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ORIGINAL ARTICLE

A hybrid sufficient performance measure approach to improve robustness and efficiency of reliability-based design optimization

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Abstract
The stable convergence and efficiency of reliability-based design optimization (RBDO) using performance measure approach (PMA) are the major issue to develop the reliability methods based on modified chaos control (MCC), hybrid chaos control (HCC) and finite-step length adjustment (FSL). However, these methods may be inefficient for RBDO problems with convex and concave probabilistic constraints. In this paper, an adaptive modified chaos control (AMC) is proposed to provide the robust and efficient results in RBDO. The proposed AMC is adjusted using dynamical chaos control factor, which is extracted using sufficient descent condition for PMA. Using sufficient criterion, the proposed AMC is adaptively combined with advanced mean value (AMV) to improve the performance of PMA, named as hybrid adaptive modified chaos control (HAMC). Considering the robustness and efficiency, the proposed HAMC is compared with several existing reliability methods by three nonlinear structural/mathematical performance functions and two RBDO problems. The results indicate that the proposed HAMC with sufficient descent condition provides superior convergences in terms of both robustness and efficiency, compared to existing PMA methods using AMV, MCC, HCC and FSL.

Keywords Reliability-based design optimization · Sufficient criterion · Hybrid adaptive modified chaos control · Performance measure approach

1 Introduction

The practical engineering problems such as mechanical and structural systems involve uncertainties in geometrical dimensions, material property, and load [1–3]. The optimization design of structures described by the probabilistic constraints can be applied to consider these uncertainties in the optimization process. The optimization-based probabilistic constraints are provided the safe design domains which are evaluated using a reliability method. Commonly, the most probable point (MPP) search such as the first-order reliability method (FORM) [4–7] or second-order reliability method [8–10] is used for the reliability analysis and reliability-based design optimization (RBDO) problems. The FORM iterative formula can be provided the suitable performances for a compromise optimal design between efficient computational burden and structural safety. The efficiency and robustness of the FORM-based MPP search in the reliability loop are main capabilities for the RBDO methods.

The RBDO is generally classified as decoupled approaches (DA) [11–14], single-loop approaches (SLA)

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[15–19] and double-loop approaches (DLA) [20–23]. Du and Chen [12] proposed the sequential optimization and reliability assessment (SORA) for RBDO problems using sequential deterministic optimization and reliability analysis to evaluate the MPP that the boundaries of probabilistic constraints are shifted into the safe region based on the MPP obtained for reliability loop. A decoupling approached was proposed by Cheng et al. [24] based on applied sequential sub-programming for approximating optimum models as objective probabilistic constraints. An incremental shifting process-based MPP for probabilistic constant was proposed to enhance the SORA by Huang et al. [25]. For multiphase interval and probabilistic uncertainties, Huang et al. [26] developed the decoupling strategy-based RBDO. For improving the accuracy of DA-based SORA, Li et al. [27] applied the sequential sampling scheme with probabilistic and convex set in reliability loop of RBDO problems.

The single loop single vector was proposed by Liang et al. [18]; while Shan and Wang [19] proposed a reliable design space in SLA. Adaptive methods for dynamical selection of SLA or DLA have been developed to increase the accuracy of SLA and to improve the efficiency of DLA [28, 29]. The dynamical chaos control method was applied in DLA for improving the RBDO efficiency by Keshtegar and Hao [17]. The conjugate gradient search direction-based MPP search was utilized in SLA [30]; while the adaptive conjugate sensitivity vector is used to improve the robustness of SLA [31]. Meng et al. [32] proposed a stability transformation method with dynamical chaos control to improve the robustness of SLA; while Li et al. [33] applied the self-adjusted chaos control to improve the efficiency of SLA.

For DLA-based RBDO, Youn et al. [22] proposed hybrid reliability method by AMV combined with conjugate mean value (CMV) to improve the stability of performance measure approach (PMA)-based DLA. The MPP search-based reliability method is improved using angle information between successive iterations by Du et al. [34]. Yang [35] applied the chaos control (CC) using stability transformation method (STM) to improve the instability of the FORM in the reliability loop of DLA-based RBDO. Using the STM formulation, the search direction of MPP search formula using chaos control factor was improved using directional sensitivity vector by in modified chaos control (MCC) hybrid by AMV to enhance its efficiency [36]; while the adaptive chaos control factor was implemented for DLA-based RBDO by [28]. Self-adaptive modified chaos control was proposed using an adaptive directional sensitivity vector in DLA-based PMA [37]. Enhanced CC was formulated to improve the efficiency of reliability loop-based STM and was applied in the RBDO-based PMA [23]. The accelerate chaos control with dynamical formulation was proposed to solve the RBDO problems with PMA [38]. The argument step size by applying a directional sensitivity vector with two terms and adaptive step size was proposed to improve the efficiency of RBDO-based PMA [20]. The enhanced CC-based MPP search is combined by isogeometric analysis to solve the complex engineering RBDO problems [39]. The stability and computational burden of PMA were improved based on the modified FORM in reliability loop of the RBDO-based DLA as active stagey-based formulation [29], self-adjusted step size [40], enriched formulation-based self-adjusted mean value [41], hybrid descent step size combined by AMV [40], a dynamical modified search direction-based AMV [21], relaxed formulation by the adaptive step size [42]. Recently, the MPP-based reliability method using conjugate gradient method was developed for RBDO-based PMA to provide the stable results in reliability loop [37] while Zhu et al. [43] enhanced the efficiency of the conjugate sensitivity vector based on the hybrid formulation. The main effort in SLA is to provide the accurate results with stable convergence. Therefore, the modified reliability method in SLA should provide the stable results at every iteration for highly nonlinear probabilistic constraints. However, the DLA and DA are formulated based on the MPP-based search using the reliability method. Therefore, the robust iterative MPP search algorithm is strongly impotent in the reliability loop of these RBDO schemes to provide the stable results. Another challenge in RBDO-based DLA is to develop a reliability method with low computational formulation and simple application.

For reliability loop in DLA-based PMA, the first-order inverse reliability problem is used due to its efficiency and simplicity compared to reliability method in the reliability index approaches [44]. The inverse reliability method-based MPP search is divided into two main categories by the sensitivity vector as conjugate MPP search [45–47] (e.g., conjugate mean value (CMV) [22], conjugate gradient analysis (CGA) [30] and self-adaptive conjugate method [37]) and gradient-based MPP search (e.g., modified chaos control (MCC) [36], finite step-length adjustment (FSL) [5], and limited decent method [48]). The computational burdens of improved versions-based MPP search including CMV, MCC, CGA and FSL may be increased for some convex problems. Therefore, the efficiency and robustness are the main challenges of reliability analysis-based MPP in PMA. Consequently, the popular MPP search method is to establish a robust and efficient reliability method which is formulated with simple relation. Consequently, selecting an appropriate step size to compute the sensitivity vector to develop an iterative formula-based MPP search in RBDO using PMA is more important to reduce the computational burden with the stable results.

The adaptive strategy to compute the step size can be provided a dynamical relation with adjusted sensitively vector to improve the efficiency of FORM in reliability loop-based PMA. The sufficient descent criterion is applied to compute the adaptive step size which is utilized to improve the robustness of the directional sensitivity vector in PMA-based MPP search. A hybrid algorithm is proposed using sufficient descent criterion to enhance the efficiency and robustness of inverse reliability method named hybrid adaptive modified chaos control (HAMC). The adjusted MCC and AMV methods are adaptively combined in the proposed reliability method-based PMA. This paper is structured to illustrate the performances of HAMC in several sections as: The RBDO methodology is presented in Sect. 2, while the MPP search-based reliability method is described in Sect. 3. The reliability method-based PMA using proposed HAMC is formulated in Sect. 4. The ability of the proposed HAMC is investigated for both robustness and efficiency by three reliability examples and two RBDO problems in Sect. 5. The conclusions presented in Sect. 6 state that the proposed MPP search formulation provides the superior convergence performances compared to other studied methods.

2 Reliability-based design optimization methodology

2.1 RBDO model

Using objective function under probabilistic constraints in RBDO, the optimization model is typically formulated as follows [29, 49]

$$\begin{aligned}
&\text{find} \quad \boldsymbol{d}\ ,\ \boldsymbol{\mu}_{\boldsymbol{x}} \\
&\min \quad f(\boldsymbol{d},\boldsymbol{X}) \\
&\text{s.t.} \quad P_f[g_j(\boldsymbol{d},\boldsymbol{X}) \le 0\,] \le \Phi(-\beta_t^j) \quad j = 1,2,\dots,p \\
&\boldsymbol{d}^{\mathrm{L}} \le \boldsymbol{d} \le \boldsymbol{d}^{\mathrm{U}}, \quad \boldsymbol{\mu}_{\boldsymbol{x}}^{\mathrm{L}} \le \boldsymbol{\mu}_{\boldsymbol{x}} \le \boldsymbol{\mu}_{\boldsymbol{x}}^{\mathrm{U}},
\end{aligned} \tag{1}$$

where f is the cost function, p is the number of probabilistic constraints (g_j) with reliable domain of β_t^j, Φ is the standard normal cumulative function. Two categories variables as design $\boldsymbol{d} \in R^k$ and random variables $\boldsymbol{X} \in R^m$ are considered in the RBDO model. $\boldsymbol{d}^{\mathrm{L}}$ and $\boldsymbol{\mu}_{\boldsymbol{x}}^{\mathrm{L}}$ are, respectively, the lower bounds for design and lower mean for random variables; while, $\boldsymbol{d}^{\mathrm{U}}$ and $\boldsymbol{\mu}_{\boldsymbol{x}}^{\mathrm{U}}$ are, respectively, upper bounds for design and upper mean for random design. $g(\boldsymbol{d},\boldsymbol{X}) < 0$ denotes the failure domain. Therefore, the reliable failure probability (P_f) can be determined as follows [50, 51]:

$$P_f\left[g(\boldsymbol{d},\boldsymbol{X}) \le 0\right] = \int\limits_{g(\boldsymbol{d},\boldsymbol{X})\le 0} \dots \int f_X(x)\mathrm{d}\boldsymbol{X}, \tag{2}$$

where $f_X(x)$ is the joint probability density function of the random variables $\boldsymbol{X}$. Generally, the MPP-based search approaches are used to compute the optimal results of optimization problems under uncertainties due to their simplicity and efficiency in RBDO [52, 53]. The RBDO-based optimization model is solved based on two main loops as outer loop which is used as an optimization solver to search $\boldsymbol{d}$ and the inner loop to provide the safe constraints which is operated using reliability algorithm-based MPP search in DLA. PMA for evaluating the probabilistic constrains can be provided the high efficiency as computational burden for RBDO problems [54].

2.2 Performance measure approach

By applying the RBDO model in in Eq. (1), the probabilistic constraints in RBDO model-based PMA can be rewritten as

$$g_j(\boldsymbol{d},\boldsymbol{X}) = F_{g_j}^{-1}(\boldsymbol{d},\Phi(-\beta_t^j)) \ge 0, \tag{3}$$

where F_{g_j} is the cumulative distribution function for performance function g_j. Therefore, we have

$$\begin{aligned}
&\text{find} \quad \boldsymbol{d},\ \boldsymbol{\mu}_{\boldsymbol{x}} \\
&\min \quad f(\boldsymbol{d},\boldsymbol{X}) \\
&\text{S.t.} \quad g_j(\boldsymbol{d},\boldsymbol{X}) \ge 0 \quad j = 1,2,\dots,p \\
&\boldsymbol{d}^{\mathrm{L}} \le \boldsymbol{d} \le \boldsymbol{d}^{\mathrm{U}}, \quad \boldsymbol{\mu}_{\boldsymbol{x}}^{\mathrm{L}} \le \boldsymbol{\mu}_{\boldsymbol{x}} \le \boldsymbol{\mu}_{\boldsymbol{x}}^{\mathrm{U}}.
\end{aligned} \tag{4}$$

In PMA, the probabilistic constraints $g_j(\boldsymbol{d},\boldsymbol{X})$ are evaluated using MPP which is related to target reliability index (β_t^j). Consequently, the MPP using FORM is computed for jth constraint as below model:

$$\begin{aligned}
&\text{find} \quad \boldsymbol{U}^* \\
&\min \quad g_j(\boldsymbol{d},\boldsymbol{U}) \\
&\text{s.}t. \quad \|\boldsymbol{U}\| = \beta_t^j,
\end{aligned} \tag{5}$$

where $\boldsymbol{U}^*$ is MPP, $\boldsymbol{U}$ is the independent normal standard variables. In the RBDO-based PMA, the iterative formula to search MPP in reliability loop is essential key to provide the stable results with fast convergence. The HCC and FSL methods were proposed to improve the efficiency and robustness of MPP search-based FORM. In HCC, the MCC is combined with AMV using convexity criterion to improve the efficiency of MCC and robustness of AMV. Thus, the hybrid formulation with AMV can provide the efficient computational results in RBDO. The AMV is combined with modified FORM approaches in hybrid descent mean value [55], adaptive chaos control [28], hybrid mean vale [22], and hybrid self-adjusted mean value [40]. This strategy is applied to enhance the efficiency of improved versions of the MPP search approaches because the MCC and the CMV may provide the inefficient results for convex probabilistic constants.

3 MPP search methods

In this section, the HCC and FSL formulae to search MPP in reliability loop of DLA-based PMA are presented.

3.1 Hybrid modified chaos control (HCC)

In HCC algorithm, the AMV and MCC are adaptively combined based on the convexity criterion as follows [36]:

$$\varsigma_{k+1} = (\boldsymbol{n}_{k+1} - \boldsymbol{n}_k) \cdot (\boldsymbol{n}_k - \boldsymbol{n}_{k+1}), \tag{6}$$

where ς_{k+1} describes the convex type of the performance function, i.e., $\varsigma_{k+1} \le 0$ is concave and $\varsigma_{k+1} > 0$ is convex function at the point $\boldsymbol{U}_{k+1}$. $\boldsymbol{n}_k$ is the normalized sensitivity vector at point $\boldsymbol{U}_k$, which is given as follows:

$$\boldsymbol{n}_k = \frac{\boldsymbol{U}_k}{\|\boldsymbol{U}_k\|}. \tag{7}$$

The convexity criterion in Eq. (6) is a way to select the AMV or MCC for MPP search in DLA. The convexity-based function type is introduced by employing three consecutive iterative data using normalized sensitivity vectors. The convexity criterion is plotted in Fig. 1 for convex type (1a) and concave type (1b) function. It can be seen, ς_{k+1} is cosine of the angle θ_k between $\boldsymbol{n}_{k+1} - \boldsymbol{n}_k$ and $\boldsymbol{n}_k - \boldsymbol{n}_{k-1}$. If the Sign($\varsigma_{k+1}$) as shown in Fig. 1a is larger than zero (convex problem), the iterative point of the AMV method is applied; while, the iterative AMV formula may be oscillated for concave performance measure function as shown in Fig. 1b for Sign(ς_{k+1})<0. Therefore, the hybrid formula of MPP search using convexity criterion is defined to improve the robustness of AMV.

The HCC was proposed by combining AMV and MCC in terms of convexity criterion as below.

For $\varsigma_{k+1} > 0$ with convex performance function (see Fig. 1a), the AMV iterative formula is used to compute the new point as below:

$$\begin{aligned} \boldsymbol{U}_{k+1}^{\mathrm{AMV}} &= \beta_t\, \boldsymbol{n}(u_k^{\mathrm{AMV}}) \\ \boldsymbol{n}(u_k^{\mathrm{AMV}}) &= -\frac{\nabla_u g(\boldsymbol{d}, \boldsymbol{U}_k^{\mathrm{AMV}})}{\left\|\nabla_u g(\boldsymbol{d}, \boldsymbol{U}_k^{\mathrm{AMV}})\right\|}, \end{aligned} \tag{8}$$

where $\boldsymbol{n}(u_k^{\mathrm{AMV}})$ stands for the normalized steepest sensitivity vector.

For the concave performance, i.e., $\varsigma_{k+1} \le 0$, the MCC iterative formula is applied to determine the new point as follows:

$$\begin{aligned} \boldsymbol{U}_{k+1}^{\mathrm{MCC}} &= \beta_t \frac{\tilde{\boldsymbol{n}}_{k+1}}{\|\tilde{\boldsymbol{n}}_{k+1}\|} \\ \tilde{\boldsymbol{n}}_{k+1} &= \boldsymbol{U}_k^{\mathrm{MCC}} + \lambda \mathbf{C}\left[f(u_k) - \boldsymbol{U}_k^{\mathrm{MCC}}\right] \\ f(u_k) &= -\beta_t \frac{\nabla_u g(\boldsymbol{d}, \boldsymbol{U}_k)}{\left\|\nabla_u g(\boldsymbol{d}, \boldsymbol{U}_k)\right\|}, \end{aligned} \tag{9}$$

where $f(u_k)$ represents the new point computed by AMV formulation (8). $\mathbf{C}$ is the $n \times n$ dimensional involutory matrix, which only has one element in each row and each column to be 1 or − 1 and the others are 0. Actually, the unit matrix of $\mathbf{C}$ and λ (i.e., $0 < \lambda < 1$) represents the chaos control factor with a smaller value to achieve stabilization. $\tilde{\boldsymbol{n}}_{k+1}$ stands for the modified sensitivity vector. The MCC method updates the point by moving it on beta hypersphere.

3.2 Finite-step length adjustment (FLS) method

The FSL iterative formula is presented as follows [5]:

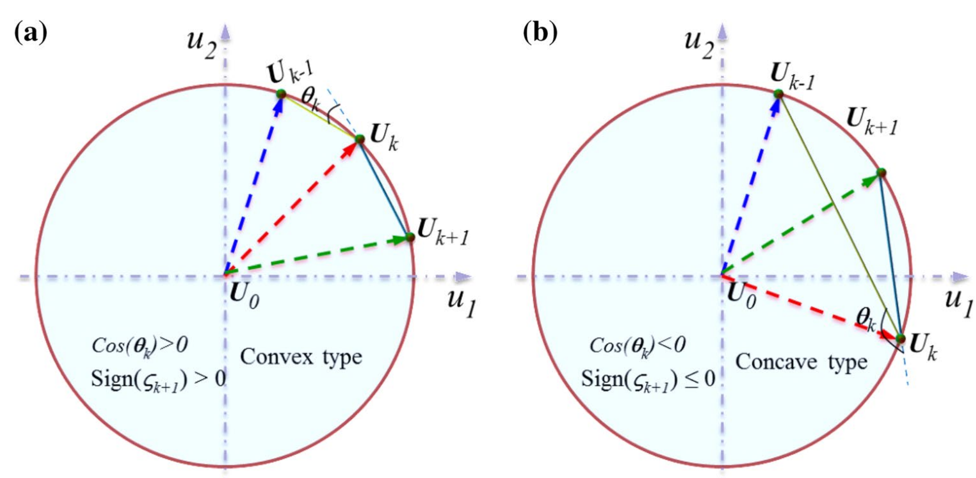

Fig. 1 Convexity criterion for performance functions **a** convex type, **b** concave type

$$\begin{aligned} U_{k+1}^{\mathrm{FSL}} &= \beta_k \frac{U_{k+1}^{\lambda}}{\left\| U_{k+1}^{\lambda} \right\|} \\ U_{k+1}^{\lambda} &= U_k^{\mathrm{FSL}} - \lambda \nabla_u g(d, U_k), \end{aligned} \tag{10}$$

where U_{k+1}^{FSL} and U_k^{FSL} are, respectively, the new and previous points computed by FSL formula. λ is the finite-step length, which is selected a larger value $\lambda > 0$. U_{k+1}^{λ} is the point along the direction of $U_k^{\mathrm{FSL}} - \lambda \nabla_u g(d, U_k)$. if $\lambda \to \infty$ then the FSL formula is similar to the AMV. Consequently, it is a criterion as $\left\| U_{k+1}^{\mathrm{FSL}} - U_k^{\mathrm{FSL}} \right\| \geq \left\| U_k^{\mathrm{FSL}} - U_{k-1}^{\mathrm{FSL}} \right\|$ is used to control instability of the FSL method for highly concave performance functions as set $\lambda = \lambda / c$ when $\left\| U_{k+1}^{\mathrm{FSL}} - U_k^{\mathrm{FSL}} \right\| \geq \left\| U_k^{\mathrm{FSL}} - U_{k-1}^{\mathrm{FSL}} \right\|$ in which c is an adjusting factor as $c \in 2.2$–2.6. A large finite-step length may be provided a slow convergence rate or unstable results for FSL method in highly nonlinear performance functions while the criterion $\left\| U_{k+1}^{\mathrm{FSL}} - U_k^{\mathrm{FSL}} \right\| \geq \left\| U_k^{\mathrm{FSL}} - U_{k-1}^{\mathrm{FSL}} \right\|$ is satisfied its convergences with stable results.

4 Hybrid sufficient modified chaos control

4.1 Sufficient descent criterion

In this section, the sufficient descent criterion is used to provide the hybrid formulation of MPP search algorithm. The sufficient descent criterion is suggested as below:

$$\left\| U_{k+1}^{\mathrm{AMV}} - U_k \right\| < \left\| U_{k-1} - U_k \right\|. \tag{11}$$

The sufficient descent criterion in Eq. (11) is evaluated using the new point computed by AMV and previous points computed by AMC.

The above criterion is simply computed using the results of iterative formula-based MPP search using points $U_{k+1}^{\mathrm{AMV}} U_k$ and U_{k-1}. The new sensitivity vector has a sufficient descent property when Eq. (11) is satisfied based on point U_{k+1}^{AMV}; thus, the AMV can be used to compute the new point. Otherwise, i.e., $\left\| U_{k+1}^{\mathrm{AMV}} - U_k \right\| \geq \left\| U_{k-1} - U_k \right\|$, the new point is obtained based on insufficient search direction, and the criteria are, respectively, plotted in Fig. 2a, b for concave and convex types at the point U_k of performance functions. It can be seen that the proposed criterion is different from the convexity criterion. Fig. 2b schematically shows that the performance function is concave at the point U_k, but the new search direction vector is sufficient descent. This means that the new steepest descent search direction vector ($\tilde{n}_{k+1}$) can be computed using the AMV approach ($n(u_k^{\mathrm{AMV}})$) based on the descent criterion. Nevertheless, the new point should be computed using the MCC-based MPP search method by applying convexity criterion (see Fig. 2b). However, the descent criterion in Fig. 2b is sufficient; thus, the AMV approach is accurately provided the sufficient search direction to compute the new point. This means that the convergence rate of hybrid method increases using the sufficient decent condition. Therefore, a hybrid formula based on the proposed sufficient descent criterion is converged faster than the HCC, which is developed based on convexity criterion.

Figure 3 illustrates insufficient descent iteration at new point of reliability method-based MPP search that this figure includes convex (Fig. 3a) and concave (Fig. 3b) functions which is given by convexity criterion. The insufficient descent condition presented in Fig. 3 showed that the improved versions of MPP search should be used in reliability loop of DLA, while AMV can be used to search MPP based on convexity criterion in HCC as showed in Fig. 3a. In this current paper, the sufficient descent condition can be used to adapt the sensitivity vector using AMV when $\left\| U_{k+1}^{\mathrm{AMV}} - U_k \right\| < \left\| U_{k-1} - U_k \right\|$ or the modified MPP search method in the iterative FORM formula.

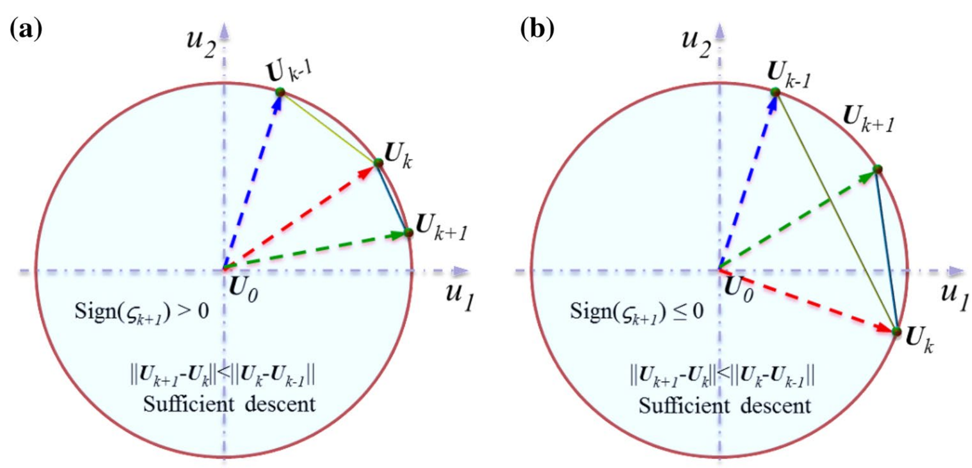

Fig. 2 Sufficient descent for performance functions **a** convex type, **b** concave type

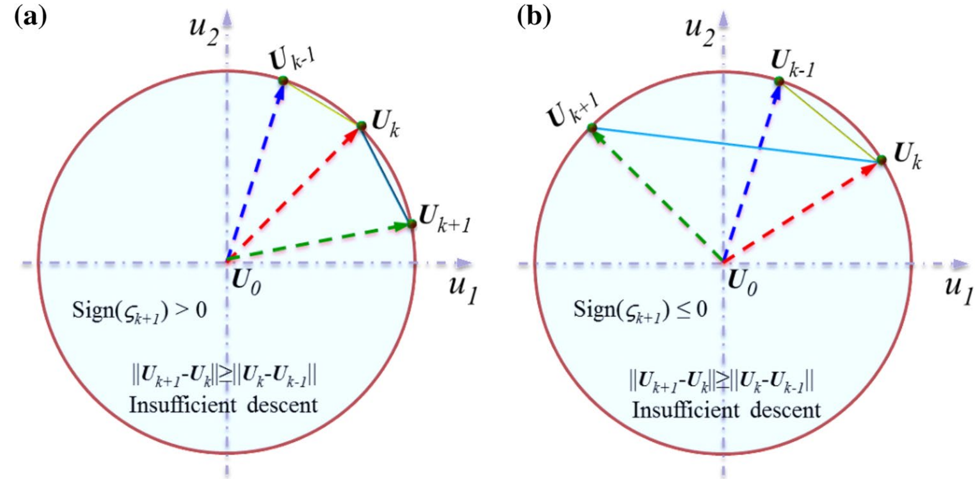

Fig. 3 Insufficient descent for performance functions **a** convex type, **b** concave type

4.2 Adaptive modified chaos control (AMC) method

It supposes that the length between the new point-based AMV and previous point as $\left\|U_{k+1}^{\mathrm{AMV}} - U_k^H\right\|$ (where U_k^H is the point computed by hybrid formulation) is less than the relative previous length as $\left\|U_k^H - U_{k-1}^H\right\|$ which are used to compute the sufficient criterion descent criterion, then the AMV has a successful iteration. This means that $\left\|U_{k+1}^{\mathrm{AMV}} - U_k^H\right\| < \left\|U_k^H - U_{k-1}^H\right\|$; thus, we have $\left\|U_k^H - U_{k-1}^H\right\| \to 0$ when $k \to \infty$. Using this creation, the stable convergence is obtained based on a fixed point as $\left\|U_k^H - U_{k-1}^H\right\| \to 0$.

It can be concluded that the robustness of hybrid algorithm based on criterion $\left\|U_{k+1}^{\mathrm{AMV}} - U_k^H\right\| < \left\|U_k^H - U_{k-1}^H\right\|$ is dependent on modified algorithm of MPP search for reliability analysis. The stable convergence of hybrid iterative formulation can be guaranteed when the modified reliability algorithm provides robust results. In this paper, a sufficient iterative method is proposed using the sufficient descent condition which is applied to determine the chaos control factor as below simple dynamical relation:

$$\lambda_k < \frac{\left\|U_k^H - U_{k-1}^H\right\|}{\left\|U_{k+1}^{\mathrm{AMV}} - U_k^H\right\|} = \frac{\left\|U_k^H - U_{k-1}^H\right\|}{\delta^k \times \left\|U_{k+1}^{\mathrm{AMV}} - U_k^H\right\|}. \tag{12}$$

It is obvious $0 < \lambda_k < \frac{1}{\delta^k}$ and $\lambda_k \to 0$ when $k \to \infty$, where δ represents the adaptive coefficient as $1.01 < \delta \leq 1.2$. Consequently, $\frac{\|U_k^H - U_{k-1}^H\|}{\|U_{k+1}^{\mathrm{AMV}} - U_k^H\|} \approx 0 \Rightarrow U_k^H \approx U_{k-1}^H$, while $\left\|U_{k+1}^{\mathrm{AMV}} - U_k^H\right\|$ is given a larger value. This means that the proposed sensitivity vector which is computed based on the sufficient descent criterion can provide stable results by considering $||U_k^H - U_{k-1}^H|| < ||U_{k-2}^H - U_{k-1}^H||$ from previous results. The iterative formula for reliability analysis using the adaptive modified chaos control (AMC) presented in Eq. (13) which is satisfied the sufficient descent condition as below:

$$\begin{aligned} U_{k+1}^{\mathrm{AMC}} &= \beta_t \frac{\tilde{n}_{k+1}^A}{\left\|\tilde{n}_{k+1}^A\right\|} \\ \tilde{n}_{k+1}^A &= U_k^{\mathrm{AMC}} + \lambda_k \left[U_{k+1}^{\mathrm{AMV}} - U_k^{\mathrm{AMC}}\right]. \end{aligned} \tag{13}$$

The $\tilde{n}^A$ represents normalized sensitivity vector which is computed using modified chaos control factor with adaptive property using sufficient descent criterion. The proposed condition is as simple as MCC and AMV, while the sufficient descent condition is applied to provide the stable results for concave and convex performance functions. The modified chaos control method as well as the adjusted chaos control in the method proposed by Li et al. [33] is used in this search direction. Unlike the chaos control factor proposed by Li et al. [33], the proposed chaos control is dynamically adjusted using sufficient descent condition in AMC. The proposed AMC is adapted using a dynamical accelerated chaos control which is the major difference between the AMC and MCC in HCC or FSL. The dynamical chaos control in AMC is adaptively accelerated between 1 and 0 for problem with insufficient properties which is given the smaller values at the final iterations. Therefore, it can provide the stable results for highly nonlinear problems, finally.

4.3 Hybrid sufficient reliability method

The proposed AMC is applied for hybrid iterative formula by applying the AMV based on sufficient descent condition. The hybrid formula-based AMC and AMV approaches are utilized to improve the robustness and efficiency of FORM-based MPP search in RBDO. The sufficient descent criterion in Eq. (11) is used to hybrid formulation of AMC and AMV. In addition, hybrid method for MPP search ($\boldsymbol{U}_{k+1}^{H}$) is given as follows:

$$\boldsymbol{U}_{k+1}^{H}=\begin{cases}\boldsymbol{U}_{k+1}^{\mathrm{AMV}} & \left\|\boldsymbol{U}_{k+1}^{\mathrm{AMV}}-\boldsymbol{U}_{k}^{H}\right\|<\left\|\boldsymbol{U}_{k}^{H}-\boldsymbol{U}_{k-1}^{H}\right\| \\ \boldsymbol{U}_{k+1}^{\mathrm{AMV}} & \left\|\boldsymbol{U}_{k+1}^{\mathrm{AMV}}-\boldsymbol{U}_{k}^{H}\right\|\geq\left\|\boldsymbol{U}_{k}^{H}-\boldsymbol{U}_{k-1}^{H}\right\|\end{cases}, \quad (14)$$

where $\boldsymbol{U}_{k+1}^{\mathrm{AMV}}$ is determined based on Eq. (8) and $\boldsymbol{U}_{k+1}^{\mathrm{AMC}}$ is computed based on the proposed AMC formula in Eq. (13). The AMC and AMV methods are applied in the proposed hybrid adaptive modified chaos control (HAMC) using sufficient descent criterion.

Based on Eq. (14), the step size in HAMC method is varied between 0 and 1 which is equal to 1 for performance functions with sufficient descent condition. AMC is used when sufficient condition is not satisfied; thus, the chaos control is adjusted to less than 1 using Eq. (12). Therefore, hybrid formulation in Eq. (14) may improve the robustness inverse reliability method.

4.4 Algorithm of hybrid adaptive modified chaos control

The iterative procedure of the proposed HAMC method is described in this section. The descent condition is applied for hybrid AMV with AMC by the following steps:

Step 1: Define performance function $g(\boldsymbol{d},\boldsymbol{X})$ and β_t. Given statistical random variables μ and σ. Set $k=0$, and ε (stopping criterion).

Step 2: Transfer random variable into normal standard space $u=\boldsymbol{\Phi}^{-1}\{F_X(x)\}$.

Step 3: Compute $\nabla_u g(\boldsymbol{d},\boldsymbol{U}_k^{\mathrm{AMV}})$ and new point $\boldsymbol{U}_{k+1}^{\mathrm{AMV}}$ using Eq. (8).

Step 4: For $k=1$, $\boldsymbol{U}_{k+1}^{H}=\boldsymbol{U}_{k+1}^{\mathrm{AMV}}$Otherwise i.e. $k>1$, If $\left\|\boldsymbol{U}_{k+1}^{\mathrm{AMV}}-\boldsymbol{U}_{k}^{H}\right\|<\left\|\boldsymbol{U}_{k}^{H}-\boldsymbol{U}_{k-1}^{H}\right\|$, then $\boldsymbol{U}_{k+1}^{H}=\boldsymbol{U}_{k+1}^{\mathrm{AMV}}$Else (i.e. $\left\|\boldsymbol{U}_{k+1}^{\mathrm{AMV}}-\boldsymbol{U}_{k}^{H}\right\|\leq\left\|\boldsymbol{U}_{k}^{H}-\boldsymbol{U}_{k-1}^{H}\right\|$), using Eq. (13) $\boldsymbol{U}_{k+1}^{H}=\boldsymbol{U}_{k+1}^{\mathrm{AMC}}$

Step 5: If $\left\|\boldsymbol{U}_{k+1}^{H}-\boldsymbol{U}_{k}^{H}\right\|/\left\|\boldsymbol{U}_{k}^{H}\right\|<\varepsilon$, then stop, print $\boldsymbol{U}^{*}=\boldsymbol{U}_{k+1}^{H}$ and $g(\boldsymbol{d},\boldsymbol{U}_{k+1}^{H})$Else $k=k+1$ and go to Step 2.

Based on the sufficient descent condition for combination of the MPP search methods, the flowchart of HAMC is plotted in Fig. 4. As can be seen, the iterative formula is as simple as the AMV, and the hybrid formula of MPP search is computed by the new and pervious results. The sufficient descent condition and the adaptive step size are used to compute the adaptive search direction vector in HAMC. However, steepest descent search direction with a constant step size (i.e., 1 or 0.1) is implemented in HCC using the convexity criterion. As seen, the step size is adjusted based on the different between $\left\|\boldsymbol{U}_{k}^{H}-\boldsymbol{U}_{k-1}^{H}\right\|$ and $\left\|\boldsymbol{U}_{k+1}^{\mathrm{AMV}}-\boldsymbol{U}_{k}^{H}\right\|$. The main differences between the proposed

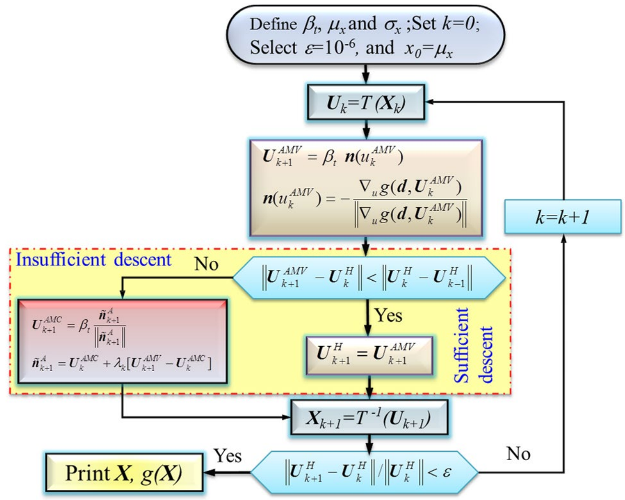

Fig. 4 Iterative framework of HAMC algorithm-based MPP search

HACM and HCC are the implementation of sufficient descent criterion and a new adaptive modified search direction in Eq. (12).

5 Illustrative examples

The robustness and efficiency of proposed HAMC with descent criterion are investigated through three nonlinear performance functions and two RBDO problems with nonlinear convex and concave probabilistic constraints which are computed based on DLA-based PMA. The adaptive coefficient in HAMC is set as $\delta=1.1$. The results of HAMC methods are compared with the AMV, MCC (with the parameters of $\mathbf{C}=\mathbf{I}$ and $\lambda=0.2$), FSL (its parameters are given to be $\lambda=30$ and $c=2.5$), and HCC (parameters are given as $\mathbf{C}=\mathbf{I}$ and $\lambda=0.2$). For this purpose, the numbers of computations of $\nabla g(\boldsymbol{U})$ (central finite difference), the performance value at MPP ($g(\boldsymbol{X}^{*})$) for each performance function, the objective and the number of evaluating probabilistic constraints for RBDO problems are used to compare reliability algorithms with stopping criterion ($\varepsilon=10^{-6}$).

5.1 Reliability examples

Three nonlinear performance functions are used as

Example 1 $g=0.3x_1^2x_2-x_2+0.8x_1+1$, in which, $x_1\sim N(0,0.55^2)$, $x_2\sim N(6,0.55^2)$ and $\beta_t=3.0$.

Example 2 $g=x_1^4+2x_2^4-20$, in which, $x_1\sim N(10,\ 5^2)$, $x_2\sim N(12,\ 5^2)$ and $\beta_t=2.5$.

Example 3 $g = 0.489x_3x_7 + 0.843x_5x_6 - 0.0432x_9x_{10} + 0.0556x_9x_{11} + 0.000786x_{11}^2 - 0.75$, in which, $i = 1 \sim 7$ $x_i \sim N(1,\ 0.005^2)$, $i = 8 \sim 9$ $x_i \sim N(0.3,\ 0.006^2)$, $i = 9 \sim 10$ $x_i \sim N(0,\ 10.0^2)$ and $\beta_t = 3.0$

The converged results of Examples 1–3 for different reliability methods are listed in Table 1. It is obvious that the proposed hybrid HAMC method by combining the AMV and AMC algorithms improves the robustness of AMV for these highly nonlinear performance functions, and it enhances the efficiency of MCC by using the adaptive chaos control. The HAMC method is accurately converged to stable results for all examples, while the AMV method is periodically yielded to unstable solutions. It is evident that the MCC and SLA are more robust than AMV. The HAMC is as robust as the MCC and HCC, but is significantly more computationally efficient. The proposed HAMC using hybrid sensitivity vector with sufficient descent criterion improves the efficiency of MPP search-based FORM formula compared to other modified methods.

Figure 5 illustrates the convergence histories of different reliability methods for Example 1. As seen, the HCC, SLA, and HAMC are converged to stable results as $g(\boldsymbol{X}^*) = -6.7116$ and $\boldsymbol{X}^* = (-0.15416, 7.64278)$. The SLA and HAMC are converged with different iterations. Consequently, it is shown that different sensitivity vectors are obtained by HAMC and SAL formulas.

By comparing the results of Table 2 for Example 2, the MCC and HCC are not converged to stable solutions based on $\lambda = 0.2$. The AMV produces periodic-2 solutions as $g(\boldsymbol{X}^*) = (10207.93, 41{,}533.52)$; while The HAMC yields stable results as 50.30983 after 20 iterations which is closely in agreement with the results extracted from Ref. [36] i.e. $g(\boldsymbol{X}^*) = 50.3096$. The proposed HAMC method is efficient, while the HCC is the more inefficient method based on the parameters $\boldsymbol{C} = \boldsymbol{I}$ and $\lambda = 0.15$ among other inverse reliability methods.

The convergence histories-based performance function for Example 3 is presented in Table 3. It can be seen that the HAMC is slightly more efficient than the MCC and HCC schemes. The HAMC is converged about three times faster than the MCC method. The AMV yields to 2-periodic unstable results as 1.395544 and 0.65651 while the proposed HAMC are efficiently converged to stable performance value as $g(\boldsymbol{X}^*) = 0.075282$ after 27 iterations which is more agreement with the extracted results from Ref. [56].

Table 1 Results of different reliability-based MPP search methods

Method	Example 1	Example 2	Example 3
AMV	Periodic-2	Periodic-2	Periodic-2
MCC	− 6.71162 (36)	50.30983 (67)[a]	0.07528 (44)
HCC	− 6.71162 (43)	50.30983 (70)[a]	0.07528 (43)
SLA	− 6.71162 (67)	50.30983 (64)	0.07528 (24)
Proposed HAMC	− 6.71121 (17)	50.30983 (20)	0.07528 (16)

[a]These methods yielded to unstable solutions as 2-periodic with step size $\lambda = 0.2$, thus to achieve stabilization, the step size is selected to be 0.15

5.2 RBDO examples

Two RBDO examples are selected herein to illustrate the performances of the proposed HAMC which is formulated using sufficient descent criterion for RBDO problem. Two examples are selected to demonstrate the performances of the proposed HAMC method with highly nonlinear constraints in terms of mathematical and structural optimization problems.

Example 4 A nonlinear mathematical RBDO problem is given as [19]:

$$
\begin{aligned}
&\text{Find} \quad \boldsymbol{d} = [d_1, d_2]^{\mathrm{T}} \\
\min \quad & f(\boldsymbol{d}) = d_1 + d_2 \\
&\text{S.t.} \quad P_f[g_j(\boldsymbol{X}) > 0] \le \Phi(-\beta_t^j), \quad j = 1, 2, 3 \\
&\text{where} \quad g_1 = 1 - \frac{x_1^2 x_2}{20} \\
&g_2 = 1 - \frac{(x_1 + x_2 - 5)^2}{30} - \frac{(x_1 - x_2 - 12)^2}{120} \\
&g_3 = 1 - \frac{80}{x_1^2 + 8x_2 + 5} \\
&0 \le d_i \le 10, \quad x_i \sim N(d_i,\ 1^2) \quad \text{for} \quad i = 1, 2 \\
&\boldsymbol{d}^0 = [5,\ 5], \quad \beta_t^1 = \beta_t^2 = \beta_t^3 = 2.5.
\end{aligned} \tag{15}
$$

This example includes three probabilistic constraints g_1, g_2, g_3 and two Gumbel random variables with means of design point and standard deviation of 1. The RBDO results of different DLA-based PMA methods are summarized in Table 4 with stopping criterion $\varepsilon = 10^{-6}$. It can be found from Table 4 that the results of different reliability methods are almost equal to the results extracted from Ref. [56]. In addition, results indicate that the AMV yields unstable solution, but the MCC, HCC, SLA, and proposed HAMC methods are robustly converged. The MCC is most inefficient among RBDO-based PMA methods. However, the HAMC is slightly more efficient than other existing PMA-based MPP search approaches. The HAMC is converged about three times faster than the MCC method. It can be concluded that the proposed PMA-based HAMC can slightly enhance the efficiency of MCC and also improves the robustness of AMV method for this nonlinear problem.

The converged optimum and number of evaluating the probabilistic constraints for different stopping criteria (ε)

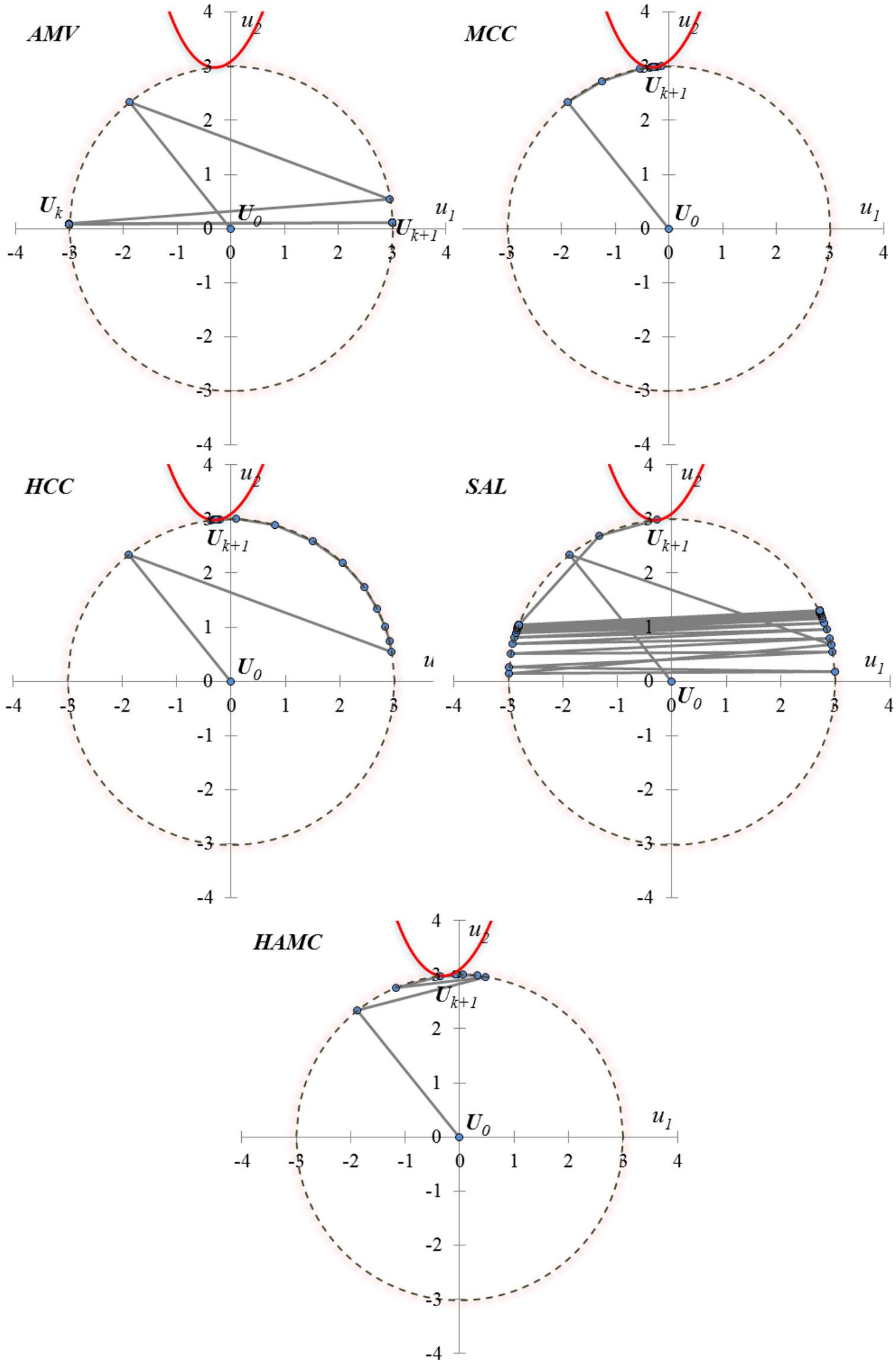

Fig. 5 Iterative histories of different reliability methods for Example

are listed in Table 5 for the RBDO example in Eq. (15). The HAMC converges faster than HCC and SLA methods, while the HCC is computationally more efficient than the MCC and SLA.

Example 5 Vehicle side impact example [19].

Based on a quadratic regression-based response surface method, the optimization model for was extracted for vehicle crashworthiness as below:

Table 2 Performance function of different reliability methods for Example 2

Iteration	AMV	MCC[a]	HCC[a]	SLA	HAMC
1	51,452	51,452	51,452	51,452	51,452
2	1793.377	1793.377	1793.377	1793.377	1793.377
3	41,491.06	449.8948	41,491.06	41,488.57	8671.156
…	…	…	…	…	…
19	41,533.52	50.33174	50.39994	41,102.38	50.30983
20	10,207.93	50.32513	50.37132	10,172.09	50.30983
…	…	…	…	…	
26	10,207.93	50.31184	50.31788	10,117.4	
27	41,533.52	50.31127	50.31563	38,879.33	
…	…	…	…	…	
63	41,533.52	50.30983	50.30983	50.30983	
64	10,207.93	50.30983	50.30983	50.30983	
65	41,533.52	50.30983	50.30983		
66	10,207.93	50.30983	50.30983		
67	41,533.52	50.30983	50.30983		
68	10,207.93		50.30983		
69	41,533.52		50.30983		
70	10,207.93		50.30983		
71	41,533.52		50.30983		
72	10,207.93				
73	41,533.52				

[a]The step size is considered as $\lambda=0.15$ to achieve the stable results for these methods

$$
\begin{aligned}
&\text{Find} \quad \boldsymbol{d} = [x_1 \sim x_9]^{\mathrm{T}} \\
&\min \quad f(\boldsymbol{d}) \\
&\text{S.t.} \quad P_f[F_{\mathrm{AL}} \leq 1\,\mathrm{kN}] \geq R_1 \\
&P_f[D_{\mathrm{up}} \leq 32\,\mathrm{cm}] \geq R_2 \\
&P_f[D_{\mathrm{mid}} \leq 32\,\mathrm{cm}] \geq R_3 \\
&P_f[D_{\mathrm{low}} \leq 32\,\mathrm{cm}] \geq R_4 \\
&P_f[\mathrm{VC}_{\mathrm{up}} \leq 0.32\,\mathrm{cm}] \geq R_5 \\
&P_f[\mathrm{VC}_{\mathrm{mid}} \leq 0.32\,\mathrm{cm}] \geq R_6 \\
&P_f[\mathrm{VC}_{\mathrm{low}} \leq 0.32\,\mathrm{cm}] \geq R_7 \\
&P_f[F_{\mathrm{ps}} \leq 4\,\mathrm{kN}] \geq R_8 \\
&P_f[V_{\text{B-pillar}} \leq 9.9\,\mathrm{m/cm}] \geq R_9 \\
&P_f[V_{\mathrm{door}} \leq 15.69\,\mathrm{m/cm}] \geq R_{10}, \\
&\text{where} \\
&\boldsymbol{\mu}^L \leq \boldsymbol{\mu}_i \leq \boldsymbol{\mu}^U, \quad i = 1 \sim 9;\ \mu_{10}, \mu_{11} = 0, \\
&\boldsymbol{d}^0 = \boldsymbol{\mu}_0, \quad \beta_t^1 \sim \beta_t^{10} = 3.0.
\end{aligned} \tag{16}
$$

With

Table 3 Performance function of different reliability methods for Example 3

Iteration	AMV	MCC	HCC	SLA	HAMC	HSLA
1	0.582	0.582	0.582	0.582	0.582	0.582
2	0.31493	0.31493	0.31493	0.31493	0.31493	0.31493
3	1.14586	0.161093	1.14586	0.582463	0.214331	0.912962
…	…	…	…	…	…	…
15	1.395544	0.075284	0.075283	0.075282	0.075282	0.075282
16	0.65651	0.075283	0.075282	0.075282	0.075282	0.075282
…	…	…	…	…		…
20	0.65651	0.075282	0.075282	0.075282		0.075282
21	1.395544	0.075282	0.075282	0.075282		0.075282
22	0.65651	0.075282	0.075282	0.075282		
23	1.395544	0.075282	0.075282	0.075282		
24	0.65651	0.075282	0.075282	0.075282		
…	…	…	…			
42	0.65651	0.075282	0.075282			
43	1.395544	0.075282	0.075282			
44	0.65651	0.075282				
…	…					
423	1.395544					
424	0.65651					
425	1.395544					
426	0.65651					

$$
\begin{aligned}
f &= 1.98 + 4.9x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7, \\
F_{\mathrm{AL}} &= 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10}, \\
D_{\mathrm{up}} &= 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.77x_7x_8 + 0.32x_9x_{10}, \\
D_{\mathrm{mid}} &= 33.86 + 2.95x_3 + 0.1792x_{10} - 5.05x_1x_2 - 11x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22x_8x_9, \\
D_{\mathrm{low}} &= 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10}, \\
\mathrm{VC}_{\mathrm{up}} &= 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} \\
&\quad + 0.08045x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11}, \\
\mathrm{VC}_{\mathrm{mid}} &= 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 \\
&\quad + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} \\
&\quad - 0.000535x_6x_{10} + 0.00121x_8x_{11} + 0.00184x_9x_{10} - 0.02x_2^2, \\
\mathrm{VC}_{\mathrm{low}} &= 0.74 + 0.61x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2, \\
F_{\mathrm{ps}} &= 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2, \\
V_{\mathrm{B\text{-}pillar}} &= 10.58 - 0.647x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10}, \\
V_{\mathrm{door}} &= 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11}^2,
\end{aligned}
\tag{17}
$$

where F_{AL} and F_{ps} are dummy abdomen load and dummy pubic symphysis force, respectively. D_{up},D_{mid} and D_{low} are the dummy upper rib, middle rib and lower rib; while $\mathrm{VC}_{\mathrm{up}}$, $\mathrm{VC}_{\mathrm{mid}}$ and $\mathrm{VC}_{\mathrm{low}}$ are the dummy upper chest, middle chest and lower chest, respectively. Also, $V_{\mathrm{B\text{ - }pillar}}$ and V_{door} represent the velocity at the middle B-pillar position and door belt line, respectively. This example involves eleven random variables, as listed in Table 6.

The optimal design point, objective function at the optimal design point, and the probabilistic constraints values, which are evaluated on the beta-hypersphere, are listed in Table 7 to compare the accuracy of PMA-based reliability methods of HAMC with extracted results from Ref. [19]. As can be seen from Table 7, the constraints g_2, g_8, and g_{10} are active in RBDO using HAMC for this example. The AMV is not converged, but the MCC, HCC and SLA are converged to stable results as well as the proposed HAMC. The HAMC formula using PMA is improved the stability of AMV, and is enhanced the computational efficiency about six, five and three times faster than the MCC, HCC and SLA methods, respectively. The MCC, HCC, SLA, HAMC for PMA can provide accurate results and more agreement with the results obtained from Ref. [19].

Table 4 The results of RBDO problem for Example 4

Method	Design variables	Objective	Iterations	Iterate $g_1\backslash g_2\backslash g_3$	F Evaluations
AMV	–	–	–	Not converged	
MCC	(3.760865, 3.691388)	7.452252	11	1545\492\3003	5040
HCC	(3.760865, 3.691388)	7.452252	11	1098\591\582	2271
SLA	(3.760864, 3.691388)	7.452252	11	354\592\990	1936
HAMC	(3.760864, 3.691386)	7.452251	11	294\717\582	1593
SMCC[a]	(3.76086, 3.69139)	7.45225	–	801/762/1959	3522

[a]Results are extracted from Ref. [56] based on self-adaptive modified chaos control method (SMCC)

Table 5 The results of RBDO problem in Example 4 for different stopping criterion (ε)

Method	$\varepsilon = 10^{-4}$	$\varepsilon = 10^{-6}$	$\varepsilon = 10^{-8}$	$\varepsilon = 10^{-10}$
AMV	–	–	–	–
MCC	7.452258 (3096)[a]	7.452252 (5040)	7.452252 (6981)	7.452252 (8917)
HCC	7.452253 (1338)	7.452252 (2271)	7.452252 (3204)	7.452252 (4116)
SLA	7.452254 (1363)	7.452252 (1936)	7.452252 (2509)	7.452252 (3103)
HAMC	7.452256 (1152)	7.452251 (1593)	7.452252 (2223)	7.452252 (3014)

[a]Objective value at the optimum point and number of evaluating probabilistic constraints

Table 6 Design and random variables for vehicle side impact

Random variables		Std. dev.	Upper bound	Nominal/initial	Lower bound
x_1	B-pillar inner	0.03	0.5	1.0	1.5
x_2	B-pillar reinforcement	0.03	0.45	1.0	1.35
x_3	Floor side inner	0.03	0.5	1.0	1.5
x_4	Cross member	0.03	0.5	1.0	1.5
x_5	Door beam	0.05	0.875	2.0	2.625
x_6	Door belt line reinforcement	0.03	0.4	1.0	1.2
x_7	Roof rail	0.03	0.4	1.0	1.2
x_8	Material of B-pillar inner	0.006	0.192	0.3	0.345
x_9	Material of floor side inner	0.006	0.192	0.3	0.345
x_{10}	Barrier height	10	− 30	0	30
x_{11}	Barrier height position	10	− 30	0	30

Table 7 Comparing the results for vehicle side impact example

Method	AMV	MCC	HCC	SLA	HAMC	RDS[a]
x_1	Not converged	0.87831	0.87831	0.81409	0.81095	0.8008
x_2		1.35	1.35	1.35	1.35	1.35
x_3		0.82011	0.82011	0.72782	0.72799	0.7134
x_4		1.5	1.5	1.5	1.5	1.5
x_5		0.92084	0.92084	0.93853	0.93850	0.875
x_6		1.2	1.2	1.2	1.2	1.2
x_7		0.4	0.4	0.4	0.4	0.4
x_8		0.345	0.345	0.345	0.345	0.345
x_9		0.192	0.192	0.192	0.192	0.192
Objective	–	29.75871	29.75871	28.83133	28.81701	28.5526
F evaluation	–	67728	52,248	30,432	10,668	N/A
Constraints						
g_1 (X^a)	–	0.5307	0.5307	0.5221	0.5259	0.5424
g_2 (X^a)	–	0	0	0	0	0
g_3 (X^a)	–	1.8565	1.8565	1.7054	1.6945	1.6307
g_4 (X^a)	–	1.8558	1.8558	1.8366	1.8251	1.871
g_5 (X^a)	–	0.0796	0.0796	0.0748	0.0745	0.0789
g_6 (X^a)	–	0.1056	0.1056	0.0727	0.0725	0.1056
g_7 (X^a)	–	0.0181	0.0181	0.0163	0.0164	0.0162
g_8 (X^a)	–	0	0	0	0	0
g_9 (X^a)	–	0.3795	0.3795	0.3763	0.3824	0.3821
g_{10} (X^a)	–	0	0	0	0	0.2013

[a]Results are extracted from Shan and Wang [19] based on reliable design space (RDS)

The proposed hybrid formula using sufficient descent condition can be applied for reliability loop in RBDO problems, successfully. The sufficient descent condition can be used to enhance the performance convergence including both efficiency and robustness of the reliability methods as well as the convexity criterion. The HAMC adaptively implemented the modified sensitivity vector based on the proposed AMC in Eq. (13) and AMV in Eq. (8). Thus, the proposed hybrid method can be applied for RBDO of real engineering problems, in future.

6 Conclusions

In this paper, a criterion named a sufficient descent is applied for adaptive modified sensitivity vector of reliability loop-based MPP search, which is used to evaluate probabilistic constraints in reliability-based design optimization (RBDO) using performance measure approach (PMA). An adaptive modified chaos control (AMC) method is proposed by developing a dynamical chaos control using sufficient descent condition. The AMV and proposed AMC methods are adaptively implemented using sufficient descent criterion in RBDO-based PMA. The results of reliability examples and RBDO problems for proposed hybrid method named HAMC are compared with several reliability methods. The results indicated that the HAMC improved the robustness of AMV for highly nonlinear functions and it enhanced the efficiency of existing modified versions of MPP search. The HAMC has top performances for accuracy and efficiency compared to other existing RBDO-based PMA. To be specific, the HAMC is more robust than the AMV and is more computationally efficient than the MCC, HCC, and SAL. Therefore, the HAMC can be implemented to evaluate the probabilistic constraints of real and complex RBDO engineering problems in future.

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