

Modelling count data with overdispersion and spatial effects

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Abstract In this paper we consider regression models for count data allowing for overdispersion in a Bayesian framework. We account for unobserved heterogeneity in the data in two ways. On the one hand, we consider more flexible models than a common Poisson model allowing for overdispersion in different ways. In particular, the negative binomial and the generalized Poisson (GP) distribution are addressed where overdispersion is modelled by an additional model parameter. Further, zero-inflated models in which overdispersion is assumed to be caused by an excessive number of zeros are discussed. On the other hand, extra spatial variability in the data is taken into account by adding correlated spatial random effects to the models. This approach allows for an underlying spatial dependency structure which is modelled using a conditional autoregressive prior based on Pettitt et al. in *Stat Comput* 12(4):353–367, (2002). In an application the presented models are used to analyse the number of invasive meningococcal disease cases in Germany in the year 2004. Models are compared according to the deviance information criterion (DIC) suggested by Spiegelhalter et al. in *J R Stat Soc B* 64(4):583–640, (2002) and using proper scoring rules, see for example Gneiting and Raftery in *Technical Report no. 463*, University of Washington, (2004). We observe a rather high degree of overdispersion in the data which is captured best by the GP model when spatial effects are neglected. While the addition of spatial effects to the models allowing for overdispersion gives no or only little improvement, spatial Poisson models with

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spatially correlated or uncorrelated random effects are to be preferred over all other models according to the considered criteria.

Keywords Bayesian inference · Count data · Overdispersion · Spatial regression models · Zero inflated models

1 Introduction

A very popular model for count data is the Poisson distribution. However, in the Poisson model equality of the variance and the mean is assumed which is too restrictive for overdispersed data where the variance in the data is higher than the expected one from the model. This paper aims to give an overview over different models for spatially indexed count data allowing for overdispersion in a Bayesian perspective. We follow two approaches for dealing with the extra variability in overdispersed data. On the one hand, we consider a wider class of models allowing for overdispersion, on the other hand spatial random effects are introduced to capture unobserved spatial heterogeneity in the data.

Overdispersion with respect to the Poisson model can be modelled by introducing an additional parameter. In particular, we consider the negative binomial (NB) distribution and the generalized Poisson (GP) distribution introduced by [Consul and Jain \(1973\)](#). Both models allow an independent modelling of the mean and the variance by the inclusion of an additional parameter.

When dealing with a data set with an excessive number of zeros, zero-inflated models might be used, see for example [Winkelmann \(2003\)](#). In contrast to the GP and the NB model, overdispersion in zero inflated models is caused by the occurrence of more zero observations than expected. Zero inflated models can be used in combination with any model for count data. Additionally to the zero observations arising from the count data model an extra proportion of zeros is incorporated. [Lambert \(1992\)](#) introduced the zero inflated Poisson regression model, a Bayesian analysis of the zero inflated Poisson model is given in [Rodrigues \(2003\)](#). Zero inflated regression models in combination with the generalized Poisson distribution have been addressed in [Famoye and Singh \(2003a,b\)](#) using maximum likelihood estimation, a Bayesian analysis without the inclusion of covariates is given in [Angers and Biswas \(2003\)](#). [Agarwal et al. \(2002\)](#) use a zero inflated Poisson regression model for spatial count data in a Bayesian framework.

The second approach for modelling unobserved data heterogeneity is the introduction of random effects. For spatially indexed data which are the focus in this paper, spatial random effects associated with each region or site may be used, allowing for the modelling of an underlying spatial dependency structure.

In this paper, we consider Poisson, NB, GP and zero inflated (ZI) regression models both including and without spatial random effects in a Bayesian context. In contrast to classical inference the Bayesian approach allows to adjust for parameter uncertainty by assigning prior distributions to the parameters. Further, a spatial correlation structure is easily incorporated in a Bayesian

setting by assuming an adequate prior distribution for the spatial random effects. In this paper, we assume a proper Gaussian conditional autoregressive spatial prior based on [Pettitt et al. \(2002\)](#) which takes the neighbourhood structure of the data into account and allows for spatial dependencies. Since this results in a high dimensional, complex posterior distribution, Markov Chain Monte Carlo (MCMC) is used for parameter estimation.

We give an application of the considered models to disease mapping. In particular, we analyse the number of invasive meningococcal disease cases registered in Germany in the year 2004. Models are compared using the deviance information criterion (DIC) suggested by [Spiegelhalter et al. \(2002\)](#) and proper scoring rules, see for example [Gneiting and Raftery \(2004\)](#). We observe a substantial degree of overdispersion in the data which is modelled best by the GP distribution if spatial effects are neglected. While the addition of spatial random effects gives no or little improvement to the models allowing for overdispersion, spatial effects turn out to be significant for the Poisson model. In particular, according to the DIC and the scoring rules a spatial Poisson models gives the best fit for these data. However, no smooth spatial pattern is modelled. Instead some isolated regions with high risk are detected by the spatial effects, indicating that the risk is not sufficiently explained by the incorporated covariates in these regions.

This paper is organized as follows. In Sect. 2 the negative binomial, the generalized Poisson and zero-inflated regression models are presented. The conditional autoregressive prior assumed for the spatial effects is discussed in Sect. 3, prior assumptions for the regression and model dependent overdispersion parameters are given in Sect. 4. The DIC and the used proper scoring rules are reviewed in Sect. 5. Finally, in Sect. 6 the presented models are applied to analyse the number of invasive meningococcal disease cases in Germany is given. Section 7 gives a summary of the results and draws conclusions.

2 Regression models for count data including overdispersion

A commonly used model for count regression data is Poisson regression, where equality of mean and variance for each observation is assumed. Since this condition is not satisfied any more if overdispersion is present in the data, we consider regression models, which allow the variance to be larger than the mean in this section. For a detailed study of various count data models see [Winkelmann \(2003\)](#).

2.1 Negative binomial (NB) regression

In a negative binomial regression model for independent count data observations $Y_i, i = 1, \dots, n$ with parameters $r > 0$ and $\mu_i > 0$, denoted by $Y_i \sim \text{NB}(r, \mu_i)$, the density is defined by

$$P(Y_i = y_i | r, \mu_i) = \frac{\Gamma(y_i + r)}{\Gamma(r)y_i!} \cdot \left(\frac{r}{\mu_i + r}\right)^r \cdot \left(\frac{\mu_i}{\mu_i + r}\right)^{y_i} \quad (1)$$

with

$$E(Y_i | r, \mu_i) = \mu_i \quad \text{and} \quad \text{Var}(Y_i | r, \mu_i) = \mu_i \left(1 + \frac{\mu_i}{r}\right).$$

The mean μ_i is assumed to depend on covariates \mathbf{x}_i and unknown regression parameters $\boldsymbol{\beta}$ by

$$E(Y_i | \mathbf{x}_i, \boldsymbol{\beta}) = \mu_i > 0.$$

The variance is the mean multiplied by the positive factor $\varphi_i := 1 + \frac{\mu_i}{r}$ and therefore greater than the mean, i.e. overdispersion can be modelled in the negative binomial distribution. We call the factor φ_i dispersion factor. Note, that the dispersion factor takes observation specific values. In the limit $r \rightarrow \infty$ the NB distribution converges to the Poisson distribution with parameter μ_i , see [Winkelmann \(2003\)](#). The negative binomial distribution also arises from a Poisson distribution where the parameter θ_i is assumed to be random and to follow a Gamma distribution with mean $E(\theta_i) = \mu_i$ and variance $\text{Var}(\theta_i) = \frac{\mu_i^2}{r}$. Therefore, overdispersion in the NB model can be interpreted by unobserved heterogeneity among observations.

2.2 Generalized Poisson (GP) regression

The generalized Poisson distribution has been introduced by [Consul and Jain \(1973\)](#) and is investigated in detail in [Consul \(1989\)](#). Independent response variables Y_i are called generalized Poisson distributed with parameters $\mu_i > 0$ and λ , denoted by $\text{GP}(\mu_i, \lambda)$, if

$$P(Y_i = y_i | \mu_i, \lambda) = \begin{cases} \mu_i [\mu_i(1 - \lambda) + \lambda y_i]^{y_i - 1} \frac{(1 - \lambda)}{y_i!} \exp[-\mu_i(1 - \lambda) - \lambda y_i], & y_i = 0, 1, 2, \dots \\ 0 & \text{for } y_i > m \text{ when } \lambda < 0 \end{cases} \quad (2)$$

where $\max(-1, -\frac{\mu_i}{m - \mu_i}) < \lambda < 1$ and $m (\geq 4)$ is the largest positive integer for which $\mu_i(1 - \lambda) + m\lambda > 0$ for negative λ . Mean and variance are given by

$$E(Y_i | \mu_i, \lambda) = \mu_i \quad \text{and} \quad \text{Var}(Y_i | \mu_i, \lambda) = \frac{\mu_i}{(1 - \lambda)^2} \quad (3)$$

with μ_i depending on covariates \mathbf{x}_i and regression parameters $\boldsymbol{\beta}$ like in the NB regression model. From (3) it is clear that $\varphi := \frac{1}{(1 - \lambda)^2}$ can be interpreted as an dispersion factor for the GP distribution. For $\lambda = 0$, the generalized Poisson distribution reduces to the Poisson distribution with parameter μ_i , for

$\lambda < 0$ underdispersion can be modelled, whereas for $\lambda > 0$ overdispersion is obtained. The focus in this paper is the modelling of overdispersion, therefore λ is assumed to take only values in the interval $[0, 1)$ in the remainder of this paper. Similar to the NB model, the GP distribution is a mixture of Poisson distributions as has been proved by Joe and Zhu (2005). While the dispersion parameter in the NB regression model depends on μ_i leading to a variance function which is quadratic in μ_i , the dispersion parameter in the GP regression model is the same for each observation and results in a linear variance function. For a closer comparison of the NB and the GP distribution see Joe and Zhu (2005) and Gschlößl (2006).

2.3 Zero inflated (ZI) regression models

For count data with an excessive number of zero observations zero inflated (ZI) models can be used. These models allow for a higher number of zeros than can be explained by standard models for count data. Additional to the zero observations arising from the supposed count data distribution, a proportion of extra zeros is assumed. ZI models have been widely used in the literature, for a short overview see Winkelmann (2003).

Let $\pi(y_i|\theta)$ be a distribution function for count data with unknown parameters θ . Then a zero inflated model with extra proportion $p \in [0, 1]$ of zeros is defined by [see Agarwal et al. (2002)]

$$P(Y_i = y_i|p, \theta) = \begin{cases} p + (1 - p)\pi(y_i = 0|\theta) & \text{if } y_i = 0 \\ (1 - p)\pi(y_i|\theta) & \text{if } y_i > 0 \end{cases} \tag{4}$$

Mean and variance are given by

$$E(Y_i|p, \theta) = (1 - p)E_\pi(Y_i|\theta) \tag{5}$$

and

$$\text{Var}(Y_i|p, \theta) = p(1 - p)[E_\pi(Y_i|\theta)]^2 + (1 - p)\text{Var}_\pi(Y_i|\theta). \tag{6}$$

The introduction of latent indicator variables $\mathbf{Z} = (Z_1, \dots, Z_n)'$ leads to a model which is easier to handle in a Bayesian context and in particular allows a Gibbs step for p . Z_i takes the value $z_i = 0$ for all observations with $y_i > 0$. For all zero observations $y_i = 0$, the latent variable takes the value $z_i = 0$ if observation i arises from the count data distribution $\pi(y_i|\theta)$ and the value $z_i = 1$ for extra zeros. Marginally, $Z_i \sim \text{Bernoulli}(p)$. Using the latent variables \mathbf{Z} , the joint

likelihood of $\mathbf{Y} = (Y_1, \dots, Y_n)'$ and \mathbf{Z} is given by

$$\begin{aligned} f(\mathbf{Y}, \mathbf{Z}|p, \boldsymbol{\theta}) &= \prod_{i=1}^n p^{z_i} [(1-p)\pi(y_i|\boldsymbol{\theta})]^{1-z_i} \\ &= \prod_{i:y_i=0} p^{z_i} [(1-p)\pi(0|\boldsymbol{\theta})]^{1-z_i} \cdot \prod_{i:y_i>0} (1-p)\pi(y_i|\boldsymbol{\theta}). \end{aligned}$$

In this paper, we will focus on the zero inflated Poisson and the zero inflated generalized Poisson regression models, which are special cases of the ZI model (4). The zero inflated negative binomial distribution will not be discussed in this paper, since the GP model turned out to be more adequate than the NB model for the application considered later on.

2.3.1 Zero inflated Poisson (ZIP) regression

Here the Poisson distribution is assumed for the underlying count data distribution, i.e. $\pi(y_i|\boldsymbol{\theta}) := \pi(y_i|\mu_i) = \mu_i^{y_i} \frac{\exp(-\mu_i)}{y_i!}$ where the parameter μ_i is again assumed to depend on covariates \mathbf{x}_i and unknown regression parameters $\boldsymbol{\beta}$. Mean and variance of the ZIP regression model, denoted by $\text{ZIP}(p, \mu_i)$, are specified by $E(Y_i|p, \mu_i) = (1-p)\mu_i$ and $\text{Var}(Y_i|p, \mu_i) = (1-p)\mu_i(\mu_i p + 1) = E(Y_i|p, \mu_i)(\mu_i p + 1)$. For $p > 0$ the dispersion factor $\varphi_i := \mu_i p + 1$ of the ZIP regression model is positive, i.e. the presence of extra zeros leads to overdispersion.

2.3.2 Zero inflated generalized Poisson (ZIGP) regression

The ZIGP regression model was already introduced by Famoye and Singh (2003b), in Famoye and Singh (2003a) a generalisation to k -inflated GP regression models is given which allow the modelling of data with an excessive number of k -values ($k = 0, 1, 2, \dots$). The ZIGP regression model, denoted by $\text{ZIGP}(p, \mu_i, \lambda)$, is obtained if the density function of the GP distribution given in (2) is chosen for $\pi(y_i|\boldsymbol{\theta})$ and a regression is performed on μ_i . The mean and the variance of the ZIGP regression model are given by $E(Y_i|p, \mu_i, \lambda) = (1-p)\mu_i$ and $\text{Var}(Y_i|p, \mu_i, \lambda) = E(Y_i|p, \mu_i, \lambda) \left[p\mu_i + \frac{1}{(1-\lambda)^2} \right]$. The dispersion factor of the ZIGP model is therefore given by $\varphi_i := p\mu_i + \frac{1}{(1-\lambda)^2}$. Here, overdispersion can both result from the overdispersion parameter λ of the GP distribution and the extra proportion of zeros p when $p > 0$.

3 Spatial effects using a Gaussian conditional autoregressive model

In addition to covariates we will incorporate spatial random effects in the regression models in order to account for spatial heterogeneity as well as spatial correlation in the data. We consider models for data aggregated in

regions. A spatial dependency structure is imposed by assuming a prior distribution for the spatial effects which takes the neighbourhood structure of the area under consideration into account. In particular we consider a special case of the Gaussian conditional autoregressive (CAR) model introduced by [Pettitt et al. \(2002\)](#). Assume the data to be distributed on J regions $\{1, \dots, J\}$. Then the vector $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_J)'$ of spatial effects is assumed to follow a multivariate normal distribution, in particular

$$\boldsymbol{\gamma} \sim N(0, \sigma^2 \mathbf{Q}^{-1}) \tag{1}$$

where the elements of the precision matrix $\mathbf{Q} = (Q_{ij})$, $i, j = 1, \dots, J$ are given by

$$Q_{ij} = \begin{cases} 1 + |\psi| \cdot N_i & i = j \\ -\psi & i \neq j, i \sim j \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

We write $i \sim j$ for regions i and j which are contiguous and assume regions to be neighbours if they share a common border. N_i denotes the number of neighbours of region i . The conditional distribution of γ_i , given all the remaining components $\boldsymbol{\gamma}_{-i}$, $i = 1, \dots, J$ is given by

$$\gamma_i | \boldsymbol{\gamma}_{-i} \sim N\left(\frac{\psi}{1 + |\psi| \cdot N_i} \sum_{j \sim i} \gamma_j, \sigma^2 \frac{1}{1 + |\psi| \cdot N_i}\right). \tag{3}$$

The parameter ψ determines the overall degree of spatial dependence, for $\psi = 0$ all regions are spatially independent, whereas for $\psi \rightarrow \infty$ the degree of dependence increases. [Pettitt et al. \(2002\)](#) show that \mathbf{Q} is symmetric and positive definite, therefore (1) is a proper distribution. Another convenient feature of this CAR model is that the determinant of \mathbf{Q} which is needed for the update of ψ in a MCMC algorithm can be computed efficiently, see [Pettitt et al. \(2002\)](#) for details. Many other authors have dealt with conditional autoregressive models. An overview about CAR models is given in the book by [Banerjee et al. \(2004\)](#) and in [Jin et al. \(2005\)](#) where also multivariate CAR models are discussed. The most popular model is probably the intrinsic CAR model introduced by [Besag and Kooperberg \(1995\)](#). The joint density for $\boldsymbol{\gamma}$ in the intrinsic CAR model is improper in contrast to model (3) described above which has a proper joint density.

[Czado and Prokopenko \(2004\)](#) consider a modification of model (3) which is a proper model as well but reduces to the intrinsic CAR model in the limit. Another modification of the intrinsic CAR model leading to a proper prior has been presented by [Sun et al. \(2000\)](#).

4 Bayesian inference

In order to account for parameter uncertainty and to allow for an underlying spatial structure we consider the count data regression models discussed in Sect. 2 in a Bayesian context. MCMC will be used for parameter estimation. For more information on Bayesian data analysis and MCMC methods see [Gilks et al., \(1996\)](#) and [Gelman et al. \(2004\)](#).

Assume the response variables $Y_i, i = 1, \dots, n$ to be observed at J regions. Besides the well known Poisson regression model $\text{Poi}(\mu_i)$ we consider the $\text{NB}(r, \mu_i), \text{GP}(\mu_i, \lambda), \text{ZIP}(p, \mu_i)$ and $\text{ZIGP}(p, \mu_i, \lambda)$ model. In each of these models the parameter $\mu_i, i = 1, \dots, n$ is specified by

$$\mu_i = t_i \exp(\mathbf{x}'_i \boldsymbol{\beta} + \gamma_{R(i)}),$$

where $\mathbf{x}_i = (1, x_{i1}, \dots, x_{ik})$ denotes the vector of covariates and t_i gives the observation specific exposure defining statistically an offset. The vector $\boldsymbol{\beta} = (\beta_0, \dots, \beta_k)$ denotes the vector of unknown regression parameters. Note, that an intercept β_0 is included in the model. To allow for geographical differences in the J regions spatial random effects $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_J)$ are introduced, $R(i) \in \{1, \dots, J\}$ denotes the region of the i th observation. The parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}, \lambda, p$ and r , respectively, are taken to be a priori independent and the following prior distributions are chosen:

- $\pi(\boldsymbol{\beta}) \sim N(0, \tau^2 \mathbf{I}_{k+1})$, with $\tau^2 = 100$
- $\pi(\boldsymbol{\gamma} | \sigma^2, \psi) \sim N(\mathbf{0}, \sigma^2 \mathbf{Q}^{-1})$ with \mathbf{Q} specified as in (2)

For the hyperparameters σ^2 and ψ the proper priors

- $\pi(\sigma^2) \sim \text{IGamma}(a, b)$ with $a = 1$ and $b = 0.005$
- $\pi(\psi) \sim \frac{1}{(1+\psi)^2}$

are assumed. For the model specific parameters the following prior distributions are chosen:

- GP Regression: $\pi(\lambda) \sim U([0, 1])$
- NB Regression : $\pi(r) \sim \text{Exponential}(b)$, i.e. $\pi(r) = be^{-rb}$, where $\pi(b) \sim \text{Exponential}(0.005)$
- ZIP/ZIGP Regression: $\pi(p) \sim U([0, 1])$

Most update steps of the MCMC algorithms are performed using a single component Metropolis Hastings (MH) step. The algorithmic scheme for a spatial regression model with an additional overdispersion parameter can be summarized as follows:

- Update of overdispersion parameter θ (i.e. $\theta = \lambda$ for GP, $\theta = r$ for NB): sample $\theta | \mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\gamma}$
- Update of regression parameters: sample $\beta_j | \mathbf{y}, \theta, \boldsymbol{\beta}_{-j}, \boldsymbol{\gamma}, \quad j = 0, \dots, k$
- Update of spatial effects
 - Sample $\frac{1}{\sigma^2} | \boldsymbol{\gamma}, \psi \sim \text{Gamma}$

- Sample $\psi|\boldsymbol{\gamma}, \sigma$
- Sample $\gamma_j|\mathbf{y}, \theta, \boldsymbol{\beta}, \boldsymbol{\gamma}_{-j}, \psi, \sigma, \quad j = 1, \dots, J$

Since the full conditional of σ^2 is Inverse Gamma, σ^2 can be sampled directly using a Gibbs step. For the remaining parameters a MH step is used. In particular, the overdispersion parameter θ , the regression parameters $\boldsymbol{\beta}$ and the spatial effects $\boldsymbol{\gamma}$ are updated component by component using an independence proposal distribution. For the independence proposal we take a t -distribution with $\nu = 20$ degrees of freedom with the same mode and the same inverse curvature at the mode as the target distribution. The spatial hyperparameter ψ is updated using a symmetric random walk proposal. For the Poisson regression model the same algorithmic scheme can be used, neglecting the overdispersion parameter.

To avoid convergence problems in the ZI models which arose in simulated data due to correlation between the intercept β_0 , p and λ , we use collapsed algorithms, in particular β_0 , p and λ are updated with the latent variables \mathbf{z} integrated out, i.e. based on model (4). Doing so convergence and mixing of the samplers was improved considerably. The algorithmic scheme for the ZIGP regression model is given by

- Updates with \mathbf{z} integrated out
 - Sample $\beta_0|\mathbf{y}, p, \lambda, \boldsymbol{\beta}_{-0}, \boldsymbol{\gamma}$
 - Sample $p|\mathbf{y}, \lambda, \boldsymbol{\beta}, \boldsymbol{\gamma}$
 - Sample $\lambda|\mathbf{y}, p, \boldsymbol{\beta}, \boldsymbol{\gamma}$
- Update of latent variables: $\forall i$ with $y_i = 0$

$$\text{Sample } z_i|\mathbf{y}, \lambda, p, \boldsymbol{\beta}, \boldsymbol{\gamma} \sim \text{Bernoulli} \left(\frac{p}{p + (1 - p) \exp(-\mu_i(1 - \lambda))} \right)$$
- Update of regression parameters: sample $\beta_j|\mathbf{y}, \lambda, \boldsymbol{\beta}_{-j}, \mathbf{z}, \boldsymbol{\gamma}, \quad j = 1, \dots, k$
- Update of spatial effects
 - Sample $\frac{1}{\sigma^2}|\boldsymbol{\gamma}, \psi \sim \text{Gamma}$
 - Sample $\psi|\boldsymbol{\gamma}, \sigma$
 - Sample $\gamma_j|\mathbf{y}, \lambda, \boldsymbol{\beta}, \mathbf{z}, \boldsymbol{\gamma}_{-j}, \psi, \sigma, \quad j = 1, \dots, J$

The latent variables \mathbf{z} can be updated using a Gibbs step. Since the full conditional of p is log concave, adaptive rejection sampling (ARS) introduced by Gilks and Wild (1992) is used to update p . The parameters $\boldsymbol{\beta}$, λ and $\boldsymbol{\gamma}$ are updated using a MH step with an independence proposal distribution as described above. For the ZIP regression model the same algorithm with λ fixed to 0 can be used.

5 Bayesian model comparison

Bayes factors based on marginal likelihood provide a method for model comparison, see Kass and Raftery (1995). Further, Bayesian model averaging (BMA), see for example Hoeting et al. (1999), which is based on Bayes factors, presents a method for model selection taking model uncertainty into account. However,

for complex hierarchical models like those considered in this paper, the computation of Bayes factors requires substantial efforts, see [Han and Carlin \(2001\)](#). Therefore we decided to use model choice criteria like the deviance information criterion and proper scoring rules which can be easily computed using the available MCMC output.

5.1 Deviance information criterion (DIC)

[Spiegelhalter et al. \(2002\)](#) suggest to use the following criterion for model comparison in Bayesian inference. Assume a probability model $p(\mathbf{y}|\boldsymbol{\theta})$. The Bayesian deviance $D(\boldsymbol{\theta})$, which is used as a measure for goodness of fit, is defined as

$$D(\boldsymbol{\theta}) = -2 \log p(\mathbf{y}|\boldsymbol{\theta}) + 2 \log f(\mathbf{y})$$

where $f(\mathbf{y})$ is some fully specified standardizing term. To measure the model complexity [Spiegelhalter et al. \(2002\)](#) introduce the effective number of parameters p_D defined by

$$p_D := E[D(\boldsymbol{\theta}|\mathbf{y})] - D(E[\boldsymbol{\theta}|\mathbf{y}]).$$

Finally they define the deviance information criterion (DIC) as the sum of the posterior mean of the deviance and the effective number of parameters

$$DIC := E[D(\boldsymbol{\theta}|\mathbf{y})] + p_D.$$

According to this criterion the model with the smallest DIC is to be preferred. p_D and DIC are readily determined using the available MCMC output by taking the posterior mean of the deviance to obtain $E[D(\boldsymbol{\theta}|\mathbf{y})]$ and the plug-in estimate of the deviance $D(E[\boldsymbol{\theta}|\mathbf{y}])$ using the posterior mean $E[\boldsymbol{\theta}|\mathbf{y}]$ of the parameter $\boldsymbol{\theta}$.

5.2 Proper scoring rules

Apart from the DIC we use proper scoring rules for categorical variables for comparing models, in particular we consider the Brier score and the logarithmic score presented for example in [Gneiting and Raftery \(2004\)](#). While [Gneiting and Raftery \(2004\)](#) use scoring rules for assessing the quality of probabilistic forecasts, our focus is model comparison based on the posterior predictive distribution. Under the probability model $p(\mathbf{y}|\boldsymbol{\theta})$ a scoring rule assigns a value $S(\mathbf{p}_i, y_i)$ for each observation $y_i, i = 1, \dots, n$ based on the posterior predictive probability vector $\mathbf{p}_i = (p_{i1}, p_{i2}, \dots, p_{im})$. Here the component $p_{ik} := P(y_i = k|\mathbf{y})$ denotes the posterior predictive probability that the i th observation takes the

value k which can be estimated by

$$\hat{p}_{ik} := \frac{1}{R} \sum_{j=1}^R p(y_i = k | \theta^j)$$

where $\theta^j, j = 1, \dots, R$, denotes the j th MCMC iterate of θ after burnin. For computational reasons we set $p_{im} := 1 - \sum_{k=1}^{m-1} p_{ik}$ where $m - 1$ gives the highest response value observed in the data. This ensures that the probability vector \mathbf{p}_i sums up to 1.

Models are then compared based on the mean score given by

$$S(\theta) = \frac{1}{n} \sum_{i=1}^n S(\mathbf{p}_i, y_i).$$

We consider positive oriented scores here, i.e. the model with the highest mean score is to be preferred. The Brier score first proposed by [Brier \(1950\)](#) is defined by

$$S(\mathbf{p}_i, y_i) = 2p_{iy_i} - 1 - \sum_{k=1}^m p_{ik}^2$$

where $p_{iy_i} = P(y = y_i | \theta)$ denotes the posterior predictive probability for the true value y_i under the considered model. The Brier score corresponds to the expression

$$-\frac{1}{n} \sum_{k=1}^m \sum_{i=1}^n (p_{ij} - \hat{p}_{ik}^{\text{emp}})^2$$

where $\hat{p}_{ik}^{\text{emp}} = \begin{cases} 1 & y_i = k \\ 0 & \text{otherwise} \end{cases}$ denotes the empirical probability that the i th observation takes the value j . Hence, according to the Brier score the model which minimizes the squared difference between the observed and the estimated probabilities is considered best.

The logarithmic score is defined by

$$S(\mathbf{p}_i, y_i) = \log p_{iy_i}$$

and therefore chooses the model which gives the highest probability for observing the true value. Both scores are proper, i.e. the highest score is obtained for the true model, see [Gneiting and Raftery \(2004\)](#) for details. Further, when parameter estimation is done using MCMC both scores are computed easily based on the MCMC output as indicated above. Ideally, in order to avoid using the data twice, parameter estimation should be based on about 75% of the data

only, whereas the scores should be computed for the remaining 25% of the data. However, since the data set analysed in the next section is rather small and our focus is on comparing models rather than prediction, we will use the same data for estimation and computation of the scores.

6 Application

6.1 Data description

In this section, the proposed models will be used to analyse the number of invasive meningococcal disease cases reported in Germany during the year 2004. Meningococcal disease is caused by bacteria and can lead to serious, perilous diseases, like for example meningitis, in which case we refer to invasive meningococcal disease. In 2004, 600 cases of invasive meningococcal disease were reported in Germany. Germany is divided into 439 regions, for each of these regions the number of invasive meningococcal disease cases is given for both men and women. However, in preliminary analyses no significant influence of gender could be detected, therefore we decided to model the total number of cases without distinguishing between men and women. This means, that we have only one observation $y_i, i = 1, \dots, 439$ for each region. Since we are interested in relative risks, population effects are eliminated by including the expected number of cases in each region as an offset in the analysis. The expected number of cases t_i in each region $i = 1, \dots, n$ is determined by the size of the population pop_i in region i times the overall observed risk, i.e. the total number of cases divided by the total population $\frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n \text{pop}_i}$. A histogram of the relative risks y_i/t_i in each region is given in Fig. 1. A high proportion (67.2%) of the data is equal to zero, the average relative risk of meningococcal disease is given by 1.06, however the variance of the relative risks y_i/t_i is given by 2.04 which is substantially higher than the mean and therefore already indicates the presence of overdispersion in the data.

On a higher aggregation level Germany consists in 16 states. Besides the modelling of overdispersion in the data, an interesting issue is to detect whether there are areas with an increased risk of contracting invasive meningococcal disease. In this case, vaccination could be strongly recommended in these risky regions. Therefore, we include the 16 states as covariates in our model, which will be modelled as factor covariates with state 1 as reference level.

Extra heterogeneity in the data, which might not be satisfactorily explained by the state factors, can be handled by the model specific dispersion parameters in the NB, the GP and the ZI models. While overdispersion in the NB and the GP model can be interpreted as unobserved heterogeneity among observations, zero inflated models would assume that part of the observations equal to zero are extra zeros, i.e. in some regions the occurrence of invasive meningococcal disease might not have been reported or detected.

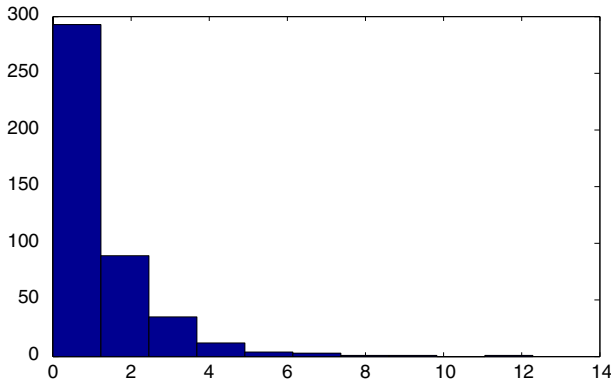


Fig. 1 Histogram of the observed relative risks $y_i/t_i, i = 1, \dots, 439$

On the other hand, heterogeneity in the data might also be taken into account by assuming a finer geographic resolution, i.e. by including a random spatial effect for each region. We will assume the CAR prior presented in Sect. 3 for these spatial effects which allows for a spatial dependency structure. In contrast to this approach, the effects of the states included as factor covariates can be seen as unstructured effects on a lower resolution, since no correlation between states is allowed.

We first analysed the data set in Splus using a Poisson model without spatial effects including an intercept and the 16 states as factor covariates. The effects of the four states Nordrhein-Westfalen, Mecklenburg-Vorpommern, Sachsen-Anhalt and Thüringen were found to be significant and only the regression indicators of these states will be included in the following. This model was used as a starting model for the MCMC algorithms of the models discussed in Sect. 2.

6.2 Models

The MCMC algorithms for the Poisson, the GP, the NB, the ZIP and the ZIGP regression models are run for 20,000 iterations. The parameter $\mu_i, i = 1, \dots, n$ is specified for all models by

$$\mu_i = t_i \exp(\mathbf{x}_i' \boldsymbol{\beta} + \gamma_i)$$

with the same covariates included in each model and t_i as defined above. All models are fitted with and without spatial effects. Since we have only one observation for each region we use the simplified notation $\gamma_{R(i)} = \gamma_i$. The first 5,000 iterations of the MCMC samplers are discarded as burnin, convergence is achieved well before for all models. After convergence the mixing of the samplers is satisfactorily good, the estimated empirical autocorrelations with lag 5 are in general well below 0.05 for the regression parameters in all non spatial models and both the regression parameters and the spatial effects in the spatial

GP and ZIGP models. Only in the spatial Poisson, ZIP and NB models a lag of about 20 is needed in order to obtain autocorrelations of the regression parameters below 0.05, for the spatial effects the autocorrelations are below 0.05 at a lag of 5 in the Poisson and ZIP models and a lag of about 10 in the NB model. The estimated posterior means and 90% credible intervals for the regression parameters are reported in Table 1 for all models. Estimation of the intercept slightly differs between the models and also changes when spatial effects are added, especially for the Poisson and ZIP models where large spatial effects are observed, see below. Estimation of the state effects is rather similar in all models. For a comparison of the estimated overdispersion in the different models, we consider the estimated dispersion factors φ_i (see Sect. 2) for the NB, GP, ZIP and ZIGP regression models. In particular, we compute the mean, minimum, maximum value and quantiles of the estimated posterior means $\hat{\varphi}_i := \frac{1}{R} \sum_{j=1}^R \hat{\varphi}_i^j$ of the dispersion factors in each model, where $\hat{\varphi}_i^j$ denotes the j th MCMC iterate for φ_i after burnin. The results are reported in Table 2. Note, that the dispersion factor in the GP regression model is the same for all observations, whereas it depends on the parameter μ_i and therefore is different for each observation in the other models. Except for the ZIP model, all models exhibit a substantial degree of overdispersion with respect to the Poisson model, regardless whether spatial effects are included or not. In the non spatial NB model the average of

Table 1 Posterior means and 90% credible intervals for the regression parameters (β_1, \dots, β_4 : effects for the states Nordrhein-Westfalen, Mecklenburg-Vorpommern, Sachsen-Anhalt and Thüringen, respectively) in the different models for the meningococcal disease data

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
No spatial effects					
Poi	-0.17 (-0.26, -0.09)	0.43 (0.30, 0.57)	0.65 (0.28, 1.02)	0.56 (0.23, 0.87)	0.56 (0.22, 0.88)
NB	-0.15 (-0.25, -0.04)	0.42 (0.23, 0.60)	0.67 (0.23, 1.10)	0.54 (0.16, 0.91)	0.48 (0.09, 0.85)
GP	-0.16 (-0.26, -0.06)	0.42 (0.26, 0.58)	0.65 (0.22, 1.06)	0.45 (0.05, 0.84)	0.50 (0.09, 0.88)
ZIP	-0.13 (-0.23, -0.03)	0.43 (0.27, 0.59)	0.69 (0.29, 1.08)	0.60 (0.25, 0.94)	0.59 (0.24, 0.95)
ZIGP	-0.15 (-0.26, -0.04)	0.42 (0.23, 0.61)	0.67 (0.22, 1.09)	0.47 (0.05, 0.87)	0.52 (0.09, 0.92)
With spatial effects					
Poi	-0.25 (-0.38, -0.14)	0.40 (0.18, 0.61)	0.68 (0.21, 1.13)	0.58 (0.19, 0.97)	0.47 (0.06, 0.86)
NB	-0.18 (-0.29, -0.06)	0.41 (0.20, 0.61)	0.68 (0.23, 1.21)	0.55 (0.17, 0.93)	0.48 (0.08, 0.87)
GP	-0.16 (-0.26, -0.06)	0.42 (0.25, 0.58)	0.65 (0.21, 1.06)	0.45 (0.04, 0.84)	0.50 (0.09, 0.88)
ZIP	-0.23 (-0.36, -0.10)	0.39 (0.16, 0.63)	0.68 (0.22, 1.14)	0.60 (0.19, 0.99)	0.49 (0.07, 0.90)
ZIGP	-0.15 (-0.26, -0.04)	0.42 (0.23, 0.61)	0.66 (0.22, 1.09)	0.48 (0.06, 0.88)	0.52 (0.10, 0.92)

Table 2 Estimated posterior means for the model specific dispersion parameters in the considered models with and without spatial effects, with the 2.5% and 97.5% quantiles given in brackets

Parameter	γ	Mean (2.5%, 97.5%)	Min	25%	$\hat{\phi}_i$		Max
					mean 50%	75%	
r in NB	Yes	6.433 (2.483, 22.552)	1.051	1.150	1.293 1.205	1.313	5.358
	No	3.836 (2.202, 6.864)	1.064	1.200	1.396 1.273	1.428	7.053
λ in GP	Yes	0.162 (0.098, 0.231)			1.432		
	No	0.163 (0.098, 0.232)			1.435		
p in ZIP	Yes	0.029 (0.001, 0.087)	1.007	1.020	1.041 1.028	1.044	1.536
	No	0.056 (0.006, 0.125)	1.013	1.041	1.056 1.089	1.089	2.226
p in ZIGP	Yes	0.019 (0.001, 0.064)			1.434		
λ in ZIGP	Yes	0.155 (0.088, 0.224)	1.412	1.421	1.426	1.436	1.819
	No	0.019 (0.001, 0.063)			1.440		
λ in ZIGP	No	0.157 (0.090, 0.227)	1.417	1.427	1.432	1.442	1.826

Further the mean (upper row), range and quantiles (lower row) of the estimated posterior means of the dispersion factors $\hat{\phi}_i$ are given

the estimated posterior means of the dispersion parameter is given by 1.396 and drops to 1.293 when spatial effects are included. The dispersion parameter in the GP model is estimated by 1.435, results are hardly affected by the inclusion of spatial effects. The extension from a GP to a ZIGP model has almost no impact on the estimation of λ and the average dispersion parameter ϕ_i , the proportion of extra zeros p is estimated very close to zero. The proportion of extra zeros p is estimated as 5.6% in the non spatial and 2.9% in the spatial ZIP model. According to the large 95% credible interval for p however, no significant degree of zero inflation seems to be present. Unobserved heterogeneity still present in the data after adjusting for covariates is captured better by the GP and NB model, whereas the assumption of extra zeros is obviously not appropriate for this data. The range of the estimated spatial effects is reported in Table 3. The largest spatial effects are obtained for the Poisson model where unexplained heterogeneity in the data is captured by the spatial effects alone. Results for the ZIP model are very close to the Poisson model which is to be expected, since no significant proportion of extra zeros was detected. In the NB model the range of the spatial effects is considerably smaller, however, part of the data variability in the NB model is explained by spatial effects as well rather than the parameter r alone. This is in contrast to the GP and ZIGP model, where the estimated spatial effects are all very close to zero. Overdispersion in

these models is captured by the parameter λ only, resulting in a high estimated dispersion parameter.

The map plot of the estimated posterior means of the spatial effects in the Poisson model, given in Fig. 2 roughly represents the spatial pattern of the observed relative risk in each region $\frac{y_i}{t_i}, i = 1, \dots, n$ which is plotted in the upper row in Fig. 3. The estimated posterior mean and median of the spatial hyperparameter ψ in the Poisson model, see Table 3 are rather small, the lower bound of the 95% credible interval is close to zero, indicating that the overall degree of spatial dependence is very small. This is reflected in the estimated spatial pattern which is not particularly smooth. Only some rather isolated regions, which are marked in black in the right map in Fig. 2, have a significant positive spatial effect according to the 80% credible intervals. In these regions the observed number of invasive meningococcal disease cases was rather high and most of them do not lie within the four states included as covariates, see Fig. 3. Therefore without spatial effects the risk in these regions is not modelled sufficiently. To investigate further that the degree of spatial smoothing is small, we conducted a nonstructured spatial analysis of the Poisson model by allowing for uncorrelated random effects for each region. Resulting posterior means and 80% credible intervals for the random effects are very similar to the maps shown in Fig. 2, thus confirming that the spatial effects capture unstructured heterogeneity effects. In addition, the DIC, Brier and logarithmic scores are similar to the ones reported in Tables 4 and 5 of the spatial Poisson model, thus resulting in a similar fit.

The estimated posterior means of the relative risk μ_i/t_i are plotted for the non-spatial and spatial Poisson model in the lower row of Fig. 3. In the non-

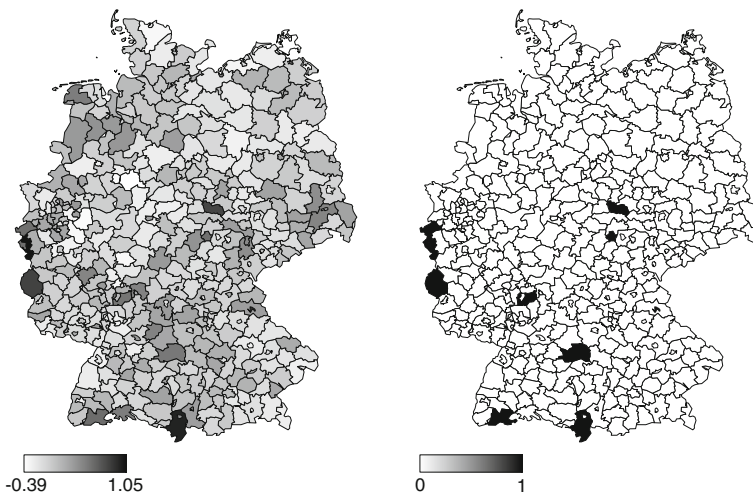


Fig. 2 Maps of the estimated posterior means (*left*) and 80% credible intervals (*white* 0 included in credible interval, *black* strictly negative credible interval) of the spatial effects in the Poisson regression model for the Meningococcal disease data

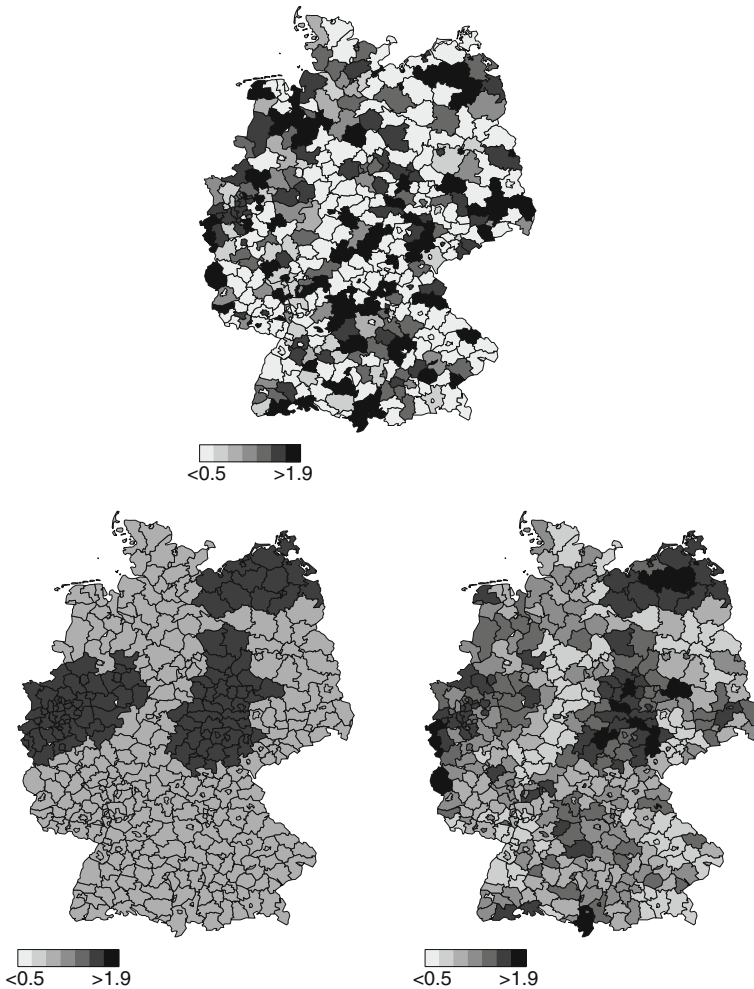


Fig. 3 Maps of the observed relative risk y_i/t_i (upper row) and the estimated posterior means of the risk μ_i/t_i in the non-spatial (lower row, left) and spatial (lower row, right) Poisson regression model for the meningococcal disease data

spatial Poisson model geographic differences are modelled by four state indicators only. Since the risk in two of the states, which are neighbours, is about the same, visually only three states can be distinguished in this plot. Note, that for a better visual comparison we have built seven classes for the relative risk, in particular, risks greater than 1.9 (the 80% percentile of the observed relative risks) are summarized in one class. When spatial random effects are included, the effects of the four risky states are still clearly visible, however the true spatial pattern is reflected more accurately. Though the estimated spatial pattern is smoother than the true one, in general regions with high or low risk are detected reasonable well.

Table 3 Range of estimated posterior means of the spatial effects as well as estimated posterior means, medians and 95% credible intervals given in brackets for the spatial hyperparameters in the considered models for the meningococcal disease data

Model	$(\min_j \hat{\gamma}_j \max_j \hat{\gamma}_j)$	$\hat{\psi}$		$\hat{\sigma}^2$	
		Mean	Median	Mean	Median
Poisson	(−0.383, 1.059)	0.394 (0.011, 0.431)	0.207	0.541 (0.149, 0.615)	0.422
NB	(−0.151, 0.345)	1.223 (0.015, 1.352)	0.588	0.227 (0.012, 0.281)	0.140
GP	(−0.009, 0.020)	2.604 (0.038, 2.199)	1.194	0.012 (0.001, 0.011)	0.005
ZIP	(−0.371, 0.998)	0.329 (0.013, 0.397)	0.235	0.477 (0.169, 0.529)	0.417
ZIGP	(−0.011, 0.019)	1.739 (0.044, 1.905)	1.154	0.018 (0.001, 0.027)	0.011

Table 4 DIC, $E[D(\theta|\mathbf{y})]$ and effective number of parameters p_D for the different models

Model	γ	DIC	$E[D(\theta \mathbf{y})]$	p_D
Poisson	No	1291.8	1286.8	5.04
NB	No	1273.9	1267.8	6.10
GP	No	1265.6	1259.6	6.01
ZIP	No	1291.8	1285.9	5.96
ZIGP	No	1267.8	1261.5	6.35
Poisson	Yes	1248.7	1159.1	89.56
NB	Yes	1270.8	1240.0	30.74
GP	Yes	1265.7	1258.3	7.32
ZIP	Yes	1255.4	1175.1	80.31
ZIGP	Yes	1267.6	1260.2	8.28

6.3 Model comparison using DIC

In order to compare the presented models the DIC, reviewed in Sect. 5.1, is considered. In Table 4, the DIC, the posterior mean of the deviance and the effective number of parameters are given for each model. Only in the Poisson

Table 5 Brier score and logarithmic score for the considered models with and without spatial effects

Model	γ	Brier score	Logarithmic score
Poisson	No	−0.6937	−1.4569
NB	No	−0.6883	−1.4363
GP	No	−0.6873	−1.4272
ZIP	No	−0.6921	−1.4549
ZIGP	No	−0.6878	−1.4291
Poisson	Yes	−0.6280	−1.2422
NB	Yes	−0.6717	−1.3779
GP	Yes	−0.6863	−1.4243
ZIP	Yes	−0.6481	−1.3251
ZIGP	Yes	−0.6900	−1.4529

regression case a well defined normalizing constant $f(\mathbf{y})$ (see Sect. 5.1) exists, while in all other models the likelihood of the saturated model depends on the unknown overdispersion parameters. Therefore we make the choice of setting the normalizing function $f(\mathbf{y})$ to 0. Consequently $E[D(\theta|\mathbf{y})]$ is based only on the unscaled deviance which cannot be directly interpreted as an overall goodness of fit measure of one specific model. However similar to likelihood ratio tests, $E[D(\theta|\mathbf{y})]$ can be used for comparing the model fit of several models when the complexity of the models, measured by the number of parameters, is the same.

For the non spatial models the lowest value of the DIC is obtained for the GP model, while the DIC for the Poisson and the ZIP model takes the highest value. Hence, according to the DIC the GP model is considered best among the non spatial models, while the Poisson and ZIP model clearly perform worse. The effective number of parameters p_D is close to the true number of parameters which is five for the Poisson regression model, six for the NB, GP and ZIP regression models and seven for the ZIGP regression model.

When spatial effects are added, the posterior mean of the deviance and the number of effective parameters in the GP and ZIGP models hardly change. As mentioned in the previous section already, spatial effects are not significant in these models, i.e. after adjusting for covariate information, there is no further spatial heterogeneity in the data which might be captured by the spatial effects. Instead any overdispersion present in the data seems to be sufficiently captured by the model specific dispersion parameter. The DIC for the spatial NB model is slightly better than for the non spatial one, hence spatial effects somewhat improve the model. However, the spatial pattern is rather smooth as can be seen from the effective number of parameters estimated by 30.74. For the Poisson and ZIP regression model in contrast, a significant drop in the DIC is observed when spatial effects are taken into account. This shows that there is some extra variability in the data which is not sufficiently explained by the covariates only in these models. Since the Poisson model does not allow for overdispersion and the heterogeneity is not of a zero inflated nature, for these two models the unexplained variability is covered by the spatial effects. According to the DIC the spatial Poisson model gives the best fit and is to be preferred to a non spatial GP model. Note, that the DIC must be used with care here, since strictly speaking the DIC is defined for distributions of the exponential family only, see [van der Linde \(2005\)](#). However, as noted earlier, the posterior mean of the deviance $E[D(\theta|\mathbf{y})]$ can also be taken into account for comparing the model fit of the non spatial models, since the number of parameters in these nonspatial models are similar. This leads to the same ranking of the models as based on the DIC.

6.4 Model checking using proper scoring rules

Apart from the DIC we also compute the Brier score and the logarithmic score presented in Sect. 5.2 for each model, results are reported in Table 5. These scores are based on the posterior predictive probabilities and the true observed

number of cases. It should be kept in mind that the same data are used both for parameter estimation and for computation of the posterior predictive probabilities. However, as already indicated in Sect. 5.2 the use of training data sets is difficult here, since our data set is very small. The results support the conclusions drawn in the previous section. For the non spatial models the GP regression model fits the heterogeneity in the data best, followed by the ZIGP and NB regression model. The use of a non spatial ZIP regression model does not seem to be appropriate, the gain in comparison to the non spatial Poisson model for which the lowest scores are obtained is very small. The scores for the GP and the ZIGP model hardly change by allowing for spatial effects, indicating that the model specific dispersion parameters capture the data heterogeneity well. Again a small improvement in the NB model is observed when spatial effects are included. The scores for the spatial Poisson and ZIP model however, are considerably smaller than for the other models. This confirms again, that there is significant evidence that heterogeneity effects vary spatially and that structured or unstructured spatial Poisson models give the best fit to the data.

7 Conclusions

We have presented several regression models for count data allowing for overdispersion. Overdispersion is either modelled by the introduction of an additional parameter as in the NB and GP model, by allowing for an extra proportion of zero observations using zero inflated models or by combining zero inflated models with overdispersed distributions.

Further, additionally spatial random effects are included in the models in order to account for unobserved spatial heterogeneity in the data. This approach allows for spatial correlations between observations.

These models were applied to analyse the number of invasive meningococcal disease cases in Germany in the year 2004. The DIC, the Brier and the logarithmic score were used for model comparison. The models allowing for overdispersion gave a significantly better fit than an ordinary non spatial Poisson regression model. Among these non spatial models, the GP model fitted the data best, while the overdispersion present did not seem to be caused by the presence of extra zeros in the data. For the GP and the ZIGP model the inclusion of spatial effects did not improve the models, in the NB model still some significant spatial variation was detected. For the Poisson model which does not allow for overdispersion and the ZIP model which is not modelling the nature of the overdispersion appropriately, the inclusion of spatial effects led to a significant improvement. According to the considered criteria the spatial Poisson model is to be preferred to all other models. But we would like to note that the spatial model fitted shows no smooth surface structure, it rather indicates isolated specific regions where the covariates provide no adequate fit. This might indicate that the occurrence of invasive meningococcal disease has only a weak spatial component and is mostly sporadic over regions.

Instead of analysing the number of cases of invasive meningococcal disease for one year only, it might be interesting to include data over several years in the analysis. Space–time interactions could be included in order to examine whether the spatial pattern changed over the years. This is the topic of future research.

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References

- Agarwal DK, Gelfand AE, Citron-Pousty S (2002) Zero-inflated models with application to spatial count data. *Environ Ecol Stat* 9:341–355
- Angers JF, Biswas A (2003) A Bayesian analysis of zero-inflated generalized Poisson model. *Comput Stat Data Anal* 42:37–46
- Banerjee S, Carlin B, Gelfand A (2004) *Hierarchical modeling and analysis for spatial data*. Chapman & Hall/CRC, New York
- Besag J, Kooperberg C (1995) On conditional and intrinsic autoregressions. *Biometrika* 82:733–746
- Brier G (1950) Verification of forecasts expressed in terms of probability. *Mon Weather Rev* 78 (1):1–3
- Consul P (1989) *Generalized Poisson distributions. Properties and Applications*. Marcel Dekker, New York
- Consul P, Jain G (1973) A generalization of the Poisson distribution. *Technometrics* 15:791–799
- Czado C, Prokopenko S (2004) Modeling transport mode decisions using hierarchical binary spatial regression models with cluster effects. Discussion paper 406, SFB 386 Statistische Analyse diskreter Strukturen [Http://www.stat.uni-muenchen.de/sfb386/](http://www.stat.uni-muenchen.de/sfb386/)
- Famoye F, Singh K (2003a) On inflated generalized Poisson regression models. *Adv Appl Stat* 3(2):145–158
- Famoye F, Singh K (2003b) Zero inflated generalized Poisson regression model (submitted)
- Gelman A, Carlin J, Stern H, Rubin D (2004) *Bayesian data analysis*, 2nd edn. Chapman & Hall/CRC, Boca Raton
- Gilks W, Wild P (1992) Adaptive rejection sampling for Gibbs sampling. *Appl Stat* 41(2):337–348
- Gilks W, Richardson S, Spiegelhalter D (1996) *Markov Chain Monte Carlo in Practice*. Chapman & Hall/CRC, Boca Raton
- Gneiting T, Raftery AE (2004) Strictly proper scoring rules, prediction and estimation. Technical Report no. 463, Department of Statistics, University of Washington
- Gschlöbl S (2006) *Hierarchical Bayesian spatial regression models with applications to non-life insurance*. PhD thesis, Munich University of Technology
- Han C, Carlin B (2001) Markov chain Monte Carlo methods for computing Bayes factors: a comparative review. *J Am Stat Assoc* 96:1122–1132
- Hoeting J, Madigan D, Raftery A, Volinsky C (1999) Bayesian model averaging: a tutorial. *Stat Sci* 14(4):382–417
- Jin X, Carlin B, Banerjee S (2005) Generalized hierarchical multivariate CAR models for areal data. *Biometrics* 61:950–961
- Joe H, Zhu R (2005) Generalized Poisson distribution: the property of mixture of Poisson and comparison with Negative Binomial distribution. *Biometric J* 47:219–229
- Kass R, Raftery A (1995) Bayes factors and model uncertainty. *J Am Stat Assoc* 90:773–795
- Lambert D (1992) Zero-inflated Poisson regression with and application to defects in manufacturing. *Technometrics* 34(1):1–14
- van der Linde A (2005) DIC in variable selection. *Statistica Neerlandica* 59(1):45–56
- Pettitt A, Weir I, Hart A (2002) A conditional autoregressive Gaussian process for irregularly spaced multivariate data with application to modelling large sets of binary data. *Stat Comput* 12(4):353–367

- Rodrigues J (2003) Bayesian analysis of zero-inflated distributions. *Commun Stat* 32(2):281–289
- Spiegelhalter D, Best N, Carlin B, van der Linde A (2002) Bayesian measures of model complexity and fit. *J R Stat Soc B* 64(4):583–640
- Sun D, Tsutakawa RK, Kim H, He Z (2000) Bayesian analysis of mortality rates with disease maps. *Stat Med* 19:2015–2035
- Winkelmann R (2003) *Econometric analysis of count data*, 4th edn. Springer, Berlin Heidelberg, Germany