

A Monte Carlo technique in safety assessment of slope under seismic condition

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Abstract In geotechnical engineering, stabilization of slopes is one of the significant issues that needs to be considered especially in seismic situation. Evaluation and precise prediction of factor of safety (FOS) of slopes can be useful for designing/analyzing very important structures such as dams and highways. Hence, in the present study, an attempt has been done to evaluate/predict FOS of many homogenous slopes in different conditions using Monte Carlo (MC) simulation technique. For achieving this aim, the most important parameters on the FOS were investigated, and finally, slope height (H), slope angle (α), cohesion (C), angle of internal friction (ϕ) and peak ground acceleration (PGA) were selected as model inputs to estimate FOS values. In the first step of analysis, a multiple linear regression (MLR) equation was developed and then it was used for evaluation and prediction by MC technique.

Generally, MC model simulated FOS of less than 1.18, lower and higher than measured and predicted FOS values, respectively. However, the results of MC simulation for the FOS values of more than 1.33, is higher than those measured and predicted FOS values. As a result, the mean of FOS values simulated by MC was very close to the mean of actual FOS values. Moreover, results of sensitivity analysis demonstrated that the (ϕ), among other parameters, is the most effective one on FOS. The obtained results indicated that MC is a reliable approach for evaluating and estimating FOS of slopes with high degree of performance.

Keywords Seismic FOS · Monte Carlo simulation · Multiple linear regression · Sensitivity analysis

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1 Introduction

The precise analysis of slope stability is an important task in the construction and designs various civil engineering structures, such as highways, dams, open pits and excavations. As defined by some researchers (e.g., [1–3]), the ratio of shear strength to driving stress generally due to gravitational force along the failure plane is defined as the factor of safety (FOS). The evaluation of the FOS is a common approach to analyze the slope stability [4]. Generally, $FOS < 1$ and $FOS > 1$ signify the slope to be unstable and stable, respectively [1]. The slice of soil mass above the failure surface is also a key parameter for the slope stability analysis [5, 6]. To assess the slope stability, different methods, such as limit analysis, limit equilibrium, boundary and finite element methods as well as finite difference method can be utilized. Among them, limit equilibrium method (LEM) is the most traditional method to estimate stability of a slope and widely used in many studies [1–3, 7]. By reviewing the previous investigations, unit weight (γ), slope height (H), slope angle (α), pore pressure ratio, cohesion (C), angle of internal friction (ϕ) and peak ground acceleration (PGA) are the most effective parameters on slope stability [8–11]. In the literature, many attempts have been done to predict the slope stability with a high degree of accuracy. It has been tried to simulate FOS using several methods, such as logistic regression approach, nonlinear failure statistical method and geographic information system (GIS) [12–15]. Apart from the mentioned methods, the use of soft computing techniques in the field of geotechnical engineering (e.g., [16–19]), especially for FOS simulation has been highlighted by various researchers [20–24]. Least square support vector machine (LSSVM) and artificial neural network (ANN) were developed to simulate FOS in the study conducted by Samui and Kothari [10]. They used γ , H , α , C , ϕ and pore water pressure coefficient as the model inputs. According to their result, LSSVM model can simulate FOS with higher level of accuracy in comparison with ANN model. A comprehensive study to predict the critical FOS of homogeneous finite slopes, was presented by Erzin and Cetin [11], using multiple regression (MR) and ANN. In their study, two different ANN models were developed. The first ANN model had five input parameters, including γ , H , α , C , ϕ , while the second ANN model had four input parameters, including H , α , C , ϕ . They showed that the first ANN model (with five input parameters) can simulate FOS better than the second one (with four input parameters). As a conclusion, γ is one of the most effective parameters on the FOS. In addition, it was found that the ANN model can be performed for FOS simulation with a greater degree of confidence in comparison with MR model. Gordan et al.

[25] employed a hybrid model based on ANN and particle swarm optimization (PSO) for the FOS simulation, using 699 datasets. The simulated values by PSO-ANN model have been compared to obtained results of ANN model. In their study, H , α , C , ϕ and peak ground acceleration (PGA) were used as input parameters. Based on their results, it was observed that PSO-ANN model shows better simulation capability than ANN model.

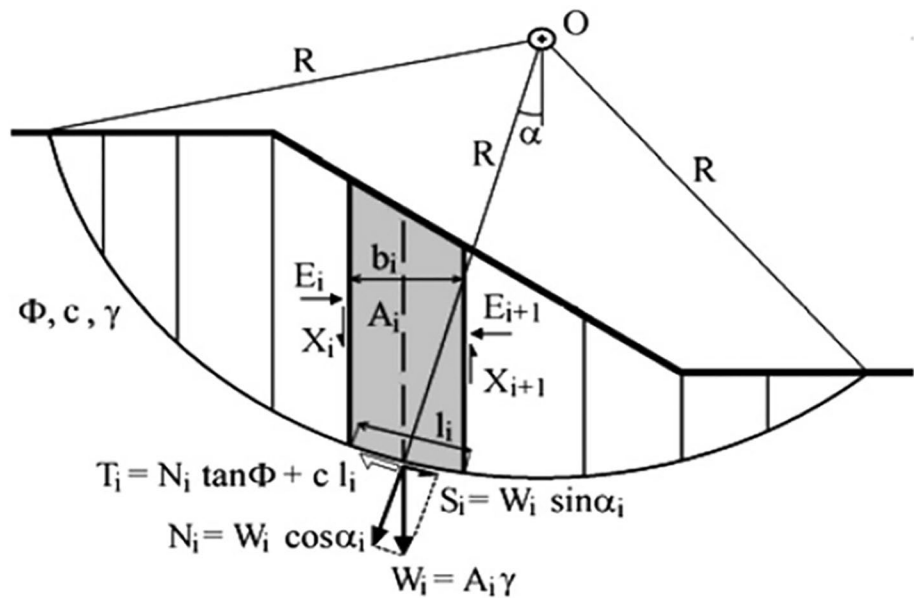
In the recent years, application of Monte Carlo (MC) approach, as a stochastic simulation method, has been expanding in the field of mining and geotechnical engineering. As an example, Sari et al. [26] developed a MC model to evaluate the blast-induced back-break at the Sungun copper mine, Iran. In their research, spacing, burden, specific charge, stemming and geometric stiffness ratios were set as input parameters. They successfully indicated that the MC is a practical tool to evaluate the blast-induced back-break. In the present research work, MC simulation is used to evaluate/predict FOS, as a common descriptor for the seismic slope stability. First, an empirical equation is developed using multiple linear regression model. Then, the developed equation was used for simulation purpose by MC.

2 Established database

To prepare a suitable database for analysis of slope stability, a series of analyses/assumptions have been done using LEM-slices methods. Among them, the simplest and most commonly used one in engineering practice is Fellenius and Bishop's method. According to Fig. 1, the soil mass above a trial failure surface is divided into slices by vertical sections. Each portion is taken as having a straight-line base, denoting mutual support between the slices. In Fellenius approach, the forces acting between each pair of slices (E , X) are neglected, so only the forces acting on the base of each slice (W , N and S) are left for consideration. In view of the soil resistance (T) and the friction action, as consequent, the safety factor can be defined as a relation between opposing and toppling moments regarding the center point of an assumed slip circle. The constant ray in moments equations can be reduced and an ultimate formulation of the safety factor comprises only resisting and sliding forces (S , T). In Bishop's approach, apart from the total moments' equilibrium conditions, an additional equation of vertical force equilibrium is taken into account individually for each of the slices. Such consideration leads to an implicit formulation of the safety factor which needs an iterative procedure. However, the general equilibrium equations as mentioned below should be persuaded according to Eq. 1.

$$\Sigma H = 0, \Sigma V = 0 \text{ and } \Sigma M = 0 \quad (1)$$

Fig. 1 The idea of slice method and definition of safety factor according to Eqs. 2–4



Finally, Equations 2, 3 and 4 show Fellenius, Bishop and Simplified Bishop methods, respectively.

$$F_s = \frac{\sum_i T_i}{\sum_i S_i} \tag{2}$$

$$F_s = \frac{\sum_i [cb_{i+} + (w_i + x_i - x_{i+1})\tan\phi] \frac{\sec\alpha_i}{1+(\tan\alpha_i \tan\phi)/F_s}}{\sum_i S_i} \tag{3}$$

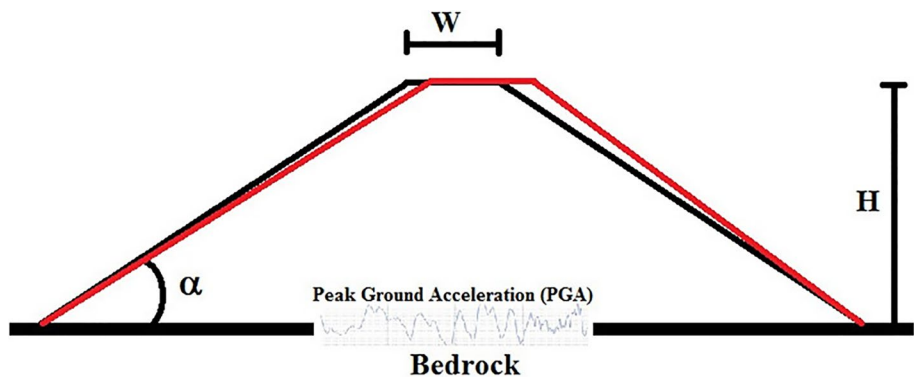
$$F_s = \frac{\sum_i [cb_{i+} + w_i \tan\phi] \frac{\sec\alpha_i}{1+(\tan\alpha_i \tan\phi)/F_s}}{\sum_i S_i} \tag{4}$$

In total, 250 slopes with different parameters were considered and analyzed using GeoStudio software which is based on LEM technique. Some of the steps in the modeling process were introducing boundary conditions, model dimensions, material properties and seismic motion. By reviewing the previous investigations, it was found that several parameters, such as slope height (H), slope angle (α),

cohesion (C), angle of internal friction (ϕ) and earthquake accelerator motion, are the most effective parameters on FOS of slopes (e.g., [10, 11, 24, 27]). Kramer [28] mentioned that peak ground acceleration (PGA) is a measure of earthquake acceleration on the ground. Therefore, in this study, for predicting seismic FOS, five effective parameters (H, α , C, ϕ and PGA) were chosen as model inputs.

In the modeling procedure, H values of 15 and 20 m, α values of 20°, 25° and 30°, C values of 20, 30, 40 and 50 kPa, ϕ values of 30°, 35° and 40° and PGA values of 0, 0.1 and 0.2 g were considered and applied. All slopes were considered as homogenous with γ of 18 kg/m³ (as a normal and logical amount). In addition, crest width of 8 m was supposed for all constructed models. All slopes were placed on the bedrock with rigid behavior. In the analysis by GeoStudio, Mohr-Coulomb failure criterion as a well-known criterion was considered. In this research, grid and radius slip surface (with 30 slices as slip surfaces) was used to calculate FOS. It is worth mentioning that in this method, the final FOS should be

Fig. 2 A schematic view of embankment deformation due to earthquake accelerator motion



suiting nearly in the middle of the grid. A schematic view of embankment deformation due to earthquake accelerator motion is displayed in Fig. 2. Finally, a database comprising 250 datasets with the mentioned input and output parameters were prepared for purposes of this study.

3 Developing a FOS predictive model

The multiple linear regression (MLR) is a widely used method for solving different engineering problems, especially for prediction purposes. In the MLR, a linear equation between one or more input (independent) parameters and one output (dependent) parameter is fitted. In recent years, application of multiple linear regression model in various engineering disciplines has been expanding [29–32]. As an example, Hasanipanah et al. [33] developed a MLR model to predict blast-induced back-break. The R^2 between measured and predicted values of BB was 0.857 that revealed the MLR is an acceptable model for the prediction of back-break induced by mine blasting. The general MLR form can be formulated as follows:

$$Y = P_0 + P_1X_1 + \dots + P_nX_n \tag{5}$$

where X_i ($i = 1, \dots, n$) and Y are input (independent) and output (dependent) parameters, respectively. In addition, P_i ($i = 0, 1, \dots, n$) are regression coefficients. As per stated in the previous studies [26, 32], for simulation of FOS using MC modeling, developing an empirical equation is required. For this purpose, MLR is used to develop an empirical equation in the present study. To develop it, H , α , C , \emptyset and PGA were used as independent parameters, while FOS was used as dependent parameter. In this regard, the prepared database comprising 250 datasets were considered and Eq. 6 was constructed using SPSS v16 software [34].

$$FOS = 0.662 - (0.0012H) - (0.0493\alpha) + (0.001C) + (0.0607\emptyset) - (0.307PGA) \tag{6}$$

In Eq. 6, H , α , C , \emptyset and PGA are slope height (m), slope angle ($^\circ$), cohesion (kPa), angle of internal friction ($^\circ$) and peak ground acceleration (m/s^2), respectively. The statistical

Table 1 Statistical information for the constructed MLR model

Predictor	Coefficient	SE	<i>t</i> Stat	<i>P</i> value
Intercept	0.662	0.0965	6.8559	5.76E–11
<i>H</i>	–0.001	0.0032	–0.380	0.7035
α	–0.049	0.002	–23.703	3.04E–65
<i>C</i>	0.001	0.0007	1.430	0.153
\emptyset	0.0607	0.0019	30.531	2.64E–85
PGA	–0.3075	0.0104	–29.382	4.17E–82

Table 2 Performance indices for the developed MLR equation

Performance index	RMSE	R^2	VAF	MEDAE
Amount	0.662	0.0965	6.8559	5.76E–11

information for developed MLR equation to estimate FOS is given in Table 1. Performance of the established equation was evaluated using coefficient of determination (R^2) and root mean squared error (RMSE).

$$R^2 = \frac{\left[\sum_{i=1}^n (x_i - x_{\text{mean}})^2 \right] - \left[\sum_{i=1}^n (x_i - x_p)^2 \right]}{\left[\sum_{i=1}^n (x_i - x_{\text{mean}})^2 \right]} \tag{7}$$

$$RMSE = \sqrt{\frac{1}{n} \times \sum_{i=1}^n [(x_i - x_p)^2]} \tag{8}$$

$$VAF = \left[1 - \frac{\text{var}(x_i - x_p)}{\text{var}(x_i)} \right] \times 100 \tag{9}$$

$$MEDAE = \text{median}(x_i - x_p) \tag{10}$$

where, n is the number of datasets, x_p and x_i are the predicted and measured values, respectively. Note that, R^2 , RMSE, VAF and MEDAE equal to 1, 0, 100% and 0, respectively, indicate the best approximation. Considering the prepared database and Eq. 6, values of 0.12, 0.886, 88.6% and 0.005 were obtained for RMSE, R^2 , VAF and MEDAE, respectively (see Table 2). These values demonstrated that the constructed equation can be used successfully for the prediction of FOS in this case study. In addition, Fig. 3 depicts the scatter plots of FOS predicted by the

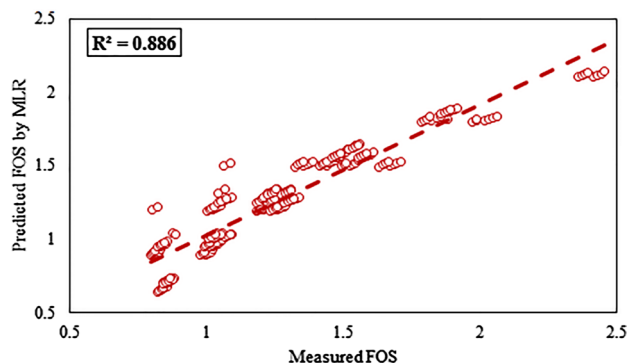


Fig. 3 Measured vs predicted values of FOS using MLR model

MLR. From this figure, it is found that the predicted values are in good agreement with the measured values.

4 Monte Carlo modeling to simulate FOS

4.1 Background

Monte Carlo (MC) simulation (known as probability simulation) is a computerized technique that considers the impact of uncertainty and risk in a variety of forecasting models on different issues, e.g., project management, financial analysis, and decision-making problems. To make a probabilistic approximation for an arithmetic equation or developed model, a repeated random sampling method is used by the MC simulation with the help of statistical analysis [35, 36]. For development of a predictive model, a series of assumptions that are related to the problem should be considered, and expected value(s) should be estimated. Mainly, this technique pursues two important goals: first, quantitative determination of variability and uncertainty when exposure of risk is simulated. Second, investigation of the most important agents of variability and uncertainty as well as the way they contribute to overall variance and the range of model results [35]. The MC simulation uses a range of estimated values as inputs and releases also a range of values as output of the system (unlike the conventional forecasting models in which only fixed values are estimated). This leads to creating a more realistic picture from a simulation model.

In the MC simulation, based on the range of estimates, a value is chosen randomly for each task or input; then, an output is computed based on the chosen values, and the obtained results are recorded. This procedure is irritated many times using a variety of values. Generally, in an MC modeling, value of 1000 is set (as the default) for repeating the simulation process. When the simulation process is implemented, numerous results are obtained as output ranges. The results help to describe the probability of achieving many results in the MC modeling [37–39]. In MC simulation, if the value taken by a variable has no effect on the value assumed by another variable, then both variables are independent. Due to this feature, the representation and analysis of uncertainty can be easier.

The MC simulator has been recently applied to the geotechnical fields of study [32, 39], and slope stability [40–42]. Based on the Kuz–Ram fragmentation model, Morin and Ficarazzo [43] applied MC simulation to simulation of the complete fragmentation size distribution. It was found that this simulator was reasonably practical in simulating the rock fragmentation. In another research, the MC simulation was used by Jahed Armağhani et al. [32] for estimating the risk of flyrock during

the operations carried out at Ulu Tiram granite quarry. Results obtained from a total of 10,000 trials in the MC simulation demonstrated the maximum distance of the flyrock. In another study, an MC modeling was also introduced by Ghasemi et al. [39], which was used for simulating the flyrock distance that occurred due to blasting operations conducted in the Sungun copper mine, Iran. The @RISK software was used as MC simulator in that study. The obtained results revealed that the MC simulation could simulate flyrock distance with a completely acceptable level of accuracy.

Calamak and Yanmaz [44] conducted a probabilistic slope stability analyses regarding the earth-fill dams. In this area of research, inputs and uncertainties, such as unit weight, hydraulic conductivity, cohesion, and the soil's internal friction angle are given to the MC model. Finally, this simulator has been shown computationally efficient because of its numerous random runs and in determining the contributions that various sources of uncertainty have. Danka [45] also used the MC simulator for computation of probability of failure of soil dikes when a flood occurs. Additionally, EL-Ramly and Morgenstern [46] showed significance of applying the probabilistic techniques to the estimation of the FOS in studies carried out on slope stability. The probabilistic techniques have been proposed for investigating the effect of uncertainty regarding the slope design reliability. For instance, Husein Malkawi et al. [47] have used the MC simulation to examine the reliability of slope stability. Furthermore, Ma and Wang [48] employed the MC simulation to carry out a probabilistic analysis to analyze the slope stability.

In the present study, an attempt has been made to develop a MC model for estimating FOS concentrate on seismic FOS using PGA as one of the input parameters. The following section describes the procedure of MC modeling in simulation of seismic FOS.

4.2 Development of MC simulation

For estimation of the FOS, as stated earlier, Eq. 6, which is an empirical equation, was developed using MLR.

Table 3 Data input for the MC simulation

Input parameter	Minimum	Maximum	Function
Slope height (H)	15	20	Uniform
Gradient (α)	20	30	Uniform
Cohesion (C)	20	50	Uniform
Friction angle (ϕ)	30	40	Uniform
Peak ground acceleration (PGA)	0	1.962	Uniform

Table 4 Correlation coefficients of Spearman's rho for model inputs

	H	α	C	\emptyset	PGA
H	1				
α	0.029257	1			
C	-0.008	0.043765	1		
\emptyset	0.022526	0.198648	0.030755	1	
PGA	0.006431	-0.04453	0.015974	0.099926	1

This section presents the way how to apply the developed MLR equation for simulating the FOS and identifying parameters with highest influence on that. According to the available data, in the stochastic model, all inputs, i.e., H , α , C , \emptyset and PGA are taken into account as continuous probability distributions (CPD). In the uniform distribution, there is constant probability, and it is considered as the probability distribution function of all of the input variables applied to the MC simulator. The inputs and their corresponding distribution functions that are employed the MC simulation are displayed in Table 3.

In the present research, the Risk Solver Platform software was used as MC simulator [36]. In this software, a user-friendly environment is provided as MS-Excel™ add-in features for the MC functions and random sampling. In general, the MC analysis comprised two sampling schemes, namely Latin hypercube sampling and simple random sampling that one of them is only employed. In the Latin hypercube sampling, a stratified sampling scheme is expected to be capable of properly representing lower and upper ends of the distributions applied to the analysis. According to the

literature (e.g., [35]), the sampling method of Latin hypercube is further preferred compared to the simple random sampling. This is because the former requires fewer simulations for generating the same level of accuracy [49]. Sampling was iterated for 10,000 times using Latin hypercube sampling to ensure all possible combinations were selected randomly. As a result, each simulation run yielded 10,000 various possible combinations of input variables. Note that the possible contributions were selected randomly from defined distributions. The modeling of MC simulation is affected deeply by the relationships that exist between input variables. For this reason, the relationships between input variables must be examined and taken into consideration (as presented in Table 4) to obtain an MC modeling improved for the simulation of FOS range. As it can be seen in Table 4, there is a relationship among the factors of slope height, friction angle, cohesion, gradient, and peak ground acceleration; thus, existing formulas generally compute one of them as a function of the other factors. In cases where the goal is to obtain meaningful combinations rather than completely random sampling in the process of input sampling, those correlations shown in Table 4 must be taken into account for performing simulation. The program comprises important rank-order correlations (Spearman's rho) with the use of a correlation matrix among the model inputs. For stochastically estimation of the amount of FOS, the following steps were performed:

1. The FOS data in different conditions were obtained using GeoStudio software.

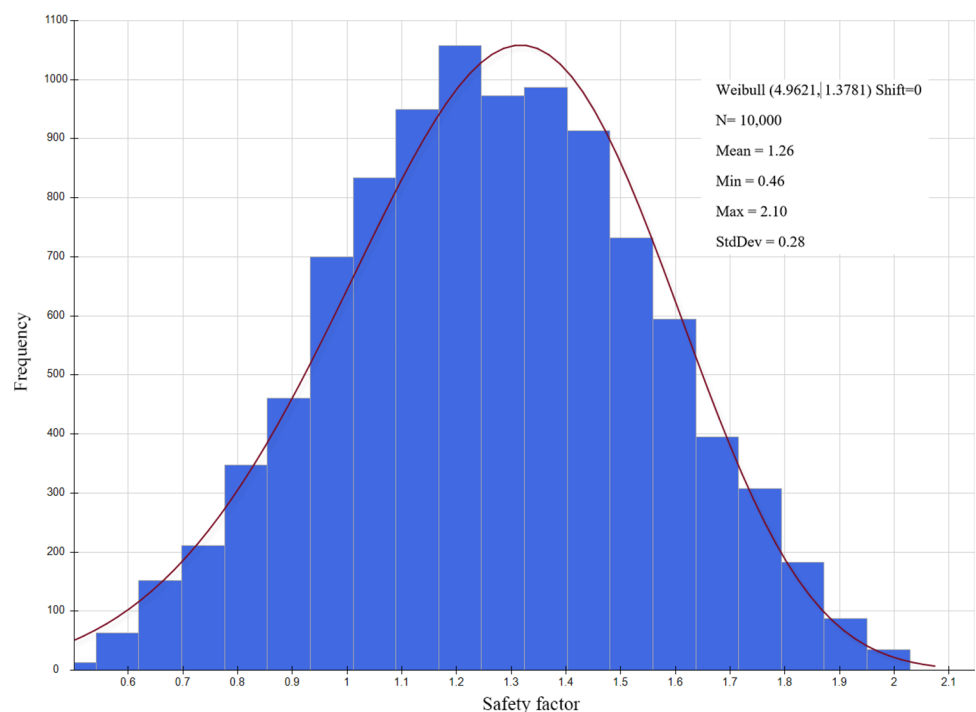
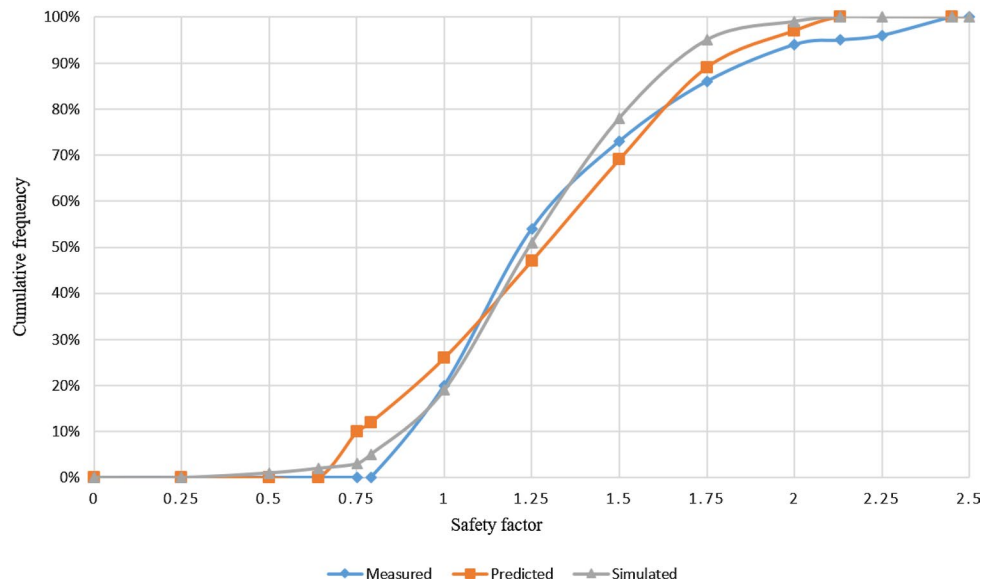
Fig. 4 Distribution model of the FOS obtained through the MC modeling

Fig. 5 Results of the measured FOS, predicted FOS by MLR model, and simulated FOS by MC modeling



2. Using the Risk Solver Platform, the best fitted distribution functions were applied to the inputs;
3. The FOS was stochastically assessed in case of each model input in Eq. 6 using the continuous probability distributions;
4. The correlations between the input variables were added to the MC modeling;
5. For statistically representing the amount of FOS, the MC simulation was iterated for 10,000 times in the spreadsheet model.

Figure 4 presents the distribution model of the FOS obtained through the MC modeling as well as a summary of statistics. The minimum, average, and maximum FOS were 0.46, 1.26, and 2.1, respectively. Moreover, for the amount of FOS, the Weibull distribution was shown the best-fitting model. As it can be seen in the results obtained from the simulated model, the FOS was shown in a wide range of variation. In addition, Fig. 5 demonstrates the results of the measured FOS and the predicted FOS obtained by MLR model, and the simulated FOS obtained by MC modeling. Figure 5 indicates that with nearly 80% confidence, the amount of FOS was more than 1. It shows that the probability of having a stable slope is 80%; however, there is 20% chance that the slope is not stable. Furthermore, Fig. 6 illustrates the scatter plots of the model inputs that were obtained using the system in the simulation process.

4.3 Sensitivity analysis

Since the FOS is dependent on all parameters discussed above, a correlation sensitivity analysis was carried out for the identification of those parameters that have most effect on this factor. In this analysis, the Risk Solver Platform was

Table 5 Sensitivity ranking of input variables according to correlation analysis

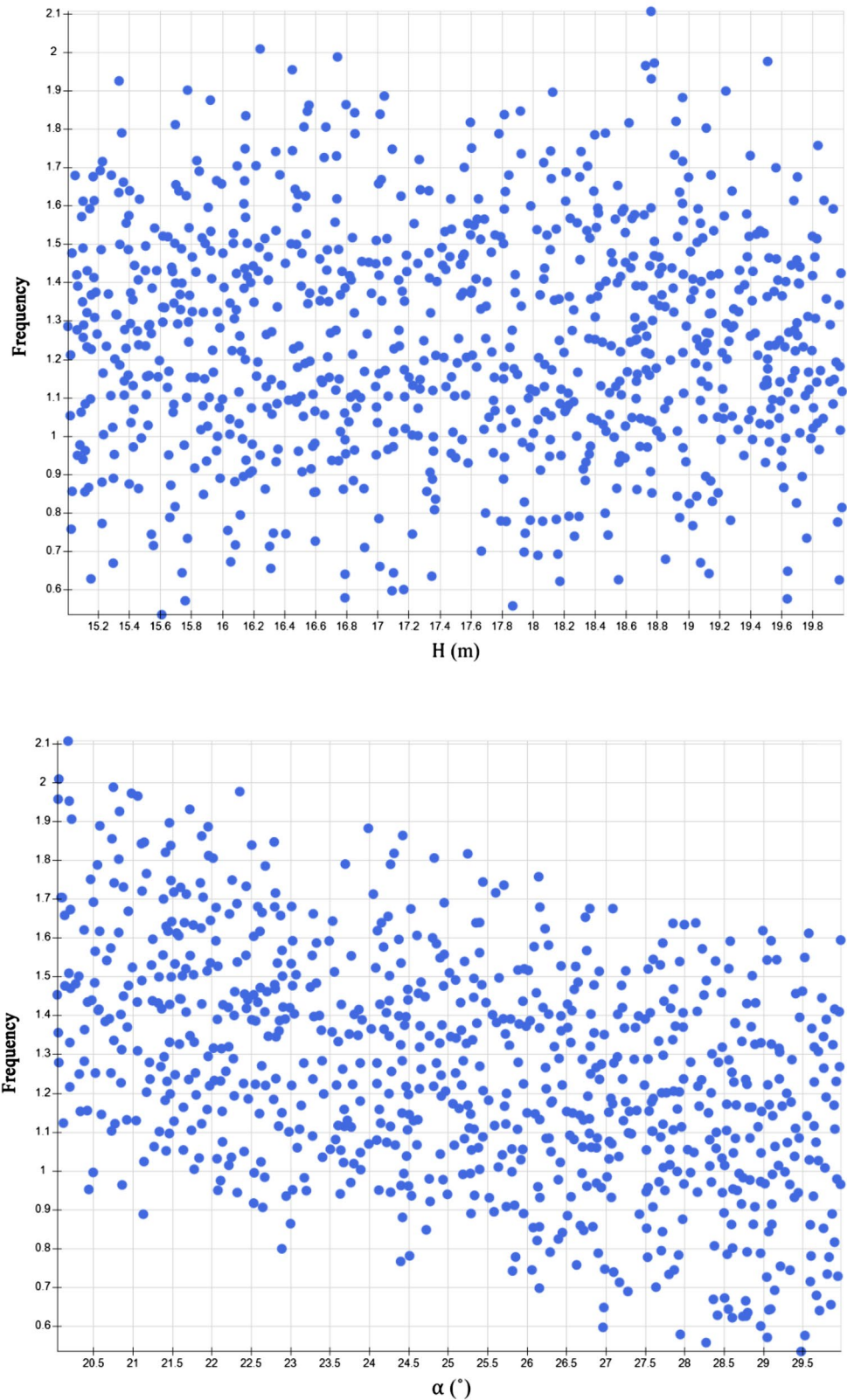
Name of variable	Correlation coefficient
Friction angle	+0.62 (1)
Peak ground acceleration	-0.61 (2)
Gradient	-0.49 (3)
Cohesion	+0.04 (4)
Slope height	+0.002 (5)

employed to explore the rank-order correlations achieved from the simulated results that were related to the model inputs and output. Relationships between two datasets can be examined using rank-order correlation and making comparison between the ranks of each value in a dataset. Note that the rank-order values for correlation, which are obtained through the use of Risk Solver Platform, can be in a range between -1 and +1. For computation of the rank values, data were sorted from smallest to largest; then, the numbers were assigned to the rank values depending on their position in the order. Table 5 shows the sensitivity level of the FOS results to the variables. The FOS, in a descendent order, is sensitive to ϕ , PGA, α , C , and H . As it can be seen, holding correlation coefficient of 0.62, ϕ among the others, is the most effective issue on the FOS.

5 Discussion and conclusions

In the presented paper, MC simulation was used to evaluate/predict FOS, as the most important descriptor of the slope stability especially in seismic situation. For this aim, a total number of 250 datasets were analyzed in the GeoStudio

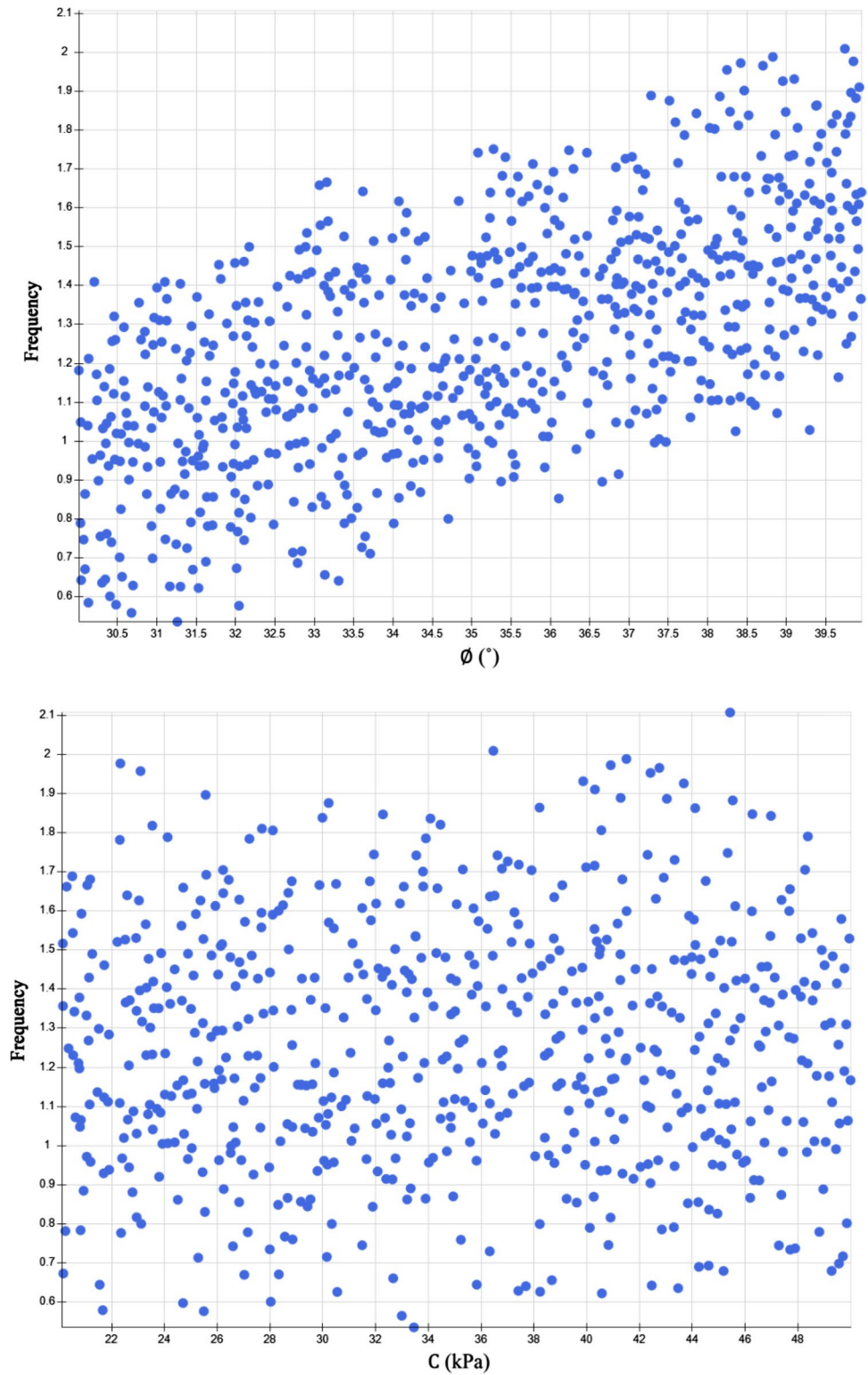
Fig. 6 Scatter plots of the inputs used in MC simulation



software environment. In this research, H , α , C , \emptyset and PGA , as the most effective parameters on the FOS, were selected and used as independent parameters. Moreover,

FOS was chosen as the dependent parameter. In the first step of MC modeling, developing an empirical equation is required. In this regard, the MLR technique was used and

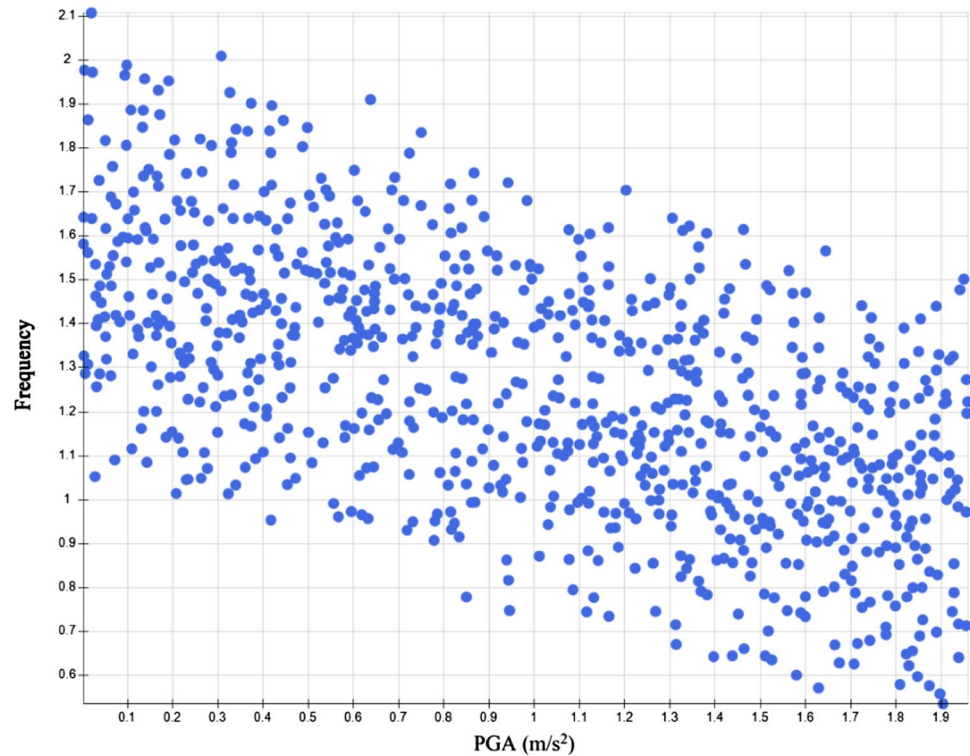
Fig. 6 (continued)



an empirical equation with $R^2=0.88$ and $RMSE=0.12$ was developed. The results showed that predicted values using the constructed equation are in good agreement with the actual data, which demonstrates the reliability of the

developed MLR model. Then, this equation was used in the MC modeling. The obtained results of predicted and simulated FOS values are very similar to the measured one in all of data points. Generally, MC model simulated FOS of less

Fig. 6 (continued)



than 1.18, lower and higher than measured and predicted FOS values, respectively. However, the results of MC simulation for the FOS values of more than 1.33, is higher than those measured and predicted FOS values. As a result, the mean of the FOS simulated by the MC simulation is 1.26, whereas it is 1.27 based on the actual FOS. Thus, findings indicated that the proposed MC modeling is capable of simulating the FOS very appropriately.

Results presented in Table 5 revealed that there is a direct relationship between the FOS and ϕ , H and C. That is, if these parameters increase, FOS increases too. On the other hand, PGA and α are in an indirect relationship with the FOS, which means that any increase in these parameters causes a decrease in the FOS. Furthermore, the sensitivity analysis results demonstrated that ϕ , among other parameters, was the most effective one on the FOS value. It is noticeable that because of the nature of the problem, the model/equation proposed in this research cannot be applied directly to other conditions and only should be used for the mentioned parameters and their ranges. To do simulation in other slopes, the process presented in this research should be reconsidered.

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