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REGULAR ARTICLE

Detecting shifts in Conway–Maxwell–Poisson profile with deviance residual-based CUSUM and EWMA charts under multicollinearity

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Abstract

Monitoring profiles with count responses is a common situation in industrial processes and for a count distributed process, the Conway–Maxwell–Poisson (COM-Poisson) regression model yields better outcomes for under- and overdispersed count variables. In this study, we propose CUSUM and EWMA charts based on the deviance residuals obtained from the COM-Poisson model, which are fitted by the PCR and *r*–*k* class estimators. We conducted a simulation study to evaluate the effect of additive and multiplicative types shifts in various shift sizes, the number of predictor, and several dispersion levels and to compare the performance of the proposed control charts with control charts in the literature in terms of average run length and standard deviation of run length. Moreover, a real data set is also analyzed to see the performance of the newly proposed control charts. The results show the superiority of the newly proposed control charts against some competitors, including CUSUM and EWMA control charts based on ML, PCR, and ridge deviance residuals in the presence of multicollinearity.

Keywords Conway–Maxwell–Poisson distribution · Principal component regression · Profile monitoring · Deviance residual · Control chart

Mathematics Subject Classification 62P30 · 62J12 · 62J07 · 62H12

1 Introduction

In statistical quality control, profile refers to the situations in which the quality of a process or product is represented by a functional relationship between a response and

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one or more predictors. When the response variable follows a distribution that belongs to the exponential family, it is called a generalized linear profile (GLP).

Because of the similarities of the profiles to the regression models, monitoring methods are typically based on regression modelling. Although the maximum likelihood (ML) approach is the traditional way of estimating parameters, alternative estimation methods are discussed in the literature due to the problems arising from the multicollinearity between the predictors. Some of these alternative estimation methods are integrated into profile monitoring.

A common way of monitoring GLPs with count response is the application of Poisson profile monitoring methods. Skinner et al. (2003) proposed a technique based on the likelihood ratio statistic and used the deviance residual from Poisson regression for monitoring purposes. Amiri et al. (2011) examined Hotelling's T^2 approach based on estimated model parameters, Asgari et al. (2014) developed a procedure that involves a mixture of log and square root link functions for the modelling process with count response. Asgari et al. (2014) monitored the process via proposed Shewhart and exponentially weighted moving average (EWMA) charts based on the standardized residuals. Qi et al. (2016) suggested a control chart based on weighted likelihood ratio statistics for monitoring GLPs. Later Marcondes Filho and Sant'Anna (2016) proposed a Shewhart-type residual control chart based on the principal component (PC) scores for the Poisson processes. The effect of the parameter estimator in Poisson profile monitoring is investigated by Maleki et al. (2019). Wen et al. (2021) proposed a regression-adjusted EWMA chart that adjusts and updates the expected values according to the situation to monitor the Poisson process. Mammadova and Özkale (2021a) studied the impact of the tuning parameter on ridge deviance-based control charts and Iqbal et al. (2022) presented homogeneously weighted moving average control charts where monitored observations are either deviance or standardized residuals of the generalized linear model (GLM).

The aforementioned approaches are extended for monitoring COM-Poisson profiles since a Poisson distribution becomes unsuitable when the data set shows signs of over or underdispersion. The flexible two-parameter COM-Poisson distribution was proposed by Conway and Maxwell (1962) to overcome the challenges caused by the difference between the mean and variance of the count data set. Park et al. (2018) adapted the principal components regression (PCR) approach for monitoring Poisson processes and constructed an r-chart for COM-Poisson profile monitoring. Park et al. (2020) examined COM-Poisson regression-based control charts and utilized randomized residuals to build a Shewhart chart. Rao et al. (2020) studied a mixed EWMA and cumulative sum (CUSUM) chart for COM-Poisson profile while Shewhart, EWMA, and CUSUM charts on the bases of ridge deviance residuals were developed by Mammadova and Özkale (2021b) for monitoring Poisson as well as the COM-Poisson profiles. Jamal et al. (2021) monitored real-time highway safety surveillance data set with CUSUM and EWMA charts and used randomized quantile and deviance residuals of the COM-Poisson regression model for the monitoring.

To address the multicollinearity problem, options other than ridge estimator were developed for GLMs. The iterative PCR estimator and the first-order approximated Liu estimator were proposed for GLMs respectively by Marx and Smith (1990) and Kurtoğlu and Özkale (2016). Özkale (2019) studied a combination of the Liu and PCR estimators, the *r–d* class estimator to minimize the effect of multicollinearity. Abbasi and Özkale (2021) developed the iterative *r–k* class estimator for GLMs by combining ridge and PCR estimators. The authors showed the superiority of proposed approaches to ML and ridge estimators in terms of the mean squared error criterion through the simulation study. Apart from the mentioned studies, several studies are specifically devoted to the examination of COM-Poisson distribution in the framework of GLM (see, Guikema and Goffelt (2008); Lord et al. (2008); Sellers and Shmueli (2010); Francis et al. (2012)). Reformulation of the distribution was suggested by Guikema and Goffelt (2008) to model the data set in order to prevail over the computational limitations. Characteristics of the COM-Poisson regression model, estimation, diagnostics, and interpretation were discussed by Guikema and Goffelt (2008); Sellers and Shmueli (2010) utilized the Bayesian technique for parameter estimation whereas Sellers and Shmueli (2010) and Francis et al. (2012) used unconstrained optimization. Abdella et al. (2019) introduced a penalized likelihood technique in the form of the ridge estimation, Mammadova and Özkale (2021b) provided an iterative closed-form solution to the ridge estimator given by Abdella et al. (2019); Sami et al. (2022b) proposed a COM-Poisson ridge regression estimator, which is a ridge estimator obtained at the final iteration of ML estimator. Recently, Sami et al. (2022a) proposed a modified one-parameter Liu estimator, and Rasheed et al. (2022) developed a modified jackknifed Liu-type estimator for COM-Poisson regression.

In this paper, we propose an extension of residual-based CUSUM and EWMA charts for identifying abnormalities in the COM-Poisson profile mean by using PCR and *r–k* class estimators. We intend to address a multicollinearity problem and optimize the monitoring process by reducing the dimension of the data set while detecting out-of-control observations as quickly as possible by utilizing CUSUM and EWMA charts based on the PCR and the *r–k* class estimator. Compared with the control charts previously proposed in the literature, our contributions can be summarized as follows:

- CUSUM and EWMA control charts, which are used to detect small changes in the process, will become effective by combining these control charts with the *r–k* class estimator, which is an effective estimator in multicollinearity.
- The CUSUM and EWMA control charts based on the *r–k* class estimator provide a general framework of CUSUM and EWMA control charts based on ridge and PCR estimators.
- The CUSUM and EWMA control charts based on the *r–k* class estimator outperform the CUSUM and EWMA control charts based on the ML estimator in the presence of multicollinearity.

The following is the outline for this paper: Sect. 2 covers a brief description of the COM-Poisson distribution and estimation methods for the COM-Poisson regression model in the case of multicollinearity. Construction of the deviance-based CUSUM and EWMA charts for monitoring GLP with correlated predictors and dispersed count response is presented in Sect. 3. Section 4 provides a performance analysis of the proposed method through a simulation study. Section 5 delivers a real-life application that is carried out in the example of the SECOM data set. The concluding remarks are given in Sect. 6.

2 COM-Poisson modelling

2.1 COM-Poisson distribution

Introduced in the early 1960 s, the COM-Poisson distribution has attracted more attention from researchers in the recent past due to its flexibility. Characterizations of the distribution is thoroughly investigated by Boatwright et al. (2003); Shmueli et al. (2005); Li et al. (2020).

The probability density function of the COM-Poisson distribution is defined as

$$f(y_i) = \frac{\mu_i^{y_i}}{(y_i!)^v} \frac{1}{Z(\mu_i, v)}, \quad y_i = 0, 1, 2, \ldots, \quad i = 1, 2, \ldots, n$$

where $\mu_i \geq 0$ is the centering parameter, $v \geq 0$ is the shape parameter, $Z(\mu_i, v) = \sum_{s=0}^{\infty} \frac{\mu_i^s}{(s!)^v}$ is a normalization parameter and v is a dispersion parameter. Overdispersion in the data set is represented by $v < 1$, equidispersion by $v = 1$, and underdispersion by $v > 1$.

Based on the values of centering and shape parameter, COM-Poisson distribution converges to three different distributions. These are

- Geometric distribution: $v = 0$ and $\mu_i < 1$;
- Poisson distribution: $v = 1$;
- Bernoulli distribution: $v \to \infty$.

When $v = 0$ and $\mu_i \geq 1$, the normalization parameter $Z(\mu_i, v)$ does not converge and the distribution is undefined (Shmueli et al. 2005).

2.2 Parameter estimation in COM-Poisson regression

Let $y_{n\times 1} = [y_1, y_2, \ldots, y_n]'$ be the response vector with COM-Poisson distribution and $X_{n\times p} = [x_1, x_2, \ldots, x_n]'$, be the predictor matrix with $x_i' = [x_{i1}, x_{i2}, \ldots, x_{ip}]$, $i = 1, 2, \ldots, n$ being the i-th observation. Log-link function can be applied for modelling the relationship between y and X as $\mu = \log(X\beta)$ where $\beta_{n\times 1} = [\beta_1, \beta_2, \ldots, \beta_p]'$ is the vector of unknown parameters.

The β parameters are estimated with the help of the log-likelihood function of the COM-Poisson distribution that is provided by Sellers and Shmueli (2010) as

$$l(\beta; y) = v \sum_{i=1}^{n} y_i \log(\mu_i) - v \sum_{i=1}^{n} \log(y_i!) - \sum_{i=1}^{n} \log\left(Z(\mu_i; v)\right). \tag{1}$$

Sellers and Shmueli (2010) and Francis et al. (2012) proposed using the iteratively reweighted least squares (IRLS) technique which was presented by Nelder and Wedderburn (1972) and Wood (2017). Then, a closed form solution for the IRLS estimator known as the ML estimator was given by Mammadova and Özkale (2021b) as

$$\hat{\beta}_{ML}^{(t)} = \left(X'\hat{W}_{ML}^{(t-1)}X\right)^{-1} X'\hat{W}_{ML}^{(t-1)}u_{ML}^{(t-1)}$$

where t is the iteration step, $u_{ML}^{(t-1)} = X\hat{\beta}_{ML}^{(t-1)} + (\hat{W}_{ML}^{(t-1)})^{-1}(y - \hat{\mu}_{ML}^{(t-1)})$ is the working response, and $\hat{W}_{ML}$ is the estimated weight matrix evaluated at $\hat{\beta}_{ML}^{(t-1)}$. After the successful iterations, $\hat{\beta}_{ML}$ at convergence is obtained as $\hat{\beta}_{ML} = \left(X'\hat{W}_{ML}X\right)^{-1} X'\hat{W}_{ML}u_{ML}$.

The weighted matrix for the COM-Poisson model was first introduced by Sellers and Shmueli (2010) and later elaborated by Francis et al. (2012) as $W = \mathrm{diag}(w_{ii})$,[1] $i = 1, 2, \ldots, n$ where

$$w_{ii} = \sum_{s=0}^{\infty} \frac{\frac{v(v-1)s^2(\exp(\mu_i))^{2s}\left(\frac{(\exp(\mu_i))^s}{s!}\right)^{v-2}}{(s!)^2} + \frac{vs^2(\exp(\mu_i))^s\left(\frac{(\exp(\mu_i))^s}{s!}\right)^{v-1}}{s!}}{\sum_{s=0}^{\infty}\left(\frac{(\exp(\mu_i))^s}{s!}\right)^{v}} - \sum_{s=0}^{\infty} \frac{\left[\frac{vs(\exp(\mu_i))^s\left(\frac{(\exp(\mu_i))^s}{s!}\right)^{v-1}}{s!}\right]^2}{\sum_{s=0}^{\infty}\left[\left(\frac{(\exp(\mu_i))^s}{s!}\right)^{v}\right]^2}. \tag{2}$$

In the case of uncorrelated predictors, it is well known that the ML estimator is a reliable method. However, multicollinearity poses challenges with the computation of the inverse matrix of $X'WX$, which is essential for ML estimation. Therefore, alternative approaches were proposed.

One of these alternatives is the iterative ridge estimator presented by Mammadova and Özkale (2021b) in COM-Poisson regression as

$$\hat{\beta}_{ridge}^{(t)} = (X'\hat{W}_{ridge}^{(t-1)}X + kI_p)^{-1}X'\hat{W}_{ridge}^{(t-1)}u_{ridge}^{(t-1)}$$

where t refers the iteration step, $u_{ridge}^{(t-1)} = X\hat{\beta}_{ridge}^{(t-1)} + (\hat{W}_{ridge}^{(t-1)})^{-1}(y - \hat{\mu}_{ridge}^{(t-1)})$ is the working response, $\hat{W}_{ridge}$ is the weight matrix in Eq. (2) evaluated at $\hat{\beta}_{ridge}^{(t-1)}$, and k is the tuning parameter.[2] The ridge estimator of β at convergence has the form of $\hat{\beta}_{ridge} = \left(X'\hat{W}_{ridge}X + kI_p\right)^{-1} X'\hat{W}_{ridge}u_{ridge}$.

Another suitable alternative to the ML estimation in case of the multicollinearity is the PCR estimation. Unlike the ridge estimator, the PCR estimator does not require a tuning parameter; instead, it addresses the multicollinearity problem by generating a new set of uncorrelated variables using the singular value decomposition (SVD) technique discussed in GLMs by Smith and Marx (1990), Aguilera et al. (2006), Özkale and Arıcan (2016), Abbasi and Özkale (2021). Jolliffe (2002) stated that the SVD is effective in terms of both computation and interpretation in PCR. They also emphasized the importance of standardizing the predictors to zero mean and unit variance to eliminate scale dependence of PCs.

[1] $\hat{W}$ represents the estimated from of W where μ is replaced by its estimator obtained by the indexed name.

[2] The tuning parameter value of the ridge estimator in COM-Poisson regression can be computed by the method given by Sami et al. (2022b), whereas many methods can be found in linear regression.

In brief, SVD can be described as follows: Let the linear predictor η be expressed as $\eta = X\beta = XTT'\beta = X^*\omega$ where $X^* = XT$ and $\omega = T'\beta$. T is an orthogonal matrix through $T'X'\hat{W}_{ML}XT = \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p)$ and $\lambda_1 = \lambda_{max} \geq \lambda_2 \geq \ldots \lambda_p = \lambda_{\min}$ are the eigenvalues of the $X'\hat{W}_{ML}X$ matrix. X^* matrix can be partitioned as $X^* = [X_r^* \quad X_{p-r}^*]$, where $X_r^* = XT_r \quad (r \leq p)$ is the matrix of the PCs that will be retained in the model. ω, T, and Λ are partitioned as $\omega = [\omega_r \quad \omega_{p-r}]$, $\quad T = [T_r \quad T_{p-r}]$, and $\Lambda = [\Lambda *_r, \Lambda *_{p-r}]$ where $\Lambda *_r = X_r^{*'}\hat{W}_{ML}X_r^* = \text{diag}(\lambda_1, \ldots, \lambda_r)$, $\Lambda *_{p-r} = X_{p-r}^{*'}\hat{W}_{ML}X_{p-r}^* = \text{diag}(\lambda_{r+1}, \ldots, \lambda_p)$ and r is to the number of PCs that will be included the model.

By using SVD, Abbasi and Özkale (2021) obtained the PCR and *r–k* class estimators in GLMs which were originally introduced by Marx and Smith (1990); Baye and Parker (1984) in linear regression. We adjust the PCR and *r–k* class estimators for the COM-Poisson model as

$$\hat{\beta}_{PCR}^{(t+1)} = T_r\left(T_r'X'\hat{W}_{ML}XT_r\right)^{-1}T_r'X'\hat{W}_{ML}u_{PCR}^{(t)} \tag{3}$$

where $u_{PCR}^{(t)} = XT_rT_r'\hat{\beta}_{PCR}^{(t)} + \left(\hat{W}_{ML}\right)^{-1}\left(y - \mu_{PCR}^{(t)}\right)$ is evaluated at $\hat{\beta}_{PCR}^{(t)}$ and

$$\hat{\beta}_{r-k}^{(t+1)} = T_r\left(T_r'X'\hat{W}_{ML}XT_r + kI_r\right)^{-1}T_r'X'\hat{W}_{ML}u_{r-k}^{(t)} \tag{4}$$

where $u_{r-k}^{(t)} = XT_rT_r'\hat{\beta}_{r-k}^{(t)} + \left(\hat{W}_{ML}\right)^{-1}\left(y - \mu_{r-k}^{(t)}\right)$.

Abbasi and Özkale (2021) can be examined for the detailed information on obtaining the PCR and *r–k* class estimators in GLMs. To summarize, the general idea behind the PCR and *r–k* class estimators is that the linear predictor is reduced another linear predictor by deleting the PCs having large variances. After then, the IRLS idea is applied on this reduced linear predictor, and the resulted estimator is transformed to the original parameter space to give the PCR estimator in GLMs. On the other hand, the *r–k* class estimator is obtained by applying the ridge idea on the reduced linear predictor and transforming the resulted estimator back to the original parameter space. The notion in Eqs. (3) and (4) is that both estimators use the same number of PCs. The main difference is that the *r–k* class estimator lowers the degree of multicollinearity a little bit more and uses the tuning parameter for this purpose.

The PCR and *r–k* class estimators at convergence are respectively as

$$\hat{\beta}_{PCR} = T_r\left(T_r'X'\hat{W}_{ML}XT_r\right)^{-1}T_r'X'\hat{W}_{ML}u_{PCR}$$

and

$$\hat{\beta}_{r-k} = T_r\left(T_r'X'\hat{W}_{ML}XT_r + kI_r\right)^{-1}T_r'X'\hat{W}_{ML}u_{r-k}.$$

2.2.1 Tuning parameter selection

The tuning parameter determines the effectiveness of the ridge estimator as well as the *r–k* class estimator since the increasing value of the tuning parameter pulls the estimator further from its actual value. Studies conducted by Hoerl and Kennard (1970), Hoerl et al. (1975), Lawless and Wang (1976), Kibria (2003), Alkhamisi et al. (2006), Alkhamisi and Shukur (2007), Månsson and Shukur (2011), Kibria et al. (2012), Algamal (2018), Zaldivar (2018) and others cover a wide range of calculating approaches for the tuning parameter of the ridge estimator in linear regression and GLMs.

Abbasi and Özkale (2021) adapted the tuning parameter selection method proposed by Hoerl and Kennard (1970) in linear regression for the estimation of the *r–k* class estimator in the GLMs. We adjust the same tuning parameter for the COM-Poisson regression models and obtain

$$k = \frac{r\upsilon}{\hat{\beta}^{(0)'} T_r T_r' \hat{\beta}^{(0)}} \tag{5}$$

where $\hat{\beta}^{(0)}$ is the initial value of the β parameter which is usually taken as the ordinary least squares (OLS) estimator.

2.3 Deviance residuals

The deviance residual represents the difference between the predicted value of a given point i (the $E(y_i)$), which is denoted as $\hat{\mu}_i$ and the actual response y_i. Dobson (2002) defined the deviance residual for the i-th observation of the GLMs as the square root of the difference between the log-likelihood of the fitted and saturated models multiplied by two. The sign of the difference between the actual response and the fitted response determines the increase and decrease in deviance residual.

The i-th deviance residual based on ML, ridge, PCR, and *r–k* class estimators can be expressed by using the log-likelihood functions of the fitted model provided in Eq. (1) as

$$d_{est,i} = \text{sign}(y_i - \widehat{E(y_i)}) \times \sqrt{2\left[l(y_i, y_i; \hat{\upsilon}) - l(\widehat{E(y_i)}, y_i; \hat{\upsilon})\right]} \tag{6}$$

where $l(\widehat{E(y_i)}, y_i; \hat{\upsilon})$ and $l(y_i, y_i; \hat{\upsilon})$ are the log-likelihood functions of the fitted and saturated models, respectively, $\widehat{E(y_i)} = \hat{\mu}_{est,i}^{1/\upsilon} - \frac{\upsilon-2}{2\upsilon}$ and the subscript *est* is used to designate the method employed for modeling, i.e. $est \equiv \{ML, ridge, PCR, r-k\}$ and $\hat{\mu}_{est,i}$ is the fitted value obtained by using the corresponding estimator at convergence. Deviance residuals in Eq. (6) are constrained such that $y_i > c$ for $\upsilon < 1/(2c+1)$, $c \in N^+$.

Table 1 Construction of the deviance and ridge-deviance residual-based CUSUM and EWMA control charts

	Control chart	Required formula
Control chartstatistics	CUSUM$_{ML}$	$C^-_{ML,i} = \min[0, \hat{\mu}^0_{ML} - K - d_{ML,i} + C^-_{ML,i-1}], i = 1, 2, \ldots, n$ $C^+_{ML,i} = \max[0, d_{ML,i} - \hat{\mu}^0_{ML} - K + C^+_{ML,i-1}], i = 1, 2, \ldots, n$ $C^-_{ML,0} = C^+_{ML,0} = 0$
	CUSUM$_{ridge}$	$C^-_{ridge,i} = \min[0, \hat{\mu}^0_{ridge} - K - d_{ridge,i} + C^-_{DR,i-1}], i = 1, 2, \ldots, n$ $C^+_{ridge,i} = \max[0, d_{ridge,i} - \hat{\mu}^0_{ridge} - K + C^+_{DR,i-1}], i = 1, 2, \ldots, n$ $C^-_{ridge,0} = C^+_{ridge,0} = 0$
	EWMA$_{ML}$	$z_{ML,i} = \lambda d_{ML,i} + (1-\lambda) z_{ML,i-1}, i = 1, 2, \ldots, n$ $z_{ML,0} = \hat{\mu}^0_{ML}$
	EWMA$_{ridge}$	$z_{ridge,i} = \lambda d_{ML,i} + (1-\lambda) z_{ridge,i-1}, i = 1, 2, \ldots, n$ $z_{ridge,0} = \hat{\mu}^0_{ridge}$
Control limits	CUSUM$_{ML}$	$C_{ML} = \pm h \hat{\sigma}^0_{ML}$
	CUSUM$_{ridge}$	$CL_{ridge} = \pm h \hat{\sigma}^0_{ridge}$
	EWMA$_{ML}$	$CL_{ML} = \hat{\mu}^0_{ML} \pm L \hat{\sigma}^0_{ML} \sqrt{((\lambda/(2-\lambda))(1-(1-\lambda))^{2i})}, i = 1, 2, \ldots, n$
	EWMA$_{ridge}$	$CL_{ridge} = \hat{\mu}^0_{ridge} \pm L \hat{\sigma}^0_{ridge} \sqrt{((\lambda/(2-\lambda))(1-(1-\lambda))^{2i})}, i = 1, 2, \ldots, n$

3 Monitoring of COM-Poisson profiles

Page (1954) proposed the CUSUM chart which utilizes the past information available from previously plotted points for effective monitoring. Later, Roberts (1959) introduced the EWMA chart, an alternative to the CUSUM chart. EWMA charts also accumulate current and past information from the observations that make the control chart sensitive to small shifts. Since then, several modifications and extensions of these control charts are investigated. Montgomery (2020) describes the CUSUM and EWMA charts as alternatives to the Shewhart chart where detecting small shifts is significant.

Mammadova and Özkale (2021b) extended the traditional CUSUM and EWMA charts to the charts by using both the deviance and ridge deviance residuals to define control chart statistics. Mammadova and Özkale (2021b) gave the formulas in Table 1 to construct CUSUM and EWMA charts based on deviance and ridge deviance residuals. In Table 1, deviance-based charts are referred to as CUSUM$_{ML}$ and EWMA$_{ML}$, whereas ridge deviance-based charts as CUSUM$_{ridge}$ and EWMA$_{ridge}$ to prevent confusion. $\hat{\mu}^0_{ML}$ and $\hat{\mu}^0_{ridge}$ are the in-control means, $\hat{\sigma}^0_{ML}$ and $\hat{\sigma}^0_{ridge}$ are the in-control standard deviations of deviance and ridge deviance residuals, respectively. K and h are the reference and the decision value of the CUSUM chart, respectively. $0 < \lambda \leq 1$ refers to the smoothing parameter of the EWMA chart while L is the EWMA control limit constant.

Although the ridge estimator is a frequently used estimator in the multicollinearity problem, the *r–k* class estimator obtained by combining the ridge and PCR estimators gives better results than the ridge estimator when there is a multicollinearity problem. For this reason, we propose a new control chart alternative to the ridge deviance based chart of Mammadova and Özkale (2021b) by defining the CUSUM and EWMA chart statistics based on PCR and *r–k* deviance residuals which we denote respectively as CUSUM$_{PCR}$, EWMA$_{PCR}$, CUSUM$_{r-k}$, EWMA$_{r-k}$. The difference between these charts with that of the one given by Mammadova and Özkale (2021b) is that the chart introduced by Mammadova and Özkale (2021b) uses the ridge deviance residuals while the newly proposed charts use respectively the PCR and *r–k* deviance residuals. So we will get control charts based on PCR deviance residual that is as good as the control charts based on the ridge deviance residual but easier to calculate because they do not depend on the tuning parameter. Furthermore, we will get control charts based on the *r–k* class estimator, which will give better results than the control charts depending on the PCR and ridge deviance residuals. Thus, we will have improved the control charts proposed by Mammadova and Özkale (2021b) to perform better in the case of multicollinearity.

3.1 The newly proposed control charts based on PCR and *r–k* deviance residuals

In this subsection, we define CUSUM and EWMA control charts based on PCR and *r–k* class deviance residuals.

We give the CUSUM$_{PCR}$ and CUSUM$_{r-k}$ charts' statistics respectively as

$$C^{-}_{PCR,i} = \min[0, \hat{\mu}^{0}_{PCR} - K - d_{PCR,i} + C^{-}_{PCR,i-1}], \quad i = 1, 2, \ldots, n$$
$$C^{+}_{PCR,i} = \max[0, d_{PCR,i} - \hat{\mu}^{0}_{PCR} - K + C^{+}_{PCR,i-1}], \quad i = 1, 2, \ldots, n$$

and

$$C^{-}_{r-k,i} = \min[0, \hat{\mu}^{0}_{r-k} - K - d_{r-k,i} + C^{-}_{r-k,i-1}], \quad i = 1, 2, \ldots, n$$
$$C^{+}_{r-k,i} = \max[0, d_{r-k,i} - \hat{\mu}^{0}_{r-k} - K + C^{+}_{r-k,i-1}], \quad i = 1, 2, \ldots, n$$

where $\hat{\mu}^{0}_{PCR}$ and $\hat{\mu}^{0}_{r-k}$ correspond to the in-control mean of the PCR and *r–k* deviance residuals, $\hat{\sigma}^{0}_{PCR}$ and $\hat{\sigma}^{0}_{r-k}$ correspond to the in-control standard deviation of the PCR and *r–k* deviance residuals, respectively. The initial values are taken as zero: $C^{-}_{PCR,0} = C^{+}_{PCR,0} = 0$, $C^{-}_{r-k,0} = C^{+}_{r-k,0} = 0$.

The control limits for the CUSUM$_{PCR}$ and CUSUM$_{r-k}$ charts are as follows

$$CL_{PCR} = \pm h\hat{\sigma}^{0}_{PCR} \tag{7}$$
$$CL_{r-k} = \pm h\hat{\sigma}^{0}_{r-k}. \tag{8}$$

We define control chart statistics for the EWMA chart based on the PCR deviance residuals (EWMA$_{PCR}$) and EWMA chart based on the *r–k* deviance residuals (EWMA$_{r-k}$) as

$$z_{PCR,i} = \lambda d_{PCR,i} + (1-\lambda) z_{PCR,i-1}, \quad i = 1, 2, \ldots, n \tag{9}$$

$$z_{r-k,i} = \lambda d_{r-k,i} + (1-\lambda) z_{r-k,i-1}, \quad i = 1, 2, \ldots, n \tag{10}$$

where the initial values $z_{PCR,0}$ and $z_{r-k,0}$ are the in-control mean of the corresponding deviance residual.

The control limits for EWMA$_{PCR}$ and EWMA$_{r-k}$ are respectively as

$$CL_{PCR} = \hat{\mu}^0_{PCR} \pm L\hat{\sigma}^0_{PCR}\sqrt{(\lambda/(2-\lambda))(1-(1-\lambda))^{2i}}, i = 1, 2, \ldots, n \tag{11}$$

$$CL_{r-k} = \hat{\mu}^0_{r-k} \pm L\hat{\sigma}^0_{r-k}\sqrt{(\lambda/(2-\lambda))(1-(1-\lambda))^{2i}}, i = 1, 2, \ldots, n. \tag{12}$$

Since the combinations of K and h affect the performance of the control charts, the selection of K and h values for the CUSUM chart as well as the λ and L combinations for the EWMA chart is a sensitive task. It is usually recommended to select the values on the basis of the pre-specified in-control average run length value.

The run length (*RL*) is the number of observations until the first out-of-control observation is identified by the control chart. The average of the run length (*ARL*) is a commonly used metric to evaluate the performance of a control chart. This metric is classified as in-control ARL (ARL_0) and out-of-control ARL (ARL_1). ARL_0 is expected to be significantly large, while it is desirable for ARL_1 to be small.

4 Simulation study

In this section, we present a simulation study to compare the performance of residual-based control charts under different settings. The simulation study is carried out in *R* software. The simulation study consists of two stages:

Stage 1: In this stage, the goal is to obtain the control chart constants that meet desired in-control ARL_0.

Stage 2: The objective of this stage is to evaluate and compare the performance of the control charts.

A description of each stage is explained on the algorithms given by Fig. 1 and details are given in Sects. 4.1 and 4.2.

4.1 Determination of control chart constants

For a fixed number of observations and correlation levels, different combinations of predictor number, dispersion levels, shift type, and shift sizes are considered for the simulation study and their values are given in Table 2.

Detailed information about the simulation settings are as follows:

i) The number of observations, dispersion and correlation levels are fixed as given in Table 2. The β vector is set with elements $\beta_i = 1, i = 1, 2, \ldots, p$. ARL_0 is determined to be considered approximately 370.

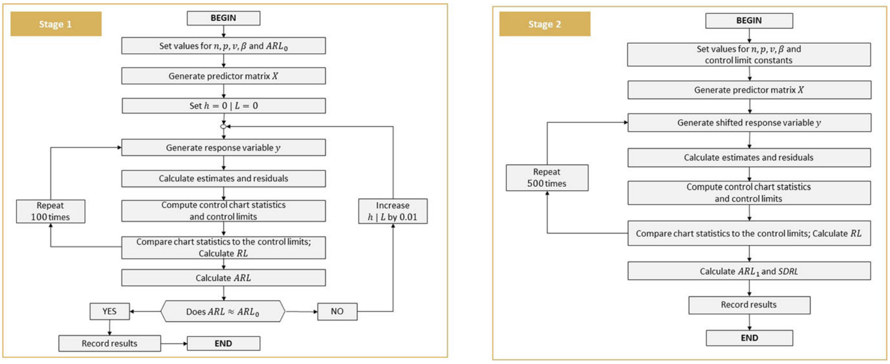

Fig. 1 Simulation algorithms for Stage 1 and Stage 2

Table 2 Simulation settings

Number of observations, n	750
Number of predictors, p	4; 7; 10
Correlation level, ρ^2	0.95
Dispersion level, v	0.75 (overdispersion) 1.00 (equidispersion) 1.50 (underdispersion)
Shift types	Additive: shift in the mean parameter Multiplicative: shift in the β coefficients
Shift size, δ	0.25 (0.25)* 1.5, 2.5,3

*This notation shows that the shift sizes from 0.25 to 1.5 are determined in increments of 0.25

ii) The correlated predictor matrix $X_{n \times p}$ is generated using the formula presented by McDonald and Galarneau (1975):

$$x_{ij} = (1 - \rho^2)^{1/2} s_{ij} + \rho s_{ip+1}, i = 1, 2, \ldots, n, \; j = 1, 2, \ldots, p$$

where s_{ij} is a random standard normal number, $\rho^2 = 0.95$ is the desired correlation between any two predictors. Then predictors are standardized by unit length standardization.

iii) The control chart constants are initialized as $h = 0$ for CUSUM and $L = 0$ for EWMA charts. Montgomery (2020) has mentioned that, $K = 0.5$ with combination corresponding h, generally provide a CUSUM chart with good ARL against 1σ shift. Lucas and Saccucci (1990) have presented optimal combinations of different λ and L that effectively minimize the ARL_1 of the EWMA chart. Montgomery (2020), on the other hand, emphasized that $0.05 \le \lambda \le 0.25$ perform efficiently in practice. Based on these results, we set the reference value for CUSUM charts as $K = 0.5$ and the smoothing parameter for EWMA charts as $\lambda = 0.05, 0.1, 0.2$ to see the impact of smoothing parameters on the overall performance.

iv) The COM-Poisson distributed response variable y is generated as $y \sim COM - Poisson(E(y), v)$, where $E(y) = \exp(X\beta)$ is the mean function.

v) Log-link function is used to model the relationship between the predictor matrix and response variable.
vi) To calculate the ML and ridge parameters of the COM-Poisson regression model, OLS estimator $\hat{\beta}^{(0)} = (X'X)^{-1}X'y$ is chosen as initial value, while for the PCR and *r*-*k* class estimators $\hat{\beta}^{(0)} = T_r T_r'(X'X)^{-1}X'y$ is chosen as the initial value. The convergence criterion for the iteration is chosen as $\|\hat{\beta}^{(t)} - \hat{\beta}^{(t-1)}\| \leq 1\times 10^{-6}$.
vii) The number of PCs is determined by the percentage of total variation (PTV) criterion which is defined by Jolliffe (2002) as the $PTV = \left(\frac{\sum_{i=1}^{r}\hat{\xi}_i}{\sum_{i=1}^{p}\hat{\xi}_i}\right) \times 100$ where $\hat{\xi}_i$, $i = 1, \ldots, p$ are the eigenvalues of the $X'WX$ matrix. This criterion determines the required number of PCs that have the highest variance for which the chosen percentage, in our case 95% is exceeded.
viii) The tuning parameter given in Eq. (5) is used in obtaining the ridge and *r*-*k* class estimators.
ix) The deviance residuals are calculated as in Eq. (6). In obtaining the deviance residuals for the saturated model, we set the normalization parameter Z equal to one when $v < 1$ and $y_i = 0, i = 1, 2, \ldots, n$, which was suggested by Sellers and Shmueli (2010).
x) We conduct the simulation study to calculate control chart statistics, then compare each control chart statistic to the control limit of the corresponding chart and obtain the RL.
xi) We reset the counter to zero and repeat the steps (iv)–(x) 100 times.
xii) We calculate the average of RL, which is ARL_0. If the desired ARL_0 is not obtained, we increase the control chart constants (h for CUSUM charts, L for EWMA charts) by 0.001 and repeat the steps (iv)–(xi) until the approximate value of desired $ARL_0 \approx 370$ is determined.

Stage 1 results of the simulation study are given in Table 3. In Table 3, the control chart constants for CUSUM and EWMA charts corresponding to the specified values of n, p, v, K, λ, ARL_0 are presented. Then the corresponding in-control mean and in-control deviation are computed that serve to compute the control limits. The control limits are then calculated by using the constants in Table 3 which will be used for Stage 2.

4.2 Performance analysis

In Stage 2, control charts are tested by adding a previously given shift to the response variable. Two types of the shifts given in Table 2 are used for the simulation. Additive shift is formulated as $\mu_1 = \mu_0 + \delta\hat{\sigma}_{ML}^0 = \exp(X\beta) + \delta\hat{\sigma}_{ML}^0$ and multiplicative shift has the form of $\mu_1 = \exp(X(\beta + \delta\hat{\sigma}_{ML}^0))$ where $\hat{\sigma}_{ML}^0$ is the in-control standard deviation of deviance residual.

The performance of the control charts is evaluated based on the RL. In Stage 2, we use ARL_1 and standard deviation of run-length (*SDRL*) to assess the performance of control charts. The chart with the lowest ARL_1 and $SDRL$ values is considered as the best.

Table 3 CUSUM and EWMA chart constants and the corresponding ARL_0 values, in-control mean and in-control standard deviations

Estimation method	p	v	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			h	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0	L	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0	L	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0	L	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0
ML	4	0.75	4.19	−0.25509	1.03614	376.35	3.44	−0.25522	1.05672	370.50	2.93	−0.26525	1.03623	375.30	2.79	−0.25848	1.03822	361.26
	4	1	4.49	−0.20624	1.05327	382.32	3.51	−0.20728	1.03610	371.51	2.82	−0.20812	1.05520	380.42	2.79	−0.21170	1.05509	380.47
	4	1.5	5.17	−0.18636	1.03368	369.47	3.12	−0.18244	1.06432	366.40	2.85	−0.18644	1.03377	382.99	3.01	−0.18302	1.03309	375.04
	7	0.75	4.23	−0.24113	1.05901	379.59	3.41	−0.23768	1.07826	377.52	2.90	−0.23615	1.05854	375.31	2.77	−0.24172	1.05966	358.72
	7	1	4.58	−0.18902	1.07166	375.15	3.42	−0.19500	1.04766	365.31	2.82	−0.19636	1.07197	372.84	2.78	−0.19870	1.06963	358.06
	7	1.5	5.12	−0.17212	1.04774	360.59	3.11	−0.16830	1.11484	373.79	2.81	−0.17708	1.04438	372.37	2.99	−0.17212	1.04774	366.76
	10	0.75	4.37	−0.20930	1.09476	382.65	3.40	−0.20851	1.11097	360.12	2.88	−0.21313	1.09792	369.78	2.73	−0.21106	1.09329	366.01
	10	1	4.76	−0.17234	1.09814	379.64	3.32	−0.17026	1.07622	355.53	2.81	−0.17110	1.09606	379.80	2.73	−0.17110	1.09606	371.09
	10	1.5	5.04	−0.15732	1.06561	378.50	3.06	−0.14945	1.03578	369.49	2.81	−0.15732	1.06561	370.32	2.91	−0.15732	1.06561	382.19
ridge	4	0.75	4.11	−0.25485	1.03677	359.63	3.44	−0.25530	1.05694	370.50	2.92	−0.26484	1.03692	366.66	2.79	−0.25822	1.03887	371.95
	4	1	4.45	−0.20736	1.05722	364.74	3.51	−0.20724	1.03196	371.51	2.81	−0.20736	1.05722	365.10	2.79	−0.21146	1.05614	378.22
	4	1.5	5.15	−0.18274	1.03309	374.32	3.11	−0.18330	1.06638	384.60	2.80	−0.18603	1.03379	361.02	2.99	−0.18606	1.03365	358.52
	7	0.75	4.24	−0.24067	1.06029	378.75	3.35	−0.23863	1.08171	359.83	2.90	−0.23762	1.06197	362.22	2.77	−0.24124	1.06090	365.18
	7	1	4.60	−0.18864	1.07330	378.15	3.37	−0.19041	1.04949	361.46	2.77	−0.19597	1.07366	367.12	2.78	−0.19829	1.07141	364.94
	7	1.5	5.14	−0.17178	1.04858	369.58	3.11	−0.17314	1.04720	370.06	2.87	−0.16849	1.04964	365.92	2.99	−0.17178	1.04858	356.05
	10	0.75	4.35	−0.21325	1.09964	363.48	3.11	−0.17017	1.10765	372.34	2.88	−0.20674	1.09723	364.24	2.79	−0.20540	1.09657	373.63
	10	1	4.68	−0.17064	1.09812	356.50	3.31	−0.20234	1.11173	361.35	2.76	−0.17064	1.09812	367.20	2.78	−0.17344	1.10313	356.71
	10	1.5	4.98	−0.15662	1.06714	361.27	3.11	−0.17012	1.07453	355.53	2.77	−0.16081	1.05900	355.93	2.86	−0.15662	1.06714	356.94

Table 3 continued

Estimation method	p	v	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			h	$\hat{\mu}^0$	$\hat{\sigma}^0$	$\boldsymbol{ARL_0}$	L	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0	L	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0	L	$\hat{\mu}^0$	$\hat{\sigma}^0$	ARL_0
PCR	4	0.75	4.14	−0.26177	1.03964	363.35	3.44	−0.25528	1.05676	370.50	2.92	−0.26492	1.03687	366.66	2.79	−0.25829	1.03883	371.96
	4	1	4.45	−0.20752	1.05652	367.27	3.51	−0.20727	1.03193	371.51	2.80	−0.20752	1.05652	363.59	2.79	−0.21163	1.05543	378.22
	4	1.5	5.11	−0.18318	1.03326	376.88	3.11	−0.18362	1.06618	379.37	2.80	−0.18654	1.03398	356.62	2.99	−0.18663	1.03383	364.58
	7	0.75	4.24	−0.24039	1.06090	378.75	3.35	−0.23860	1.08130	359.83	2.90	−0.23731	1.06263	362.21	2.78	−0.24686	1.06051	367.21
	7	1	4.60	−0.18887	1.07234	378.15	3.37	−0.19049	1.04700	361.46	2.77	−0.19620	1.07271	366.71	2.78	−0.19852	1.07046	364.90
	7	1.5	5.09	−0.17494	1.04775	355.80	3.11	−0.17067	1.10747	381.27	2.90	−0.17002	1.05163	376.09	2.91	−0.17724	1.04456	356.27
	10	0.75	4.35	−0.21229	1.10197	362.40	3.31	−0.20233	1.11109	361.35	2.83	−0.21148	1.10193	364.33	2.78	−0.20448	1.09881	369.41
	10	1	4.71	−0.17088	1.09714	369.91	3.32	−0.17024	1.07400	355.53	2.76	−0.17088	1.09714	367.20	2.77	−0.16974	1.10295	365.69
	10	1.5	5.01	−0.15723	1.06538	375.71	3.08	−0.15841	1.03587	371.73	2.82	−0.15603	1.06904	365.96	2.86	−0.15723	1.06538	364.46
r-k	4	0.75	4.11	−0.25469	1.02753	359.35	3.08	−0.15775	1.03588	379.30	2.92	−0.26467	1.02770	366.66	2.79	−0.25807	1.02963	371.95
	4	1	4.45	−0.20736	1.04773	364.74	3.31	−0.25531	1.05694	370.50	2.80	−0.20736	1.04773	356.10	2.79	−0.21146	1.04665	378.22
	4	1.5	5.14	−0.18617	1.02415	374.72	3.10	−0.20724	1.03195	371.51	2.80	−0.18613	1.02430	358.29	2.98	−0.18668	1.02438	363.49
	7	0.75	4.24	−0.24014	1.05190	378.76	3.11	−0.18332	1.06644	380.52	2.90	−0.23704	1.05364	362.21	2.78	−0.24660	1.05152	367.21
	7	1	4.60	−0.18864	1.06382	378.15	3.35	−0.23864	1.08171	359.83	2.77	−0.19597	1.06418	367.12	2.78	−0.19829	1.06194	364.94
	7	1.5	5.14	−0.17199	1.03883	367.75	3.11	−0.19041	1.04719	361.46	2.87	−0.16868	1.03990	365.92	2.99	−0.17199	1.03883	356.05
	10	0.75	4.35	−0.21205	1.09304	362.40	3.30	−0.17021	1.10787	372.34	2.83	−0.21124	1.09300	364.43	2.78	−0.20424	1.08988	374.34
	10	1	4.68	−0.17063	1.08867	359.20	3.31	−0.20236	1.11173	361.35	2.76	−0.17063	1.08867	367.20	2.78	−0.17343	1.09369	356.71
	10	1.5	4.98	−0.15693	1.05680	364.59	3.11	−0.17012	0.00000	355.53	2.80	−0.15575	1.06052	365.98	2.86	−0.15693	1.05680	364.46

Stage 2 results of the simulation study are presented in Appendix as Tables 14, 15, 16, and 17, and the key findings are given in the following paragraphs based on the shift type for each control chart:

Results for CUSUM charts

- *Additive shift, in terms of ARL_1*
  - In all combinations, the CUSUM$_{r-k}$ chart outperformed the other control charts. For most of the combinations, CUSUM$_{ML}$ shows the worst performance. ARL_1 values of the CUSUM$_{PCR}$ chart are followed by either CUSUM$_{r-k}$ or CUSUM$_{ridge}$ chart. The CUSUM$_{PCR}$ chart has the highest ARL_1 values, especially when $p = 7, 10$, and $\delta \geq 2$ for the overdispersed values. CUSUM$_{ridge}$ has similar performance to CUSUM$_{PCR}$, it mostly outperforms CUSUM$_{ML}$ except for a few combinations when $p = 4, 7$ and $v = 1, 1.5$.
  - At fixed shift size, as the number of predictors increases, the performance of all CUSUM charts improves for $v < 1$. For fixed δ and p, ARL_1 values decrease as dispersion level increases. The performance of the CUSUM charts increases as the shift size increases in all combinations of $p = 4, 7$. When $p = 4$, an analogous increase in performance is valid for $\delta \geq 1$.
  - An increase in p has a positive effect on performance by reducing the ARL_1 values. Also, there is an increase in the performance of CUSUM charts when $p = 7$ for all δ and when $p = 4, 10$ for $\delta \geq 0.5$.

- *Additive shift, in terms of SDRL*
  - The *SDRL* results of the control charts exhibit an essentially similar pattern to those of ARL_1. For fixed p and δ, the *SDRL* values of all CUSUM charts decrease as the value of v increases, and for fixed p and v, the *SDRL* values of all CUSUM charts increase as the shift size increases. CUSUM$_{r-k}$ outperforms other CUSUM charts in most combinations. In rare cases, CUSUM$_{r-k}$ is exceeded by CUSUM$_{ridge}$ or CUSUM$_{PCR}$ charts.
- *Multiplicative shift, in terms of ARL_1*
  - With very little variation in the ARL_1 values, the performances of CUSUM charts are similar to the results when the response is additively shifted. CUSUM$_{r-k}$ outperforms other CUSUM charts in all cases. CUSUM$_{ridge}$ and CUSUM$_{PCR}$ charts also show similar performance. The exceptions are $p = 7$ and $v = 1.5$ for CUSUM$_{ridge}$ and $p = 4, 7$, and $v = 0.75$ for CUSUM$_{PCR}$. In these combinations, ARL_1 values of the control charts are the highest.
  - The increased dispersion level results in increased ARL_1 values across all CUSUM charts regardless of the number of predictors and shift size.
  - Regardless of the dispersion level, the performance of CUSUM charts increases as the number of predictor increases. For $v = 0.75$, with very minor changes,

performances of the CUSUM charts increase as shift size increases, except for $p = 7$.

- *Multiplicative shift, in terms of SDRL*
  - Like the ARL_1 results, the CUSUM$_{r-k}$ chart outperforms at least one of the CUSUM charts in terms of *SDRL* and is often followed by either CUSUM$_{ridge}$ or CUSUM$_{PCR}$. Both CUSUM$_{ridge}$ and CUSUM$_{PCR}$ often have smaller *SDRL* values compared to CUSUM$_{ML}$. The *SDRL* values of CUSUM charts increase as v increases. In general, the *SDRL* performance of the control charts tends to improve as the number of predictor increases. This is particularly evident when the response is underdispersed.

Results for EWMA charts

- *Additive shift, in terms of ARL_1*
  - In the sense of ARL_1, EWMA$_{r-k}$ surpasses other EWMA charts regardless of the smoothing parameter. In some combinations of $v \geq 1$ with $p = 4, 7$, EWMA$_{r-k}$ ($\lambda = 0.05$) outperformed by other EWMA charts. On the contrary, EWMA$_{r-k}$ ($\lambda = 0.1$) and EWMA$_{r-k}$ ($\lambda = 0.2$) are the best or second-best control chart. In cases where either EWMA$_{r-k}$ ($\lambda = 0.1$) or EWMA$_{r-k}$ ($\lambda = 0.2$) is the second-best, EWMA$_{PCR}$ is outperformed by all EWMA type control charts. EWMA$_{ridge}$ typically follows EWMA$_{r-k}$ except for $\lambda = 0.1$, $p = 10$ and $\lambda = 0.2$, $p = 4, 7$, both for the dispersion level is greater than or equal to one.
  - For cases where the response is overdispersed, the performances of EWMA charts increase as the shift size increases, except for $p = 7$. Apart from EWMA$_{r-k}$, ARL_1 values increase as the smoothing parameter for the EWMA chart gets larger, regardless of p, v, and δ.
  - While EWMA($\lambda = 0.05$) charts perform better for $p = 4$ with a significantly small shift size, EWMA charts with $p = 10$ perform much better in remaining combinations for $v = 1$ in terms of ARL_1. An increase in the performance of EWMA charts in terms of ARL_1 is especially noticeable when $p = 10$. An increase in the number of predictors increases the ARL_1 performance of the EWMA charts when $\delta \leq 2$.

- *Additive shift, in terms of SDRL*
  - In terms of *SDRL*, EWMA$_{r-k}$($\lambda = 0.05$) commonly surpasses other EWMA charts, although EWMA$_{ML}$ has the highest results compared to others. The performance of EWMA$_{r-k}$ is followed by that of EWMA$_{ridge}$ or EWMA$_{PCR}$ chart. The number of cases where EWMA$_{ML}$ has the lowest *SDRL* result values is high for $\lambda = 0.1$ compared to the result values of EWMA charts with $\lambda = 0.05, 0.2$. Based on the *SDRL* values, the performances of the control charts decrease as the dispersion increases and increase as the shift size increases in fixed δ and $p = 10$ combinations, regardless of the smoothing parameter. With

the increase of the number of predictors and $\delta \geq 0.75$, the result for EWMA ($\lambda = 0.05$) charts becomes smaller. When the response is overdispersed and $\lambda = 0.1, 0.2$, the performance of the EWMA charts gets better as p increases. These results are inconsistent when $\upsilon \leq 1$.

- *Multiplicative shift, in terms of* ARL_1
  - Even though the performance of EWMA charts is not particularly affected on a pattern-wise by increasing the value of the smoothing parameter, EWMA charts with $\lambda = 0.2$ often have better results than corresponding charts with $\lambda = 0.05, 0.1$. However, when $\lambda = 0.1$, the number of cases where EWMA$_{ML}$ outperforms other charts is high. Some combinations with $\lambda = 0.2$, EWMA$_{ridge}$ and EWMA$_{PCR}$ charts have the same performance. When $\lambda = 0.05, 0.1$, EWMA$_{r-k}$ is either the best or the second-best control chart in terms of ARL_1. This control chart has the highest results when $\lambda = 0.2$, $\upsilon = 1.5$, $p = 4, 7$. EWMA$_{ridge}$($\lambda = 0.05$) is frequently positioned second or third, except where $p = 10$, $\upsilon = 0.75$, and $\delta \leq 0.5$. EWMA$_{PCR}$ ($\lambda = 0.05$), on the other hand, has high ARL_1 values when $p = 7$, the response is overdispersed and $p = 7, 10$, when the response is underdispersed. Regardless of the smoothing parameter, EWMA$_{ML}$ is the control chart with the worst performance.
  - For EWMA charts with $\lambda = 0.05$, ARL_1 values grow as the value of the υ increases when $p = 4, 10$. When $\lambda = 0.1, 0.2$, this pattern is observed in all combinations regardless of the p.
  - Regardless of the υ value, the performance of the EWMA charts with $\lambda = 0.1, 0.2$ increases as the number of predictors increases. Moreover, the performance of the EWMA charts increases as shift size increases. When $p = 10$, the ARL_1 values of all EWMA charts get larger as the dispersion level goes from 0.75 to 1.5.
- *Multiplicative shift, in terms of* $SDRL$
  - While in several combinations of simulation inputs, either EWMA$_{r-k}$ ($\lambda = 0.05$) or EWMA$_{r-k}$ ($\lambda = 0.1$) has high values, it exceeds at least two EWMA charts in terms of *SDRL*. Except for a few combinations when $p = 10$, EWMA$_{r-k}$ outperforms other EWMA charts when $lambda = 0.2$. In contrast, EWMA$_{ML}$($\lambda = 0.05$) and EWMA$_{ML}$($\lambda = 0.2$) provide the poorest outcomes in the majority of combinations. EWMA$_{PCR}$($\lambda = 0.2$) outperforms at least one EWMA chart in terms of *SDRL*. For fixed υ and δ, *SDRL* values of EWMA charts decrease as the number of predictors increase. An increase in shift size also increases the *SDRL* performance. This change has a greater influence on the outcomes, especially when $\lambda = 0.2$ and $p = 10$. An increasing value of υ causes an increase in *SDRL* values for $p = 10$.

The control charts with the best performance in terms of ARL_1 and $SDRL$ are presented in Table 4. Table 4 can be used as a reference to determine the best control chart for each scenario considered in the simulation.

Table 4 The control charts with the best performance based on the simulation results

Metric	p	Dispersion	CUSUM, $K=0.5$ Additive shift	CUSUM, $K=0.5$ Multiplicative shift	EWMA, $\lambda=0.05$ Additive shift	EWMA, $\lambda=0.05$ Multiplicative shift	EWMA, $\lambda=0.1$ Additive shift	EWMA, $\lambda=0.1$ Multiplicative shift	EWMA, $\lambda=0.2$ Additive shift	EWMA, $\lambda=0.2$ Multiplicative shift
ARL_1	4	$v<1$			EWMA$_{r-k}$				EWMA$_{r-k}$	
		$v=1$	CUSUM$_{r-k}$		EWMA$_{r-k}$ ($\delta<1.5$)/ EWMA$_{ridge}$	EWMA$_{r-k}$	EWMA$_{r-k}$			
		$v>1$			EWMA$_{ridge}$ / EWMA$_{r-k}$	EWMA$_{r-k}$			EWMA$_{r-k}$	EWMA$_{ridge}$
	7	$v<1$			EWMA$_{r-k}$		EWMA$_{r-k}$		EWMA$_{r-k}$	
		$v=1$	CUSUM$_{r-k}$							
		$v>1$			EWMA$_{r-k}$/EWMA$_{PCR}$ /EWMA$_{ridge}$	EWMA$_{r-k}$	EWMA$_{r-k}$ ($\delta<1.25$)	EWMA$_{PCR}$	EWMA$_{r-k}$/EWMA$_{PCR}$	EWMA$_{PCR}$/EWMA$_{r-k}$
	10	$v<1$			EWMA$_{PCR}$	EWMA$_{r-k}$	EWMA$_{ridge}$	EWMA$_{r-k}$	EWMA$_{r-k}$	EWMA$_{ridge}$
		$v=1$	CUSUM$_{r-k}$		EWMA$_{ridge}$	EWMA$_{r-k}$	EWMA$_{r-k}$		EWMA$_{PCR}$($\delta<1.25$)	EWMA$_{r-k}$/EWMA$_{ridge}$
		$v>1$			EWMA$_{ridge}$				EWMA$_{r-k}$	EWMA$_{PCR}$
$SDRL$	4	$v<1$			EWMA$_{r-k}$	EWMA$_{PCR}$	EWMA$_{r-k}$			
		$v=1$	CUSUM$_{r-k}$		EWMA$_{r-k}$				EWMA$_{r-k}$	
		$v>1$					EWMA$_{r-k}$	EWMA$_{ridge}$/EWMA$_{PCR}$		
	7	$v<1$			EWMA$_{r-k}$	EWMA$_{ridge}$/EWMA$_{r-k}$	EWMA$_{r-k}$			
		$v=1$	CUSUM$_{r-k}$		EWMA$_{r-k}$				EWMA$_{r-k}$	
		$v>1$	CUSUM$_{r-k}$/ CUSUM$_{PCR}$		EWMA$_{r-k}$	EWMA$_{r-k}$/EWMA$_{ridge}$	EWMA$_{r-k}$	EWMA$_{ridge}$/EWMA$_{r-k}$		
	10	$v<1$			EWMA$_{r-k}$				EWMA$_{r-k}$/EWMA$_{PCR}$/ EWMA$_{ridge}$	EWMA$_{ridge}$
		$v=1$	CUSUM$_{r-k}$				EWMA$_{r-k}$		EWMA$_{ridge}$	EWMA$_{r-k}$
		$v>1$			EWMA$_{r-k}$	EWMA$_{ridge}$			EWMA$_{r-k}$/EWMA$_{PCR}$	EWMA$_{r-k}$

5 Real life application

In this section, the proposed method for monitoring the COM-Poisson profile is illustrated via a case study by analyzing the SECOM data set. This data set is obtained from a semiconductor manufacturing process and is available on the UCI machine learning repository. The data set was provided by McCann and Johnston (2008).

5.1 Data information

A modern semiconductor manufacturing process is equipped with high technology. The monitoring of the process is carried out on a continuous basis through the collection of signals/variables from sensors and/or process measurement points. Each type of signal can be considered a feature to improve the quality of semiconductors.

The data set presented in this section is generated from a similar process. After being measured by the 590 sensors, each of the 1567 instances of the production line was subjected to a Pass/Fail test. Test result associated with a specific date time stamp is either -1 or 1, where -1 stands for pass and 1 for failure. The null values are described using 'NaN'. The final data set is the 1567×591 matrix which consists of 590 features and a class. Moldovan et al. (2017), Moldovan et al. (2018), and Kim et al. (2017) used a data set to illustrate different machine learning algorithms for classification problems. Cao et al. (2020) applied a mixture of two refinement methods to detect quality issues in the data set. Via data set, Takahashi et al. (2019) demonstrated simplified machine learning implementation to reduce the time spent on failure prediction in manufacturing processes. Kwon and Kim (2020) adapted iterative feature selection for failure prediction and preferred the data set to evaluate the performance of the proposed method.

Inspired by the aforementioned studies, we designed the analysis process of the SECOM data set with deviation residual-based control charts. Our purpose is to determine if the CUSUM and EWMA charts based on PCR and *r–k* deviance residuals outperform the CUSUM and EWMA charts based on ML and ridge deviance residuals. The analysis of the data is conducted in *R* by using *"COMPoissonReg"* and *"SPC"* packages.

5.2 Data reconstruction

In this subsection, we rearranged the data and made it usable for our purposes. For this case, we follow the steps described below. The original data with a binary response variable was collected on certain days, and the time of data points collected was noted, so we followed steps 3–5 to convert it to count data.

1. Null values are replaced with previous value which results in the X matrix as 1567×590;
2. All categorical predictors are removed and only continuous predictors are kept. Then, the predictor matrix X is as 1567×444;
3. The 24-h period was divided into 8 different groups which is called as time frame. Time frame variable created based on the time stamp and the corresponding time

Table 5 The correlation matrix

	S-93	S-95	S-98	S-103	S-116	S-198	S-470	S-523
S-93	1	***0.94867***	0.04749	0.04077	0.03234	0.04463	0.07528	0.03443
S-95	***0.94867***	1	0.08150	0.07547	0.03402	0.02144	0.04957	0.03896
S-98	0.04749	0.08150	1	***0.95109***	0.01027	0.00308	0.00345	0.00034
S-103	0.04077	0.07547	***0.95109***	1	0.00975	0.01316	0.01059	0.00347
S-116	0.03234	0.03402	0.01027	0.00975	1	0.00025	0.01112	***0.94833***
S-198	0.04463	0.02144	0.00308	0.01316	0.00025	1	***0.94820***	0.01797
S-470	0.07528	0.04957	0.00345	0.01059	0.01112	***0.94820***	1	0.01611
S-523	0.03443	0.03896	0.00034	0.00347	***0.94833***	0.01797	0.01611	1

intervals are $1 \rightarrow 01:00-03:59, 2 \rightarrow 04:00-06:59, 3 \rightarrow 07:00-09:59, 4 \rightarrow 10:00-12:59, 5 \rightarrow 13:00-15:59, 6 \rightarrow 16:00-18:59, 7 \rightarrow 19:00-21:59, 8 \rightarrow 22:00-01:59(nextday)$;

4. The data and the time frame are combined to create "date and time frame (DTF)", e.g. 19-7-2008-4 stands for date 19-7-2008 and time frame 4;
5. The number of passes on each DTF is summed to create the response. The new predictor variable values are obtained by taking the average of each predictor variable in the corresponding DTF. By this way, a new data set based on DTF is created; the resulting matrix X is 476×444 dimensional;
6. Within the data set arranged in steps 1–5, features/variables with the absolute value of pairwise correlation value between 0.948 and 0.952 were selected to be included in the analysis.[3] Then, we get the predictor matrix X in dimension 476×8 where the predictors in the model are values recorded by sensors numbered 93, 95, 98, 103, 116, 198, 470, and 523. The pairwise correlation matrix is than obtained as seen in Table 5.

5.3 Data processing

In this subsection, we first determine the distribution of the data set created in Sect. 5.2, which we obtained with the eight predictors with 467 samples. Since Kim et al. (2017) mentioned that the data distribution is very irregular in each feature, prior to the modeling data set, we scaled the X matrix. Then, the ML estimator of regression coefficients is obtained which is then used for the multicollinearity diagnostics, the tuning parameter selection and the number of PCs selection.

Three different count data modelling techniques are utilized to determine the best-fitting distribution for the response variable: Log-likelihood, Akaike information criterion (AIC), and Bayesian information criterion (BIC). The log-likelihood, AIC, and BIC are than computed for the COM-Poisson, Negative Binomial, and Poisson models and the exact results are given in Table 6. The COM-Poisson distribution is recognized as the best fitting distribution due to its high log-likelihood and low

[3] Multicollinearity degree of about 0.95 is targeted to support the simulation study.

Table 6 Diagnostic analysis of best-fitted distribution for response variable

	Log-likelihood	AIC	BIC
COM-Poisson	−1028.77204	2075.54409	2106.86743
Negative Binomial	−1331.30338	2680.60676	2711.93010
Poisson	−1666.44804	3342.89608	3382.21942

Table 7 Comparison of deviance residuals calculated from the models with ML, ridge, PCR, and *r*–*k* class estimators

	[Min. values, Max. values]	First 10 observations with the highest differences
$\mid d_{ML} - d_{ridge} \mid$	[0.000000008102, 0.00898]	(73, 155, 156, 234, 278, 323, 359, 362, 406, 423)
$\mid d_{ML} - d_{PCR} \mid$	[0.00000001433, 0.04814]	(11, 39, 89, 99, 128, 248, 355, 384, 400, 461)
$\mid d_{ML} - d_{r-k} \mid$	[0.0000001098, 0.04512]	(11, 39, 89, 99, 128, 248, 384, 400, 452, 461)
$\mid d_{ridge} - d_{PCR} \mid$	[0.00000006621, 0.04977]	(11, 39, 89, 99, 128, 188, 248, 355, 400, 461)
$\mid d_{ridge} - d_{r-k} \mid$	[0.00000002929, 0.04675]	(11, 39, 89, 99, 128, 248, 384, 400, 452, 461)
$\mid d_{PCR} - d_{r-k} \mid$	[0.0000000668, 0.01509]	(73, 99, 156, 234, 278, 323, 359, 362, 406, 423)

AIC and BIC values. An estimated dispersion for the response variable is computed as $v = 0.2019$, which indicates overdispersion.

After it is seen that the response variable follows COM-Poisson distribution, we parameterized the distribution. ML estimator is first obtained utilizing the OLS estimator as an initial value and $\|\hat{\beta}^{(t)} - \beta^{(t-1)}\| \leq 1 \times 10^{-6}$ is used as convergence criterion.

Scaled information matrix is examined for the multicollinearity. We calculated the eigenvalues of scaled information matrix as 64.271407, 56.797600, 54.648692, 48.305966, 1.584542, 1.432938, 1.418611, 1.326779 and the variance inflation factor (VIF) values as 28.49139, 28.50746, 29.49110, 29.49507, 28.37309, 28.51614, 28.56399, 28.34829. Since all VIF values are larger than 10, they indicate that all the predictors are involved in multicollinearity, supporting step (6) in Sect. 5.2.

The tuning parameter for ridge and *r*–*k* class estimators is obtained as $k = 0.001536528$ by using Eq. (5). The number of PCs that explain approximately 99% of the overall variance is obtained as 7. Then the ML, ridge, PCR, and *r*-*k* deviance residuals are calculated as in Eq. (6).

The deviance residuals with respect to each other are compared to see the difference between residuals. The absolute value of the differences of the residuals was taken, and the minimum and maximum values of these obtained values and the top ten highest values were tabulated in Table 7. Table 7 shows how the different estimation methods affect the residuals and in which observations they are most effective relative to each other.

Figure 2 shows while the closest residual values to each other are between d_{PCR} and d_{r-k}, it is followed by d_{ML} and d_{ridge} and it is clear that residual differences are larger than these two. The highest range is between d_{ML} and d_{r-k}.

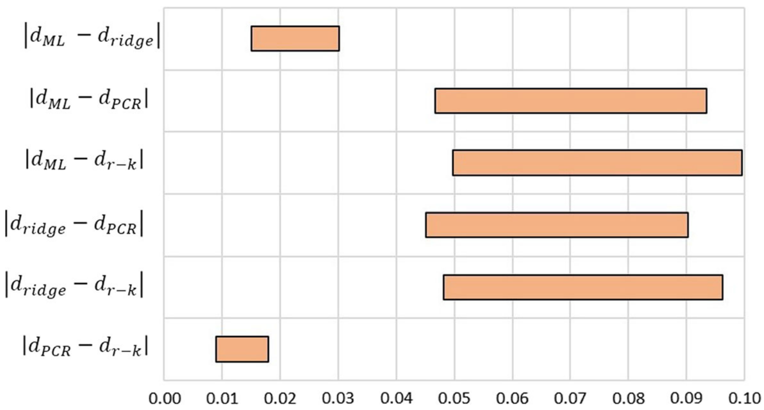

Fig. 2 Difference intervals for deviance residuals

5.4 Monitoring process: performed for CUSUM and EWMA separately

After we see in Sect. 5.3 that SECOM data set follows COM-Poisson distribution with correlated predictors and difference occurs between the residuals when different estimation methods are used, we switch to the profile monitoring.

Due to the lack of prior information on the status of the data set, we split the data set into two sets: training and test sets. The first 25% of the data set (first 119 observations) forms the training set and is used to create the data set of the in-control state, while the last 75% of the data set (last 357 observations) form the test set is used to examine the performance of the control charts.

After we split the data as training and test sets, we used the training set to construct the control limits from in-control data, and the test set for the analysis. The steps followed in each data set can be explained as follows.

1. Creating in-control data from training set
   (a) Utilizing in-built functions of the *"SPC"* package with the mean and standard deviation of deviance residuals of the training set, we obtained the control limits that met the $ARL_0 \approx 200$ criterion;
   (b) We calculated the CUSUM chart statistics with $K = 0.5$ and EWMA chart statistics with smoothing parameters of $\lambda = 0.05, 0.1, 0.2$ of ML, ridge, PCR, and *r–k* deviance-based control charts;
   (c) We compared the control chart statistics with the control limits of the each control chart. Results are given in Tables 8 and 9 for CUSUM and EWMA charts respectively.
   Table 8 shows that the 80 observations exceeded the upper control limit of the CUSUM$_{ML}$ and CUSUM$_{ridge}$ charts while 79 observations for the CUSUM$_{PCR}$ and CUSUM$_{r-k}$ charts. On the other hand, Table 9 shows that EWMA ($\lambda = 0.05$) detected two (2nd and 3rd) observations while EWMA($\lambda = 0.1$) detected only one (2^{nd}) observation as out-of-control observation. EWMA$_{ML}$ and EWMA$_{ridge}$ detected one (2^{nd}) observation whereas

Table 8 Control limits and the number of out-of-control observations identified by each CUSUM chart

	Control limits	Number of out-of-control observations
CUSUM_{ML}	$\pm$ 1.697833	80
CUSUM_{ridge}	$\pm$1.698043	80
CUSUM_{PCR}	$\pm$1.696883	79
CUSUM_{r-k}	$\pm$1.697169	79

Table 9 Control limit constants and the number of out-of-control observations identified by each EWMA chart

	Control limit constant			Number of out-of-control observations		
	$\lambda = 0.05$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.05$	$\lambda = 0.1$	$\lambda = 0.2$
EWMA_{ML}	4.839887	4.274365	3.84603	2	1	1
EWMA_{ridge}	4.839496	4.274089	3.845839	2	1	1
EWMA_{PCR}	4.843432	4.276866	3.84776	2	1	2
EWMA_{r-k}	4.842814	4.27643	3.847458	2	1	2

Table 10 CUSUM chart limits, in-control mean and standard deviation of training set

	Control limits	$\hat{\mu}_0$	$\hat{\sigma}_0$
CUSUM_{ML}	$\pm$3.011428	0.615378	0.271395
CUSUM_{ridge}	$\pm$3.009625	0.615453	0.271221
CUSUM_{PCR}	$\pm$3.004797	0.615778	0.270584
CUSUM_{r-k}	$\pm$3.001579	0.61591	0.270273

$\text{EWMA}_{PCR}(\lambda = 0.2)$ and $\text{EWMA}_{r-k}(\lambda = 0.2)$ detected two (2nd and 88th) observations.

(d) We pooled the out-of-control observations detected by ML, ridge, PCR, and *r–k* deviance-based control charts and eliminated the unique ones to create the in-control training set. 80 observations were eliminated for monitoring the CUSUM chart and three observations were eliminated for the EWMA chart;

(e) By obtaining the control limits meeting $ARL_0 = 200$, we calculated the mean and standard deviation of each in-control training set such that the results are given in Tables 10 and 11.

2. Analysis for the test set

(a) We calculated the control chart statistics for each chart by using test data;

(b) We compared the control limits obtained from the training set with the control chart statistics obtained in (a);

(c) We analyzed the performance of the control chart in terms of terms of ARL_1 where ARL_1 values are obtained by using the appropriate functions of the *"SPC"* package and the results are presented in Table 12.

Table 11 EWMA chart limit constants, in-control mean and standard deviation of training set

	$\lambda = 0.05$	$\lambda = 0.1$	$\lambda = 0.2$	$\hat{\mu}_0$	$\hat{\sigma}_0$
EWMA_{ML}	2.651935	2.776787	2.856777	0.549335	0.328883
EWMA_{ridge}	2.651731	2.776636	2.856673	0.549269	0.328982
EWMA_{PCR}	2.650594	2.775801	2.856099	0.549929	0.328395
EWMA_{r-k}	2.650289	2.775577	2.855944	0.549825	0.328533

Table 12 ARL_1 results of CUSUM and EWMA charts

	ML	*ridge*	*PCR*	*r–k*
CUSUM	5.232221	5.230022	5.212231	5.208366
EWMA, $\lambda = 0.05$	26.04584	26.04308	25.87346	25.87023
EWMA, $\lambda = 0.1$	37.30165	37.29657	37.04754	37.04124
EWMA, $\lambda = 0.2$	54.93462	54.92688	54.58413	54.57429

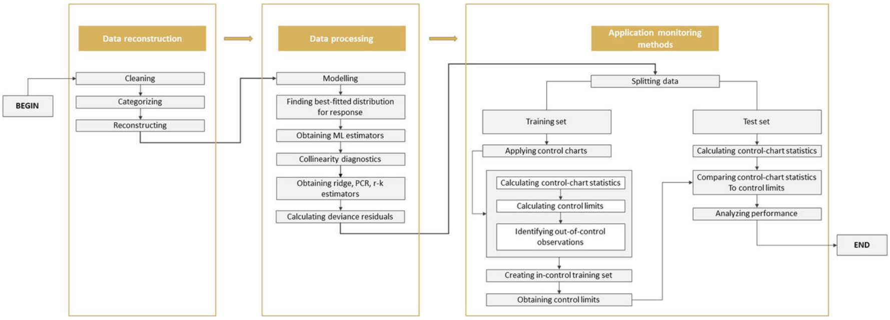

Fig. 3 Analyzing SECOM data set

The process steps from Sects. 5.3 to 5.4 can be summarized with the flowchart given in Fig. 3.

5.4.1 Performance evaluation of the control charts

To see the direction of the shift, differences between the mean of residuals in the in-control training set and the mean of residuals in the test data set are computed and the results are given by Table 13. It was observed that the mean residual of the test set decreased when compared with the mean of the residuals under the in-control training set. According to Table 13 results, it is possible to say that a negative shift has occurred. Negative change is particularly evident in Fig. 4 since control chart statistics are positioned on the bottom part of the zero line and in Figs. 5, 6 and 7 since they show a downward trend.

Table 12 shows that both CUSUM_{r-k} and EWMA_{r-k} are the best in comparison to the corresponding control charts based on the ML, ridge, and PCR deviance residuals in terms of ARL_1. The CUSUM_{PCR} and EWMA_{PCR} charts follow CUSUM_{r-k} and

Table 13 Mean change $\hat{\mu}^1_{est} - \hat{\mu}^0_{est}$

	dev_{ML}	dev_{ridge}	dev_{PCR}	dev_{r-k}
CUSUM	−0.24073	−0.24081	−0.24015	−0.24029
EWMA	−0.17469	−0.17463	−0.1743	−0.17421

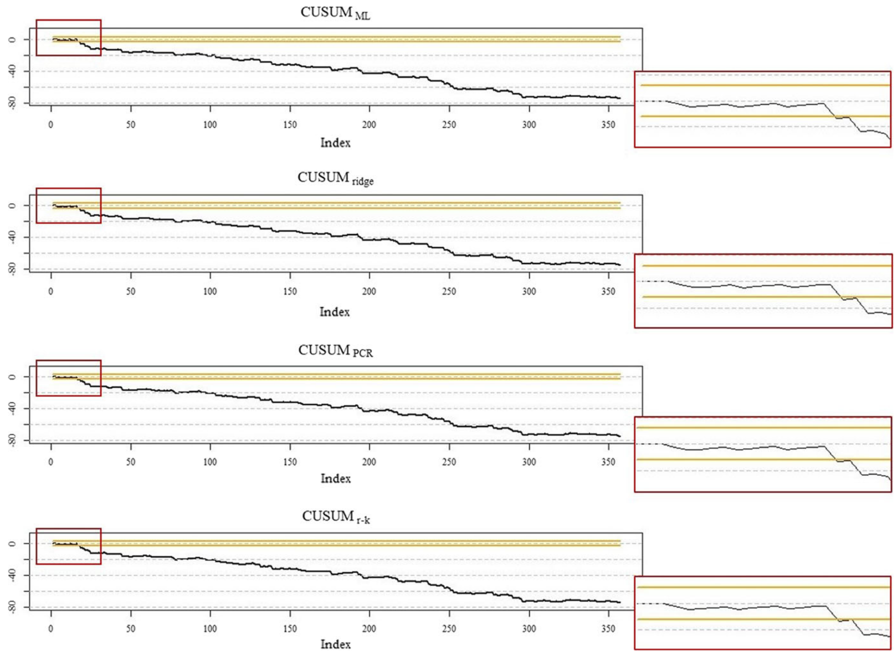

Fig. 4 Analysis of test set with CUSUM charts

EWMA$_{r-k}$, respectively and the CUSUM$_{ML}$ and EWMA$_{ML}$ charts have the highest ARL_1 values.

The visualized analysis of the test set is conducted by plotting CUSUM and EWMA chart statistics against the observation number for the test set, as well as the control limits calculated from Tables 10 and 11 and we show the outcomes with Figs. 4, 5, 6, and 7. Figures 5, 6 and 7 show the effect of the smoothing parameter for the EWMA chart.

6 Conclusions

The traditional way to monitor count data in statistical process control is to use approaches designed for Poisson distribution. However, the Poisson distribution is used in the case of equidispersion and in real-life data sets, the underdispersed or overdispersed state may occur. If indications of under or overdispersion are present in the data set, it is more appropriate to use the COM-Poisson distribution.

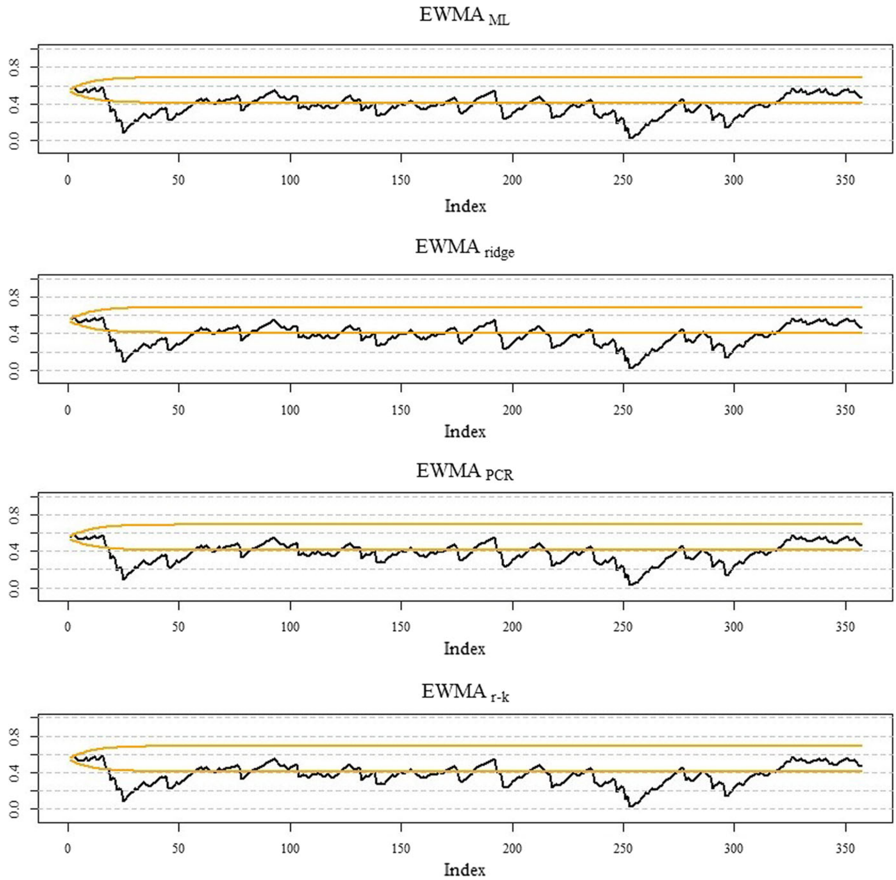

Fig. 5 Analysis of test set with EWMA charts with $\lambda = 0.05$

This study presents CUSUM$_{PCR}$, CUSUM$_{r-k}$, EWMA$_{PCR}$, and EWMA$_{r-k}$ control charts based on deviance residuals to detect the out-of-control observations in the COM-Poisson profile, addressing the issue of multicollinearity in profile monitoring. The proposed control charts are compared to the CUSUM and EWMA control charts based on the deviance and ridge deviance residuals through a simulation study and real-life data set analysis and their performances are evaluated in terms of the ARL_1 and *SDRL* statistics and.

Results showed that the *r–k* deviance-based charts mainly outperform the other control charts, while the deviance-based charts show the worst results in most combinations in the case of multicollinearity. Moreover, CUSUM charts show better results compared to the EWMA charts. It is to be noted that the choice of the smoothing parameter affects the performance of the EWMA charts. Both types of charts are more effective in detecting additive shifts than multiplicative shifts in the majority of combinations considered in the simulation. Furthermore, for fixed dispersion value and shift size, the changes in the result values are more plain and consistent in cases with high

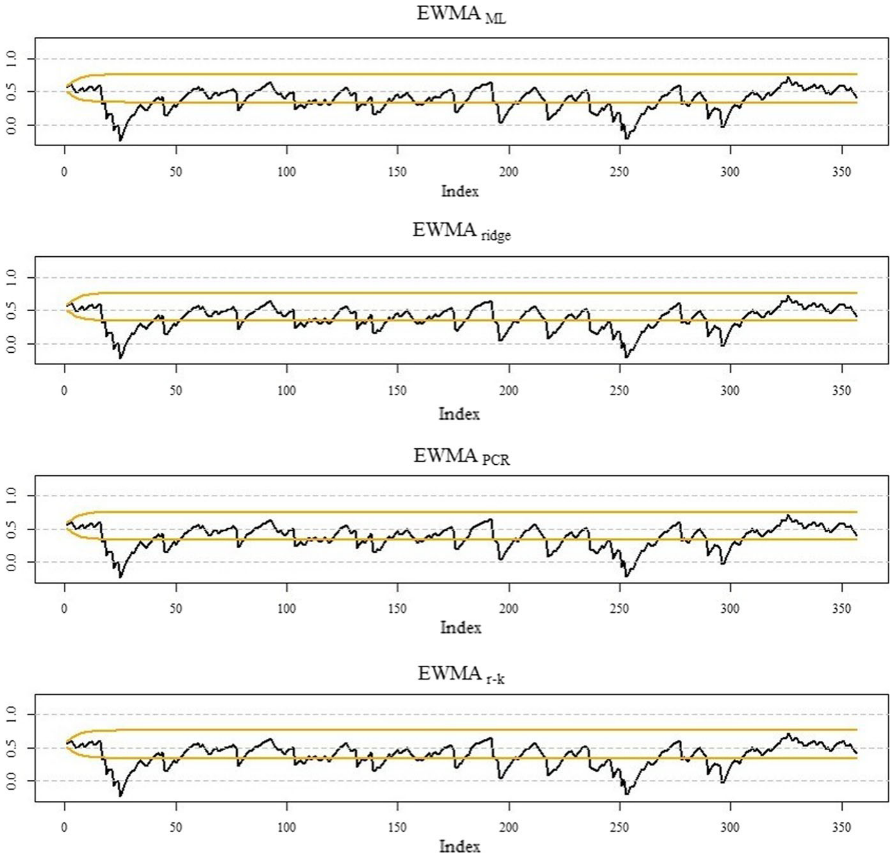

Fig. 6 Analysis of test set with EWMA charts with $\lambda = 0.1$

predictor numbers. The results are generally incompatible when the response is under-dispersed. In summary, in the case of multicollinearity, CUSUM_{r-k} and EWMA_{r-k} charts based on deviance residuals perform better than alternative CUSUM and EWMA charts in determining small shifts in the process. The good performance indicator varies according to parameters, such as the number of predictors, dispersion level and shift size.

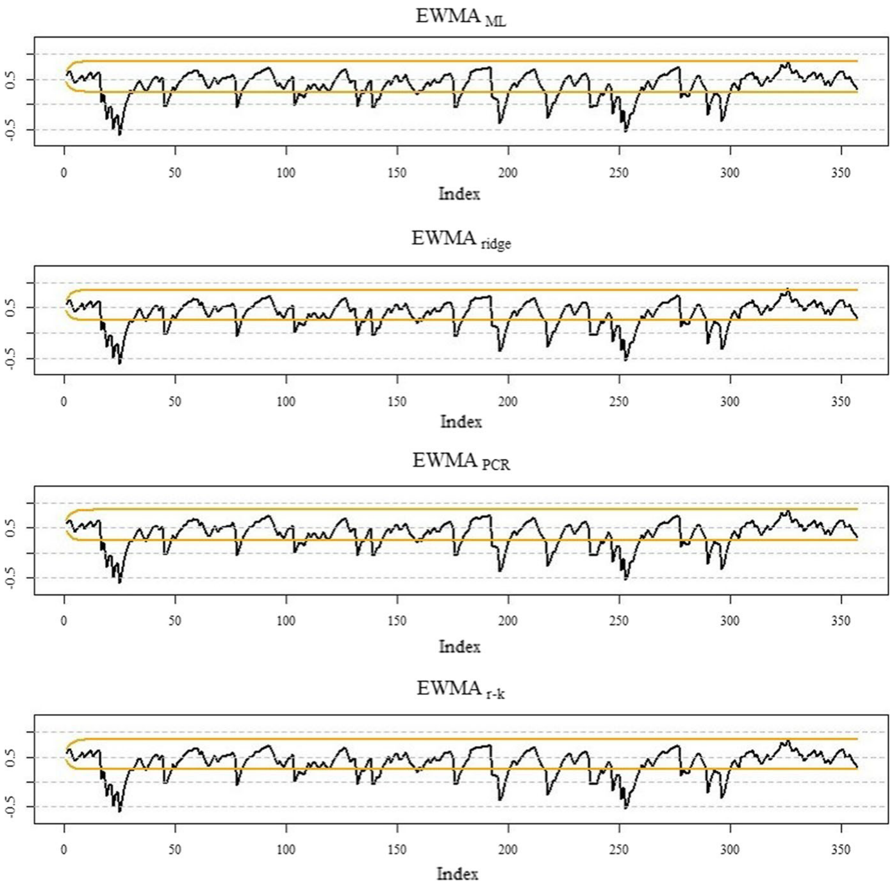

Fig. 7 Analysis of test set with EWMA charts with $\lambda = 0.2$

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Appendix: Simulation study results

Table 14 ARL_1 results when additive shift is present

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
4	0.75	0.25	23.872	22.992	23.816	22.360	157.584	157.496	157.584	148.528	192.056	190.104	190.104	130.900	199.620	199.172	198.392	192.648
4	1	0.25	25.880	25.892	25.788	25.436	160.940	160.940	160.940	85.752	192.012	191.484	190.268	186.100	196.840	196.184	196.840	187.408
4	1.5	0.25	32.092	31.012	30.784	30.348	171.768	170.104	171.112	199.300	180.856	172.840	173.280	170.716	208.336	204.072	204.388	194.588
7	0.75	0.25	22.588	22.544	22.556	22.228	164.076	163.400	162.392	153.048	151.148	188.768	188.768	117.932	167.208	166.924	167.968	158.072
7	1	0.25	25.924	25.912	25.912	25.604	161.104	157.308	157.308	89.076	190.280	179.428	179.580	171.392	177.392	177.096	177.820	169.568
7	1.5	0.25	31.404	31.680	31.028	31.004	165.156	164.972	164.788	209.160	177.824	192.020	193.444	186.400	210.364	209.820	192.384	205.268
10	0.75	0.25	18.432	18.456	18.504	18.332	78.204	50.332	71.592	69.224	113.636	114.524	102.544	53.536	94.372	93.976	94.580	90.800
10	1	0.25	22.140	21.516	21.604	21.460	90.352	79.248	90.352	79.248	121.680	114.336	115.472	110.264	117.612	117.632	118.332	113.336
10	1.5	0.25	26.992	26.296	26.632	26.064	157.912	193.440	156.760	193.440	139.284	129.268	145.920	134.308	133.176	127.116	128.016	125.288
4	0.75	0.5	24.152	22.792	22.828	22.668	148.688	149.360	148.688	146.664	180.032	178.240	180.012	123.628	189.176	188.136	189.176	185.080
4	1	0.5	26.272	26.104	26.096	25.956	150.884	150.884	150.884	75.268	177.228	176.760	175.684	170.812	168.683	168.498	168.984	160.984
4	1.5	0.5	32.464	32.188	31.236	31.048	166.848	164.092	165.468	190.008	164.072	157.700	158.660	155.284	189.496	183.136	184.352	173.820
7	0.75	0.5	21.404	21.408	21.572	21.336	155.060	150.540	150.376	142.632	140.124	173.144	173.444	110.476	150.792	151.080	151.484	145.676
7	1	0.5	25.284	25.344	25.348	25.136	159.660	155.196	155.196	86.800	172.472	163.172	163.072	152.660	160.440	160.756	160.760	155.156
7	1.5	0.5	30.852	30.660	30.584	30.064	160.632	157.832	160.164	198.944	159.788	175.924	178.956	170.436	194.984	193.216	172.416	188.184
10	0.75	0.5	16.684	16.572	16.640	16.444	70.856	47.344	67.796	63.696	99.068	100.936	91.224	47.200	74.788	82.568	83.772	80.444
10	1	0.5	20.852	19.796	20.144	19.608	85.228	71.452	85.228	71.452	111.676	103.192	104.076	102.104	100.540	107.952	108.704	104.836
10	1.5	0.5	27.832	26.896	27.580	26.064	153.596	186.248	149.028	186.248	124.932	113.412	131.720	120.452	125.384	117.472	118.772	112.840
4	0.75	0.75	23.940	23.100	23.744	22.420	137.964	137.904	137.900	134.784	156.652	155.612	156.624	106.992	174.956	174.268	174.952	172.236
4	1	0.75	26.100	26.016	25.996	25.360	149.244	149.244	149.244	71.956	159.872	158.256	155.500	150.532	148.416	148.552	148.552	144.860
4	1.5	0.75	33.720	33.172	32.916	32.804	161.652	159.136	159.300	176.204	150.852	142.524	143.584	138.780	177.192	168.948	170.868	160.332

Table 14 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
7	0.75	0.75	20.748	20.672	20.780	20.492	145.264	139.620	139.620	133.384	128.220	160.476	160.496	101.280	138.352	139.272	138.324	130.344
7	1	0.75	24.000	24.176	24.176	23.528	151.760	148.988	148.988	78.636	159.248	150.196	148.120	141.104	141.892	143.460	143.460	139.472
7	1.5	0.75	29.616	30.012	29.464	28.884	147.124	144.232	145.440	178.460	142.092	157.920	163.180	153.860	180.400	180.060	155.756	173.664
10	0.75	0.75	15.952	15.516	15.712	15.248	60.984	43.624	57.952	56.084	88.116	91.264	84.020	43.012	64.044	75.408	75.484	71.568
10	1	0.75	20.388	19.676	19.716	19.336	79.756	68.936	79.756	68.936	99.004	90.312	91.200	89.984	88.228	93.904	95.736	90.444
10	1.5	0.75	26.608	26.108	26.980	25.824	141.432	169.640	135.620	169.640	113.592	106.436	116.084	110.732	113.860	105.924	106.284	99.288
4	0.75	1	23.560	22.376	22.604	22.124	133.764	133.712	133.700	132.024	142.968	139.536	141.328	96.792	154.684	154.244	154.168	147.860
4	1	1	25.592	25.224	25.228	24.952	141.228	141.228	141.228	63.868	143.864	143.364	141.924	137.124	134.176	134.176	134.176	131.436
4	1.5	1	32.600	32.524	32.340	31.924	143.420	138.792	140.216	159.460	133.680	125.836	126.016	121.860	159.996	154.256	155.704	146.728
7	0.75	1	19.684	19.876	19.872	19.452	135.088	127.744	127.732	121.284	121.092	151.612	152.640	95.512	119.828	119.488	117.784	110.420
7	1	1	23.204	23.280	23.264	22.940	137.912	134.176	134.176	71.560	137.696	127.840	127.564	120.444	123.148	124.820	123.864	120.768
7	1.5	1	28.972	28.908	28.712	28.706	137.796	136.448	136.456	166.308	127.076	139.928	147.848	137.820	163.064	163.076	138.772	156.624
10	0.75	1	16.036	15.888	16.020	15.596	56.288	44.232	53.916	51.664	80.772	84.216	78.156	38.300	60.352	68.604	68.616	67.224
10	1	1	19.632	19.352	19.436	19.028	70.564	62.256	70.564	62.256	87.892	80.720	81.596	78.988	78.708	84.300	85.756	82.296
10	1.5	1	26.076	25.636	26.104	25.336	128.880	153.532	124.140	153.528	101.040	94.120	104.504	98.728	103.088	94.348	94.032	89.428
4	0.75	1.25	21.952	21.240	21.496	20.956	119.364	119.360	118.252	118.064	124.392	123.232	123.548	84.344	139.532	138.860	140.704	135.536
4	1	1.25	24.940	24.768	24.760	24.468	130.152	130.152	130.152	58.724	120.840	120.792	120.436	116.016	122.588	122.588	122.588	118.024
4	1.5	1.25	31.292	31.236	30.544	30.246	131.720	128.084	129.204	146.324	115.576	110.736	110.424	108.724	143.440	137.896	138.608	134.492
7	0.75	1.25	19.240	19.248	19.252	19.140	120.308	116.436	115.512	109.900	101.924	126.352	126.416	80.152	103.240	104.176	102.848	97.284
7	1	1.25	22.192	22.232	22.228	22.152	129.028	126.252	126.252	65.052	122.500	113.684	113.688	106.616	112.112	113.444	113.076	108.104
7	1.5	1.25	27.168	27.412	27.060	27.020	122.284	121.524	121.456	150.372	115.804	130.236	138.892	126.152	152.064	151.320	128.520	145.988
10	0.75	1.25	15.368	15.280	15.352	14.944	51.508	38.400	49.704	47.028	71.452	74.556	69.440	32.604	56.008	62.260	62.400	60.420
10	1	1.25	19.092	18.656	18.880	18.208	67.164	55.460	67.156	55.464	77.456	71.920	73.604	70.872	70.972	76.456	77.508	75.164
10	1.5	1.25	24.148	23.756	24.108	23.672	115.408	139.472	111.484	139.240	88.156	79.864	92.008	84.524	93.564	87.460	86.944	84.048

Table 14 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
4	0.75	1.5	20.280	20.004	20.104	19.616	106.892	106.144	104.960	102.772	108.168	106.812	107.352	71.008	120.568	119.992	120.336	112.848
4	1	1.5	22.952	22.880	22.776	22.608	121.888	121.432	121.432	48.728	102.168	101.696	100.168	98.712	110.768	110.104	109.768	107.140
4	1.5	1.5	31.000	31.000	30.696	30.264	119.996	117.504	117.760	131.924	98.600	97.216	96.764	95.820	130.916	126.464	127.812	122.708
7	0.75	1.5	18.708	18.736	18.728	18.644	109.612	102.868	102.860	97.300	86.860	108.360	108.328	68.696	90.744	91.120	88.560	86.196
7	1	1.5	21.776	21.836	21.876	21.696	117.712	114.752	114.684	54.800	105.876	96.340	96.296	91.560	97.504	97.772	97.936	94.392
7	1.5	1.5	26.441	26.208	25.980	25.784	115.968	114.240	114.884	143.560	101.008	115.556	125.700	111.996	136.700	135.564	117.976	132.804
10	0.75	1.5	15.256	15.148	15.404	15.108	49.960	29.912	47.788	42.656	64.416	65.956	60.292	30.596	52.728	58.648	58.668	56.776
10	1	1.5	18.424	17.304	17.672	17.048	59.704	50.024	59.704	50.024	71.804	65.548	66.364	64.236	64.628	69.452	71.992	68.340
10	1.5	1.5	23.728	23.108	23.364	22.700	101.644	122.672	96.212	122.672	82.128	73.668	83.184	76.752	86.136	78.328	78.804	75.444
4	0.75	2	18.532	17.864	18.012	17.428	83.744	83.736	83.456	83.440	78.508	78.212	78.508	55.944	92.796	93.908	94.524	88.956
4	1	2	21.124	20.848	20.840	20.448	105.096	105.080	105.084	35.128	82.380	82.372	80.580	78.860	85.052	83.972	83.968	80.748
4	1.5	2	28.364	28.612	28.292	28.088	95.564	91.928	92.900	91.832	78.248	76.488	76.304	74.992	107.912	104.592	105.304	100.760
7	0.75	2	18.052	18.140	18.200	17.968	79.852	70.764	70.752	68.008	64.780	77.048	77.932	51.344	66.116	66.220	66.116	61.204
7	1	2	19.856	19.924	19.916	19.784	97.640	96.220	96.220	40.776	77.944	73.184	72.828	70.252	72.828	73.308	72.940	68.104
7	1.5	2	24.592	25.084	24.544	24.366	100.152	98.844	98.456	115.608	76.784	90.012	96.388	85.700	117.544	118.656	92.624	107.448
10	0.75	2	13.652	13.608	13.684	13.376	34.092	21.380	31.720	30.032	47.192	48.156	43.588	22.720	42.844	47.204	47.300	46.148
10	1	2	17.636	16.628	16.876	16.356	44.468	37.684	44.192	37.684	56.384	52.308	53.104	51.332	51.484	56.812	57.956	54.616
10	1.5	2	21.668	21.484	21.548	21.224	78.912	95.048	75.556	95.048	62.456	57.076	65.244	61.080	69.208	64.360	64.600	61.308
4	0.75	2.5	17.268	16.704	16.724	16.392	63.180	63.476	63.456	62.480	57.144	56.820	56.928	41.716	66.820	68.568	69.316	64.252
4	1	2.5	20.552	20.300	20.296	19.736	76.064	76.064	76.064	24.764	61.772	61.768	60.828	58.048	69.625	69.327	69.028	65.824
4	1.5	2.5	26.328	26.348	26.100	25.988	76.096	72.460	73.920	78.332	63.676	61.520	61.472	60.120	94.276	91.864	91.620	86.904

Table 14 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
7	0.75	2.5	16.848	16.912	16.940	16.616	56.944	51.392	51.912	48.032	46.848	54.088	55.204	34.564	50.760	51.096	50.620	48.400
7	1	2.5	18.756	19.148	19.092	18.716	72.960	68.332	68.332	31.048	56.736	52.364	52.360	50.556	56.088	56.596	55.944	55.500
7	1.5	2.5	23.872	23.944	23.804	23.456	72.168	69.848	70.392	83.652	57.488	68.716	71.048	63.092	90.212	90.856	72.464	83.388
10	0.75	2.5	12.576	12.572	12.788	12.500	28.716	18.564	25.256	24.128	35.296	35.896	32.152	18.536	32.124	35.084	35.152	34.116
10	1	2.5	15.900	15.536	15.716	14.996	34.176	26.428	34.172	26.428	45.120	41.804	41.804	40.100	45.148	48.700	49.216	46.372
10	1.5	2.5	20.968	20.408	20.616	20.188	62.656	74.908	58.916	74.904	50.584	47.684	52.116	49.716	60.980	56.060	56.684	53.844
4	0.75	3	15.980	15.616	15.744	15.444	43.884	42.916	42.900	42.060	40.408	39.884	39.884	27.684	50.808	50.252	50.800	48.476
4	1	3	18.312	18.240	18.232	18.080	58.628	58.628	58.628	20.368	47.440	47.440	46.408	44.768	52.452	52.128	52.124	50.164
4	1.5	3	24.688	24.788	24.516	24.316	63.176	60.528	60.488	64.992	52.516	48.032	48.900	47.384	76.764	73.856	73.836	69.020
7	0.75	3	15.744	15.824	15.940	15.668	39.516	37.536	37.536	34.064	30.440	37.224	37.224	23.912	37.628	37.444	37.192	36.036
7	1	3	17.552	17.752	17.692	17.444	55.376	53.828	53.828	23.852	40.696	37.316	37.304	36.108	42.396	42.224	42.224	39.712
7	1.5	3	22.508	22.472	22.128	22.108	57.236	55.840	55.324	64.668	46.848	52.648	55.488	49.724	72.812	72.568	56.252	68.360
10	0.75	3	11.996	11.844	12.000	11.688	19.068	14.296	17.384	16.568	24.328	24.856	22.652	13.104	24.012	26.592	26.952	25.916
10	1	3	15.008	14.668	14.940	14.088	25.752	21.756	25.744	21.756	31.784	30.816	30.812	29.728	37.436	40.856	40.884	39.560
10	1.5	3	19.052	18.452	18.704	18.180	49.056	59.236	46.592	59.236	42.964	38.132	44.592	41.204	52.380	47.904	48.268	46.608

Table 15 ARL_1 results when multiplicative shift is present

p	v	δ	CUSUM, $K=0.5$				EWMA, $\lambda=0.05$				EWMA, $\lambda=0.1$				EWMA, $\lambda=0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r-k*
4	0.75	0.25	23.688	23.128	23.456	22.748	199.592	198.680	198.880	135.968	212.124	211.360	212.060	206.688	167.248	167.248	167.248	154.028
4	1	0.25	25.184	24.884	24.868	24.656	211.072	208.080	206.840	202.636	212.644	212.888	212.644	205.580	166.720	166.720	166.720	91.008
4	1.5	0.25	33.556	33.180	33.024	32.364	199.488	191.340	191.376	188.924	225.648	220.396	220.908	212.940	186.260	181.628	183.936	216.848
7	0.75	0.25	22.868	22.960	23.048	22.696	165.084	202.204	203.240	129.148	173.016	175.764	176.432	169.160	167.204	166.660	166.660	152.736
7	1	0.25	26.132	27.280	26.996	25.876	197.496	187.144	187.144	179.604	190.980	190.556	190.984	184.016	163.780	161.616	161.616	100.592
7	1.5	0.25	32.840	33.236	32.820	32.628	185.524	204.196	207.076	196.744	221.124	220.952	203.832	217.348	166.868	167.384	166.552	217.040
10	0.75	0.25	17.884	17.936	18.140	17.740	118.932	118.916	114.996	54.112	94.216	101.488	102.200	99.776	78.052	51.064	70.740	68.268
10	1	0.25	21.184	20.572	20.896	20.184	124.892	116.852	117.740	115.424	109.832	119.856	119.112	116.304	92.908	83.332	92.908	83.332
10	1.5	0.25	27.972	27.424	28.172	27.152	147.600	140.376	154.776	145.080	139.160	133.552	133.912	131.064	166.380	204.652	164.460	204.652
4	0.75	0.5	23.744	23.228	23.712	22.792	202.196	200.188	200.000	136.080	208.776	207.468	208.548	202.428	165.860	165.860	165.860	151.944
4	1	0.5	25.020	24.948	24.948	24.732	207.048	204.956	203.212	200.336	207.464	207.708	207.464	201.852	165.724	165.724	165.724	91.008
4	1.5	0.5	33.512	32.976	32.912	32.664	197.960	191.076	191.112	188.580	225.444	220.448	221.300	213.204	186.708	184.296	185.068	218.060
7	0.75	0.5	22.200	22.260	22.356	22.120	158.028	195.328	196.328	122.728	167.724	169.864	171.408	163.912	166.780	164.888	164.888	152.148
7	1	0.5	26.180	26.452	26.424	25.712	199.416	185.796	185.792	177.208	187.732	187.404	187.736	179.684	162.280	160.124	160.124	97.820
7	1.5	0.5	32.216	32.768	31.752	31.084	185.100	203.120	206.000	196.736	220.968	220.820	200.428	216.736	164.316	165.404	164.576	216.072
10	0.75	0.5	17.412	17.320	17.348	17.248	113.056	113.920	110.168	51.400	82.996	89.744	90.416	87.796	71.552	48.052	62.928	61.328
10	1	0.5	20.432	20.252	20.320	20.008	123.348	115.304	115.712	113.992	104.364	113.588	113.608	110.180	86.488	78.584	86.488	78.584
10	1.5	0.5	27.764	27.560	27.704	27.104	139.072	133.508	145.848	137.840	131.908	127.420	127.428	125.480	163.836	200.624	161.816	200.624
4	0.75	0.75	23.740	23.096	24.096	22.660	200.052	199.812	199.800	135.504	207.660	206.592	207.144	201.528	165.832	165.832	165.832	151.936
4	1	0.75	25.008	24.892	24.888	24.664	205.020	203.644	201.164	198.368	205.112	205.548	205.536	197.268	165.244	165.236	165.244	89.004
4	1.5	0.75	32.468	32.268	32.156	31.772	196.020	190.244	191.220	189.756	224.860	219.216	219.912	211.652	185.096	181.772	183.052	216.304

Table 15 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r-k*
7	0.75	0.75	21.836	21.836	22.124	21.690	156.200	190.232	192.500	122.112	162.812	165.004	165.420	159.052	166.620	164.928	164.928	151.696
7	1	0.75	26.164	26.620	26.620	25.852	197.524	181.392	181.388	176.368	180.476	180.576	180.908	174.356	160.000	158.620	158.620	97.628
7	1.5	0.75	33.336	32.832	32.084	31.748	183.044	199.776	205.592	194.540	218.752	218.580	198.104	214.524	164.072	164.096	163.088	213.468
10	0.75	0.75	16.820	16.696	16.704	16.668	107.832	108.424	103.616	47.068	79.364	87.056	86.436	83.812	66.064	45.184	59.348	55.876
10	1	0.75	19.852	19.012	19.436	18.888	119.512	109.728	109.728	107.996	102.940	111.776	111.036	108.576	83.888	74.200	83.888	74.200
10	1.5	0.75	25.832	25.460	25.544	25.196	137.852	131.320	143.888	136.440	126.868	120.228	120.248	118.696	159.312	196.044	157.880	196.040
4	0.75	1	24.128	23.160	24.196	22.844	197.920	196.792	196.780	135.356	202.148	201.216	201.632	197.184	165.232	165.224	165.224	151.932
4	1	1	25.624	25.532	25.304	24.896	204.592	203.216	200.768	198.328	201.892	202.084	202.316	195.112	165.252	165.244	165.252	90.948
4	1.5	1	32.040	31.552	31.628	31.271	194.872	189.280	190.252	188.396	223.872	217.676	220.324	209.740	183.724	180.420	182.348	216.312
7	0.75	1	22.564	22.512	22.572	22.444	147.672	183.028	184.836	115.020	161.324	161.928	162.648	154.416	166.264	162.624	162.624	148.428
7	1	1	26.004	26.248	26.248	25.752	193.152	177.152	177.140	171.436	178.716	178.720	179.148	172.528	160.264	158.864	158.864	96.316
7	1.5	1	32.584	33.008	31.816	31.232	179.996	195.716	201.224	191.920	214.460	214.288	193.776	210.200	163.776	163.104	162.052	213.360
10	0.75	1	16.072	15.896	15.952	15.816	105.708	106.784	103.004	45.280	71.972	79.508	79.592	76.808	64.764	44.680	60.524	56.468
10	1	1	19.640	19.052	19.300	18.620	114.424	105.480	105.464	103.636	106.920	106.136	105.396	103.488	78.792	66.344	78.792	66.352
10	1.5	1	25.264	24.692	24.812	24.528	132.852	126.728	138.668	131.628	125.224	116.628	116.668	115.096	154.972	190.816	152.700	190.120
4	0.75	1.25	24.000	23.076	24.216	22.684	196.356	195.268	195.456	134.388	202.468	201.840	202.724	197.692	163.132	163.124	163.124	146.644
4	1	1.25	25.496	25.348	25.340	25.036	202.672	202.080	200.752	197.640	197.408	197.600	197.832	191.668	165.248	165.240	165.248	90.676
4	1.5	1.25	31.880	31.604	31.488	31.484	192.080	184.832	185.804	183.944	219.508	214.520	217.164	207.720	182.992	179.680	181.608	214.220
7	0.75	1.25	22.820	22.728	22.844	22.452	146.292	183.172	184.984	115.840	153.964	153.948	154.660	148.176	167.640	160.644	160.644	144.944
7	1	1.25	25.548	25.592	25.592	25.396	189.568	173.544	173.544	167.196	173.480	175.156	175.584	166.256	159.080	157.676	157.676	96.348
7	1.5	1.25	32.872	33.088	31.944	31.156	178.548	194.040	199.420	188.120	213.372	213.200	190.776	208.004	163.628	162.500	161.948	212.484
10	0.75	1.25	14.868	14.696	14.700	14.444	100.432	101.056	96.048	41.948	66.396	73.388	73.936	71.228	62.780	43.160	58.304	56.164
10	1	1.25	19.312	18.900	19.168	18.428	109.256	99.344	99.344	97.756	90.624	97.616	96.876	95.392	71.908	62.508	71.908	62.508
10	1.5	1.25	24.912	24.648	24.836	24.592	130.472	121.640	134.872	128.792	120.656	112.316	113.708	109.704	151.184	184.476	148.172	183.780

Table 15 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r-k*
4	0.75	1.5	23.948	23.212	24.224	22.784	195.148	194.060	194.248	132.392	202.348	200.836	202.604	197.996	162.832	162.824	162.824	147.804
4	1	1.5	25.560	25.428	25.424	24.920	202.628	202.036	200.712	197.948	196.416	196.608	196.840	190.548	165.236	165.236	165.236	91.028
4	1.5	1.5	32.240	31.880	31.876	31.129	191.744	185.536	185.880	185.240	215.328	209.888	211.828	201.592	182.972	180.444	181.828	214.216
7	0.75	1.5	22.188	22.080	22.208	21.892	140.868	176.836	178.236	112.348	152.212	152.204	152.152	146.284	163.900	160.252	160.252	144.616
7	1	1.5	24.548	25.204	25.216	24.472	185.300	170.932	170.936	166.552	169.996	170.676	171.108	163.552	160.596	159.196	159.196	97.304
7	1.5	1.5	32.476	32.668	31.936	31.856	175.500	190.536	197.212	183.944	208.600	208.592	183.840	201.456	163.500	162.384	161.784	211.132
10	0.75	1.5	14.536	14.392	14.396	14.168	92.836	93.844	89.116	36.556	60.116	68.656	68.912	64.948	58.748	39.692	55.136	51.972
10	1	1.5	19.388	18.988	19.172	18.068	103.980	94.080	94.488	92.424	84.472	90.108	90.548	88.384	67.248	57.868	67.248	57.868
10	1.5	1.5	25.056	24.484	24.696	24.360	124.204	116.048	128.112	124.056	116.244	107.148	108.564	105.092	145.460	180.216	139.000	180.212
4	0.75	2	23.920	23.332	24.024	23.028	189.836	189.132	189.132	130.312	198.544	198.468	198.792	194.356	161.880	160.888	161.880	150.852
4	1	2	25.804	25.584	25.584	25.208	200.376	199.792	198.428	194.328	194.024	194.836	194.448	186.712	164.828	164.828	164.828	89.760
4	1.5	2	31.768	32.228	32.164	31.504	191.040	180.624	184.136	177.600	217.148	211.160	211.176	202.140	182.156	179.808	181.068	211.312
7	0.75	2	20.676	20.676	20.672	20.568	140.712	172.952	173.784	110.772	141.396	141.152	140.560	134.860	158.256	156.732	156.732	141.652
7	1	2	24.504	24.744	24.668	24.248	180.880	166.488	167.324	162.012	163.740	163.736	163.740	160.624	161.432	158.688	158.688	95.192
7	1.5	2	30.400	30.552	29.840	29.072	168.348	192.596	198.644	183.324	200.944	200.940	181.900	197.540	161.512	159.604	161.356	212.064
10	0.75	2	13.768	13.684	13.740	13.468	78.368	80.092	74.916	28.676	50.128	57.652	57.404	55.068	51.976	34.892	48.736	41.408
10	1	2	19.840	18.896	19.112	18.720	89.904	84.368	85.092	82.284	73.756	79.896	79.908	78.268	57.632	47.648	57.632	47.648
10	1.5	2	24.644	24.540	24.620	24.380	119.596	108.968	123.684	117.664	110.900	101.696	103.204	99.144	144.052	175.292	137.916	175.292
4	0.75	2.5	24.928	24.292	24.392	23.540	189.120	187.856	187.676	130.952	197.372	197.648	197.380	191.884	159.496	159.496	159.496	147.128
4	1	2.5	26.392	26.216	26.376	25.708	199.324	197.752	195.788	191.560	190.524	190.104	191.592	183.264	164.820	164.820	164.820	87.044
4	1.5	2.5	32.672	32.468	32.316	31.856	187.188	178.732	180.856	174.272	212.112	208.080	207.376	196.276	181.524	179.832	179.696	179.596
7	0.75	2.5	19.984	19.984	19.984	19.784	132.904	164.596	165.420	105.012	129.428	129.196	128.924	121.576	153.204	151.352	151.352	140.104
7	1	2.5	23.500	23.612	23.612	23.336	173.208	157.752	158.584	155.380	156.416	156.744	156.420	154.088	158.340	154.720	154.720	87.180
7	1.5	2.5	30.712	30.076	30.156	29.856	166.448	185.552	190.928	179.232	195.424	194.524	172.680	189.824	159.572	157.932	159.376	157.164

Table 15 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r-k*
10	0.75	2.5	12.728	12.712	12.716	12.616	65.400	65.840	61.568	23.028	42.092	46.464	46.144	44.588	42.080	29.292	39.528	33.236
10	1	2.5	18.056	17.484	17.720	17.180	88.388	82.772	82.772	79.868	64.692	69.664	69.680	67.496	50.888	44.308	50.888	44.308
10	1.5	2.5	24.928	24.376	24.644	24.276	111.288	102.772	113.736	108.396	105.336	95.780	97.264	93.484	169.320	161.240	163.908	161.200
4	0.75	3	25.288	24.568	24.780	24.048	189.300	189.060	188.880	127.328	196.688	194.720	195.620	190.644	160.896	160.896	160.896	148.612
4	1	3	25.752	25.600	25.572	25.496	194.800	193.592	193.332	190.500	189.600	188.164	189.656	181.348	163.280	163.272	163.284	85.376
4	1.5	3	32.940	32.332	32.308	32.296	185.108	178.212	179.396	174.468	205.968	200.960	201.244	191.324	177.660	175.680	175.862	175.904
7	0.75	3	20.024	20.124	20.152	19.972	126.960	159.044	159.048	97.208	118.228	118.228	117.044	110.908	149.432	146.276	146.276	136.256
7	1	3	23.540	23.612	23.628	23.444	167.616	151.976	152.800	148.860	146.676	146.672	146.688	141.508	157.240	154.524	154.528	87.068
7	1.5	3	29.920	29.320	29.360	28.924	162.928	177.196	185.188	171.336	186.384	185.480	164.336	180.324	158.252	157.568	158.056	157.164
10	0.75	3	12.288	12.252	12.300	12.192	57.264	57.300	53.316	19.500	35.296	40.028	39.524	37.376	36.312	25.400	35.696	29.812
10	1	3	16.248	15.884	16.176	15.716	81.988	76.500	75.684	72.560	59.376	64.920	64.924	62.944	47.364	41.528	47.364	41.528
10	1.5	3	24.424	23.436	24.204	22.960	99.056	91.456	102.204	95.732	94.036	86.400	87.640	83.416	148.600	147.726	147.640	147.284

Table 16 *SDRL* results when additive shift is present

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
4	0.75	0.25	18.5333	17.4214	17.3698	16.4535	149.5916	149.1343	149.1343	149.1324	127.4033	128.0968	127.5088	126.5029	170.6420	170.4064	170.6420	166.3885
4	1	0.25	19.8484	19.4662	19.4159	19.4990	140.8532	140.5056	140.6240	140.5843	130.5549	130.7841	130.5549	129.3869	170.1208	170.1208	170.1208	142.1964
4	1.5	0.25	24.9500	24.6198	24.6068	24.4466	145.6712	143.9357	144.3831	142.9936	130.7308	130.3705	129.7072	129.2856	162.7421	162.5640	162.2512	156.8857
7	0.75	0.25	16.6869	16.7103	16.7246	16.5537	149.3127	144.2767	144.2767	144.6975	120.5152	121.1058	122.0082	119.5631	169.4020	168.8587	168.4946	165.3236
7	1	0.25	19.1933	19.1414	19.1414	19.1255	141.6921	140.0789	140.0453	138.3043	131.2955	130.9882	131.6832	128.2750	170.2843	169.5546	169.5546	145.2811
7	1.5	0.25	23.1499	23.5618	22.8170	23.3330	144.6834	144.8009	144.4634	144.2094	128.4524	128.4127	128.2131	128.2573	163.2685	162.8370	162.9694	155.8233
10	0.75	0.25	18.7960	18.2898	18.2843	18.3090	95.7904	96.2212	93.4034	84.6702	75.1720	80.4090	81.0398	78.3973	114.6067	93.5658	109.2094	108.4279
10	1	0.25	21.3545	21.1219	21.1146	21.1341	99.3937	98.0145	98.3950	95.5926	83.2471	86.3684	87.0606	85.0820	136.2776	125.9492	136.2776	125.9492
10	1.5	0.25	23.5578	23.2325	23.1828	22.9012	119.8701	115.5036	123.6875	116.4467	105.0988	102.7300	102.8172	100.5679	150.2574	145.7062	150.2689	145.7062
4	0.75	0.5	18.3055	15.8376	15.8104	15.8108	146.6299	146.2170	146.6370	146.1267	126.0972	126.4250	126.0920	125.4579	166.4451	166.2022	166.4451	167.6669
4	1	0.5	19.2176	19.1066	19.1147	19.1119	138.8140	138.8103	138.5016	137.9935	124.7578	125.0253	125.0253	121.9858	166.9771	166.9771	166.9771	133.0052
4	1.5	0.5	24.3873	24.6761	23.7705	23.9030	142.1236	139.0773	139.6542	138.0820	130.4330	128.0613	127.8804	125.4602	160.7184	160.0701	160.1646	155.1088
7	0.75	0.5	14.2025	14.1974	14.2470	14.1957	142.6252	140.2520	140.5466	139.6649	119.6886	120.0726	120.4907	119.2856	166.1573	164.1412	163.9614	161.4097
7	1	0.5	17.9274	17.9187	17.9187	17.8295	137.2129	134.1444	134.7967	130.6425	125.8620	125.4818	125.4876	123.8069	168.1715	167.4536	167.4536	143.1998
7	1.5	0.5	21.9585	21.6639	21.7391	21.0333	141.3111	141.8473	142.1324	141.1355	130.0168	129.7094	127.8429	129.7859	161.5733	159.9872	160.6005	154.5173
10	0.75	0.5	15.6234	15.5516	15.5672	15.5325	90.8316	91.1963	88.0609	76.9567	69.6089	75.6664	76.5675	74.2482	108.2682	87.1671	106.5312	103.5919
10	1	0.5	19.4056	18.0205	18.4226	17.9966	96.9451	95.1557	95.2788	94.4211	80.9205	83.5923	84.3888	82.5189	132.2763	118.7197	132.2763	118.7197
10	1.5	0.5	26.1263	24.5832	26.1019	23.4117	114.0328	106.6502	117.6927	111.2277	102.9994	99.9382	100.8377	95.5874	147.8164	143.6282	146.7820	143.6282
4	0.75	0.75	18.7583	18.2148	18.5878	17.5621	140.9031	140.4202	140.9060	139.5053	122.9353	123.1216	122.9334	122.1751	161.8962	161.9331	161.8919	167.0645
4	1	0.75	19.5401	19.5263	19.5300	18.8309	132.2431	131.2678	130.8418	130.5178	120.1309	120.1240	120.1240	117.2507	166.2700	166.2700	166.2700	129.8693
4	1.5	0.75	24.2498	24.4148	24.1303	23.8631	138.0891	134.1205	134.9049	132.2946	128.8241	125.9784	126.1201	123.3638	158.7081	158.2197	158.0712	153.2127

Table 16 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
7	0.75	0.75	13.7446	13.3147	13.7550	13.3203	136.5141	136.0347	136.0494	132.0336	117.5608	118.5539	117.8226	114.9037	161.5303	159.5017	159.5017	157.7403
7	1	0.75	17.0388	17.5510	17.5510	16.5001	131.6753	128.2859	127.8112	126.4342	120.2534	121.0841	121.0841	119.8963	164.6960	164.2581	164.2581	135.0951
7	1.5	0.75	19.9890	20.5990	20.0761	19.6897	139.8993	139.0390	139.9979	137.9236	129.8291	129.3594	124.2759	128.2112	156.4135	155.1967	155.6166	153.1797
10	0.75	0.75	14.2232	13.2599	13.7860	13.2277	85.4404	87.3048	83.4648	71.4706	62.2805	70.7818	70.9361	68.5980	98.9577	83.3326	96.5779	94.9176
10	1	0.75	19.3092	18.8936	18.8678	18.7643	90.1363	87.4776	87.7420	87.5733	77.2797	80.5111	81.7027	78.2608	125.9245	115.0097	125.9245	115.0097
10	1.5	0.75	23.9710	23.9013	25.6608	23.7474	108.3825	104.0856	109.7793	105.8930	97.0820	93.0351	92.5967	86.1656	142.4110	140.1812	141.2032	140.1812
4	0.75	1	18.1835	17.2451	17.1432	17.0322	134.1979	132.2120	133.3618	129.2367	118.5379	119.3167	119.3608	115.4097	159.2417	159.2895	159.2388	164.9793
4	1	1	19.4610	19.4980	19.5020	19.3589	126.0891	126.0839	125.9495	123.9550	115.5516	115.5516	115.5516	114.0647	161.1465	161.1465	161.1465	119.0087
4	1.5	1	24.2024	24.2259	24.2249	23.8362	131.8738	127.8238	127.6085	124.7474	124.0685	122.6559	123.0773	120.2190	151.8693	150.1708	150.5164	149.0322
7	0.75	1	12.1511	12.0931	12.0586	11.8591	135.6540	135.7146	136.5270	129.2884	109.8290	109.9849	109.0527	105.9942	157.5727	153.8722	153.8773	150.9435
7	1	1	16.4525	16.4145	16.4205	16.3879	126.6143	122.1313	122.0645	120.2390	113.0376	114.2935	113.3325	112.9168	158.9609	158.0538	158.0538	126.0533
7	1.5	1	19.8375	19.1263	19.1136	19.1106	134.2062	132.6690	134.8718	131.6374	125.5320	125.1497	119.1075	124.5202	152.1038	151.4911	151.4913	149.8390
10	0.75	1	15.2274	15.2270	15.2478	15.0196	80.3523	82.5799	80.1240	67.1251	59.9260	65.0897	65.4467	64.9937	93.9665	83.6500	90.8449	89.6592
10	1	1	18.8713	18.7710	18.7339	18.5960	86.2820	84.3306	84.6891	83.7638	73.4766	76.5071	77.6699	75.4802	116.3663	108.0822	116.3663	108.0822
10	1.5	1	23.7518	23.7823	23.7898	23.1751	102.2207	96.6863	104.1363	99.4037	90.7334	85.1565	83.6635	80.3953	138.0108	137.6641	135.5001	137.6656
4	0.75	1.25	16.4831	15.8507	16.3683	15.7963	126.3470	125.7651	125.7772	120.1200	112.5643	111.3929	112.9622	108.9270	151.9192	151.9197	151.2409	162.6445
4	1	1.25	19.7621	19.7078	19.7030	19.6952	114.6274	114.6403	114.6412	112.5980	109.2073	109.2073	109.2073	106.2951	154.3915	154.3915	154.3915	112.1265
4	1.5	1.25	23.3418	23.4352	22.7459	22.7995	120.1956	116.8994	116.3255	115.0693	118.4414	117.3644	117.2567	116.6214	146.4824	144.9830	145.0417	143.3861
7	0.75	1.25	11.5790	11.5760	11.5767	11.5608	122.5872	123.8206	123.8296	116.5886	100.0843	100.0901	98.7867	96.5994	149.6728	147.5610	146.8139	144.4384
7	1	1.25	14.7785	14.7878	14.7918	14.7689	119.2226	114.3264	114.3315	111.6648	107.3858	108.5819	108.0125	105.7792	153.7573	152.8735	152.8735	118.7829
7	1.5	1.25	18.6971	18.7374	18.4805	18.4268	130.4779	127.7107	130.3771	126.0185	122.1778	121.8488	115.8842	121.7730	142.9684	142.3389	142.5282	143.8521

Table 16 continued

p	v	δ	CUSUM, $K=0.5$				EWMA, $\lambda=0.05$				EWMA, $\lambda=0.1$				EWMA, $\lambda=0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
10	0.75	1.25	14.2463	14.2532	14.2360	13.8271	75.2750	77.1127	74.2121	57.6707	58.0337	62.3114	62.2990	61.2300	87.9390	75.9213	86.5386	84.4083
10	1	1.25	18.5671	18.2334	18.6098	17.6485	81.0756	79.2211	80.2455	78.7000	70.0113	73.0102	73.6780	72.6182	111.7448	100.7388	111.7449	100.7395
10	1.5	1.25	21.3343	21.3339	21.2909	21.2357	92.8099	85.1688	95.9322	88.4068	85.4507	80.6815	79.1263	77.4562	130.8515	131.4111	128.2511	131.1722
4	0.75	1.5	14.8002	14.6891	14.7382	14.3983	116.8830	115.2169	116.0398	106.5943	105.6965	107.3140	107.4831	103.1883	142.9119	143.0211	142.0755	155.1636
4	1	1.5	16.8172	16.7869	16.7864	16.7867	101.6092	101.3447	100.1872	98.9247	100.1124	100.0739	100.1124	98.7462	149.4250	148.8985	148.8985	97.8439
4	1.5	1.5	23.0054	22.9805	22.8843	22.7801	106.8132	105.7065	104.7725	104.0723	113.5196	112.2066	112.8711	112.0658	137.9985	137.4835	136.7583	134.3250
7	0.75	1.5	11.2408	11.2347	11.2385	11.2042	112.3840	115.2006	114.9680	106.2255	93.5581	93.1564	90.7358	89.9400	141.5398	137.1481	137.1443	134.5739
7	1	1.5	14.8917	14.9245	14.9092	14.8082	110.1882	104.8174	104.8197	102.4881	99.5754	100.3343	99.5993	98.0087	147.4825	146.4091	146.3574	107.6070
7	1.5	1.5	18.3721	18.3314	18.1260	18.1392	123.4668	118.1265	123.2078	117.8251	120.5946	119.9059	113.1091	119.4256	138.6026	136.4134	137.3942	139.7764
10	0.75	1.5	14.5577	14.3955	15.0510	14.3943	70.6032	71.0531	68.1526	55.5707	55.8515	60.0822	60.1216	59.4347	83.8730	61.6683	81.8413	78.2765
10	1	1.5	17.7080	15.9746	16.6916	15.9404	76.7290	72.5140	73.2114	71.7767	66.5342	68.6837	70.8270	68.4816	102.4284	91.1487	102.4284	91.1487
10	1.5	1.5	21.1895	20.7757	20.8711	20.4479	89.7866	80.9965	89.9780	82.6999	82.5242	74.8151	74.8180	73.1728	120.1665	122.7505	116.0303	122.7505
4	0.75	2	12.6123	12.3149	12.3490	11.7638	94.0172	93.2842	94.0670	87.9552	89.0370	89.7477	89.6252	87.9470	123.0257	123.0171	123.0288	142.8372
4	1	2	15.2049	15.1604	15.1641	14.8063	88.5555	88.5325	86.7958	86.4259	77.9455	78.7989	78.8015	76.6842	136.0249	136.0195	136.0192	75.7873
4	1.5	2	20.9401	20.7919	20.7545	20.7092	91.3942	88.6192	88.6398	88.2277	102.5154	100.0941	101.1536	97.7219	119.5599	116.8894	116.9583	115.4479
7	0.75	2	11.8274	11.1528	11.2091	11.1119	92.4048	92.3444	92.8064	86.3814	72.0989	72.0488	72.0989	67.0209	116.2158	107.2421	107.2170	106.3794
7	1	2	13.0759	13.0339	13.0392	13.0312	87.2884	86.6137	85.9642	84.6763	79.3640	80.4703	79.5126	74.8291	131.7523	130.8590	130.8590	85.4998
7	1.5	2	17.9740	17.2449	16.9629	17.1640	108.6999	104.6449	108.2276	103.2339	112.8861	113.2900	94.3105	106.2970	125.4657	124.5804	124.3501	126.1216
10	0.75	2	12.5296	12.4631	12.5338	12.4601	57.9523	58.3274	55.3823	42.5248	50.0546	53.8303	53.9211	53.3588	63.8883	47.8770	60.2831	59.5104
10	1	2	15.8785	15.0747	15.2552	15.0065	63.7308	61.7388	62.6834	60.7659	53.7180	55.8836	56.9781	55.0953	80.0384	71.5612	80.0523	71.5612
10	1.5	2	19.0454	19.0654	19.0430	19.0297	72.3064	68.3536	76.0602	70.4281	70.5605	66.3805	65.9420	63.6319	101.0604	105.2786	99.4764	105.2786

Table 16 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
4	0.75	2.5	11.6598	11.3876	11.3687	11.3411	71.4094	71.3279	71.4143	69.1021	65.1390	68.0392	68.4626	61.8774	102.1237	103.7260	103.7304	120.8451
4	1	2.5	15.4739	15.2958	15.2945	14.5795	71.7447	71.7457	71.5097	66.8303	68.6421	70.1486	69.4171	67.0159	110.2563	110.2563	110.2563	53.1587
4	1.5	2.5	18.8618	18.6750	18.6892	18.6561	78.4731	76.4732	76.4262	74.4050	89.9113	89.1005	88.9497	85.0853	100.4306	96.4079	98.0450	95.2775
7	0.75	2.5	9.9776	9.9428	9.9551	9.8844	68.6816	67.9902	68.9603	61.4098	57.8066	58.0150	57.7148	55.1661	91.6966	84.8203	85.3141	83.1697
7	1	2.5	12.0367	12.2464	12.2808	12.0364	70.9712	66.5130	66.5157	65.5432	63.9121	64.4857	63.4997	63.0676	107.7036	102.7805	102.7805	69.3397
7	1.5	2.5	17.2577	17.3198	17.2875	16.7906	90.4859	88.0429	90.1531	82.8265	94.2752	94.4814	77.6514	88.7322	104.0166	101.8151	102.2188	106.7479
10	0.75	2.5	10.1838	10.1847	10.5864	10.1742	46.6624	47.9738	42.2369	36.4665	37.7706	40.8966	40.8668	39.5904	55.5429	41.6065	51.8758	51.3789
10	1	2.5	14.4892	14.2070	14.4539	13.5261	55.4823	52.2279	52.2279	49.6335	50.8697	52.8365	53.9505	50.8300	66.7425	50.0923	66.7398	50.0923
10	1.5	2.5	18.7789	18.5673	18.5131	18.5932	61.0466	59.1495	63.0678	60.2760	63.6258	58.6687	58.6224	57.4411	88.4203	91.7016	84.2274	91.6996
4	0.75	3	11.7062	10.8706	11.0532	10.5617	49.2629	48.4123	48.4123	44.8651	47.4307	47.0733	47.0636	45.8452	77.9723	75.6230	77.9938	97.9140
4	1	3	12.7068	12.7437	12.7461	12.6759	57.4907	57.4907	54.8290	50.9148	51.7601	51.5407	51.5392	48.1995	92.2378	92.2378	92.2378	45.6400
4	1.5	3	18.0983	18.1133	18.0636	18.0343	67.3908	60.0282	62.8054	59.7975	74.3067	72.0878	72.4148	68.1251	86.8030	83.7071	83.4185	84.4338
7	0.75	3	9.2542	9.2293	9.4372	9.2241	45.4538	47.2969	47.2870	43.0804	39.7326	40.0860	39.7677	39.3929	70.3661	68.5913	68.5913	66.3068
7	1	3	10.8885	11.0257	11.0426	10.6684	52.5577	48.7951	48.7974	47.2937	48.0598	48.0860	48.0860	44.5372	89.1903	87.7315	87.7315	53.9039
7	1.5	3	16.2818	16.1923	15.4517	16.1047	75.8288	72.8018	75.4469	69.5737	81.8058	81.8157	61.5286	75.6730	89.2776	88.1129	87.4854	88.9781
10	0.75	3	9.0055	8.9722	9.0173	8.6613	31.6369	32.0123	29.2483	24.6626	26.2105	29.4725	29.7681	28.8201	38.2472	34.1064	36.3096	36.1019
10	1	3	13.8488	13.5717	13.8389	12.4173	41.6339	40.7921	40.7878	38.7880	44.5260	45.4458	45.4618	45.3490	51.9053	44.8767	51.8888	44.8767
10	1.5	3	16.5868	16.4738	16.6369	16.2859	55.0370	49.6582	58.0880	52.2934	56.5909	52.5409	52.9725	52.0924	73.8184	77.9188	71.6974	77.9188

Table 17 *SDRL* results when multiplicative shift is present

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
4	0.75	0.25	17.6736	17.2966	17.8919	17.1976	150.3529	150.2109	150.1220	155.5543	128.6099	129.1385	128.7730	129.3042	172.1568	172.1568	172.1568	166.7558
4	1	0.25	20.2553	20.0363	20.0209	20.0218	142.6041	141.9547	142.2013	141.3255	131.7208	131.9184	131.7208	132.0811	171.0524	171.0524	171.0524	147.1205
4	1.5	0.25	25.6398	25.3944	25.2412	25.0144	146.1528	144.9848	145.1550	144.3803	129.9073	130.0650	129.9798	131.4462	164.4071	163.9065	164.1672	156.4856
7	0.75	0.25	16.8422	16.9321	16.9304	16.6764	154.9043	146.2349	146.5884	151.9150	124.3410	124.8986	125.5873	123.7121	170.7963	170.3429	170.3429	165.0079
7	1	0.25	19.9037	21.9357	21.9539	19.7391	142.2299	141.6646	141.6646	140.2673	132.4771	132.1361	132.4784	131.4006	170.9705	170.4418	170.4418	153.5434
7	1.5	0.25	26.5813	27.3404	26.5095	26.7004	146.5592	146.5582	146.5211	146.5189	127.5648	127.5431	130.2876	128.1726	165.2070	165.8711	165.4744	156.4482
10	0.75	0.25	16.6951	16.7050	16.9784	16.7395	96.2741	96.2807	96.3126	86.7991	81.0815	84.0033	84.4726	83.7861	114.1205	93.0431	109.5721	108.4111
10	1	0.25	18.4846	17.7968	18.2910	17.5178	99.0524	97.9167	97.9096	97.1773	84.8816	87.0815	86.9519	85.7983	138.7788	130.3641	138.7788	130.3641
10	1.5	0.25	24.7267	24.4567	25.1063	24.4481	121.3303	119.4049	125.3023	119.7702	103.7879	103.2889	103.0208	102.3230	153.3790	147.2033	153.0220	147.2033
4	0.75	0.5	17.8829	17.5986	18.1209	17.4360	150.1207	149.9101	149.7437	155.5372	128.1821	128.3726	128.3053	128.5648	172.0814	172.0814	172.0814	166.1012
4	1	0.5	20.2650	20.2047	20.2047	20.0731	142.3876	141.5549	141.9634	141.0615	131.4980	131.7056	131.4980	131.3210	170.7135	170.7135	170.7135	147.1225
4	1.5	0.5	25.6173	25.3236	25.2809	25.2693	146.8788	145.6329	145.8024	145.0796	130.5675	130.7257	130.3938	131.8724	164.7882	164.5895	164.1946	156.2417
7	0.75	0.5	15.8115	15.7271	15.8483	15.7328	152.4045	146.5898	146.9712	148.1919	124.4302	124.6425	125.5447	123.4562	170.4598	169.7279	169.7279	165.0677
7	1	0.5	20.3175	20.4197	20.4378	20.1290	142.4137	141.2255	141.2274	139.4948	132.1170	131.8745	132.1184	131.0367	170.2599	169.7124	169.7124	151.1523
7	1.5	0.5	25.0641	25.8814	25.1725	25.1707	146.9127	146.7121	146.3623	146.2938	128.1077	128.0814	130.9868	128.4455	164.4672	165.1723	164.7665	156.3817
10	0.75	0.5	16.1308	16.0876	16.0863	16.0849	96.2439	96.3855	96.1009	84.6253	76.0394	80.5295	81.2695	79.6109	108.0929	89.2324	101.3146	100.3808
10	1	0.5	18.1901	18.1984	18.1816	17.8777	99.1224	97.2201	97.4779	97.3198	83.5760	85.1603	85.0250	84.3775	133.7930	126.0183	133.7930	126.0183
10	1.5	0.5	26.4672	26.3508	26.3640	26.1565	117.2935	115.4266	120.4987	116.8564	100.0901	100.2747	99.3657	99.0514	151.7941	146.2701	151.3747	146.2701
4	0.75	0.75	17.9882	17.4726	18.1822	17.3417	149.8479	149.8973	149.5409	155.4092	128.0940	128.1386	128.2458	128.6592	172.0898	172.0898	172.0898	166.1037
4	1	0.75	20.2890	20.2162	20.2177	20.0876	141.5080	141.1040	141.3369	140.4368	130.9942	131.3836	130.8076	130.5439	170.4125	170.4154	170.4125	145.2649
4	1.5	0.75	24.8013	24.6975	24.9292	24.5426	146.9248	145.7519	145.9712	145.4475	130.2795	130.6534	130.1906	131.4936	164.5450	164.0231	163.9851	156.3824

Table 17 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
7	0.75	0.75	15.3041	15.3136	15.6411	15.5296	150.5990	144.7204	145.2268	146.3039	123.8390	123.9749	123.9062	122.4587	170.3128	169.7288	169.7288	165.2772
7	1	0.75	19.9668	20.0696	20.0696	19.7370	142.0518	140.4369	140.4387	139.2502	130.4786	130.5965	130.8602	129.1408	169.6923	169.5484	169.5484	151.1850
7	1.5	0.75	27.4095	27.1695	26.2777	27.1393	142.8421	145.4259	145.9978	145.6946	127.2638	127.2388	129.7430	127.5317	164.8066	165.1869	164.5946	156.6697
10	0.75	0.75	16.5640	16.5623	16.5627	16.5619	95.3615	95.6472	94.6017	80.5103	75.1176	79.3168	79.2032	77.4216	103.3181	87.6541	98.2136	95.9982
10	1	0.75	17.7030	16.6663	17.4118	16.5418	97.9514	94.5246	94.5246	94.2448	84.2064	85.5378	85.3343	85.1441	132.2348	122.2442	132.2348	122.2442
10	1.5	0.75	23.3147	23.0189	22.9297	22.9132	117.3705	115.1955	120.8004	117.1159	97.9399	96.0094	95.9805	95.3579	151.0384	146.3032	150.8665	146.3027
4	0.75	1	18.1272	17.6483	18.2911	17.5264	149.3845	149.1856	148.8214	155.2360	128.2039	128.6043	128.3331	128.6821	171.7149	171.7180	171.7180	166.1082
4	1	1	21.0314	21.0531	20.5950	20.3693	141.9957	141.5889	141.8300	140.8568	130.5395	130.7221	130.3629	130.0695	170.4035	170.4064	170.4035	147.0452
4	1.5	1	24.3227	24.2113	24.6622	24.3133	146.8263	145.4854	145.7151	145.3255	129.8766	129.8820	130.2450	131.2680	163.8843	163.3523	163.6437	156.3761
7	0.75	1	15.9963	16.0495	16.0583	15.9381	147.5642	142.9414	143.8748	142.0342	123.7962	123.3593	123.6880	121.3518	170.0237	169.2930	169.2930	164.1734
7	1	1	19.1583	19.2893	19.2130	19.1411	141.4329	139.2178	139.2239	137.5163	130.2549	130.3182	130.7045	128.7739	169.8584	169.7057	169.7057	150.3226
7	1.5	1	27.2191	27.7470	26.4527	27.0876	145.8584	145.6333	145.7416	145.6467	128.9332	128.9027	129.9414	128.6549	165.0649	164.8071	164.2308	155.8911
10	0.75	1	15.3370	15.2778	15.3331	15.3330	94.0835	94.1849	93.9229	78.5389	72.6418	75.6212	75.5469	75.1210	102.2553	86.2562	98.6841	96.2687
10	1	1	17.8904	17.5357	17.8408	16.7730	96.3024	93.5616	93.5476	93.2453	82.6305	85.0956	84.8413	84.2471	126.4450	113.1652	126.4450	113.1688
10	1.5	1	22.2246	21.8142	21.9043	21.8106	114.4665	112.1324	117.9428	114.3731	95.0210	92.1179	92.1062	91.3776	148.8874	144.5594	148.1772	144.2096
4	0.75	1.25	17.8662	17.4100	18.0898	17.2911	149.5898	149.3507	149.1653	155.3398	128.5034	129.1505	128.7106	129.1478	171.2213	171.2243	171.2243	164.1736
4	1	1.25	20.9020	20.5098	20.5139	20.3229	142.1379	141.9928	142.0117	140.9856	130.4529	130.6423	130.2909	130.1027	170.4052	170.4080	170.4052	146.6224
4	1.5	1.25	24.3045	24.0085	24.4337	24.4252	146.3272	145.1248	145.3852	144.9334	130.5338	130.3413	130.7713	131.5345	163.8860	163.3406	163.6409	157.0234
7	0.75	1.25	16.5359	16.4627	16.5270	15.9465	146.9067	142.6290	143.5624	142.1169	121.5087	121.4966	121.8805	119.8412	170.1033	168.7167	168.7167	162.7763
7	1	1.25	18.3345	18.3941	18.3941	18.3398	141.7174	139.3009	139.3009	137.7766	128.4261	129.4047	129.8055	126.4135	169.0136	168.8782	168.8782	150.3711
7	1.5	1.25	25.9936	26.3383	24.9602	26.1941	146.7137	146.6179	146.5163	146.3262	130.6723	130.6406	130.5756	130.5609	165.3838	164.7654	164.5214	156.1845

Table 17 continued

p	v	δ	CUSUM, $K=0.5$				EWMA, $\lambda=0.05$				EWMA, $\lambda=0.1$				EWMA, $\lambda=0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
10	0.75	1.25	12.9389	12.9394	12.9355	12.9316	91.7456	92.3360	90.9602	74.4255	69.0553	72.0875	72.7153	71.0707	99.4869	84.7046	96.3346	95.4857
10	1	1.25	17.6193	17.4018	17.7273	16.6235	92.4230	89.9930	89.9930	90.0187	79.5132	80.7689	80.4216	79.9680	117.5072	108.4242	117.5072	108.4242
10	1.5	1.25	21.5897	21.5748	21.5976	21.5361	113.0129	108.5528	115.2078	112.2284	94.1563	91.6939	92.0757	90.2810	145.8877	142.1490	144.9814	141.7617
4	0.75	1.5	17.6073	17.2429	17.7769	17.0719	149.7312	149.4834	149.2997	154.1561	128.6044	129.2061	128.8116	129.5996	171.2729	171.2759	171.2759	164.5784
4	1	1.5	20.7492	20.7347	20.7368	20.4658	142.0511	141.9057	141.9271	141.1653	130.2358	130.4270	130.0768	130.2751	170.4095	170.4095	170.4095	147.1492
4	1.5	1.5	23.8630	24.2756	24.6229	23.9832	146.5392	145.4541	145.9091	145.2493	130.4832	130.0753	130.7119	130.7274	163.8594	163.6892	163.6478	157.0321
7	0.75	1.5	14.5169	14.5201	14.5181	14.4572	145.0598	141.8488	142.4462	139.8746	120.5729	120.5676	120.4105	118.8415	168.5724	168.3247	168.3247	162.4185
7	1	1.5	17.2375	18.9133	18.9183	17.2749	140.9438	138.4296	138.4253	136.8823	127.1427	127.5854	128.0062	125.5658	169.4056	169.2779	169.2779	150.2833
7	1.5	1.5	25.9410	25.9410	25.0349	25.0288	146.5789	146.1949	146.2621	145.8028	129.5904	129.5869	128.8489	128.9022	165.1893	164.5654	164.3429	155.5586
10	0.75	1.5	12.9341	12.9499	12.9596	12.9218	88.9978	89.5276	87.0228	67.2732	66.2356	70.4809	70.8641	68.5132	95.2397	80.3267	92.1163	90.5514
10	1	1.5	18.0912	17.8577	18.1482	16.3946	90.7124	87.6147	88.0003	87.5103	76.3074	77.1782	77.6499	76.6628	111.6482	103.2768	111.6482	103.2768
10	1.5	1.5	22.3290	21.5465	21.5792	21.5491	110.3701	106.6589	112.8839	111.1798	90.3843	86.8587	87.0237	85.3768	144.3489	142.2002	142.3393	142.1992
4	0.75	2	17.6585	17.4406	17.4051	17.3664	148.8955	148.1254	148.1254	152.8130	129.2099	129.6607	129.4321	129.8572	170.5803	170.2049	170.5803	166.0830
4	1	2	21.2966	21.3656	21.3656	21.2108	141.3388	141.1775	141.2195	140.5214	130.4026	130.9659	130.2517	129.7742	170.6771	170.6771	170.6771	146.7188
4	1.5	2	23.5106	24.3184	24.1726	23.6800	147.2393	144.5494	145.7705	144.0210	129.9882	129.9616	130.3328	130.8366	163.7685	163.5273	163.3592	156.4063
7	0.75	2	12.6165	12.5601	12.5597	12.0606	144.6856	141.3555	141.7873	139.0866	116.4890	116.5458	115.2612	114.2665	167.3059	166.4914	166.4914	160.7895
7	1	2	17.5638	17.5339	17.5619	17.4192	138.8954	136.3880	136.8651	135.3529	124.8668	124.8655	124.8648	124.0786	169.1544	168.8044	168.8044	148.2188
7	1.5	2	22.4426	22.5318	22.1827	22.2758	141.5849	146.3193	146.6359	145.7218	129.1440	129.1410	129.7333	129.0463	164.5008	163.7392	164.3397	155.5511
10	0.75	2	11.2934	11.2987	11.3113	11.2866	83.3631	84.2077	82.2009	58.0902	58.8211	64.3855	64.3263	62.2802	90.6664	74.9088	87.1908	81.6610
10	1	2	20.8410	19.7383	19.9028	19.7217	86.1359	84.0597	84.4048	83.0738	72.0817	74.4946	74.4853	73.6915	103.1049	89.3149	103.1049	89.3149
10	1.5	2	21.9311	21.8873	21.8984	21.8317	106.8072	99.5875	110.3803	105.9992	87.1459	83.1611	83.4599	82.2664	142.1869	140.6807	140.6740	140.6807

Table 17 continued

p	v	δ	CUSUM, $K = 0.5$				EWMA, $\lambda = 0.05$				EWMA, $\lambda = 0.1$				EWMA, $\lambda = 0.2$			
			ML	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*	*ML*	*Ridge*	*PCR*	*r–k*
4	0.75	2.5	19.1876	18.9133	18.8932	17.4143	149.1579	148.2811	148.1109	153.3222	128.4262	128.7408	128.4438	127.7776	170.0770	170.0770	170.0770	165.1224
4	1	2.5	22.2020	22.0560	22.1334	21.9187	141.0420	140.5816	140.6251	140.5199	130.2920	130.4299	130.4962	129.4116	170.6823	170.6823	170.6823	145.0363
4	1.5	2.5	24.5773	24.2041	24.4981	24.3337	146.1287	143.8245	144.5398	142.4982	129.4653	129.5626	129.6490	130.0032	163.8123	163.1746	163.0464	156.5723
7	0.75	2.5	11.2896	11.3431	11.3431	11.2195	139.7286	138.4103	138.8975	133.6574	111.7945	111.8386	110.9825	109.0332	165.3877	164.7284	164.7284	160.1004
7	1	2.5	15.3087	15.3717	15.3717	15.0605	138.9135	135.8571	136.3847	136.1183	124.5862	124.9925	124.5818	123.8214	168.2044	167.3831	167.3831	142.3499
7	1.5	2.5	23.7482	22.6127	23.3635	22.6552	140.3788	144.4848	145.1484	143.8481	127.5704	127.8452	127.2599	126.8278	163.3566	162.8242	163.2298	155.1120
10	0.75	2.5	10.2546	10.2617	10.2620	10.2539	76.3537	77.0482	74.2709	51.4010	51.7920	55.5908	55.4891	53.9088	82.1426	69.0126	79.6507	74.1881
10	1	2.5	19.1423	18.2774	18.7959	17.7676	86.0813	83.5540	83.5540	82.7562	69.0337	71.2777	71.2670	69.9505	93.4229	86.1109	93.4229	86.1109
10	1.5	2.5	22.7467	22.7609	22.6984	22.7118	102.1418	97.8586	103.5457	101.4176	82.7366	78.6893	78.4686	77.6728	132.5099	132.8848	131.9531	131.8183
4	0.75	3	19.4794	17.9094	19.7039	17.5316	148.8790	148.2235	148.0543	150.2679	128.5669	128.2556	128.0266	127.6258	170.1970	170.1970	170.1970	166.1621
4	1	3	21.8205	21.8934	21.9101	21.8807	139.8023	139.6775	139.8467	139.7692	130.5036	130.3720	130.4606	128.9191	169.9517	169.9547	169.9502	143.2618
4	1.5	3	23.7392	23.6926	24.0619	23.6768	145.5643	143.7293	144.1984	142.3321	129.8618	129.0982	129.3380	129.1574	163.7185	163.0399	162.9083	157.1003
7	0.75	3	11.4034	11.4629	11.4720	11.3155	136.4560	137.0695	137.0661	127.2079	105.8064	105.8064	105.6539	103.5325	163.7897	162.1084	162.1084	158.0039
7	1	3	16.2381	16.2810	16.3078	16.2563	138.3255	133.8012	134.3712	133.6885	122.6790	122.6792	122.6705	120.9339	167.5929	167.1932	167.1967	141.3554
7	1.5	3	22.3050	21.3378	21.8946	21.2549	139.1987	142.3999	143.0607	141.3433	127.6235	127.8392	125.1217	125.5766	162.2624	162.3018	162.1331	154.8933
10	0.75	3	10.5114	10.5212	10.5346	10.4965	71.0238	70.6666	68.0586	45.7383	44.2667	49.9585	49.5686	47.1139	76.0362	63.4014	74.9421	69.4728
10	1	3	15.8067	15.1698	15.8177	14.9003	84.1577	82.1580	81.8611	80.1253	64.9276	68.3179	68.3206	66.7012	90.0773	81.9266	90.0773	81.9266
10	1.5	3	23.3511	22.9991	23.1695	22.3383	94.7661	89.9670	97.1426	91.3972	77.8024	75.1789	75.3191	73.8445	126.2742	128.9034	124.9285	124.9085

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