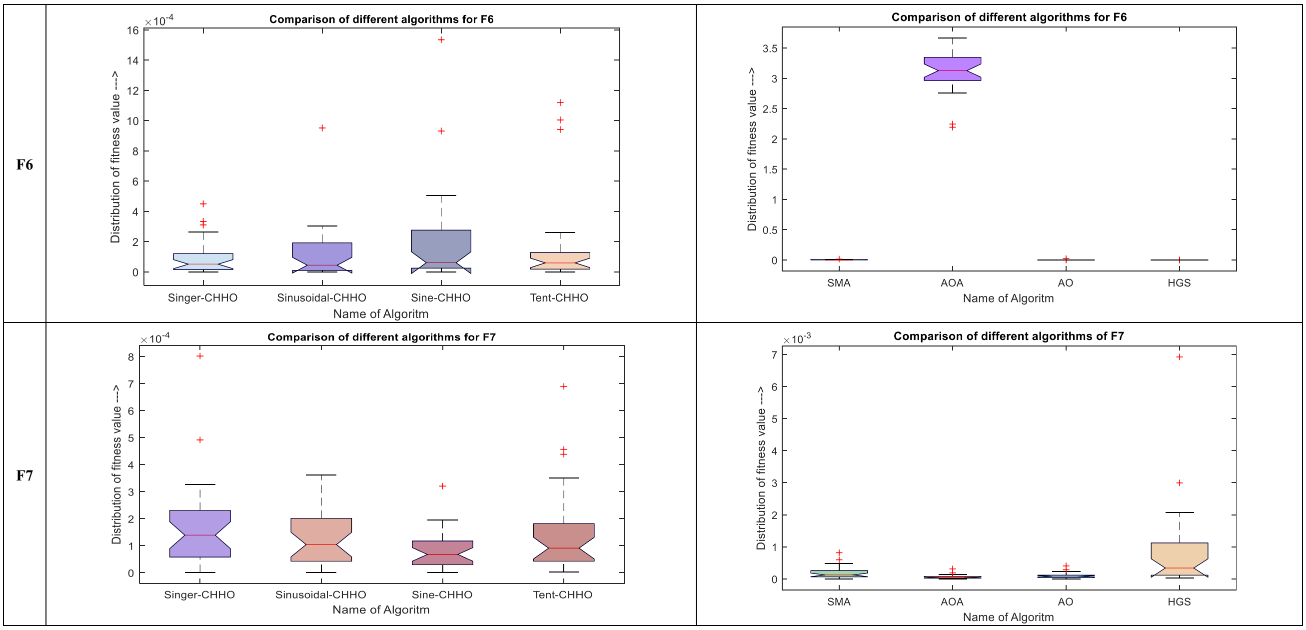


Fig. 8 (continued)

$$\text{if } \mathrm{DIM} \le 25.4\,\mathrm{mm} \quad C_D = 3.647 f_C N^{2/3} \mathrm{DIM}_B^{1.4} \tag{16.6a}$$

$$\text{if } \mathrm{DIM} \ge 25.4\,\mathrm{mm} \tag{16.6b}$$

Subjected to;

$$r_1(x) = \frac{\theta_0}{2\sin^{-1}\left(\frac{\mathrm{DIM}_B}{\mathrm{DIM}_{MAX}}\right)} - N + 1 \ge 0 \tag{16.6c}$$

$$r_2(x) = 2\mathrm{DIM}_B - K_{\mathrm{DIM}_{\mathrm{MIN}}}(\mathrm{DIM} - \mathrm{dim}) \ge 0 \tag{16.6d}$$

$$\theta_0 = 2\pi - 2\cos^{-1}\left(\frac{\left[\{(\mathrm{DIM}-\mathrm{dim})/2 - 3(t/4)\}^2 + \left(\mathrm{DIM}/2 - t/4 - DIM_B\right)^2 - \{\mathrm{dim}/2 + t/4\}^2\right]}{2\{(\mathrm{DIM}-\mathrm{dim})/2 - 3(t/4)\}\left\{D/2 - t/4 - \mathrm{DIM}_B\right\}}\right)$$

$$r_3(x) = K_{\mathrm{DIM}_{\mathrm{MAX}}}(\mathrm{DIM} - \mathrm{dim}) \ge 0 \tag{16.6e}$$

$$r_4(x) = \beta B_W - \mathrm{DIM}_B \le 0 \tag{16.6f}$$

$$r_4(x) = \mathrm{DIM}_{\mathrm{MAX}} - 0.5(DIM + \mathrm{dim}) \ge 0 \tag{16.6g}$$

$$r_5(x) = \mathrm{DIM}_{\mathrm{MAX}} - 0.5(\mathrm{DIM} + \mathrm{dim}) \ge 0 \tag{16.6h}$$

$$r_6(x) = (0.5 + re)(\mathrm{DIM} + \mathrm{dim}) \ge 0 \tag{16.6i}$$

$$r_7(x) = 0.5(\mathrm{DIM} - \mathrm{DIM}_{\mathrm{MAX}} - \mathrm{DIM}_B) - \alpha\mathrm{DIM}_B \ge 0 \tag{16.6j}$$

$$r_8(x) = f_I \ge 0.515 \tag{16.6k}$$

$$r_9(x) = f_0 \ge 0.515 \tag{16.6l}$$

$$\text{where } f_c = 37.91\left[1 + \left\{1.04\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{1.72}\left(\frac{f_I(2f_0-1)}{f_0(2f_I-1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3} \times \left[\frac{\varepsilon^{0.3}(1-\varepsilon)^{1.39}}{(1+\varepsilon)^{1/3}}\right]\left[\frac{2f_I}{2f_I-1}\right]^{0.41}$$

$$\varepsilon = \frac{\mathrm{DIM}_B}{\mathrm{DIM}_{\mathrm{MAX}}}, f_I = \frac{R_I}{\mathrm{DIM}_B}, f_0 = \frac{R_0}{\mathrm{DIM}_B}, t = \mathrm{DIM} - \mathrm{dim} - 2\mathrm{DIM}_B$$

$$\mathrm{DIM} = 160, \mathrm{dim} = 90, B_W = 30, R_I = R_0 = 11.033$$

$$0.5(DIM + \mathrm{dim}) \le \mathrm{DIM}_{\mathrm{MAX}} \le 0.6(\mathrm{DIM} + \mathrm{dim}), 0.15(\mathrm{DIM} - \mathrm{dim}) \le \mathrm{DIM}_B \le 0.45(\mathrm{DIM} - \mathrm{dim}), 4 \le N \le 50$$

$$0.515 \le f_I \text{ And } f_0 \le 0.6$$

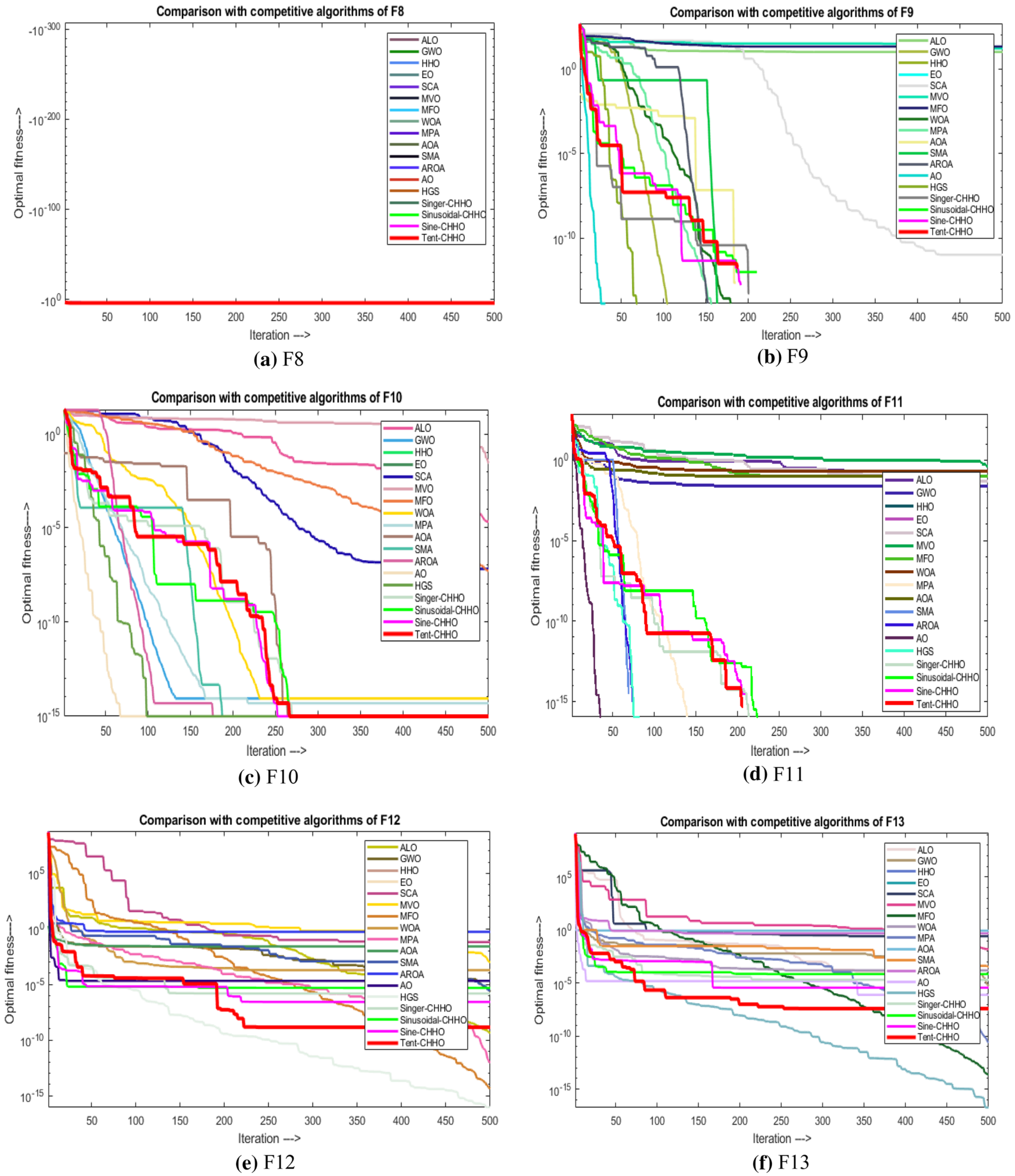


Fig. 9 Convergence curve for multi-modal test function showing comparison of CHHO with other algorithms

Table 13 Testing of multi-modal test function using CHHO

Function	Objective function fitness					Wilcoxon rank sum test	*T* test	
	Mean	STD	Best	Worst	Median	*p* value	*t* value	*h* value
F8	− 12,569.1	0.661731	− 12,569.5	− 12,566.1	− 12,569.3	1.7344E-06	2.05E-28	1
F9	0	0	0	0	0	1	0.0	1
F10	8.88E-16	0	8.88E-16	8.88E-16	8.88E-16	4.320460E-09	0.0	1
F11	0	0	0	0	0	1	0.0	1
F12	5.78E-06	5.92E-06	1.41E-09	1.86E-05	3.38E-06	1.7344E-06	0.00057	1
F13	8.42E-05	9.09E-05	3.99E-08	0.000374	5.94E-05	1.7344E-06	0.00062	1

Table 14 Statistical analysis for multi-modal benchmark functions

Functions	No. of trial run	Minimum fitness	Maximum fitness	Mean fitness	Median fitness	First quartile (25th Percentile)	Second quartile (50th Percentile)	Third quartile (75th Percentile)	Semi Interquartile Deviation	Number of outliers	Standard Deviation
F8	30	− 12,569.48662	− 12,566.13995	− 12,569.08411	− 12,569.2847	− 12,569.44548	− 12,569.2847	− 12,569.04039	0.202544201	2	0.661731
F9	30	0	0	0	0	–	0	–	–	0	0
F10	30	8.88178E-16	8.88178E-16	8.88178E-16	8.88178E-16	–	8.8818E-16	–	–	0	0
F11	30	0	0	0	0	–	0	–	–	0	0
F12	30	1.40679E-09	1.8613E-05	5.77656E-06	3.38446E-06	6.73282E-07	3.3845E-06	9.24973E-06	4.28822E-06	0	5.92E-06
F13	30	3.98977E-08	0.000374371	8.42071E-05	5.94155E-05	1.51592E-05	5.9416E-05	0.000113677	4.9259E-05	1	9.09E-05

Table 15 Dunn's test for Multi-modal benchmark function

Function	*SS*	*df*	MS	Chi-sq	Prob > Chi-sq	Error			Total	
						SS	*df*	*MS*	*SS*	*df*
F8	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F9	0	0	NaN	0	NaN	0	29	0	0	29
F10	0	0	NaN	0	NaN	0	29	0	0	29
F11	0	0	NaN	0	NaN	0	29	0	0	29
F12	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F13	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29

Table 16 Simulation time for multi-modal using **CHHO**

Functions	Mean time	Best time	Worst time
F8	0.457	0.375	1.140
F9	0.395	0.343	1.031
F10	0.409	0.359	1.015
F11	0.515	0.453	1.234
F12	0.968	0.890	1.687
F13	0.959	0.890	1.703

$$0.4 \leq K_{\mathrm{DIM}_{\mathrm{MIN}}} \leq 0.5, 0.6 \leq K_{\mathrm{DIM}_{\mathrm{MAX}}} \leq 0.7, 0.3 \leq re \leq 0.1, 0.02 \leq re \leq 0.1, 0.6 \leq \beta \leq 0.85$$

6.7 EF7—welded beam design

In welded beam design, welding is carried out by fusing different sections by molten metal as presented in Fig. 21 [37]. The major feature of this kind of design is to minimize

Table 17 Comparison of Multi-modal test function

Algorithms	Parameters	Multi-modal test function					
		F8	F9	F10	F11	F12	F13
GWO [1]	STD	− 4.0900E+02	4.740E+01	7.7800E-03	6.6600E-04	2.0700E-03	4.470E-03
	Mean	− 6.1200E+02	3.1100E-02	1.0600E-14	4.4900E-04	5.3400E-03	6.5400E-02
PSO [3]	STD	1.1500E+04	1.160E+01	5.090E-01	7.7200E-04	2.6300E-03	8.9100E-04
	Mean	− 4.8400E+04	4.670E+01	2.760E-01	9.2200E-04	6.9200E-04	6.6800E-04
GSA [109]	STD	4.930E+02	7.470E+00	2.360E-01	5.040E+00	9.510E-01	7.130E+00
	Mean	− 2.820E+03	2.600E+01	6.210E-02	2.770E+01	1.800E+00	8.900E+00
MFO [6]	STD	7.260E+02	1.620E+01	7.300E-01	2.170E-02	8.810E-01	1.930E-01
	Mean	− 8.500E+03	8.460E+01	1.260E+00	1.910E-02	8.940E-01	1.160E-01
ALO [105]	STD	3.14E+02	8.45E-06	1.50E-15	9.55E-03	9.33E-12	1.13E-11
	Mean	− 1.61E+03	7.71E-06	3.73E-15	1.86E-02	9.75E-12	2.00E-11
GA [110]	STD	2.470E+00	8.160E-01	8.080E-01	2.180E-01	2.150E-03	6.890E-02
	Mean	− 2.090E+03	6.590E-01	9.560E-01	4.880E-01	1.110E-01	1.290E-01
MVO [18]	STD	9.370E+02	3.930E+01	5.500E+00	6.000E-02	7.900E-01	9.000E-02
	Mean	− 1.170E+04	1.180E+02	4.070E+00	9.400E-01	2.460E+00	2.200E-01
SCA [19]	STD	3.600E-03	7.300E-01	1.000E+00	5.100E-03	0.000E+00	0.000E+00
	Mean	1.000E+00	0.000E+00	3.800E-01	0.000E+00	0.000E+00	0.000E+00
DA [107]	STD	3.840E+02	9.480E+00	4.870E-01	7.350E-02	9.830E-02	4.630E-03
	Mean	− 2.860E+03	1.600E+01	2.310E-01	1.930E-01	3.110E-02	2.200E-03
SSA [43]	STD	8.090E-01	0.000E+00	1.530E-01	6.510E-02	5.570E-01	7.060E-01
	Mean	5.570E-02	0.000E+00	1.950E-01	0.000E+00	1.420E-01	8.320E-02
CSMA[106]	STD	0.319584	0	0	0	0.006237	0.00989
	Mean	− 12,569.1	0	8.88E-16	0	0.003937	0.00664
TENT_CHHO	STD	**0.661731**	**0**	**0**	**0**	**5.92E-06**	**9.09E-05**
	Mean	**− 12,569.1**	**0**	**8.88E-16**	**0**	**5.78E-06**	**8.42E-05**

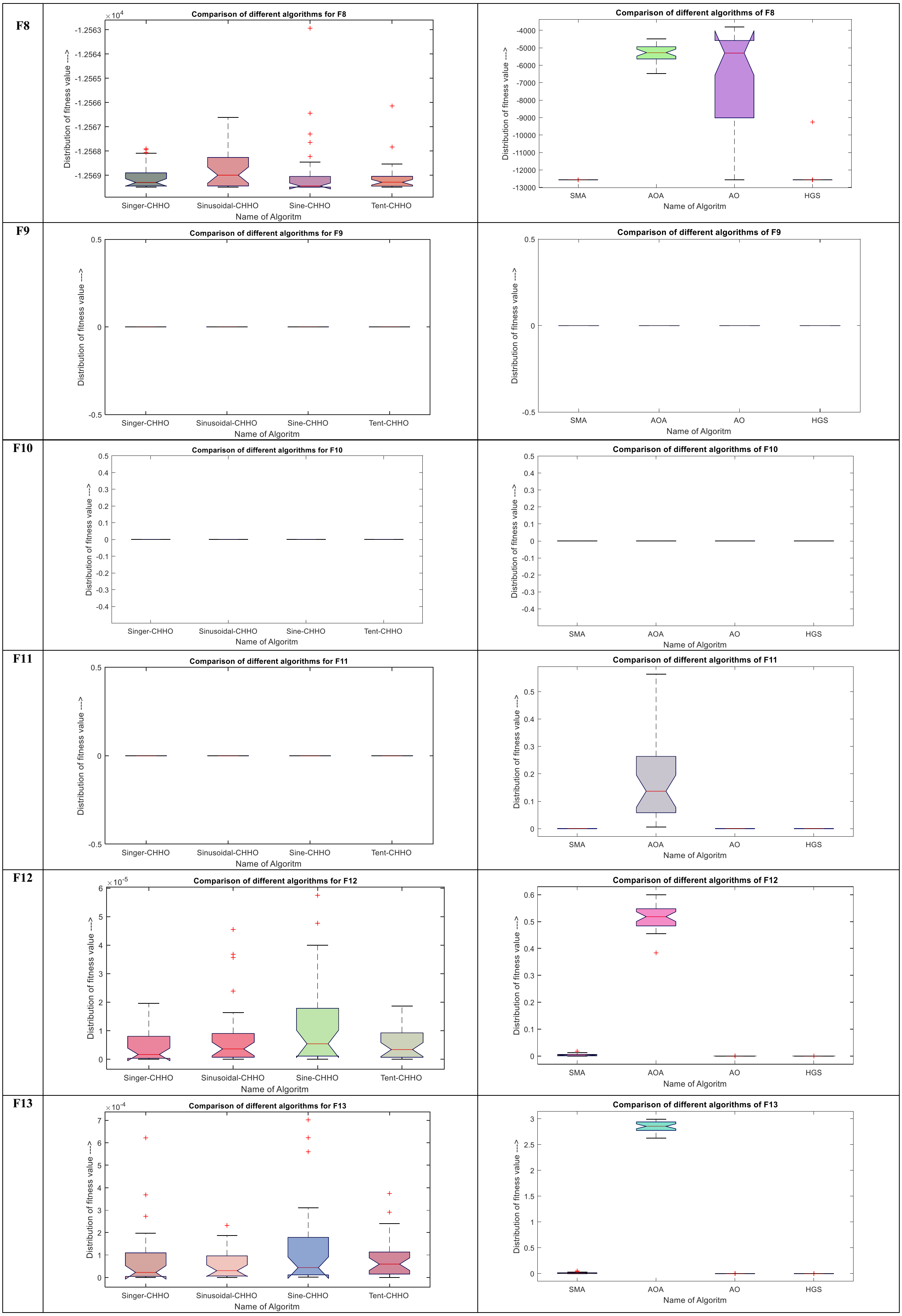


Fig. 10 Box Plot for Trial runs of F8 to F13 functions compared with other competitive algorithms

Table 18 Simulation results for fixed dimension using **CHHO**

Function	Objective function fitness					Wilcoxon rank Sum Test	*T* test	
	Mean	STD	Best	Worst	Median	*p* value	*t* value	*h* value
F14	1.590491	1.501409	0.998004	5.928845	0.998004	1.7344E-06	6.42E-07	1
F15	0.000368	0.00019	0.000308	0.001364	0.000327	1.7344E-06	2E-10	1
F16	− 1.03163	1.38E-09	− 1.03163	− 1.03163	− 1.03163	1.7344E-06	2.1E-260	1
F17	0.397892	1E-05	0.397887	0.397929	0.397888	0	0	0
F18	3	7.27E-08	3	3	3	1.7344E-06	1.2E-191	1
F19	− 3.85926	0.006377	− 3.86278	− 3.83785	− 3.86186	1.7344E-06	4.68E-89	1
F20	− 3.11029	0.126286	− 3.28658	− 2.7957	− 3.13992	1.7344E-06	8.66E-45	1
F21	− 5.21858	0.912798	− 10.0515	− 5.04081	− 5.05327	1.7344E-06	2.49E-21	1
F22	− 4.97575	0.593354	− 5.08766	− 1.83423	− 5.08605	1.7344E-06	3.45E-18	1
F23	− 5.1241	0.00409	− 5.12839	− 5.11147	− 5.12541	1.7344E-06	3.84E-24	1

Table 19 Statistical analysis for fixed-modal benchmark functions

Functions	No. of trial run	Minimum fitness	Maximum fitness	Mean fitness	Median fitness	First quartile (25th Percentile)	Second quartile (50th Percentile)	Third quartile (75th Percentile)	Semi Interquartile Deviation	Number of outliers	Standard Deviation
F14	30	0.998003838	5.928845125	1.590490673	0.998003838	0.998003838	0.99800384	0.998003839	4.83693E-10	6	1.501409
F15	30	0.000307517	0.001364383	0.000367799	0.000327107	0.000315653	0.00032711	0.000344676	1.45112E-05	1	0.00019
F16	30	− 1.031628453	-1.031628448	− 1.031628453	− 1.031628453	− 1.031628453	− 1.03162845	− 1.031628453	3.3138E-11	7	1.38E-09
F17	30	0.397887358	0.397929005	0.397892479	0.397888088	0.3978874	0.39788809	0.397892523	2.56121E-06	4	1E-05
F18	30	3	3.000000251	3.000000045	3.000000002	3	3	3.00000007	3.5124E-08	2	7.27E-08
F19	30	− 3.862777487	− 3.837853992	− 3.859258237	− 3.861858627	− 3.862545259	− 3.86185863	-3.859259843	0.001642708	3	0.006377
F20	30	− 3.286582121	− 2.795698313	− 3.110285607	− 3.139915329	− 3.194319753	− 3.13991533	− 3.048184656	0.073067549	0	0.126286
F21	30	− 10.05149728	− 5.040814398	− 5.218584119	− 5.053274403	− 5.05454562	− 5.0532744	− 5.050711933	0.001916844	2	0.912798
F22	30	− 5.087661207	− 1.834229065	− 4.97574976	− 5.086047761	− 5.087032773	− 5.08604776	− 5.081765327	0.002633723	1	0.593354
F23	30	− 5.128391462	− 5.111472638	− 5.124104306	− 5.125409432	− 5.127074191	− 5.12540943	− 5.122258727	0.002407732	1	0.00409

Table 21 Simulation time for fixed dimension using Chaotic HHO

Functions	Mean time	Best time	Worst time
F14	2.5083333	2.39062	3.2343
F15	0.3911458	0.32812	1
F16	0.3744791	0.32812	0.96875
F17	0.00	0.00	0.00
F18	0.3322916	0.28125	0.875
F19	0.4453125	0.375	1.10937
F20	0.4442708	0.39062	1.0625
F21	1.0484375	0.95312	1.79687
F22	1.2630208	1.1875	2
F23	1.6984375	1.48437	2.59375

overall cost of beam by optimizing four design variables subjected to seven constraints. The four variables are (*i*) bar thickness (*b*), (ii)bar length (*l*), (iii) weld thickness (*h*) and (iv) the bar height (*h*). The mathematical equations are formulated depending on prerequisites of variables and constraints illustrated through Eq. (16.7, 16.7a, 16.7b, 16.7c, 16.7d, 16.7e, 16.7f, 16.7g, 16.7h, 16.7i, 16.7j, 16.7k, 16.7l, 16.7m, 16.7n). The CHHO results are compared with HHO, GSA, HS, GA random, simplex and approximate methods and are listed in Table 32. The comparative analysis reveals that proposed method is competent for handling beam design problem more precisely.

$$\text{Consider, } \vec{y} = \left[y_1 y_2 y_3 y_4\right] = [hltb] \tag{16.7}$$

Minimize,

$$f(\vec{y}) = 1.10471 y_1^2 y_2 + 0.04811 y_3 y_4 \left(14.0 + y_2\right) \tag{16.7a}$$

Subject to

$$g_1(\vec{y}) = \tau(\vec{y}) - \tau_{\text{maxi}} \le 0, \tag{16.7b}$$

$$g_2(\vec{y}) = \sigma(\vec{y}) - \sigma_{\text{maxi}} \le 0, \tag{16.7c}$$

$$g_3(\vec{y}) = \delta(\vec{y}) - \delta_{\text{maxi}} \le 0, \tag{16.7d}$$

$$g_4(\vec{y}) = y_1 - y_4 \le 0, \tag{16.7e}$$

$$g_5(\vec{y}) = P_i - P_c(\vec{y}) \le 0, \tag{16.7f}$$

$$g_6(\vec{y}) = 0.125 - y_1 \le 0, \tag{16.7g}$$

$$g_7(\vec{y}) = 1.10471 y_1^2 + 0.04811 y_3 y_4 (14.0 + y_2) - 5.0 \le 0 \tag{16.7h}$$

Variable range $0.1 \le y_1 \le 2$, $0.1 \le y_2 \le 10$, $0.3 \le y_3 \le 10$, $0.1 \le y_4 \le 2$.where

$$\tau(\vec{y}) = \sqrt{(\tau^{/})^2 + 2\tau^{/}\tau^{//}\frac{y_2}{2R} + (\tau^{//})^2}, \tag{16.7i}$$

$$\tau^{/} = \frac{P_i}{\sqrt{2} y_1 y_2}, \tau^{//} = \frac{MR}{J}, M = P_i\left(L + \frac{y_2}{2}\right), \tag{16.7j}$$

$$R = \sqrt{\frac{y_2^2}{4} + \left(\frac{y_1 + y_3}{2}\right)^2}, \tag{16.7k}$$

$$J = 2\left\{\sqrt{2} y_1 y_2 \left[\frac{y_2^2}{4} + \left(\frac{y_1 + y_3}{2}\right)^2\right]\right\}, \tag{16.7l}$$

$$\sigma(\vec{y}) = \frac{6P_i L}{y_4 y_3^2}, \delta(\vec{y}) = \frac{6P_i L^3}{E y_2^2 y_4}, \tag{16.7m}$$

Table 20 Dunn's test for fixed-modal benchmark function

Function	*SS*	*df*	MS	Chi-sq	Prob > Chi-sq	error			Total	
						SS	*df*	*MS*	*SS*	*df*
F14	0	0	NaN	0	NaN	1663.5	29	57.3621	1663.5	29
F15	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F16	0	0	NaN	0	NaN	785	29	27.069	785	29
F17	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F18	0	0	NaN	0	NaN	2064.5	29	71.1897	2064.5	29
F19	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F20	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F21	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F22	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29
F23	0	0	NaN	0	NaN	2247.5	29	77.5	2247.5	29

Table 22 Comparative test results of fixed dimension

Algorithms	Parameters	Fixed dimension									
		F14	F15	F16	F17	F18	F19	F20	F21	F22	F23
GSA [109]	STD	3.83	0.00	0.00	0.00	0.00	0.00	0.02	3.74	2.01	0.00
	Mean	5.86	0.00	− 1.03	0.40	3.00	− 3.86	− 3.32	− 5.96	− 9.68	− 10.54
GWO [1]	STD	4.25	0.00	− 1.03	0.40	3.00	− 3.86	− 3.25	− 9.14	− 8.58	− 8.56
	Mean	4.04	0.00	− 1.03	0.40	3.00	− 3.86	− 3.29	− 10.15	− 10.40	− 10.53
PSO	STD	2.56	0.00	0.00	0.00	0.00	0.00	0.06	3.02	3.09	1.78
	Mean	3.63	0.00	− 1.03	0.40	3.00	− 3.86	− 3.27	− 6.87	− 8.46	− 9.95
DE [14]	STD	0.00	0.00	0.00	0.00	0.00	N/A	N/A	0.00	0.00	0.00
	Mean	1.00	0.00	− 1.03	0.40	3.00	N/A	N/A	− 10.15	− 10.40	− 10.54
CSMA [106]	STD	9.26E-13	0.000244	1.51E-09	6.82E-08	8.43E-12	4.21E-07	0.060654	0.000274	0.000208	0.000299
	Mean	0.998004	0.00055	− 1.03163	0.397887	3	− 3.86278	− 3.25824	− 10.1528	− 10.4026	− 10.536
TENT_CHHO	STD	**1.501409**	**0.00019**	**1.38E-09**	**1.00E-05**	**7.27E-08**	**0.006377**	**0.126286**	**0.912798**	**0.593354**	**0.00409**
	Mean	**1.590491**	**0.000368**	**− 1.03163**	**0.397892**	**3**	**− 3.85926**	**− 3.11029**	**− 5.21858**	**− 4.97575**	**− 5.1241**

$$P_c(\vec{y}) = \frac{4.013E\frac{\sqrt{y_3^2 y_4^6}}{36}}{L^2}\left(1 - \frac{y_3}{2L}\sqrt{\frac{E}{4G}}\right), \tag{16.7n}$$

$P_i = 6000lb, L = 14in, \delta_{\max i} = 0.25in,$
$E = 30 \times 1^6 psi, G = 12 \times 10^6 psi,$
$\tau_{\text{maxi}} = 13600psi, \sigma_{\text{maxi}} = 3000psi$

6.8 EF8—Belleville spring design

This type of design problem is shown in Fig. 22 [116]. The major consideration of this method is to minimize overall weight while satisfying various constraints. In this method, four types of designed variables, such as, internal spring diameter (DIMI), outer spring diameter (DIME), spring height (SH) and spring width (ST), are required to be optimized. The formulations of equations for spring design are described through Eq. (16.8, 16.8a, 16.8b, 16.8c, 16.8d, 16.8e, 16.8f, 16.8g). To check the validity of proposed method, a comparative result analysis is shown in Table 33. It is seen from the comparison results that proposed method gives more precise results compared to other methods.

Minimizing;

$$f(x) = 0.07075\pi(\text{DIM}_E^2 - \text{DIM}_I^2)t \tag{16.8}$$

Subject to:

$$b_1(x) = G - \frac{4P\lambda_{\max}}{(1-\delta^2)\alpha\text{DIM}_E}\left[\delta\left(S_H - \frac{\lambda \max}{2}\right) + \mu t\right] \geq 0 \tag{16.8a}$$

$$b_2(x) = \left(\frac{4P\lambda_{\max}}{(1-\delta^2)\alpha\text{DIM}_E}\left[\left(S_H - \frac{\lambda}{2}\right)(S_H - \lambda)t + t^3\right]\right)_{\lambda_{\max}} - P_{\text{MAX}} \geq 0 \tag{16.8b}$$

$$b_3(x) = \lambda_1 - \lambda_{\max} \geq 0 \tag{16.8c}$$

$$b_4(x) = H - S_H - t \geq 0 \tag{16.8d}$$

$$b_5(x) = \text{DIM}_{\text{MAX}} - \text{DIM}_E \geq 0 \tag{16.8e}$$

$$b_6(x) = \text{DIM}_E - \text{DIM}_I \geq 0 \tag{16.8f}$$

$$b_7(x) = 0.3 - \frac{S_H}{\text{DIM}_E - \text{DIM}_I} \geq 0 \tag{16.8g}$$

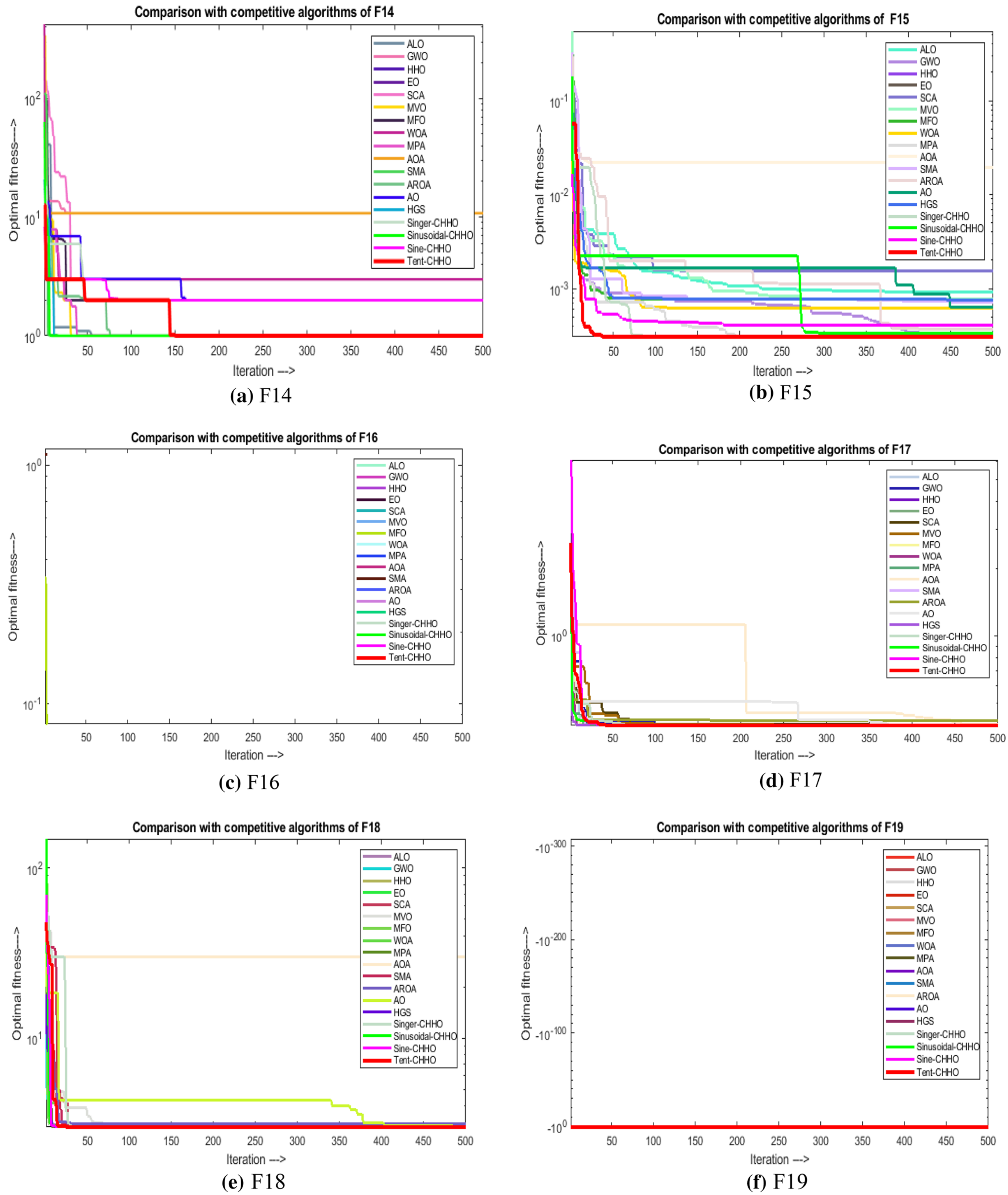
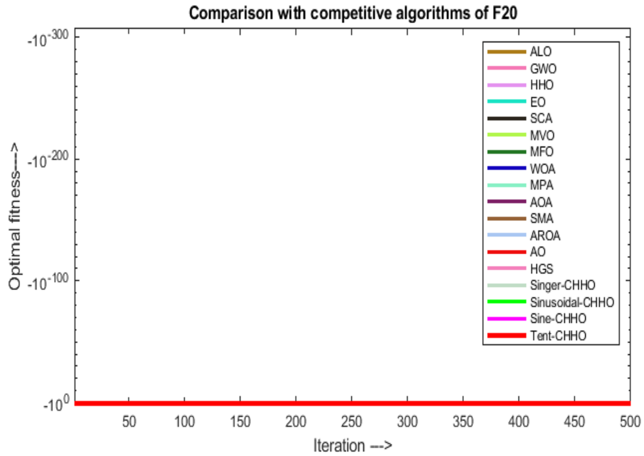
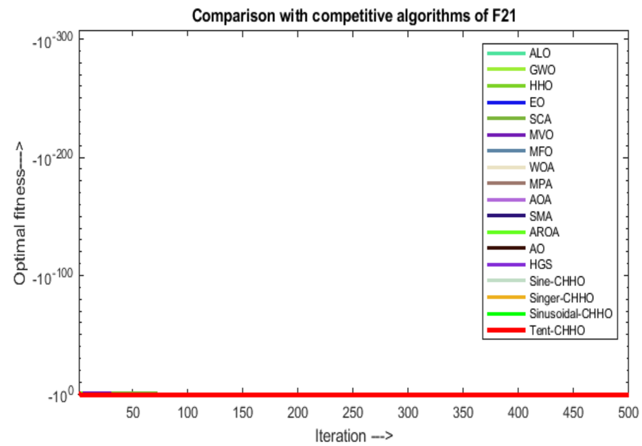


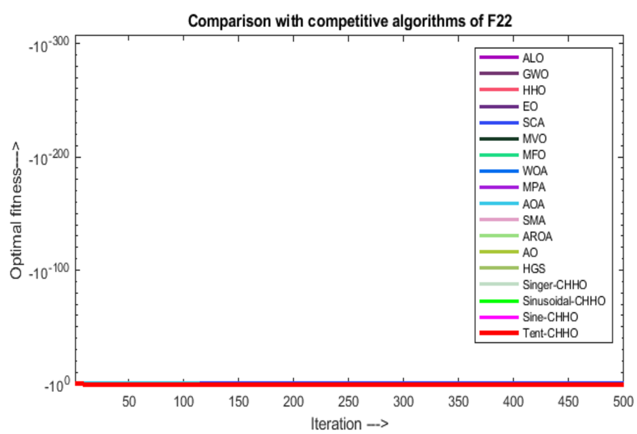
Fig. 11 Convergence curve for fixed-modal test function showing comparison of CHHO with other algorithms



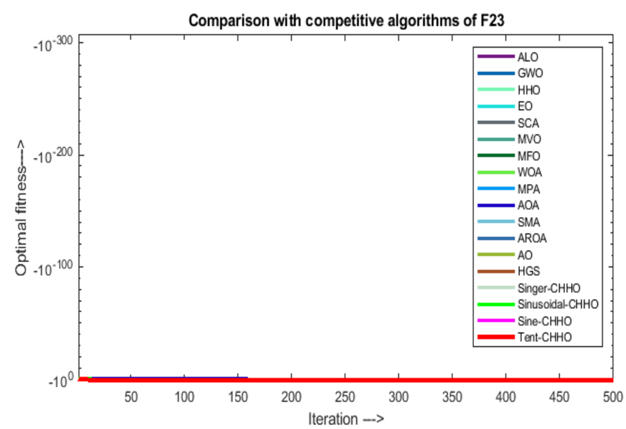
(g) F20



(h) F21



(i) F22



(j) F23

Fig. 11 (continued)

$$\text{where, } \begin{aligned} \alpha &= \frac{6}{\pi \ln J}\left(\frac{J-1}{J}\right)^2 \\ \delta &= \frac{6}{\pi \ln J}\left(\frac{J-1}{\ln J} - 1\right) \\ \mu &= \frac{6}{\pi \ln J}\left(\frac{J-1}{2}\right) \\ P_{\mathrm{MAX}} &= 5400lb \end{aligned}$$

$$P = 30e6\,\text{psi},\ \lambda_{\max} = 0.2\,\text{in},\ \delta = 0.3,\ G = 200\,\text{Kpsi},$$

$$H = 2\,\text{in},\ \mathrm{DIM}_{\mathrm{MAX}} = 12.01\,\text{in},$$

$$J = \frac{\mathrm{DIM}_E}{\mathrm{DIM}_I},\ \lambda_1 = f(a)a,\ a = \frac{S_H}{t}$$

6.9 EF9–gear train design

In this method, the four variables g_1, g_2, g_3, and g_4 are reformed to diminish the scalar value and teeth ratio as shown in Fig. 23 [116]. Teeth on each gear are the decision variables in designing process. The design aspects are expressed through Eq. (16.9a) to Eq. (16.9b). The simulation results shown in Table 34 reveal that optimum fitness has improved to greater extend compared with HHO and other methods.

Considering;

$$\vec{g} = \left[g_1 g_2 g_3 g_4\right] = \left[M_A M_B M_C M_D\right] \tag{16.9a}$$

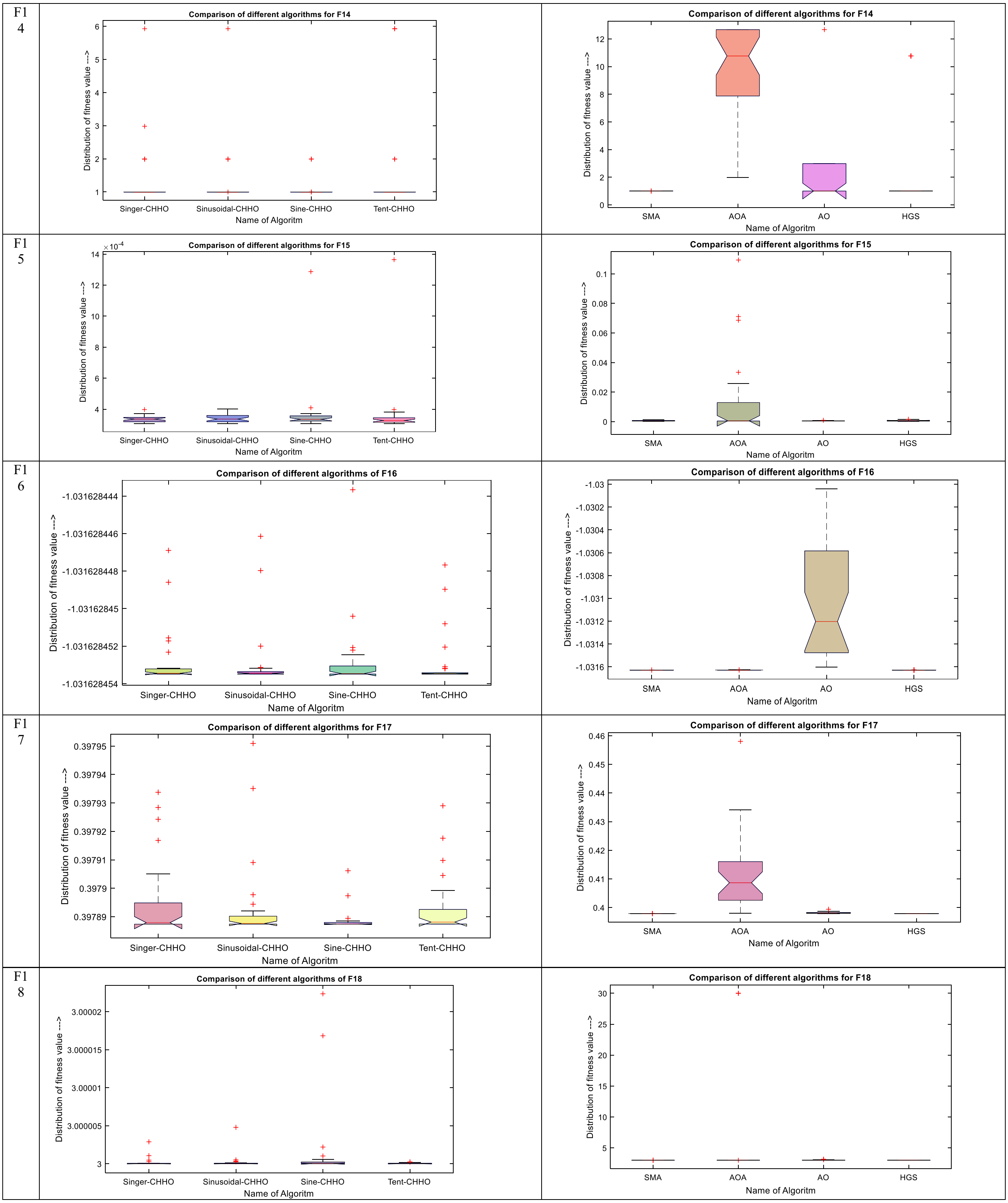


Fig. 12 Box plot of trial runs of fixed dimensions benchmark function compared with other competitive algorithms

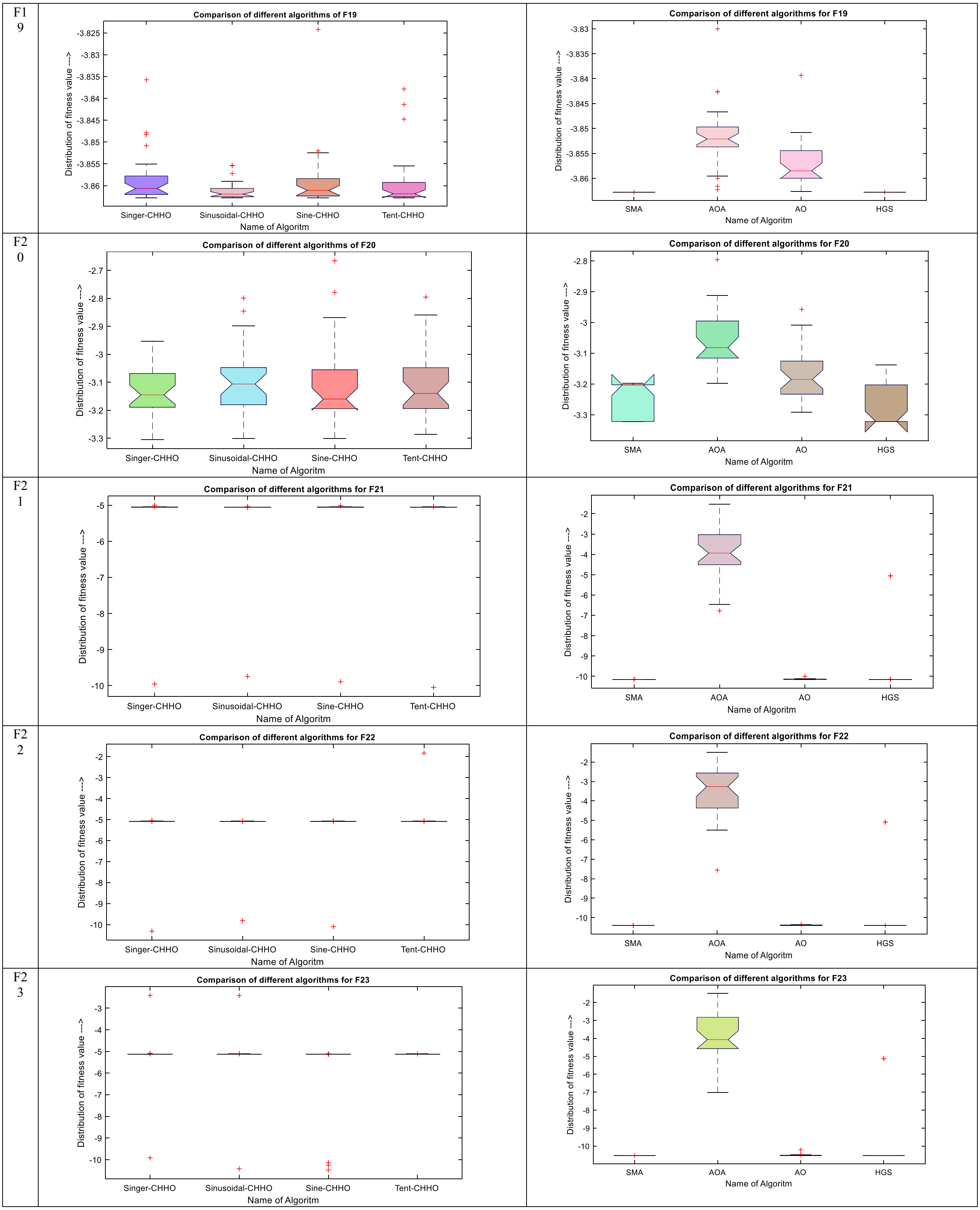


Fig. 12 (continued)

Table 23 Engineering design problem statistics data

Engineering Function(EF)	Design	Discrete-variables	Constraints
EF1	3-Bar truss problem	2	3
EF2	Speed reducer problem	7	11
EF3	Pressure vessel	4	4
EF4	Cantilever beam design	5	1
EF5	Compression spring design	3	4
EF6	Rolling element bearing	10	9
EF7	Welded beam	4	7
EF8	Belleville spring		
EF9	Gear train	4	1
EF10	Multiple Disk Clutch Brake (Discrete) variables)	5	8

Minimizing;

$$f(\vec{g}) = \left(\frac{1}{6.931} - \frac{g_3 g_4}{g_1 g_4}\right)^2 \tag{16.9b}$$

Subject to: $12 \le g_1, g_2, g_3, g_4 \le 60$.

6.10 EF10–multidisc clutch brake design

Brake design is one of the most crucial problems in engineering design. This type of design problem is illustrated in Fig. 24 [27]. The clutch design problem is mainly fabricated to minimize the overall weight. Its five design variables are inner surface radius (Rin), outer surface radius (Ro), thickness of discs (Th), actuating force (Fac) and count of friction surface (Sf). The multi-clutch design problem is mathematically formulated through Eq. (16.10, 16.10a, 16.10b, 16.10c, 16.10d, 16.10e, 16.10f). In Table 35, test results of proposed method are compared with HHO and other optimization methods. It is observed that CHHO gives better fitness as compared to HHO and other methods in terms of cost minimization.

Table 25 Computation time for EF1 to EF10 Function (EF) using CHHO

Functions	Best time	Mean time	Worst time
EF1	0.328125	0.388541667	0.96875
EF2	0.359375	0.463020833	1.015625
EF3	0.375	0.484375	1.234375
EF4	0.3125	0.4203125	0.953125
EF5	0.40625	0.566145833	1.34375
EF6	0.578125	0.7828125	1.640625
EF7	0.296875	0.510416667	1.109375
EF8	0.265625	0.35	0.796875
EF9	0.3125	0.492708333	1.078125
EF10	0.328125	0.449479167	1

Minimizing;

$$f\left(R_{in}, R_O, S_f, Th\right) = \pi Th\gamma\left(R_0^2 - R_{in}^2\right)\left(S_f + 1\right) \tag{16.10}$$

where

$R_{in} \in 60, 61, 62....80;$

$R_o \in 90, 91, ...110;$

$Th \in 1, 1.5, 2, 2.5, 3;$

$F_{ac} \in 600, 610, 620, 1000;$

$S_f \in 2, 3, 4, 5, 6, 7, 8, 9$

Subjected to,

$$m_1 = R_0 - R_{in} - \Delta R \ge 0 \tag{16.10a}$$

$$m_2 = L_{\text{MAX}} - (S_f + 1)(Th + \alpha) \ge 0 \tag{16.10b}$$

Table 24 Results engineering design problems using CHHO

Engineering functions(EF)	Mean	STD value	Best value	Worst value	Median value	p value
EF1	2.64E+02	3.19E-01	2.64E+02	2.65E+02	2.64E+02	1.73E-06
EF2	4.47E-01	4.64E-02	3.90E-01	5.71E-01	4.42E-01	1.73E-06
EF3	3.75E+03	7.37E+02	3.01E+03	5.26E+03	3.36E+03	1.73E-06
EF4	2.14E+00	3.61E-01	1.74E+00	3.49E+00	2.04E+00	1.73E-06
EF5	7.02E+03	3.63E+02	6.29E+03	7.71E+03	7.09E+03	1.73E-06
EF6	2.65E+22	2.70E+22	1.98E+00	5.30E+22	2.65E+22	1.29E-06
EF7	1.40E-02	1.38E-03	1.27E-02	1.78E-02	1.33E-02	1.73E-06
EF8	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00
EF9	− 6.74E+04	1.63E+04	− 8.41E+04	-4.24E+04	− 7.37E+04	1.73E-06
EF10	1.31E+00	1.68E-03	1.30E+00	1.31E+00	1.31E+00	1.73E-06

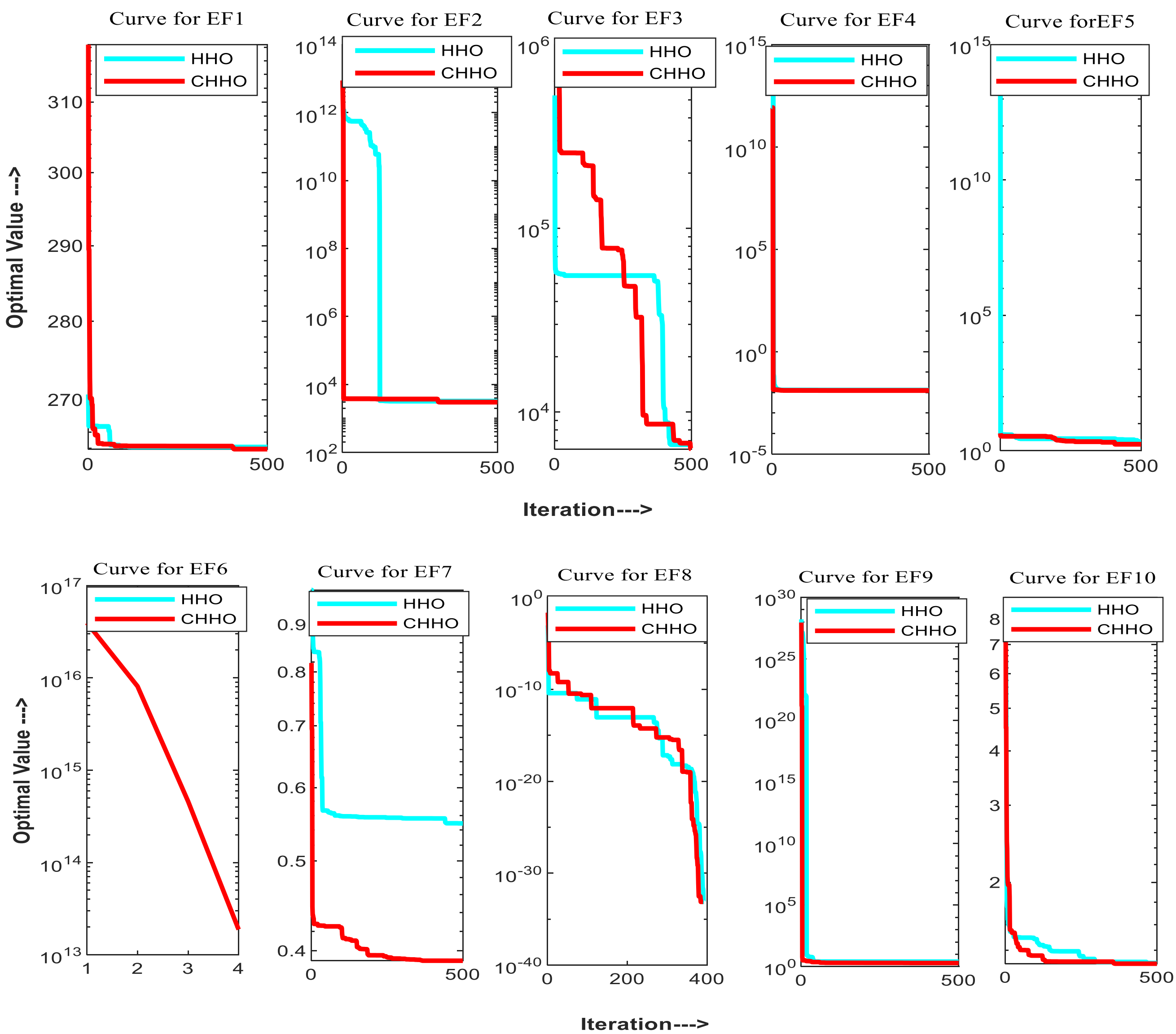


Fig. 13 Engineering design (EF1 to EF10) convergence curve comparison of CHHO with standard HHO

$$m_3 = PM_{\text{MAX}} - PM_{\pi} \geq 0 \tag{16.10c}$$

$$m_4 = PM_{\text{MAX}} Y_{\text{MAX}} + PM_{\pi} Y_{SR} \geq 0 \tag{16.10d}$$

$$m_5 = Y_{SR_{\text{MAX}}} - Y_{SR} \geq 0 \tag{16.10e}$$

$$m_6 = t_{\text{MAX}} - t \geq 0 \tag{16.10f}$$

$$m_7 = DC_h - DC_f \geq 0 \tag{16.10g}$$

$$m_8 = t \geq 0 \tag{16.10h}$$

$$PM_{\pi} = \frac{F_{ac}}{\Pi\left(R_0^2 - R_{in}^2\right)}$$

$$\text{where, } Y_{SR} = \frac{2\pi n\left(R_0^3 - R_{in}^3\right)}{90\left(R_0^2 - R_{in}^2\right)}.$$

$$t = \frac{i_x \pi n}{30\left(DC_h + DC_f\right)}$$

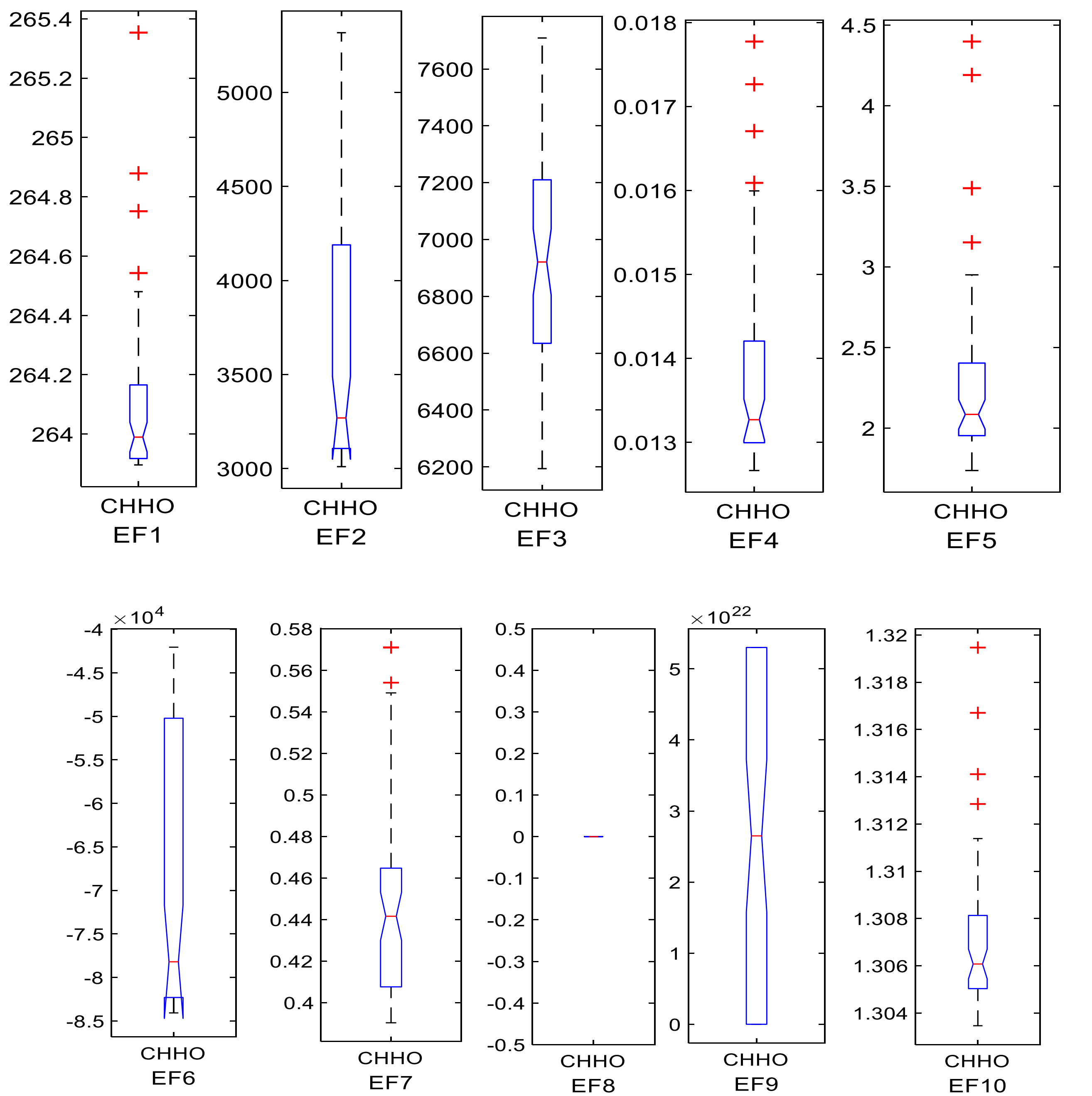


Fig. 14 Trial run for engineering design (EF1 to EF2)

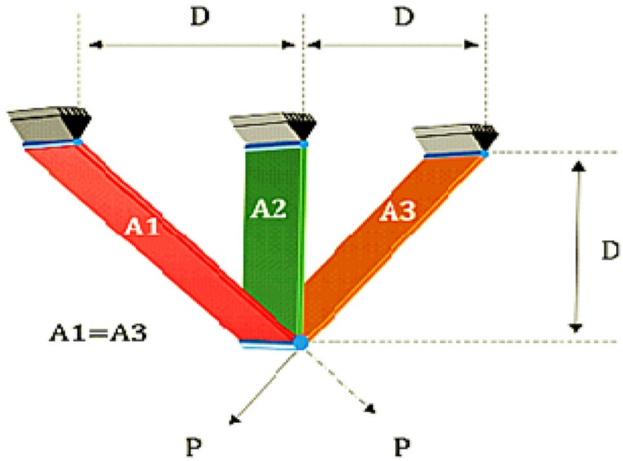


Fig. 15 Truss design

Table 26 CHHO results compared with other methods for 3-bar truss design

Algorithm	CHHO	HHO	CS [113]	Ray and Sain [114]	TSA [115]
Optimal values for variables					
*y*1	0.786672	0.78866	0.789	0.795	0.788
y2	0.413943	0.408283	0.409	0.395	0.408
Optimal weight	263.898	263.895	263.972	264.3	263.68

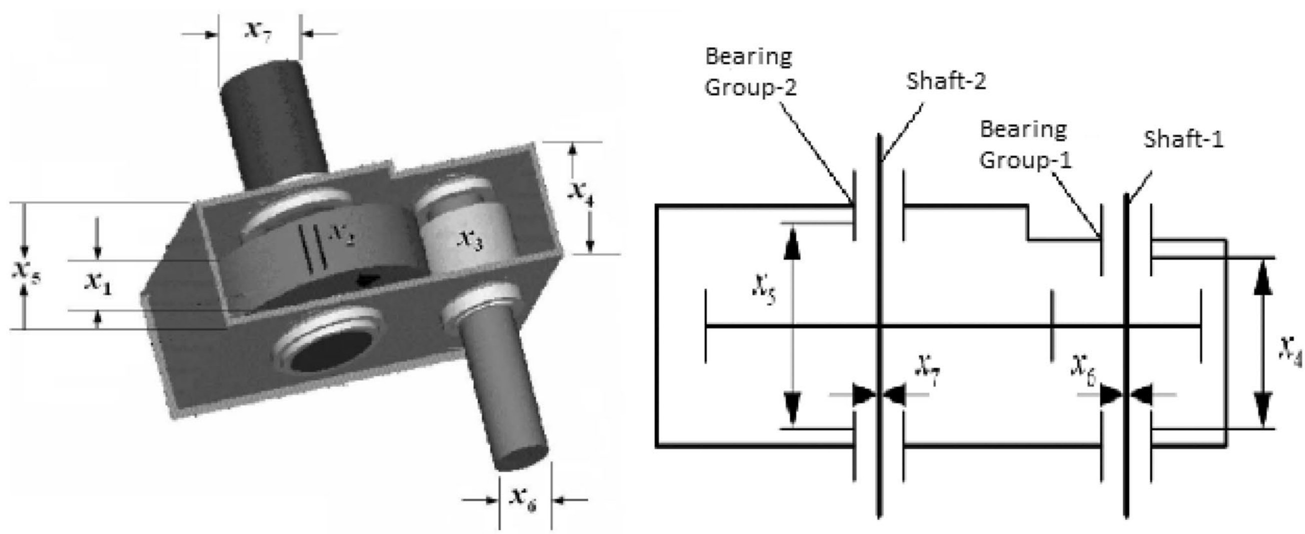


Fig.16 Speed reducer engineering design problem

Table 27 Comparison cost of speed reducer problem with other methods

Method	CHHO	HHO	MDE [117]	PSO-DE [118]	MBA [116]
Fitness values for variables					
*z*1	**3.5**	3.56	3.50001	3.50	3.5
*z*2	**0.7**	0.7	0.7	0.7	0.7
*z*3	**17**	17	17	17	17
*z*4	**7.3**	8.0186	7.300156	7.3	7.300033
*z*5	**7.715418**	8.01891	7.800027	7.8	7.715772
*z*6	**3.350215**	3.4948	3.350221	3.350214	3.350218
*z*7	**5.286655**	5.2867	5.286685	5.286683	5.286654
Optimum Cost	**2994.4737**	3060.372	2996.3566	2996.3481	2994.4824

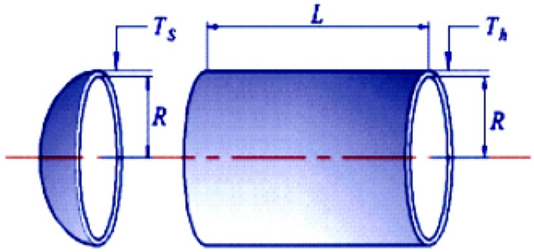


Fig. 17 Pressure vessel design

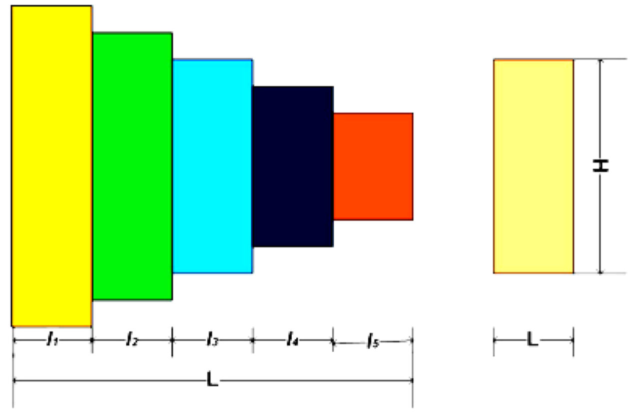


Fig. 18 Cantilever beam design

Table 28 Comparative analysis of CHHO with classical heuristic algorithms

Algorithm	CHHO	HHO	GWO [119]	GSA [109]	PSO [38]	GA [120]	DE [121]	ACO [79]	Branch-bound [48]
Optimum value									
Ts	**0.886494**	0.817583	0.84806	1.125	0.8125	0.8125	0.8125	0.8125	1.125
Th	**0.486049**	0.4312	0.4345	0.625	0.4375	0.4345	0.4375	0.4375	0.625
R	**45.85021**	42.09174	42.8279	55.9887	42.0913	40.3239	42.0984	42.1036	47.7
L	**135.1251**	167.8369	176.7587	84.4542	176.7465	200	176.6377	176.5727	117.701
Optimum Cost	**6193.94**	6286.337	7016.962	6051.5	8538.84	6061.0	6059.7	6059.734	7198.043

Table 29 Results of beam problem compared with other techniques

Method	CHHO	HHO	CS [113]	ALO [105]	SOS [53]	MMA [122]	GCA_I [122]
Optimal values for variables							
l1	6.058512	5.9374	6.0089	6.0181	6.0188	6.01	6.01
l2	4.838525	4.9199	5.3049	5.3114	5.3034	5.3	5.304
l3	4.460903	4.3780	4.5023	4.4884	4.4959	4.49	4.49
l4	3.471295	3.5468	3.5077	3.4975	3.499	3.49	3.498
l5	2.113283	2.1648	2.1504	2.1583	2.1556	2.15	2.15
Optimum weight	**1.30328**	1.3037	1.33996	1.33996	1.33995	1.33999	1.34

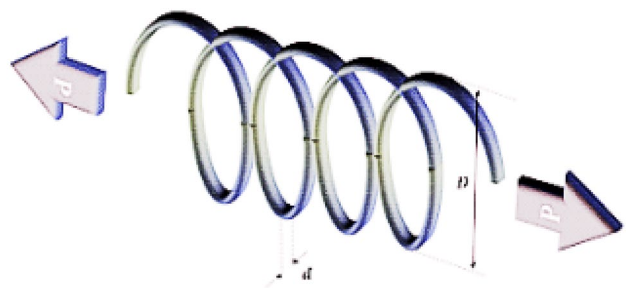


Fig. 19 Design of compression spring

7 Conclusion and future scope

In the proposed research, tent chaotic strategy has been combined with classical HHO for achieving a better exploitation search capabilities. The proposed algorithm is tested using 23 standard benchmark functions and also on universal design problems to justify the effectiveness of the proposed method. In the set of experiments, CHHO was compared with basic HHO. The experimental results revealed that proposed CHHO algorithm improves convergence for most

Table 30 Comparison of CHHO with other methods

Method	CHHO	HHO	GWO [1]	GSA [1]	CPSO [123]	ES [124]	GA [125]	HS [126]	DE [127]
Optimized value for variables									
‘*d*’	0.05170	0.05179	0.0516	0.0503	0.0517	0.052	0.0515	0.0512	0.0516
‘*D*’	0.35712	0.35930	0.3567	0.3237	0.3576	0.364	0.3517	0.3499	0.3547
‘*N*’	11.2652	11.1388	11.2889	13.5254	11.2445	10.890	11.632	12.076	11.4108
Optimum weight	**0.01266**	0.01269	0.01195	0.01267	0.0127	0.0126	0.0126	0.0127	0.01267

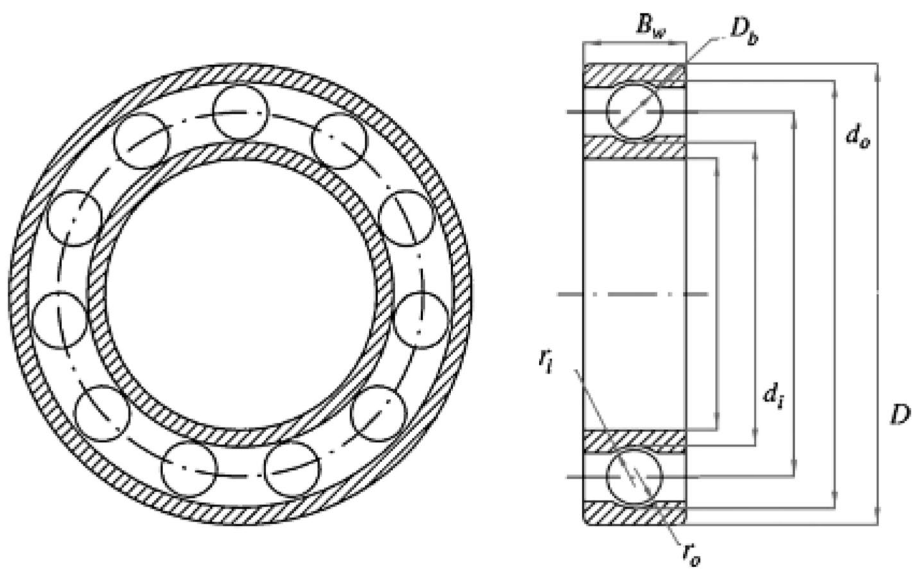


Fig. 20 Bearing deign for rolling elements

Table 31 Assessment of rolling element design variables with other methods

Method	CHHO	HHO	WCA [128]	SCA [129]	MFO [6]	MVO [18]
Values for variables						
*r*1	**125.7227**	125	125.72	125	125	125.6002
*r*2	**21.4233**	21.0745	21.42300	21.03287	21.03287	21.32250
*r*3	**11.00116**	11.0764	10.01030	10.96571	10.96571	10.97338
*r*4	**0.515**	0.515	0.515000	0.515	0.515	0.515
*r*5	**0.515**	0.515	0.515000	0.515	0.515000	0.515000
*r*6	**0.4944**	0.4055	0.401514	0.5	0.5	0.5
*r*7	**0.6986**	0.606	0.659047	0.7	0.67584	0.68782
*r*8	**0.3**	0.3	0.300032	0.3	0.300214	0.301348
*r*9	**0.03346**	0.0844	0.040045	0.027780	0.02397	0.03617
*r*10	**0.60049**	0.6	0.600000	0.62912	0.61001	0.61061
Optimum fitness	**83,455.825**	**84,072.584**	85,538.48	83,431.11	84,002.524	84,491.266

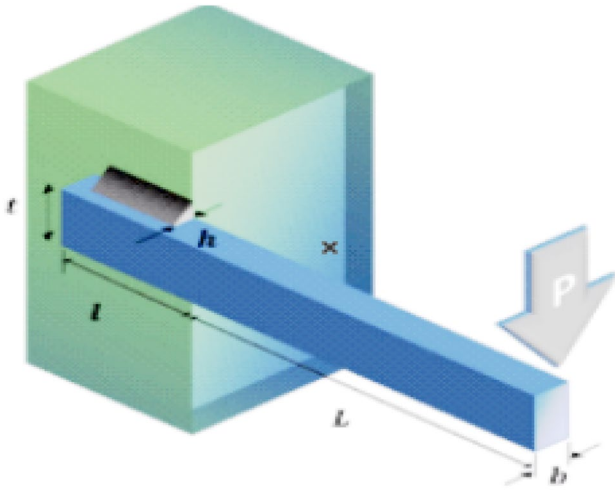


Fig. 21 Design of welded beam

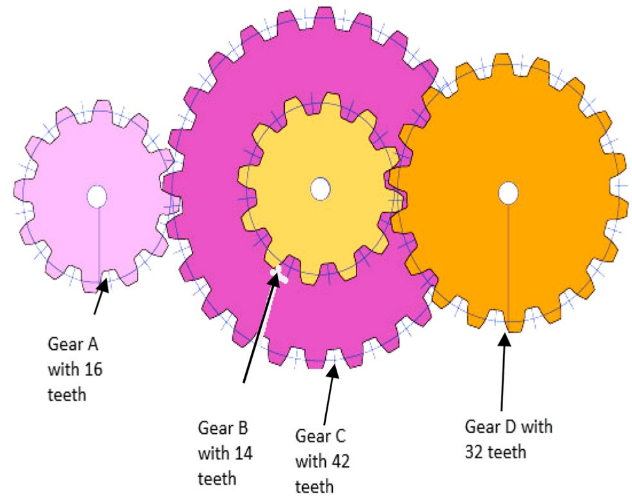


Fig. 23 Gear train problem

Table 32 Relative investigation of Welded beam Design with other methods

Method	CHHO	HHO	GSA [48]	HS [130]	GA [125]	Random [131]	Simplex [131]	APPROX [131]
Optimum variables								
h	**0.202886**	0.204039	0.2442	0.1821	0.2489	0.4575	0.2792	0.2444
l	**3.545256**	3.531061	6.2231	3.857	6.173	4.7313	5.6256	6.2189
t	**9.005042**	9.027463	8.2915	10	8.1789	5.0853	7.7512	8.2915
b	**0.207302**	0.206147	0.2443	0.2024	0.2533	0.66	0.2796	0.2444
Optimal cost	**1.7369**	1.75835	1.88	2.3807	2.4331	4.1185	2.5307	2.3815

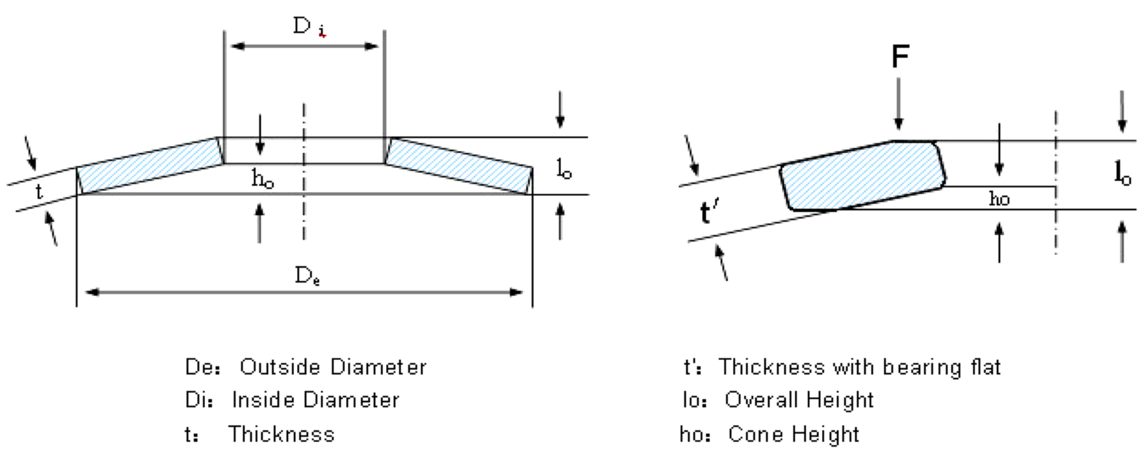


Fig. 22 Belleville spring design

Table 33 Relative analysis of design variables with other algorithms

Method	CHHO	HHO	TLBO [21]	MBA [116]
Values for variables				
×1	**11.98694**	12.0060	12.01	12.01
×2	**10.00147**	10.0254	10.0304	10.0304
×3	**0.204191**	0.2041	0.20414	0.20414
×4	**0.2**	0.2	0.2	0.2
Optimum fitness	**1.9813**	1.9801	1.9896	1.9896

of the Standard benchmark functions. Thus, the resultant chaotic Harris Hawk's optimization (CHHO) is capable of giving more optimistic and convergent results. Therefore, it is observed that the proposed CHHO may be a good choice for solving numerical optimization problems and in future may be considered to solve power system dispatch and unit commitment problems considering electric and hybrid electric vehicles including uncertainty of wind and solar power.

Table 34 Relative analysis of gear train problem with other methods

Method	CHHO	HHO	GeneAS [120]	Kannan and Kramer [120]	Sandgren [120]
Optimal values for variables					
*g*1	41	56	50	41	60
*g*2	47	58	33	33	45
*g*3	16	22	14	15	22
*g*4	17	21	17	13	18
Optimum fitness	**0.1434**	0.14563	0.144242	0.144242	0.144124

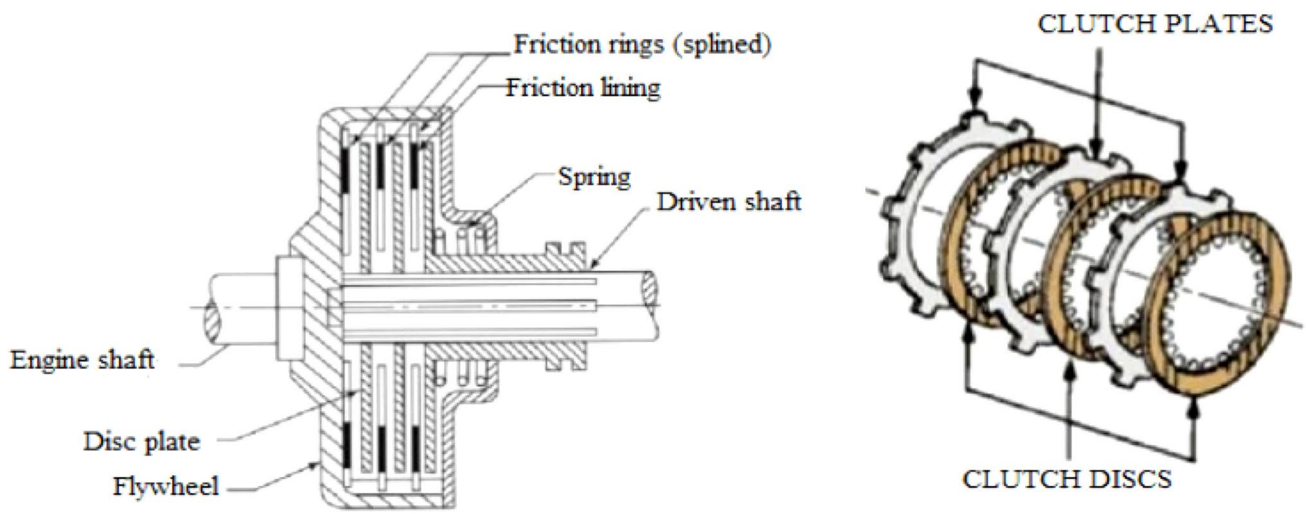


Fig. 24 Multidisc clutch break design

Table 35 Relative analysis of CHHO for multiple disk clutch brake design with other algorithms

Method	CHHO	HHO	WCA [8]	TL-BO [132]	PVS [133]
Fitness variables					
*x*1	**69.99991**	69.9999	70	70	70.00
*x*2	**90**	90	90	90	90
*x*3	**2.318123**	2.31278	3	3	3
*x*4	**1.5**	1	1	1	1
*x*5	**997.7027**	1000	910	810	880
Optimum fitness	0.24697	0.2569	0.3166	0.31365	0.323656

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