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A set theoretical shuffled shepherd optimization algorithm for optimal design of cantilever retaining wall structures

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Abstract

In this paper, a recently developed meta-heuristic algorithm, shuffled shepherd optimization algorithm (SSOA), is employed for optimal design of reinforced concrete cantilever retaining wall structures under static and seismic loading conditions. The concepts of set theory are employed to express the SSOA in a set theoretical term. The Rankine and Coulomb theories are utilized in order to estimate the lateral earth pressures under the static loading condition, whereas the Mononobe–Okabe method is employed for the seismic one. Optimization aims to minimize the cost of cantilever retaining wall while satisfying some constraints on stability and strength limits. The design is based on the requirements of ACI 318-05. In order to investigate the efficiency of the SSOA, one benchmark cantilever retaining wall problem is considered from the literature. Comparing the optimization results obtained by the SSOA with those of other meta-heuristics shows the efficient performance of the SSOA in both aspects of accuracy and convergence rate.

Keywords Shuffled shepherd optimization algorithm · Set theory · Cantilever retaining wall structures · Structural optimization · Meta-heuristic algorithms

1 Introduction

Retaining wall structures are designed and constructed for the purpose of supporting vertical or near-vertical slopes of soil or other loose materials [1]. These structures are used for design situations in which there is an abrupt difference in the ground level of adjacent areas of land. Retaining walls are among the most common geotechnical structures used in many locations, including railways embankments, roads, culverts, bridge abutments, etc. Therefore, it is very important to design a safe and low-cost retaining wall structure. Retaining wall structures can be categorized into two main groups of gravity retaining walls and cantilever retaining walls. Gravity retaining walls are usually built of plain concrete and rely solely on their own weight for stability against sliding and overturning. This kind of retaining walls is economical for height up to 2 m [2]. Cantilever retaining walls are built of reinforced concrete and generally consist

of a vertical stem, a base slab, and a shear key. Different types of forces act on a cantilever retaining wall, including self-weight of the wall, surcharge loads, lateral pressures due to soil and surcharge, etc. [3]. The design process of a cantilever retaining wall includes two distinct steps. The first step is focused on the estimation of forces acting on the wall. In this step, the structure is controlled for stability against overturning, sliding, and bearing capacity failure. In the second step, each component of the retaining wall in checked for strength and the required reinforcement of each component is calculated. Therefore, proper design of a cantilever retaining wall requires an accurate estimation of the acting forces, especially lateral earth pressures. The Coulomb and Rankine theories are two well-known classical theories for determining lateral earth pressures under static loading condition. Mononobe–Okabe method is still the first choice of geotechnical engineers to estimate lateral earth pressures under seismic loading condition [4].

Optimization approaches can be categorized into two main groups of mathematical programming techniques and meta-heuristic algorithms. Mathematical programming techniques are analytical and employ the techniques of differential calculus for finding the optimum points [5]. These techniques are useful only in cases where the objective

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function is continuous and differentiable. Due to the limitations of mathematical programming techniques, some non-traditional optimization methods, known as meta-heuristic algorithms, have been developed over the past few decades [6]. Meta-heuristic algorithms, which are conceptually different from mathematical programming techniques, try to integrate rule-based theories and randomization with the aim of exploring effectively the search space of the optimization problem [7].

Different meta-heuristic algorithms have been employed for optimal design of cantilever retaining wall structures, including big bang–big crunch (BB–BC) algorithm [8], gravitational search algorithm (GSA) [9], charged system search (CSS) algorithm [10], hybrid firefly algorithm [11], swarm intelligence techniques [12], dolphin echolocation optimization (DEO) algorithm [13], teaching–learning-based optimization [14], evolutionary algorithms [15], biogeography-based optimization (BBO) algorithm [16], simulated annealing (SA) algorithm [17], gray wolf optimization (GWO) algorithm [18], neural network, and bee colony techniques [19, 20]. Kaveh et al. [21] optimized cost design of cantilever retaining walls by means of eleven population-based meta-heuristics including artificial bee colony (ABC), big bang–big crunch (BB–BC), cuckoo search (CS), charged system search (CSS), imperialist competitive algorithm (ICA), ray optimization (RO), tug of war optimization (TWO), and water evaporation optimization (WEO). Mergos and Mantoglou [22] applied the flower pollination algorithm (FPA), for the first time, to the optimum design of reinforced concrete cantilever retaining walls. Kazemzadeh Azad and Akış [23] performed optimum cost design of mechanically stabilized earth walls using adaptive dimensional search algorithm.

Shuffled shepherd optimization algorithm (SSOA) is a new population-based meta-heuristic developed by Kaveh and Zaerreza [24]. The SSOA algorithm is inspired by the herding behavior of shepherds who takes care of and guards a flock of sheep. SSOA has recently been utilized to solve truss layout optimization problems, and the optimal design results confirm its efficient performance in both aspects of accuracy and convergence speed [25].

This study aims to optimal design of cantilever retaining wall structures utilizing a recently developed population-based meta-heuristic algorithm called shuffled shepherd optimization algorithm (SSOA). The optimization goal is to minimize the cost of cantilever retaining walls while satisfying some constraints on stability and strength. The design is based on the requirements of American Concrete Institute code for structural concrete (ACI 318-05) [26]. A benchmark cantilever retaining wall problem is investigated to demonstrate the performance of the SSOA algorithm. The design is performed under static and seismic loading conditions. The Rankine and Coulomb theories are employed for the static loading condition, whereas the Mononobe–Okabe method is utilized for the seismic one.

The rest of this paper is organized as follows: In Sect. 2, the shuffled shepherd optimization algorithm (SSOA) is presented briefly. The set theoretical shuffled shepherd optimization algorithm (STSSOA) is presented in Sect. 3. The optimization problem is defined in Sect. 4. Section 5 is devoted to analysis of reinforced concrete cantilever retaining wall structures. In Sect. 6, the optimization results are discussed in detail. Eventually, the last section concludes the paper.

2 Shuffled shepherd optimization algorithm (SSOA)

The shuffled shepherd optimization algorithm (SSOA) is a novel population-based meta-heuristic that mimics the herding behavior of shepherds. Humans have learned over time that they can use animal abilities to achieve their goals. For instance, shepherds have learned to use fast-ridden horses to herd animals like domesticated sheep and cows. Shepherds try to steer their herds to the right direction. For this purpose, shepherds usually put animals like horses or herding dogs in their herds and use the herding instinct of these animals to direct the herd and guard it from predation and theft. This behavior is the basis for obtaining the step size of sheep in the SSOA algorithm [24]. In the SSOA algorithm, the set of candidate solutions is considered as a herd of sheep. There are many herds that use a common pasture. Hence, in the SSOA algorithm, the population of candidate solutions (the main herd) is divided into some subpopulations (smaller herds) with the same number of sheep. SSOA starts with a randomly generated herd of sheep. The sheep are evaluated and sorted in ascending order of their penalized objective function values. Next, the herd of sheep is divided into a predetermined number of smaller herds with the same number of sheep. The division is done in such a way that the smaller herds are close to each other in terms of average quality. In the next step, a unique step size is calculated for each sheep. For this purpose, each sheep is selected and considered as a shepherd. Naturally, the selected sheep (shepherd) belongs to one of the herds. In the herd containing the selected sheep (shepherd), obviously there are some better and worse sheep compared to the selected sheep. The better and worse sheep are called horses and sheep, respectively. A horse and a sheep are selected randomly with respect to the selected sheep (shepherd). The shepherd rides the horse and moves from the current position to his new position. Therefore, the current position is obtained and evaluated. The current position is updated in a greedy manner so that the better position is considered as the current position. This process is repeated for the sheep of all herds. Next, the herds merge together and the sheep are sorted in ascending order of their

penalized objective function values. Again, the sheep are divided into some smaller herds with equal cardinality, and the above-mentioned process is repeated until the stopping criterion is satisfied.

3 Set theoretical shuffled shepherd optimization algorithm (STSSOA)

In this section, the concepts of set theory are employed to generalize the concepts of shuffled shepherd optimization algorithm (SSOA). Taking a close look at the shuffled shepherd optimization algorithm, one can find an analogy between the concepts of set theory and the SSOA. The set of candidate solutions (a flock of sheep) can be considered as a set of elements. The weight of an element is defined as the value of its penalized objective function. Similarly, the weight of a subset is defined as sum of the weights of its elements. In each iteration of the SSOA, the sheep (elements) are sorted in ascending order of weight. Next, the flock of sheep (the set of elements) is divided into a predetermined number of smaller flocks (subsets). The subsets have the same cardinality. The subsets are generated in such a way that their weights are close to each other. Next, a unique step size of movement is defined for each element as follows: A horse (a better element, i.e., an element with smaller weight) and a sheep (a worse element, i.e., an element with larger weight) are randomly chosen for the element under consideration (shepherd). These better and worse elements are chosen from the subset of which the element under consideration belongs. Therefore, the set of new elements is generated. The old elements are compared and replaced with their corresponding newly generated elements in a simple greedy manner so that the element with smaller weight will be preferred. Again, the subsets are generated and the process is repeated until the stopping criterion is satisfied. The set theoretical shuffled shepherd optimization algorithm (STSSOA) is stated in five steps as follows:

Step One (Forming the Initial Elements) Like other population-based meta-heuristics, shuffled shepherd optimization algorithm (SSOA) starts with a set of randomly generated initial candidate solutions named as elements. In other words, in SSOA, the initial candidate solutions (initial population) can be considered as a set with nEL elements. The set of elements (EL) can be generated by the following equation:

$$EL = Lb + (Ub - Lb) \times rand(nEL, nV) \qquad (1)$$

where nEL is the number of elements. In addition, nV is the number of design variables. Ub and Lb are the upper and lower bounds of the design variables, respectively.

It is worse to mention that the computational efficiency of the algorithm can be improved by seeding the initial population with feasible solutions [27].

Step Two (Forming the Subsets) Initially, the elements are sorted in ascending order of their penalized objective function (PFit). Next, the set of initial candidate solutions (initial set) is divided into k subsets with the same cardinality. For this purpose, k null subsets are considered. In the first step of forming the subsets, the first k best elements of the sorted set are removed from the sorted set, and each element is put in a subset randomly. Consequently, all k subsets have an element at this step. At the next step, the next k best elements of the sorted set are removed from the sorted set and each of them is put in a subset randomly. Consequently, all k subsets have two elements at this step. This process is repeated until no more element remains in the initial set. At the end of the process (the last step), all k subsets have an equal number of m elements. The subsets are close to each other in terms of average penalized objective function values. It is obvious that the first and last elements of a subset have the lowest and highest values of penalized objective function among the elements of the subset, respectively. It can be said that

$$n\text{EL} = k \times m \qquad (2)$$

where nEL is the number of elements. In addition, k and m are the number of subsets and the number of elements of each subset, respectively.

The average penalized objective function value of the ith subset (mean_{s_i}) can be calculated as:

$$\text{mean}_{s_i} = \frac{1}{m}\left(\text{PFit}_{i,1} + \text{PFit}_{i,2} + \cdots + \text{PFit}_{i,m}\right); \quad i = 1, 2, \dots, k \qquad (3)$$

where k and m are the number subsets and the number of elements of each subset, respectively. In addition, $\text{PFit}_{i,j}$ is the value of penalized objective function of the jth element of the ith subset.

Step Three (Elements Movement) In this step, a unique step size of movement is defined for each element of each subset as follows. For this purpose, the first element of the first subset is selected. Next, a better element and a worse one are chosen with respect to the selected element. These better and worse elements are chosen from the subset of which the selected element belongs. For instance, for jth element of a subset, there are $j - 1$ better elements and $m - j$ worse elements. It is clear that there is no better element for the first element of a subset. Furthermore, there is no worse element for the last element of a subset.

$$
\begin{aligned}
\text{stepsize}_{i,j} = {} & \beta \times \text{rand}_1 \times \left(\text{EL}_\text{B} - \text{EL}_{i,j}\right) + \alpha \times \text{rand}_2 \\
& \times \left(\text{EL}_\text{W} - \text{EL}_{i,j}\right); \quad i = 1, 2, \dots, k \quad j = 1, 2, \dots, m
\end{aligned}
\qquad (4)
$$

where $EL_{i,j}$ is the element under consideration. In addition, EL_B and EL_W are the better and worse elements compared to the element under consideration. Figure 1 illustrates the movement of an element to its new position. It is again emphasized that the better and worse elements are chosen from the subset of which the element under consideration belongs. $rand_1$ and $rand_2$ are random numbers chosen from the continuous uniform distribution on the [01] interval. For the first element of each subset, the first term of the above equation will be equal to zero. In addition, for the last element of each subset, the second term of the above equation will be equal to zero. According to the above equation, when a good element attracts the element under consideration, the exploitation ability for the algorithm is provided, and vice versa, if a bad element attracts the element under consideration, the exploration is provided. In addition, β and α are the factors that control the exploitation and exploration, respectively. An efficient optimization algorithm should perform good exploration in early iterations and good exploitation in the final iterations [7]. Thus, β and α are increasing and decreasing functions, respectively, and are defined as:

$$\beta = \beta_0 + \frac{\beta_{Max} - \beta_0}{MaxIter} \times Iter \qquad (5)$$

$$\alpha = \alpha_0 - \frac{\alpha_0}{MaxIter} \times Iter \qquad (6)$$

The new position of the jth element of the ith subset is calculated by:

$$newEL_{i,j} = EL_{i,j} + stepsize_{i,j} \qquad (7)$$

Step Four (Replacement Strategy) Each new element is evaluated and compared with its corresponding old element based on the penalized objective function. The old element is replaced with the newly generated one in a simple greedy manner so that the element with smaller penalized objective function is preferred. In this way, the new set of elements is formed.

Step Five (Termination Criteria) If the number of iteration of the algorithm (Iter) becomes larger than the maximum number of iterations (MaxIter), the algorithm terminates. Otherwise go to Step two.

The pseudo-code of the STSSOA algorithm is provided as follows:

- Define the algorithm parameters: nEL, k, m, β_0, β_{Max}, α_0, and MaxIter.
- Generate random initial solutions or elements (EL).
- Evaluate the initial population (set of elements) and form its corresponding vectors of the objective function (Fit) and penalized objective function (PFit).
- While Iter < MaxIter.
- Sort the set of elements in an ascending order based on their penalized objective function (PFit).
- Form the subsets based on the method described in step two.
- Determine the new movement matrix (stepsize) using Eq. (4).
- Generate new elements (newEL) using the stepsize matrix based on Eq. (7).
- Evaluate the new elements.
- Apply replacement strategy between old and new elements based on the method described in step four.
- Update the number of algorithm iterations (Iter).

Fig. 1 Movement of an element to the new position
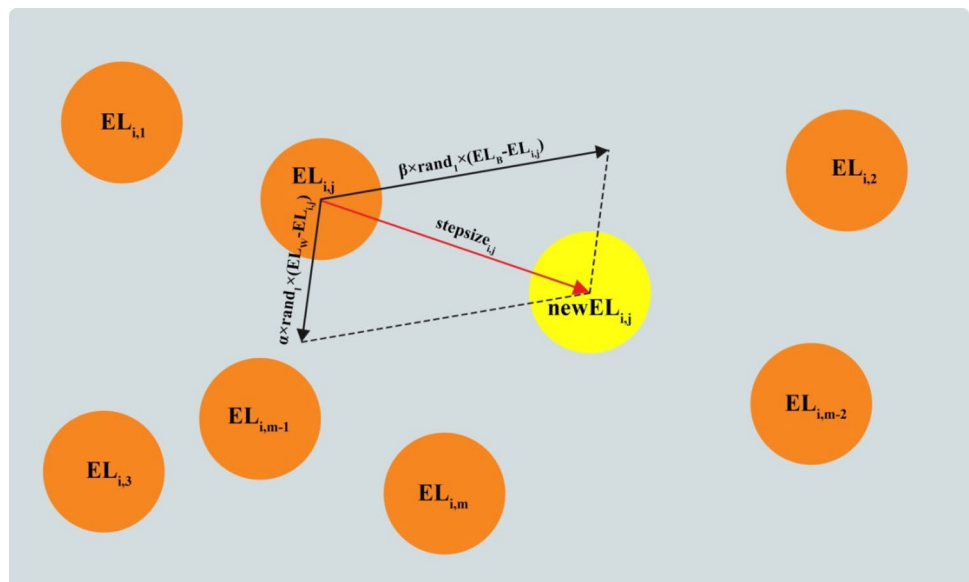
- Monitor the best element found by the algorithm so far.
- End While.

The flowchart of STSSOA is shown in Fig. 2. As Fig. 2 suggests, it is not possible explicitly to distinguish between exploration and exploitation phases of the algorithm because the STSSOA involves these two abilities together in the elements movement step (step three) using movement toward better and worse elements. Figure 3 illustrates the elements of STSSOA at different steps of the algorithm.

4 Definition of the optimization problem

The aim of this study is optimal design of cantilever retaining walls structures. The optimal design is defined as the one which results in the least possible cost while satisfying some stability and strength constraints. Figure 4 shows that the continuous design variables of the problem consist of seven geometric parameters related to the configuration of cantilever retaining wall. The design variables
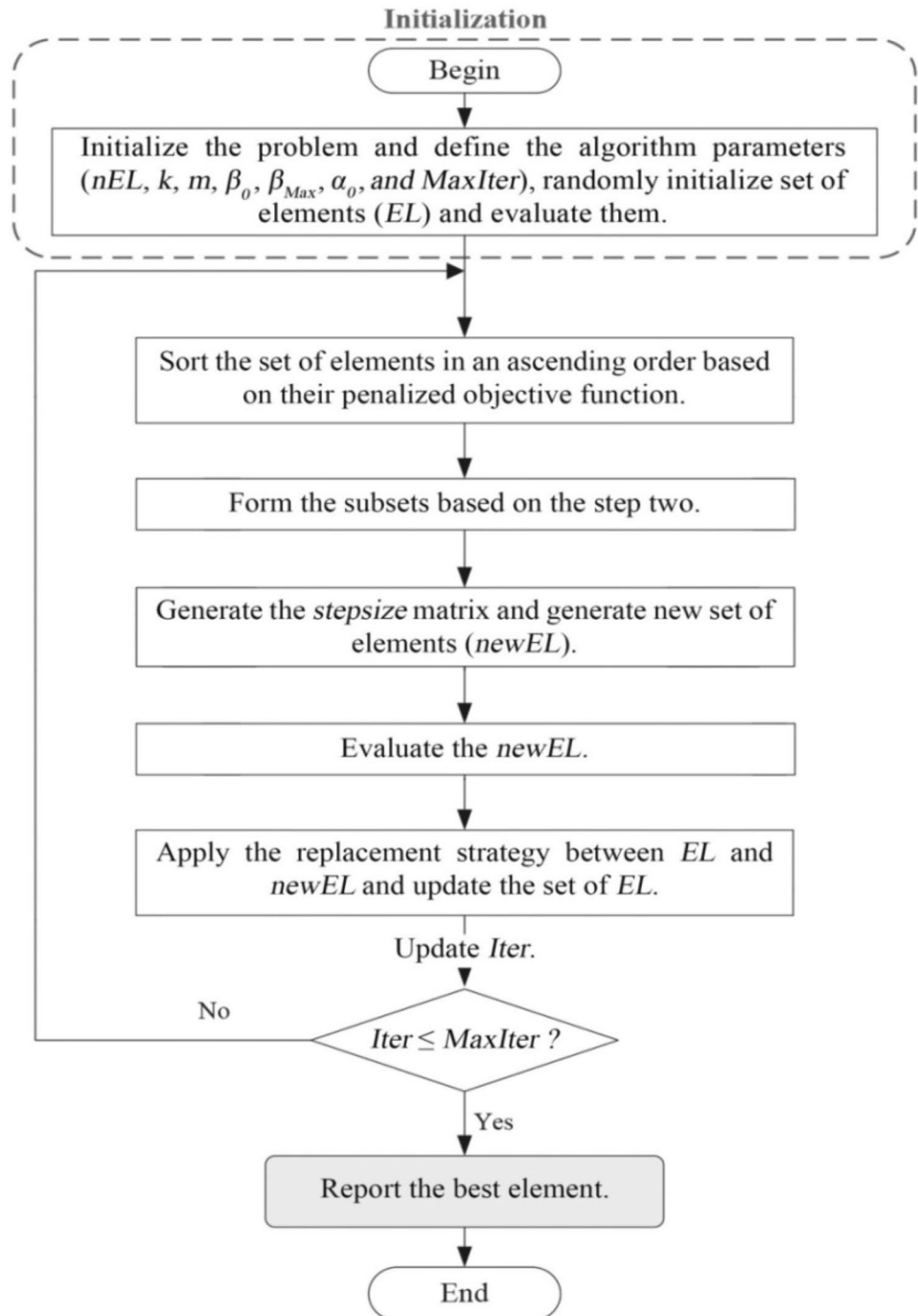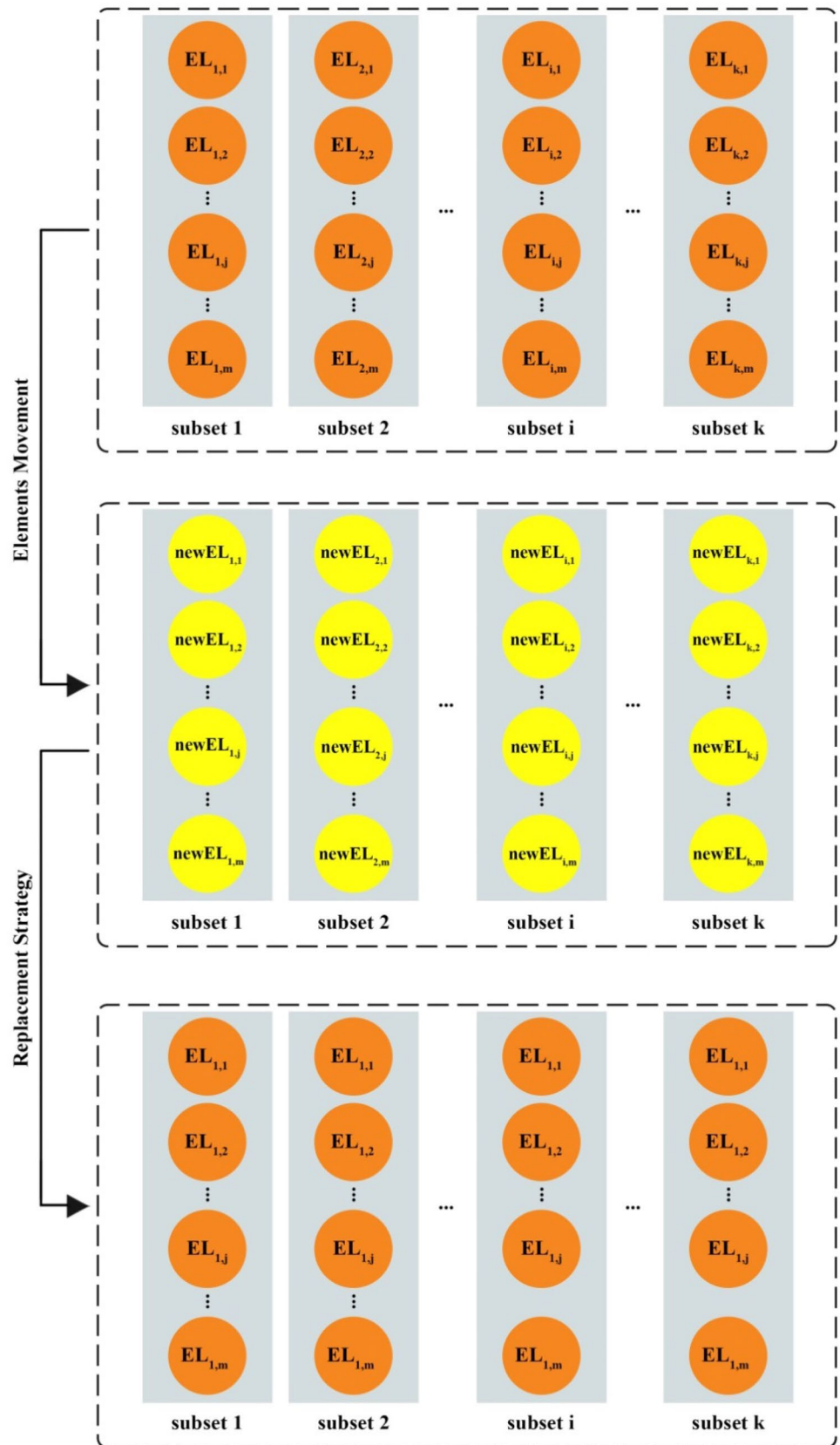
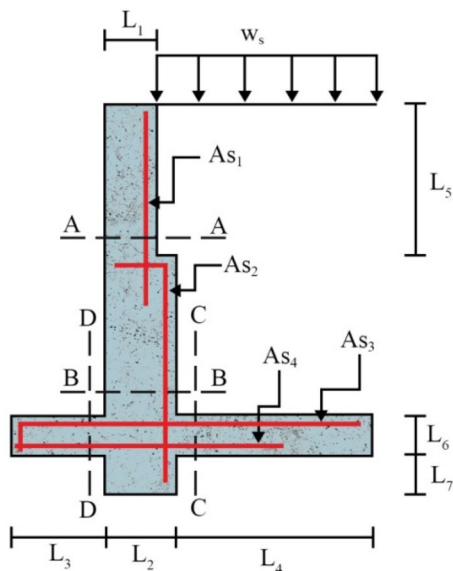**Fig. 2** Flowchart of the STSSOA algorithm

Fig. 3 Elements of the STSSOA algorithm

Fig. 4 Design variables of the reinforced concrete cantilever retaining wall

Fig. 5 Forces acting on a cantilever retaining wall

include the thickness of top stem (L_1), the thickness of key and stem (L_2), the toe width (L_3), the heel width (L_4), the height of top stem (L_5), the footing thickness (L_6), and the key depth (L_7). The wall is analyzed and designed with respect to these geometric dimensions. The optimization problem can be formulated as follows [13]:

$$\text{Find } \{L\} = \{L_1, L_2, \ldots, L_7\} \tag{8}$$

to minimize

$$\text{Cost}(\{L\}) = V_C + W_S \times \left(\frac{C_3 + C_4}{C_1 + C_2} \right) \tag{9}$$

subjected to:
$$\begin{cases} \text{SF}_o \geq 1.5 & (10) \\ \text{SF}_s \geq 1.5 & (11) \\ \text{SF}_b \geq 2 & (12) \\ M_u / (\phi_b \bar{M}_n) \leq 1 & (13) \\ V_u / (\phi_v \bar{V}_n) \leq 1 & (14) \end{cases}$$

where $\{L\}$ is the vector of design variables which determines the configuration of the wall; L_i is the ith design variable; $\text{Cost}(\{L\})$ denotes the cost function of the structure; V_C represents the volume of concrete per unit length of the structure; and W_S is the weight of reinforcing steel per unit length of the structure. Additionally, $C_1, C_2, C_3,$ and C_4 are the costs of concrete, reinforcing steel, concreting, and reinforcement, respectively. The value of the parameter $\frac{C_3+C_4}{C_1+C_2}$ (cost parameter) depends on many factors, such as country, economic conditions, type of structure, and work conditions, but studies show that a value in the range of 0.035–0.045 can be appropriate for it [13]. In this study, similar to [10], the value
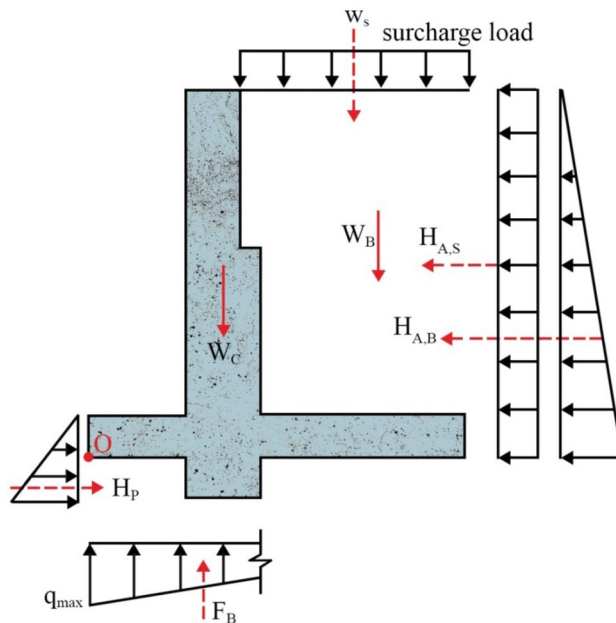
of cost parameter is considered to be equal to 0.04. Equations (10)–(14) represent constraints of the optimization problem, including stability and strength constraints. SF_o, SF_s, and SF_b are the factors of safety against overturning about the toe, sliding, and bearing capacity failure, respectively. The factors of safety against sliding and overturning failure under seismic loading condition may be reduced to 75% of them under static loading condition. Furthermore, the allowable soil pressure may be increased by 33% under seismic loading condition [28]. V_u, $\bar{V}_n$, M_u, and $\bar{M}_n$ denote the maximum shear force, nominal shear strength, maximum bending moment, and nominal flexural strength, respectively. In addition, ϕ_b and ϕ_v are the strength reduction factors for flexure and shear, respectively. Equations (13) and (14) must be controlled for all critical sections of the structure. These sections are given in Fig. 4.

5 Analysis of reinforced concrete cantilever retaining wall structure

Many forces act on a cantilever retaining wall, including surcharge load, lateral pressures due to the soil and surcharge, soil pressures acting on the footing, weight of the wall, weight of the soil above the base, etc. Figure 5 shows the forces acting on a cantilever retaining wall. W_S is the surcharge load, W_B is weight of the backfill above the heel, W_C is weight of the wall, $H_{A,S}$ is the active force due to the surcharge, $H_{A,B}$ is the active force due to the soil, H_P is the passive force acting on the wall, and F_B is the resultant vertical

load. In addition, q_{max} and q_{min} are the pressures of the toe and heel sections, respectively.

5.1 Active and passive earth pressures

There are various relationships to determine the active and passive pressures acting on a cantilever retaining wall. In this study, the structure has been investigated under the static and seismic loading conditions. The Rankine and Coulomb theories are employed to estimate the lateral earth pressures under static loading condition, whereas the Mononobe–Okabe method is utilized for seismic loading condition. According to the Rankine theory, the active and passive earth pressure coefficients are estimated through the following equations [29]:

$$k_{A,R} = \cos \alpha \left(\frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \right) \tag{15}$$

$$k_{P,R} = \cos \alpha \left(\frac{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}} \right) \tag{16}$$

where α is angle of the backfill soil with respect to the horizontal, and ϕ is angle of internal friction of the backfill soil. In addition, $k_{A,R}$ and $k_{P,R}$ are the Rankine active and passive earth pressure coefficients, respectively.

Based on the Coulomb earth pressure theory, the active and passive earth pressure coefficients can be calculated through the following equations [29]:

$$k_{A,C} = \frac{\sin^2 (\beta + \phi)}{\sin^2 (\beta) \sin (\beta - \delta) \left[1 + \sqrt{\frac{\sin (\phi+\delta) \sin (\phi-\alpha)}{\sin (\beta-\delta) \sin (\alpha+\beta)}} \right]^2} \tag{17}$$

$$k_{P,C} = \frac{\sin^2 (\beta - \phi)}{\sin^2 (\beta) \sin (\beta + \delta) \left[1 - \sqrt{\frac{\sin (\phi+\delta) \sin (\phi+\alpha)}{\sin (\beta+\delta) \sin (\alpha+\beta)}} \right]^2} \tag{18}$$

where δ is angle of friction between the soil and the base slab, and β is angle of the back face of the retaining wall with respect to the horizontal. In addition, $k_{A,C}$ and $k_{P,C}$ are the Coulomb active and passive earth pressure coefficients, respectively.

According to the Mononobe–Okabe method, the active and passive earth pressure coefficients under seismic loading condition can be calculated as [30]:

$$k_{A,M} = \frac{\sin^2 (\phi + \beta - \theta)}{\cos(\theta) \sin^2 (\beta) \sin (\beta - \delta - \theta) \left[1 + \sqrt{\frac{\sin (\phi+\delta) \sin (\phi-\alpha-\theta)}{\sin (\beta-\delta-\theta) \sin (\alpha+\beta)}} \right]^2} \tag{19}$$

$$k_{P,M} = \frac{\sin^2 (\beta - \phi - \theta)}{\cos (\theta) \sin^2 (\beta) \sin (\beta + \delta - \theta) \left[1 - \sqrt{\frac{\sin (\phi+\delta) \sin (\phi+\alpha-\theta)}{\sin (\beta+\delta-\theta) \sin (\alpha+\beta)}} \right]^2} \tag{20}$$

where $k_{A,M}$ and $k_{P,M}$ are seismic earth pressure coefficients according to the Mononobe–Okabe method for active and passive states, respectively. θ can be calculated as [30]:

$$\theta = \tan^{-1} \left[\frac{k_h}{(1 - k_v)} \right] \tag{21}$$

In the above equation, k_h and k_v are the horizontal and vertical acceleration coefficients, respectively. These coefficients can be expressed as [30]:

$$k_h = \frac{\text{horizontal earthquake acceleration component}}{\text{acceleration due to gravity, g}} \tag{22}$$

$$k_v = \frac{\text{vertical earthquake acceleration component}}{\text{acceleration due to gravity, g}} \tag{23}$$

5.2 Stability control

The factor of safety against overturning about the toe (about point O in Fig. 5) can be calculated by the following equation [29]:

$$SF_o = \frac{\sum M_R}{\sum M_O} \tag{24}$$

where $\sum M_R$ is sum of the moments of forces resisting overturning about the toe, and $\sum M_O$ is sum of the moments of forces tending to cause overturning about the toe. Weight of the wall, weight of the soil above the base, and surcharge load are those forces that contribute to the resisting moment, whereas lateral pressures due to the soil and surcharge are those contribute to the overturning moment. The factor of safety against sliding can be calculated as [29]:

$$SF_s = \frac{\sum F_R}{\sum F_D} = \frac{H_P + \sum V \times \tan(\delta)}{H_{A,S} + H_{A,B}} \tag{25}$$

where $\sum F_R$ is sum of the horizontal resisting forces, and $\sum F_D$ is sum of the horizontal driving forces. In addition, $\sum V$ is sum of the vertical forces, including surcharge load, weight of the backfill above the heel, and weight of the wall. $\sum V$ can be expressed as:

$$\sum V = W_B + W_C + W_S \tag{26}$$

The factor of safety against bearing capacity failure may be expressed as [29]:

$$SF_b = \frac{q_u}{q_{max}} \qquad (27)$$

where q_u is the ultimate bearing capacity of the soil, and q_{max} is the maximum pressure occurring at the end of the toe section. The maximum and minimum pressures can be determined by the following equation [29]:

$$q_{max,min} = \frac{\sum V}{B}\left(1 \pm \frac{6e}{B}\right); \quad e \le \frac{B}{6} \qquad (28)$$

where e is the eccentricity of the resultant vertical load, and B is width of the base slab. If the value of eccentricity becomes greater than $\frac{B}{6}$, q_{min} will be negative, which means there will be some tensile stress at the end of the heel section. In such cases, the value of q_{max} is equal to [31]:

$$q_{max} = \frac{4\sum V}{3(B - 2e)}; \quad e > \frac{B}{6} \qquad (29)$$

The magnitude of e can be calculated as:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \qquad (30)$$

6 Results and discussion

In order to investigate the efficiency of the SSOA algorithm, the algorithm is employed for optimal cost design of reinforced concrete cantilever retaining wall structures. The cantilever retaining wall is designed under static and seismic loading conditions. The Coulomb and Rankine theories are employed to model the static loading condition, whereas the seismic loading condition is modeled by the Mononobe–Okabe method. For the seismic loading condition, the optimization is carried out for different values of horizontal and vertical acceleration coefficients, as shown in Table 1. Table 2 lists

Table 1 Horizontal and vertical acceleration coefficients

Case number	Horizontal acceleration coefficient (k_h)	Vertical acceleration coefficient (k_v)
Case 1	0	0
Case 2	0.15	0
Case 3	0.15	0.15
Case 4	0.15	0.075

the lower and upper bounds of design variables. The specified parameters of the problem, including soil parameters, design parameters, and retaining wall parameters are listed in Table 3. It should be mentioned that our problem is not a large-scale one. For large-scale problems one can use special methods as discussed in [32, 33].

Table 4 shows the statistical results of SSOA in 50 independent runs. The initial population of each run is generated in a random manner. For all cases, the best, worst, and average optimized costs and the standard deviation of optimized costs are reported. Results in Table 4 indicate that the cantilever retaining wall structures designed based on the Coulomb theory are heavier compared to those designed based on the Rankine theory. This result was predictable, because the Coulomb theory takes the friction between the soil and the base slab into consideration, whereas the Rankine theory does not. Therefore, the maximum active and minimum passive earth pressures against the wall are related to the Coulomb theory and Rankine theory, respectively. The results show that the horizontal acceleration coefficient has a direct effect on the cost design of the cantilever retaining wall, while the vertical acceleration coefficient has a reverse effect on it. For all cases, the shear and stability capacity ratios are calculated at four critical sections of the structure and the results are listed in Table 5. A close examination of Table 5 reveals that the design is controlled by two factors, which are the shear strength of critical section of the toe slab, and bearing strength of the soil under the toe slab. Tables 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17 present the optimal design results obtained by the SSOA algorithm and those previously reported by Kaveh et al. [21]. The results obtained utilizing the Coulomb theory are listed in Tables 6 and 7, whereas those obtained utilizing the Rankine theory are listed in Tables 8 and 9. In addition, the results obtained by the Mononobe–Okabe method for different values of horizontal and vertical acceleration coefficients are listed in Tables 10, 11, 12, 13, 14, 15, 16, and 17. For all cases, the maximum number of objective function evaluation is considered equal to 5000 as the stopping criterion. Comparison of the optimal design results obtained by the SSOA algorithm and those of the other considered meta-heuristics indicates that the SSOA algorithm has better performance in the aspects of best optimized cost, average optimized cost, and the standard deviation of optimized costs. For example, as Table 6 demonstrates, the best optimized cost obtained

Table 2 Lower and upper bounds of design variables

Design variable	L_1	L_2	L_3	L_4	L_5	L_6	L_7
Lower bound (cm)	30	30	45	180	150	30	20
Upper bound (cm)	60	60	120	300	610	90	90

Table 3 Parameters of the optimization problem

Parameter	Symbol	Unit	Value
Factor of safety against overturning	SF_o	–	1.5
Factor of safety against sliding failure	SF_s	–	1.5
Factor of safety against bearing capacity failure	SF_b	–	2
Surcharge load	W_S	kN/m^2	10
Height of stem	H	m	6.1
Allowable soil pressure	q_a	kN/m^2	300
Concrete cover	d_c	cm	5
Unit weight of concrete	γ_c	kN/m^3	24
Unit weight of reinforcement steel	γ_s	kN/m^3	78
Unit weight of backfill soil	γ_b	kN/m^3	22
Angle of the backfill soil with respect to the horizontal	α	°	0
Angle of internal friction of the backfill soil	ϕ	°	35
angle of friction between the soil and the base slab	δ	°	$\frac{1}{2}\phi \leq \delta \leq \frac{2}{3}\phi$
Angle of the back face of the retaining wall with respect to the horizontal	β	°	90
Base friction coefficient	μ	–	$\tan(\delta)$
Height of soil in front of wall	h_p	m	0
Yield strength of rebar	f_c	MPa	300
Yield strength of concrete	f_y	MPa	25
Strength reduction factors for flexure	ϕ_b	–	0.9
Strength reduction factors for shear	ϕ_v	–	0.75

Table 4 Statistical results of SSOA for the cantilever retaining wall problem

Method	Best cost	Average cost	Worst cost	SD
Rankine	7.370	7.760	17.630	1.927
Coulomb	6.941	6.949	7.220	0.042
Mononobe–Okabe (Case 1)	6.219	6.228	6.593	0.053
Mononobe–Okabe (Case 2)	8.676	10.204	21.759	3.553
Mononobe–Okabe (Case 3)	7.811	7.821	8.250	0.062
Mononobe–Okabe (Case 4)	8.241	8.986	18.093	2.504

by the SSOA algorithm is 6.941, while it is 6.946, 6.946, 6.951, 6.970, 6.961, 6.968, 6.957, and 6.963 for ABC, BB–BC, CS, CSS, ICA, RO, TWO, and WEO, respectively. Furthermore, the standard deviation of the optimized costs obtained by the SSOA algorithm is 0.171, while it is 1.337, 0.346, 1.161, 0.506, 1.187, 1.071, 1.014, and 0.906 for ABC, BB–BC, CS, CSS, ICA, RO, TWO, and WEO, respectively. Figures 6, 7, 8, 9, 10, and 11 show the convergence histories of the SSOA algorithm and those obtained by other considered meta-heuristics. A zoomed part is attached to the convergence histories in order to simplify the comparison of convergence curves. A close examination of the convergence curves shows that the convergence rate of the SSOA algorithm is considerably higher than those of other considered meta-heuristics. As the curves suggest, the convergence of the SSOA algorithm is almost achieved for all cases within the first

Table 5 Capacity ratios of optimum designs obtained by SSOA

Method	Shear capacity (%)				Stability capacity (%)		
	A:A	B:B	C:C	D:D	Overturning	Sliding	Bearing
Rankine	40.49	72.71	22.56	100.00	42.12	62.82	100.00
Coulomb	39.18	67.85	21.95	100.00	41.85	56.61	100.00
Mononobe–Okabe (case 1)	41.88	67.47	26.55	100.00	35.89	44.11	100.00
Mononobe–Okabe (case 2)	41.42	77.14	29.13	100.00	37.10	48.33	100.00
Mononobe–Okabe (case 3)	41.35	73.59	28.64	100.00	36.82	47.58	100.00
Mononobe–Okabe (case 4)	41.31	75.81	28.93	100.00	36.97	47.98	100.00

Table 6 Comparison of optimum designs utilizing Coulomb theory

Design variable	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
L_1 (cm)	30.00	30.00	30.00	30.05	30.00	30.00	30.00	30.00	30.00
L_2 (cm)	49.10	50.49	48.04	53.75	47.59	49.30	50.86	47.03	49.83
L_3 (cm)	109.98	109.72	110.49	109.32	111.32	110.40	109.77	110.56	109.57
L_4 (cm)	230.83	230.32	230.17	228.25	228.78	231.07	230.58	231.78	230.96
L_5 (cm)	325.44	324.88	325.38	337.00	325.49	333.20	324.80	325.83	325.04
L_6 (cm)	45.62	45.55	45.76	45.47	45.96	45.74	45.55	45.83	45.52
L_7 (cm)	20.09	20.00	20.02	20.24	20.00	20.00	21.37	20.00	20.00
As_1 (cm^2/m)	11.24	11.20	11.23	12.37	11.24	12.01	11.20	11.28	11.20
As_2 (cm^2/m)	36.51	35.22	37.55	32.56	38.02	36.31	34.90	38.62	35.82
As_3 (cm^2/m)	18.49	18.46	18.56	18.42	18.65	18.55	18.46	18.59	18.44
As_4 (cm^2/m)	18.49	18.46	18.56	18.42	18.65	18.55	18.46	18.59	18.44
Best cost	6.946	6.946	6.951	6.970	6.961	6.968	6.957	6.963	6.941
Average cost	7.270	7.042	7.343	7.072	7.384	7.169	7.234	7.243	6.986
SD	1.337	0.346	1.161	0.506	1.187	1.071	1.014	0.906	0.171
No. of analyses	5000	5000	5000	5000	5000	5000	5000	5000	5000

Table 7 Optimized costs at different stages of optimization (Coulomb theory)

No. of analyses	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
1000	7.136	6.984	7.303	7.105	7.465	7.024	7.147	7.247	6.960
2000	6.967	6.957	7.004	7.003	7.272	7.000	7.065	7.072	6.942
3000	6.958	6.950	6.955	6.983	6.980	6.968	6.994	7.012	6.941
4000	6.954	6.947	6.952	6.983	6.966	6.968	6.962	6.966	6.941
5000	6.946	6.946	6.951	6.970	6.961	6.968	6.957	6.963	6.941

Table 8 Comparison of optimum designs utilizing Rankine theory

Design variable	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
L_1 (cm)	30.00	30.02	30.00	30.00	30.00	30.00	30.02	30.00	30.00
L_2 (cm)	49.54	52.35	51.86	50.39	52.29	51.15	51.81	51.18	51.03
L_3 (cm)	117.84	117.20	117.45	117.79	117.39	117.19	117.31	117.52	117.29
L_4 (cm)	241.74	241.21	240.90	241.78	240.52	242.29	241.30	241.90	241.89
L_5 (cm)	313.66	313.81	314.28	313.65	313.64	308.74	315.06	312.79	313.64
L_6 (cm)	48.39	48.22	48.35	48.33	48.27	48.29	48.26	48.25	48.24
L_7 (cm)	20.00	20.27	20.00	20.00	20.00	20.00	20.00	20.00	20.00
As_1 (cm^2/m)	11.20	11.21	11.26	11.20	11.20	11.20	11.33	11.20	11.20
As_2 (cm^2/m)	40.00	37.26	37.71	39.13	37.32	38.38	37.76	38.35	38.49
As_3 (cm^2/m)	19.78	19.70	19.77	19.75	19.73	19.74	19.72	19.72	19.71
As_4 (cm^2/m)	19.78	19.70	19.77	19.75	19.73	19.86	19.72	19.76	19.71
Best cost	7.376	7.375	7.379	7.374	7.374	7.390	7.375	7.375	7.370
Average cost	7.850	7.430	7.688	7.476	7.684	7.691	7.709	7.589	7.404
SD	1.412	0.185	0.942	0.572	1.379	1.324	1.330	0.749	0.143
No. of analyses	5000	5000	5000	5000	5000	5000	5000	5000	5000

Table 9 Optimized costs at different stages of optimization (Rankine theory)

No. of analyses	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
1000	7.976	7.399	7.761	7.424	7.519	7.481	7.576	7.539	7.381
2000	7.618	7.383	7.556	7.380	7.382	7.455	7.449	7.403	7.372
3000	7.420	7.380	7.452	7.377	7.375	7.424	7.406	7.394	7.370
4000	7.380	7.375	7.384	7.374	7.374	7.400	7.383	7.390	7.370
5000	7.376	7.375	7.379	7.374	7.374	7.390	7.375	7.375	7.370

Table 10 Comparison of optimum designs utilizing Mononobe–Okabe method (Case 1)

Design variable	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
L_1 (cm)	30.00	30.00	30.01	30.09	30.00	30.00	30.00	30.00	30.00
L_2 (cm)	49.79	52.30	49.16	48.52	48.57	51.49	49.92	51.31	50.08
L_3 (cm)	69.78	69.22	69.84	70.11	70.08	69.59	69.66	69.42	69.63
L_4 (cm)	225.04	224.29	225.57	225.38	225.37	224.42	225.18	224.56	225.11
L_5 (cm)	337.38	336.34	338.98	339.47	337.37	335.60	337.91	337.48	337.41
L_6 (cm)	39.08	38.91	39.12	39.22	39.20	38.98	39.04	38.94	39.02
L_7 (cm)	20.00	20.00	20.00	20.15	20.00	20.00	20.01	20.04	20.00
As_1 (cm^2/m)	11.20	11.20	11.35	11.35	11.20	11.20	11.25	11.21	11.20
As_2 (cm^2/m)	33.54	31.52	34.09	34.67	34.63	32.15	33.43	32.29	33.29
As_3 (cm^2/m)	15.44	15.40	15.46	15.50	15.50	15.39	15.42	15.37	15.41
As_4 (cm^2/m)	15.44	15.36	15.46	15.50	15.50	15.39	15.42	15.37	15.41
Best cost	6.222	6.229	6.225	6.232	6.226	6.228	6.220	6.220	6.219
Average cost	6.335	6.286	6.330	6.368	6.380	6.322	6.341	6.433	6.257
SD	0.373	0.207	0.258	0.304	0.344	0.270	0.304	0.530	0.164
No. of analyses	5000	5000	5000	5000	5000	5000	5000	5000	5000

Table 11 Optimized costs at different stages of optimization (Mononobe–Okabe method, Case 1)

No. of analyses	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
1000	6.481	6.249	6.393	6.394	6.463	6.310	6.365	6.410	6.226
2000	6.240	6.240	6.253	6.235	6.250	6.269	6.242	6.247	6.221
3000	6.223	6.231	6.237	6.233	6.230	6.254	6.224	6.227	6.220
4000	6.222	6.230	6.230	6.233	6.226	6.228	6.221	6.221	6.219
5000	6.222	6.229	6.225	6.232	6.226	6.228	6.220	6.220	6.219

2000 analyses. This result is concluded from Tables 7, 9, 11, 13, 15, and 17 too. Figures 12, 13, 14, 15, 16, and 17 show final design results of SSOA in 50 independent runs. The success rate of the SSOA algorithm can be obtained through these figures. For example, the success rate of the SSOA algorithm in 96% for the Coulomb and Rankine theories.

Table 12 Comparison of optimum designs utilizing Mononobe–Okabe method (Case 2)

Design variable	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
L_1 (cm)	30.00	30.00	30.01	30.00	30.24	30.01	30.02	30.00	30.00
L_2 (cm)	58.15	59.49	59.07	59.39	56.42	60.000	57.85	60.00	59.34
L_3 (cm)	103.63	103.52	103.42	102.93	104.53	102.81	103.66	103.19	103.32
L_4 (cm)	290.47	289.95	290.21	291.25	291.37	290.91	290.71	289.78	290.06
L_5 (cm)	279.10	278.43	280.05	279.12	283.45	287.83	280.89	279.68	279.15
L_6 (cm)	54.60	54.52	54.53	54.33	54.82	54.25	54.61	54.43	54.48
L_7 (cm)	20.00	20.05	20.00	21.22	20.03	20.20	20.00	20.03	20.01
As_1 (cm^2/m)	11.20	11.20	11.31	11.20	11.58	12.22	11.40	11.27	11.21
As_2 (cm^2/m)	48.18	46.80	47.23	46.91	50.10	46.30	48.50	46.30	46.95
As_3 (cm^2/m)	22.68	22.64	22.65	23.19	22.78	23.19	22.69	22.62	22.62
As_4 (cm^2/m)	22.68	22.64	22.65	22.56	22.78	22.52	22.69	22.60	22.62
Best cost	8.679	8.683	8.679	8.697	8.704	8.696	8.682	8.677	8.676
Average cost	8.893	8.756	8.993	8.819	9.049	8.804	9.009	8.782	8.719
SD	0.851	0.216	1.047	0.660	0.995	0.362	1.171	0.464	0.196
No. of analyses	5000	5000	5000	5000	5000	5000	5000	5000	5000

Table 13 Optimized costs at different stages of optimization (Mononobe–Okabe method, Case 2)

No. of analyses	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
1000	8.713	8.721	9.070	8.768	9.018	8.731	9.140	8.713	8.699
2000	8.681	8.688	8.777	8.730	8.922	8.705	8.769	8.704	8.678
3000	8.680	8.685	8.700	8.710	8.748	8.705	8.700	8.680	8.677
4000	8.680	8.683	8.685	8.697	8.718	8.702	8.690	8.678	8.676
5000	8.679	8.683	8.679	8.697	8.704	8.696	8.682	8.677	8.676

Table 14 Comparison of optimum designs utilizing Mononobe–Okabe method (Case 3)

Design variable	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
L_1 (cm)	30.00	30.00	30.00	30.19	30.00	30.00	30.03	30.01	30.00
L_2 (cm)	59.99	58.18	56.13	54.81	56.47	60.00	58.34	53.88	56.35
L_3 (cm)	91.57	91.78	92.03	92.45	93.86	91.65	91.49	92.65	91.93
L_4 (cm)	265.86	267.26	268.59	269.09	264.26	266.65	267.69	269.91	268.44
L_5 (cm)	314.66	296.06	307.61	297.65	297.72	299.11	300.86	297.01	296.02
L_6 (cm)	49.12	49.19	49.39	49.42	50.03	49.08	49.08	49.47	49.26
L_7 (cm)	20.00	20.00	20.00	20.02	20.08	20.00	20.09	20.00	20.00
As_1 (cm^2/m)	13.40	11.25	12.55	11.32	11.43	11.58	11.76	11.34	11.24
As_2 (cm^2/m)	39.13	40.64	42.53	43.84	42.20	39.12	40.50	44.82	42.32
As_3 (cm^2/m)	20.12	20.16	20.25	20.26	20.55	20.11	20.21	20.44	20.19
As_4 (cm^2/m)	20.12	20.16	20.25	20.26	20.55	20.11	20.11	20.28	20.19
Best cost	7.838	7.817	7.831	7.822	7.858	7.828	7.819	7.828	7.811
Average cost	8.185	7.894	8.146	7.965	8.062	8.012	8.142	8.157	7.851
SD	1.117	0.272	0.938	0.421	0.334	0.577	1.056	0.841	0.126
No. of analyses	5000	5000	5000	5000	5000	5000	5000	5000	5000

Table 15 Optimized costs at different stages of optimization (Mononobe–Okabe method, Case 3)

No. of analyses	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
1000	8.286	7.840	8.427	7.974	8.147	8.000	8.194	8.161	7.818
2000	7.992	7.827	7.912	7.881	8.003	7.876	7.896	7.992	7.813
3000	7.870	7.826	7.847	7.834	7.960	7.840	7.833	7.888	7.811
4000	7.843	7.821	7.835	7.824	7.867	7.828	7.825	7.834	7.811
5000	7.838	7.817	7.831	7.822	7.858	7.828	7.819	7.828	7.811

Table 16 Comparison of optimum designs utilizing Mononobe–Okabe method (Case 4)

Design variable	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
L_1 (cm)	30.00	30.00	30.00	30.07	30.00	30.00	30.03	30.00	30.00
L_2 (cm)	58.39	59.99	57.06	54.54	57.56	60.00	55.61	58.81	57.55
L_3 (cm)	97.54	97.19	97.66	98.30	97.11	97.19	98.07	98.11	97.70
L_4 (cm)	279.07	278.43	280.04	280.94	280.77	278.66	280.46	278.92	279.47
L_5 (cm)	287.02	290.03	287.38	288.10	287.44	294.32	288.14	295.81	287.01
L_6 (cm)	51.84	51.73	51.88	52.12	51.68	51.69	52.04	51.93	51.90
L_7 (cm)	20.00	20.04	20.00	20.0	20.00	20.00	20.04	20.00	20.00
As_1 (cm^2/m)	11.20	11.54	11.24	11.29	11.25	12.04	11.31	12.21	11.20
As_2 (cm^2/m)	44.14	42.67	45.46	48.18	44.95	42.66	46.98	43.75	44.96
As_3 (cm^2/m)	21.39	21.38	21.61	21.54	22.08	21.51	21.51	21.43	21.42
As_4 (cm^2/m)	21.39	21.34	21.41	21.52	21.31	21.32	21.49	21.43	21.42
Best cost	8.241	8.247	8.245	8.254	8.253	8.253	8.249	8.260	8.241
Average cost	8.633	8.300	8.359	8.313	8.440	8.416	8.524	8.701	8.263
SD	1.516	0.181	0.271	0.186	0.617	0.509	0.878	1.378	0.062
No. of analyses	5000	5000	5000	5000	5000	5000	5000	5000	5000

Table 17 Optimized costs at different stages of optimization (Mononobe–Okabe method, Case 4)

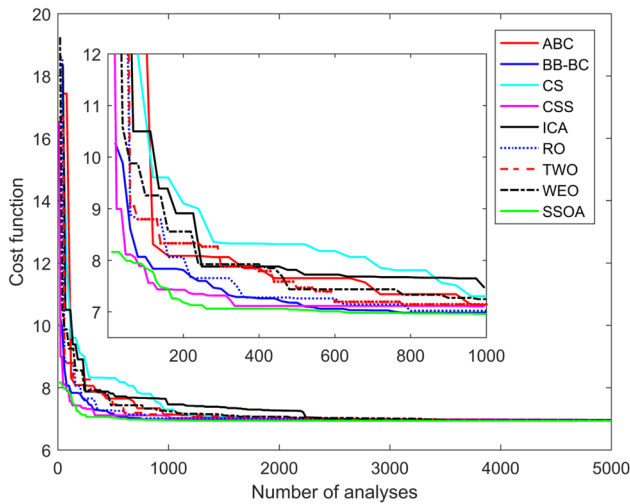| No. of analyses	Kaveh et al. [21]								Present work
	ABC	BB–BC	CS	CSS	ICA	RO	TWO	WEO	
1000	8.652	8.277	8.312	8.283	8.462	8.394	8.601	8.789	8.266
2000	8.299	8.277	8.263	8.283	8.297	8.293	8.331	8.449	8.244
3000	8.250	8.254	8.251	8.282	8.268	8.253	8.260	8.322	8.241
4000	8.243	8.249	8.246	8.255	8.258	8.253	8.255	8.261	8.241
5000	8.241	8.247	8.245	8.254	8.253	8.253	8.249	8.260	8.241

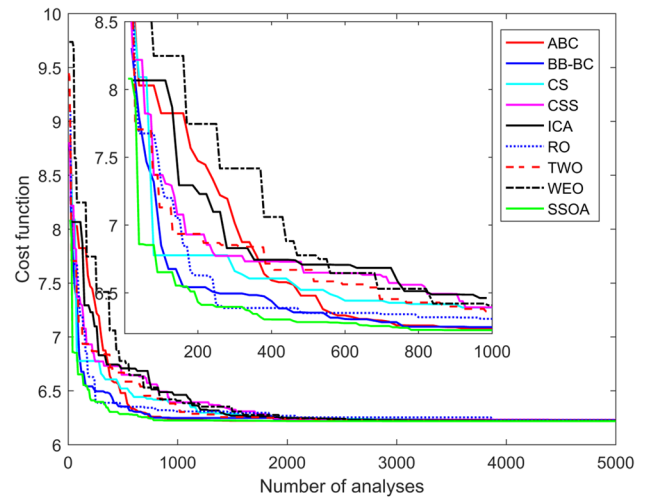**Fig. 6** Comparison of convergence histories utilizing Coulomb theory

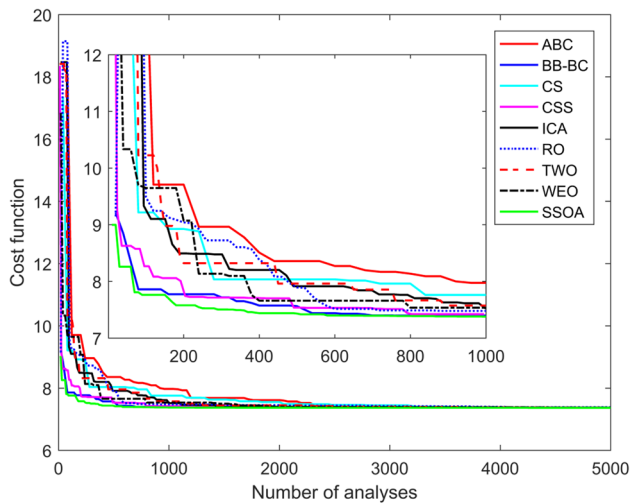**Fig. 8** Comparison of convergence histories utilizing Mononobe–Okabe method (Case 1)

Fig. 7 Comparison of convergence histories utilizing Rankine theory
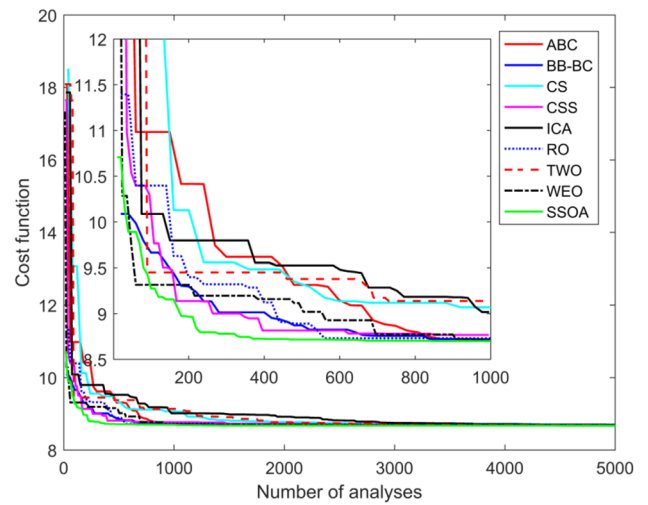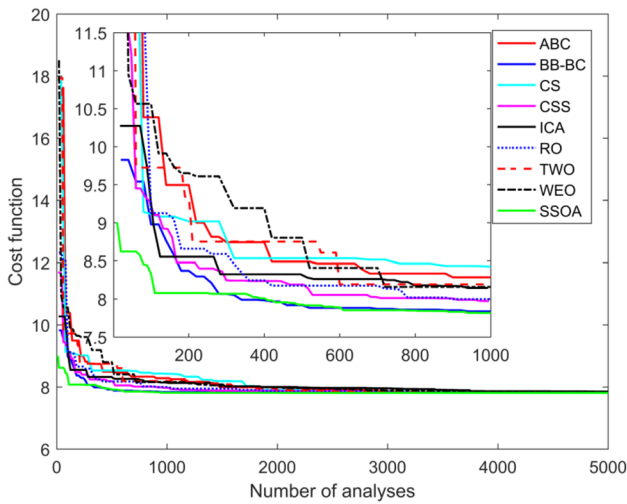

Fig. 9 Comparison of convergence histories utilizing Mononobe–Okabe method (Case 2)

7 Conclusion

This study introduced a recently developed population-based meta-heuristic, the shuffled shepherd optimization algorithm (SSOA). The concepts of set theory are employed to generalize the SSOA algorithm, which has led to the set theoretical shuffled shepherd optimization algorithm (STSSOA). The SSOA algorithm is examined for optimal design of reinforced concrete cantilever retaining walls under static and seismic loading

Fig. 10 Comparison of convergence histories utilizing Mononobe–Okabe method (Case 3)
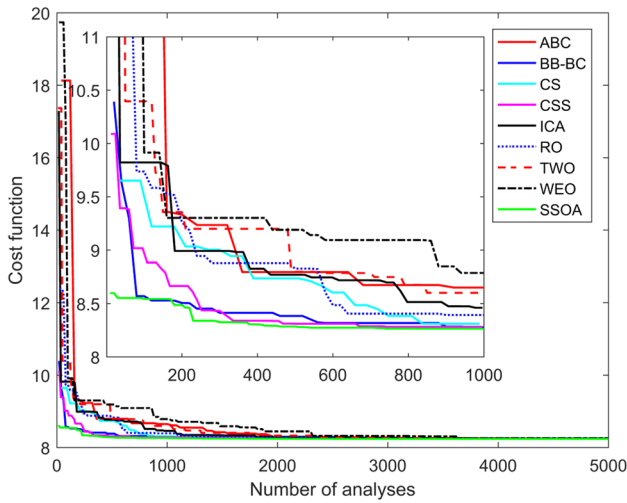
Fig. 12 Final results of the cantilever retaining wall obtained by SSOA (50 runs)

Fig. 11 Comparison of convergence histories utilizing Mononobe–Okabe method (Case 4)

Fig. 13 Final results of the cantilever retaining wall obtained by SSOA (50 runs)

conditions. Optimization aims to minimize the cost of cantilever retaining wall structures while satisfying some constraints on stability and strength. The static loading condition is modeled by the Rankine and Coulomb theories, whereas the Mononobe–Okabe method is utilized for the seismic loading

condition. The results indicate that the design based on the Coulomb theory leads to heavier cantilever retaining wall structures compared to those designed based on the Rankine theory. Furthermore, the horizontal and vertical acceleration coefficients have direct and reverse effects on the cost design of
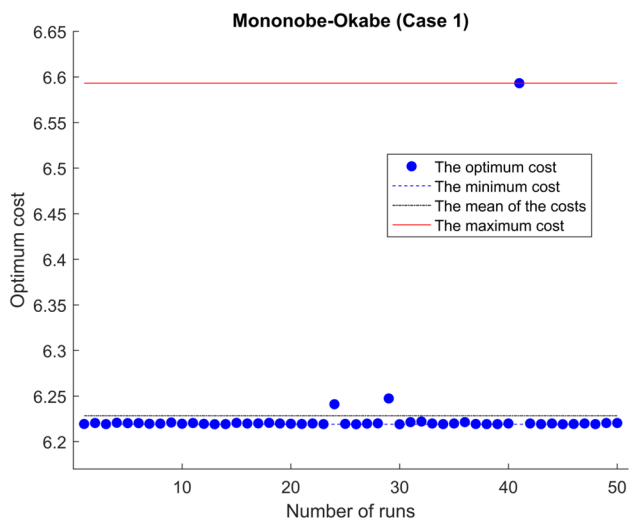
Fig. 14 Final results of the cantilever retaining wall obtained by SSOA (50 runs)
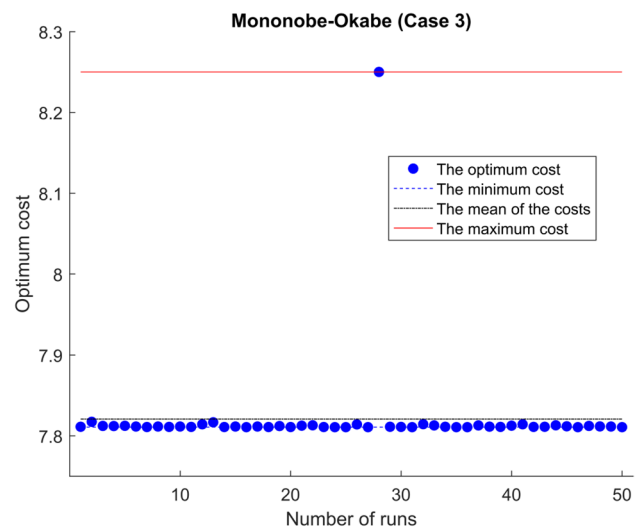

Fig. 16 Final results of the cantilever retaining wall obtained by SSOA (50 runs)
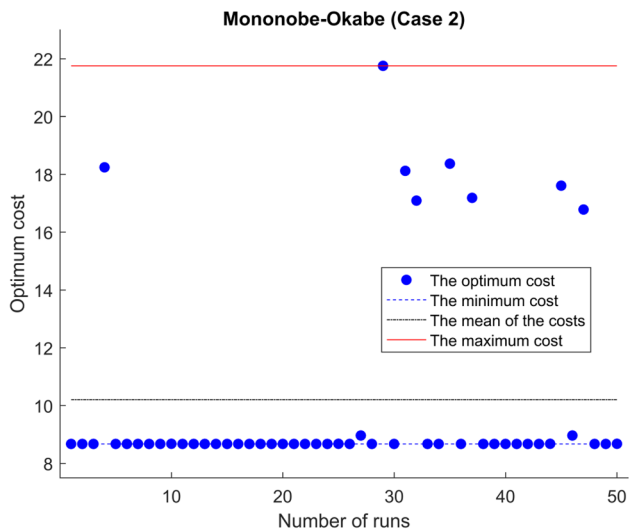

Fig. 15 Final results of the cantilever retaining wall obtained by SSOA (50 runs)
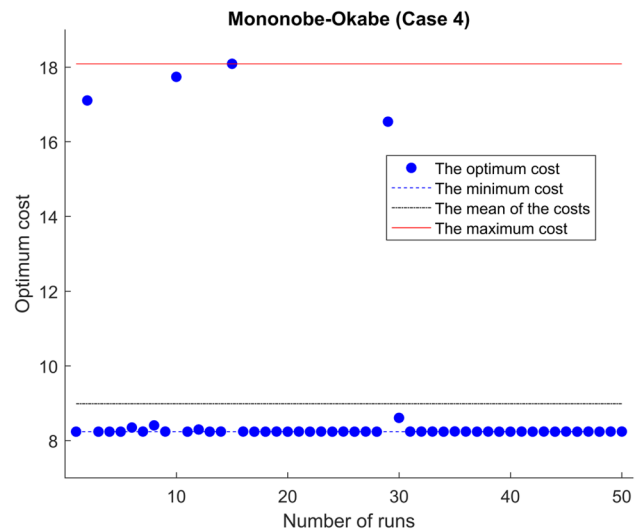

Fig. 17 Final results of the cantilever retaining wall obtained by SSOA (50 runs)

the cantilever retaining wall, respectively. It can be concluded from the optimal design results that the design controlling factors of the problem are the shear capacity of the toe slab and bearing capacity of the soil under the toe slab. The optimal design results show the superiority of the SSOA algorithm over the other investigated meta-heuristics and confirm the efficient performance of the SSOA algorithm for the optimal cost design of cantilever retaining wall structures.

Compliance with ethical standards

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