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Some Relationships Between Cronbach's Alpha and the Spearman-Brown Formula

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Abstract: Cronbach's alpha is an estimate of the reliability of a test score if the items are essentially tau-equivalent. Several authors have derived results that provide alternative interpretations of alpha. These interpretations are also valid if essential tau-equivalency does not hold. For example, alpha is the mean of all split-half reliabilities if the test is split into two halves that are equal in size. This note presents several connections between Cronbach's alpha and the Spearman-Brown formula. The results provide new interpretations of Cronbach's alpha, the stepped down alpha, and standardized alpha, that are also valid in the case that essential tau-equivalency or parallel equivalency do not hold. The main result is that the stepped down alpha is a weighted average of the alphas of all subtests of a specific size, where the weights are the denominators of the subtest alphas. Thus, the stepped down alpha can be interpreted as an average subtest alpha. Furthermore, we may calculate the stepped down alpha without using the Spearman-Brown formula.

Keywords: Stepped up alpha; Stepped down alpha; Standardized alpha; Reliability; Coefficient alpha; Psychometrics.

1. Introduction

An important concept in psychometrics and test theory is the reliability of a test score. In general, a test score is said to have high reliability if it produces similar values for a participant when administration conditions are kept consistent. More formally, in classical test theory reliability is defined as the ratio of the true score variance and the total score variance (Lord and

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Novick 1968; Thorndike 1971; McDonald 1999). Since the true score variance cannot be directly observed, the reliability of a test score must be estimated. In practice there is usually only a single test administration, rather than two or more. In this case a standard approach to reliability estimation is the internal consistency reliability method. The most commonly used internal consistency coefficient is coefficient alpha (Cronbach 1951; Cortina 1993; Osburn 2000; Sijtsma 2009; Furr and Bacharach 2013; Field 2013).

Coefficient alpha was proposed by Guttman (1945) as lambda3 and later popularized as alpha by Cronbach (1951). The coefficient has been applied in thousands of research studies and Cronbach's paper has been cited numerous times (Cortina 1993; Sijtsma 2009). Suppose we have a test that consists of $m \geq 2$ items. Let σ_{ij} denote the covariance between items i and j with $1 \leq i, j \leq m$, and let σ_T^2 denote the variance of the total test score. Cronbach's alpha is defined as

$$\alpha = \frac{m}{m-1} \cdot \frac{\sum_{i \neq j} \sigma_{ij}}{\sigma_T^2}. \tag{1}$$

Various authors have criticized the use of alpha. Examples have been presented that show that alpha is not a measure of the one-dimensionality of a test (Cronbach 1951; Grayson 2004; Sijtsma 2009). Furthermore, there are several coefficients that are higher lower bounds to the reliability of a test than alpha (Revelle and Zinbarg 2009; Sijtsma 2009).

However, most critics and reviewers of alpha agree that it is likely that the coefficient will continue to be a standard tool in reliability estimation in the near future (Cortina 1993; Sijtsma 2009). Moreover, many years after Cronbach's paper, alpha is still a hot topic in current research. For example, the derivation of alpha is based on several assumptions from classical test theory (Lord and Novick 1968; McDonald 1999; Graham 2006). Robustness of alpha to violations of its two major assumptions, essential tau-equivalency and uncorrelated errors, has been documented in Graham (2006), Green and Hershberger (2000), and Green and Yang (2009), while robustness to non-normal data has been studied in Sheng and Sheng (2012).

Once the reliability of a test score is estimated (e.g. using Crobach's alpha), it is frequently desirable to predict the reliability of a test score of a test that measures the same construct but has a different length. For example, in recent years there is an increasing interest in short or shortened tests (Kruyen, Emons, and Sijtsma 2012). A recent literature review is presented in Kruyen, Emons, and Sijtsma (2013). In practice there are limitations on the available time and resources and a short test may therefore be more efficient. To predict the reliability of a test score that is based on a different test length, psychometricians commonly use the Spearman-Brown prediction formula (S-B formula). Let ρ denote the reliability of a test score before adjustment and let ρ^* denote the adjusted reliability. The Spearman-Brown formula is given by (Spearman 1910; Brown 1910)

$$\rho^* = \frac{N\rho}{1 + (N-1)\rho}, \tag{2}$$

where $N > 0$ is the extension factor.

Formulas (1) and (2) both stem from classical test theory (Lord and Novick 1968; Thorndike 1971). However, compared to essential tau-equivalency, the S-B formula is based on the stricter assumption of parallel equivalency. In the context of the split-half approach to reliability estimation the S-B formula is sometimes called the 'step up' formula. The split-half method is another approach to estimating the reliability of a test when there is only one test administration (Revelle and Zinbarg 2009; Field 2013; Furr and Bacharach 2013). If we are interested in the reliability of a subtest, the adjusted alpha will be referred to as the stepped down alpha. If we are interested in the reliability of a longer test, the adjusted alpha will be referred to as the stepped up alpha.

Several authors have attempted to give meaning to Cronbach's alpha, and to its estimate from a sample, when the items are not essentially tau-equivalent (McDonald 1999, p. 93). Cronbach (1951) showed that alpha is the mean of all (Flanagan-Rulon) split-half reliabilities, if a test is split into two halves that are equal in size. The problem with the split-half approach is that there are multiple ways to divide the items of a test into two halves. The estimate therefore depends on the way the split is made (Callender and Osburn 1977; Field 2013). Cronbach (1951) showed that if a test is split into two subtests of equal size, then alpha for the full test is the mean of all the split-half reliabilities. Using alpha instead of the split-half estimate removes, in a way, the arbitrariness of how to split a test. Cronbach's result was generalized by Raju (1977) to the case that a test can be split into any number of parts that are equal in size. These properties of alpha are important because they provide a proper interpretation of alpha (Cortina 1993). In this note we present several relationships between Cronbach's alpha and the S-B formula. The results are algebraic and provide new interpretations of alpha, the stepped down alpha, and standardized alpha. Following Cronbach (1951) and Raju (1977), the interpretations are also valid in the case that essential tau-equivalency and parallel equivalency do not hold.

The note is organized as follows. In Section 2 we present the theorems for Cronbach's alpha. This section includes the following result. The problem with a subtest of a test is that there are multiple ways to remove items from the test. In general, each subtest has a different associated alpha, which is an estimate of the reliability of the subtest score. For a specific subtest size we may consider all possible subtests and the associated subtest alphas. It is shown that the stepped down alpha is a weighted average of the alphas of all subtests of a specific size, where the weights are the denominators of the subtest alphas. This shows that the stepped down alpha can be interpreted as an average subtest alpha. It also shows that we can find the stepped down alpha without using the S-B formula. In Section 3 we present an existence theorem that is a consequence of the main theorem. It is shown that, in general, Cronbach's alpha can be decreased by shortening a test. In Section 4 we formulate the results from Sections 2 and 3 for standardized alpha. Section 5 contains a conclusion.

2. Cronbach's Alpha

Before presenting the theorems, we first introduce some additional notation and definitions. Suppose we have a test that consists of $m \geq 2$ items. Let $\overline{\text{cov}}_m$ and $\overline{\text{var}}_m$ denote, respectively, the average covariance and average variance between the m items. Cronbach's alpha can be defined as

$$\alpha_m = \frac{m\overline{\text{cov}}_m}{\overline{\text{var}}_m + (m-1)\overline{\text{cov}}_m}. \tag{3}$$

The subscript m of α_m denotes that alpha is defined on m items. For the results below, Formula (3) is more convenient to work with than Formula (1).

Suppose we want to shorten the m-item test to a k-item subtest where $1 \leq k < m$. On the one hand we could predict the reliability of the k-item subtest using α_m in the S-B formula. The corresponding stepped down alpha is given by

$$\alpha_m^* = \frac{k\overline{\text{cov}}_m}{\overline{\text{var}}_m + (k-1)\overline{\text{cov}}_m}. \tag{4}$$

Formula (4) is obtained by using $\rho = \alpha_m$ and $N = k/m$ in Formula (2), and multiplying the numerator and denominator of the result by $\overline{\text{var}}_m + (m-1)\overline{\text{cov}}_m$. Alternatively, we can remove $m-k$ items from the m-item test. In this case the associated coefficient alpha is given by (Raju 1977)

$$\alpha_k = \frac{k\overline{\text{cov}}_k}{\overline{\text{var}}_k + (k-1)\overline{\text{cov}}_k}, \tag{5}$$

where $\overline{\text{cov}}_k$ and $\overline{\text{var}}_k$ denote, respectively, the average covariance and average variance between the k items of the subtest. The number of ways we can pick a subtest of k items out of m items is given by the binomial coefficient

$$\binom{m}{k} = \frac{m!}{k!(m-k)!}. \tag{6}$$

Hence, we have $\binom{m}{k}$ distinct k-item subtests and as many versions of α_k.

The subtest alphas α_k are related to the overall alpha α_m and the stepped down alpha α_m^* in several ways. We first present the result mentioned in the introduction. Theorem 1 shows that the stepped down alpha α_m^* is a weighted average of the subtest alphas α_k.

Theorem 1. *Coefficient α_m^* is a weighted average of the α_k, where the weights are the denominators of the α_k.*

Proof: Since Formula (4) presents an expression of α_m^*, we must determine an expression of the weighted average. The numerator of the weighted average is equal to the sum of the $k\overline{\text{cov}}_k$. Let cov_m and cov_k denote the sum of the covariances of, respectively, the total test and a given k-item subtest. For a given k-item subtest we have the identity

$$\overline{\text{cov}}_k = \frac{2}{k(k-1)}\,\text{cov}_k. \tag{7}$$

If we consider all k-item subtests, the number of times a pair of items is part of a k-item subtest is given by the binomial coefficient

$$\binom{m-2}{k-2} = \frac{(m-2)!}{(k-2)!(m-k)!}. \tag{8}$$

Thus, the sum of all the cov_k is given by

$$\binom{m-2}{k-2}\text{cov}_m = \binom{m-2}{k-2} \cdot \frac{m(m-1)}{2} \cdot \overline{\text{cov}}_m. \tag{9}$$

Using (7) and (9), the sum of the $k\overline{\text{cov}}_k$, i.e., the numerator of the weighted average, is given by

$$\binom{m-2}{k-2} \cdot \frac{2}{k(k-1)} \cdot \frac{m(m-1)}{2} \cdot k\overline{\text{cov}}_m = \binom{m}{k} k\overline{\text{cov}}_m. \tag{10}$$

The denominator of the weighted average is equal to the sum of the weights $\overline{\text{var}}_k + (k-1)\overline{\text{cov}}_k$. It consists of two parts. If we consider all k-item subtests, the number of times a single item is part of a k-item subtest is given by the binomial coefficient

$$\binom{m-1}{k-1} = \frac{(m-1)!}{(k-1)!(m-k)!}. \tag{11}$$

Using (11), the part of the denominator involving the item variances is given by

$$\binom{m-1}{k-1} \cdot \frac{1}{k} \cdot m \cdot \overline{\text{var}}_m = \binom{m}{k}\overline{\text{var}}_m, \tag{12}$$

while the part involving the item covariances is, using (8), equal to

$$\binom{m-2}{k-2}\cdot\frac{2}{k(k-1)}\cdot\frac{m(m-1)}{2}\cdot(k-1)\overline{\text{cov}}_m=\binom{m}{k}(k-1)\overline{\text{cov}}_m. \quad (13)$$

Thus, using (10), (12) and (13) the weighted average of the α_k is given by

$$\frac{(10)}{(12)+(13)}=\frac{\binom{m}{k}k\overline{\text{cov}}_m}{\binom{m}{k}\overline{\text{var}}_m+\binom{m}{k}(k-1)\overline{\text{cov}}_m},$$

which is equivalent to Formula (4).
■

It follows from Theorem 1 that the stepped down alpha α_m^* can be interpreted in terms of the subtest alphas α_k, namely, α_m^* is an average of the subtest alphas α_k. Since the stepped down alpha is a weighted average, its value lies between the minimum and maximum values of the subtest alphas. Hence, similar to an ordinary average, the stepped down alpha can be interpreted as an average subtest alpha. This interpretation of the stepped down alpha always holds, even when parallel equivalency or essential-tau equivalency do not hold. It also follows that we may find the stepped down alpha without using the S-B formula.

Theorem 1 can also be formulated in terms of the overall alpha α_m, instead of the stepped down alpha α_m^*. Since coefficient (4) is a weighted average of the alphas of the k-item subtests (Theorem 1), it reflects the average reliability of a test of length k. Coefficient (4) can be stepped up using the S-B formula. Using $\rho=\alpha_m^*$ and $N=m/k$ in Formula (2), and multiplying the numerator and denominator of the result by $\overline{\text{var}}_m+(k-1)\overline{\text{cov}}_m$, we obtain the overall alpha α_m. Hence, we have the following corollary.

Corollary 2. *Coefficient α_m is equal to the stepped up weighted average of the α_k, where the weights are the denominators of the α_k.*

We have a second result for the overall alpha α_m. Since the subtest alphas α_k in (5) estimate the reliabilities of the k-item subtests, we could also step up each α_k to obtain $\binom{m}{k}$ different reliability estimates for a m-item test. Theorem 3 shows that the overall alpha is a weighted average of the stepped up subtest alphas, where the weights are the denominators of the stepped up subtest alphas.

Theorem 3. *Coefficient α_m is equal to a weighted average of the stepped up α_k, where the weights are the denominators of the stepped up α_k.*

Proof: Since Formula (3) presents an expression of α_m, we must determine an expression of the weighted average of the stepped up subtest alphas. We first determine an expression of a stepped up alpha α_k. Using $\rho = \alpha_k$ (Formula (5)) and $N = m/k$ in Formula (2), and multiplying the numerator and denominator of the result by $\overline{\text{var}}_k + (k-1)\overline{\text{cov}}_k$, we obtain

$$\alpha_k^* = \frac{m\overline{\text{cov}}_k}{\overline{\text{var}}_k + (m-1)\overline{\text{cov}}_k}. \tag{14}$$

Next, we determine the weighted average of the α_k^* in (14), using the denominators $\overline{\text{var}}_k + (m-1)\overline{\text{cov}}_k$ as weights. The numerator of the weighted average is equal to the sum of the $m\overline{\text{cov}}_k$. Using (8), this sum is given by

$$\binom{m-2}{k-2} \cdot \frac{2}{k(k-1)} \cdot \frac{m(m-1)}{2} \cdot m\overline{\text{cov}}_m = \binom{m}{k} m\overline{\text{cov}}_m. \tag{15}$$

The denominator of the weighted average is equal to the sum of the weights $\overline{\text{var}}_k + (m-1)\overline{\text{cov}}_k$. It consists of two parts. The part involving the item variances is given in (12). The part involving the item covariances is equal to

$$\binom{m-2}{k-2} \cdot \frac{2}{k(k-1)} \cdot \frac{m(m-1)}{2} \cdot (m-1)\overline{\text{cov}}_m = \binom{m}{k} (m-1)\overline{\text{cov}}_m. \tag{16}$$

Thus, using (15), (12) and (16) the weighted average of the α_k^* is given by

$$\frac{(15)}{(12)+(16)} = \frac{\binom{m}{k} m\overline{\text{cov}}_m}{\binom{m}{k}\overline{\text{var}}_m + \binom{m}{k}(m-1)\overline{\text{cov}}_m},$$

which is equivalent to Formula (3).
■

3. An Existence Theorem

In this section we present an existence theorem for Cronbach's alpha. Theorem 4 shows that the value of alpha can, in general, be decreased by removing some of the items from the test. The theorem follows from Theorem 1 and an inequality between the stepped down alpha α_m^* and the overall alpha α_m.

Theorem 4. *There exists a subtest alpha α_k such that $\alpha_k \leq \alpha_m$, with equality if and only if $\overline{\text{cov}}_m = \overline{\text{var}}_m$.*

Proof: We first derive the inequality $\alpha^*_m \leq \alpha_m$. We have $\alpha^*_m \leq \alpha_m \Leftrightarrow$

$$\frac{k}{\overline{\text{var}}_m + (k-1)\overline{\text{cov}}_m} \leq \frac{m}{\overline{\text{var}}_m + (m-1)\overline{\text{cov}}_m}$$
$$\Updownarrow$$
$$k\overline{\text{var}}_m + k(m-1)\overline{\text{cov}}_m \leq m\overline{\text{var}}_m + m(k-1)\overline{\text{cov}}_m$$
$$\Updownarrow$$
$$(m-k)\overline{\text{cov}}_m \leq (m-k)\overline{\text{var}}_m. \tag{17}$$

Since $k < m$, inequality (17) is equivalent to $\overline{\text{cov}}_m \leq \overline{\text{var}}_m$. The latter inequality is, e.g., presented in Winer (1971, p. 272).

Finally, it follows from Theorem 1 that α^*_m is a weighted average of the subtest alphas. The assertion then follows from the fact that the value of α^*_m lies between the minimum and maximum values of the subtest alphas α_k, and from the fact that α^*_m never exceeds α_m.
■

Thus, Cronbach's alpha can in general be decreased by removing some of the items from the test. Moreover, because k satisfies $1 \leq k < m$, there is no restriction on the number of items that may be removed. In general, alpha can be decreased by removing one, two or even $m-1$ items. Theorem 4 does not tell us which items these are, just that they exist if $\overline{\text{cov}}_m < \overline{\text{var}}_m$. Of course, the latter inequality usually holds in practice. Theorem 4 shows in an alternative way that the value of alpha depends on the number of items of a test. Since alpha tends to decrease when items are removed, alpha will be lower for shorter tests.

The inequality $\overline{\text{cov}}_m < \overline{\text{var}}_m$ is closely related to intraclass correlation

$$ICC(3,1) = \frac{\overline{\text{cov}}_m}{\overline{\text{var}}_m}$$

from Shrout and Fleiss (1979, p. 423). Intraclass correlations are often used when two or more raters classify the same number of targets on a numerical scale. We have $\overline{\text{cov}}_m = \overline{\text{var}}_m$ if and only if $ICC(3,1) = 1$. Hence, if $ICC(3,1) < 1$ it is possible to decrease Cronbach's alpha by removing items of the test.

4. Standardized Alpha

In this section we formulate the results from the previous sections for standardized alpha. Suppose we have a test that consists of $m \geq 2$ items. Let $\overline{\text{cor}}_m$ denote the average correlation between the m items. Standardized alpha can be defined as

$$\alpha_m^s = \frac{m\overline{\text{cor}}_m}{1 + (m-1)\overline{\text{cor}}_m}. \tag{18}$$

Formula (18) was proposed by Cronbach (1951, p. 321) for the case that the variances of the items are not at hand. Nowadays, standardized alpha is mostly used when there are substantial differences in the variances of the items.

Suppose we want to shorten the m-item test to a k-item subtest where $1 \leq k < m$. On the one hand, we could predict the reliability of the k-item subtest using α_m^s in the S-B formula. The corresponding stepped down standardized alpha is given by

$$\alpha_m^{s*} = \frac{k\overline{\text{cor}}_m}{1 + (k-1)\overline{\text{cor}}_m}. \tag{19}$$

Formula (19) is obtained by using $\rho = \alpha_m^s$ and $N = k/m$ in Formula (2), and multiplying the numerator and denominator of the result by $1 + (m - 1)\overline{\text{cor}}_m$. Alternatively, we can remove $m - k$ items from the m-item test. Standardized alpha for a k-item subtest is given by

$$\alpha_k^s = \frac{k\overline{\text{cor}}_k}{1 + (k-1)\overline{\text{cor}}_k}, \tag{20}$$

where $\overline{\text{cor}}_k$ denotes the average correlation between the k items.

Since we have not used any properties of covariances or variances, the arguments in Theorem 1, Corollary 2, Theorem 3 and Theorem 4 are still valid if we replace $\overline{\text{cov}}_k$ by $\overline{\text{cor}}_k$ and $\overline{\text{var}}_k$ by 1. Hence, we have the following analogous results for standardized alpha.

Corollary 5. *Coefficient α_m^{s*} is a weighted average of the α_k^s, where the weights are the denominators of the α_k^s.*

Corollary 6. *Coefficient α_m^s is equal to the stepped up weighted average of the α_k^s, where the weights are the denominators of the α_k^s.*

Corollary 7. *Coefficient α_m^s is equal to a weighted average of the stepped up α_k^s, where the weights are the denominators of the stepped up α_k^s.*

Corollary 8. *There exists a subtest alpha α_k^s such that $\alpha_k^s \leq \alpha_m^s$, with equality if and only if $\overline{\text{cor}}_m = 1$.*

5. Conclusion

Cronbach's alpha is an estimate of the reliability of a test score if the items are essentially tau-equivalent. This assumption usually does not hold in practice. Therefore, several results have been derived that give meaning to alpha when the items of a test are not essentially tau-equivalent. Cronbach (1951) showed that alpha is the mean of all (Flanagan-Rulon) split-half reliabilities if the test is split into two halves that are equal in size. Raju (1977) generalized this result to the case that a test can be split into any number of parts that are equal in size. These properties of alpha are important because they provide a proper interpretation of alpha (Cortina 1993). In this note we presented several new properties of Cronbach's alpha, the stepped down alpha, and standardized alpha. The results are algebraic and provide new interpretations of the internal consistency reliability coefficients. The interpretations are also valid if the underlying assumptions of the coefficients do not hold.

A common practice is to use the S-B formula to calculate alpha for a test of different length. The main result of the note is a new interpretation of the stepped down alpha. By removing different items of a test we obtain different subtests that have different associated alphas. Theorem 1 shows that the stepped down alpha is a weighted average of the alphas of all subtests of a specific size, where the weights are the denominators of the subtest alphas. Thus, the stepped down alpha can be interpreted as an average subtest alpha. It also follows that we can calculate the stepped down alpha without using the S-B formula.

We also presented two new interpretations of Cronbach's alpha. The overall alpha of a test is equal to a stepped up weighted average of the subtest alphas, where the weights are the denominators of the subtest alphas (Corollary 2). Furthermore, the overall alpha is also equal to a weighted average of all stepped up subtest alphas (Theorem 3). Both weighted averaging and stepping up can be defined as a function on a space of alpha coefficients: a weighted averaging function and a step up function. Corollary 2 and Theorem 3 show that the final result, the overall alpha, is not affected by the order of the functions. In other words, the two functions commute with respect to function composition.

The three results (Theorem 1, Corollary 2, and Theorem 3) were also formulated for standardized alpha (Corollaries 5, 6 and 7). Since the properties presented in Cronbach (1951) and Raju (1977) only apply to Cronbach's alpha, Corollaries 5, 6 and 7 are the first algebraic properties that have been derived for standardized alpha. These descriptions of standardized alpha are valid, even when the assumptions from classical test theory associated with standardized alpha do not hold.

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