

Optimal decision rules for fixed-size committees in polychotomous choice situations

Ruth Ben-Yashar, Jacob Paroush

Department of Economics, Bar-Ilan University, 52900 Ramat-Gan, Israel
(e-mail: benyasr@mail.biu.ac.il)

Received: 10 November 1999/Accepted: 13 April 2000

Abstract. This paper derives optimal decision rules for fixed-size committees in polychotomous choice situations. Earlier studies focus on the dichotomous choice model and thus the present extension broadens the scope of applications of the theory of collective decision making.

1 Introduction

The subject of optimal group decision-making in a committee of fixed size that is subject to human fallibility has attracted a great deal of attention in the past couple of decades or so. The results obtained by several studies are applicable to a variety of economic fields such as Labor Economics (Nitzan and Paroush 1980 and Karotkin and Paroush 1995), Management (Gradstein et al. 1990; Berg and Paroush 1998 and Ben-Yashar and Nitzan 1998), Investment and Reliability Theory (Sah and Stiglitz 1988 and Sah 1990, 1991), Social Choice and Game Theory (Fishburn and Gehrlein 1997; Karotkin 1998; Karotkin and Nitzan 1996; Paroush 1998; and Young 1995) and Industrial Organization (Paroush 1985). Results of several studies also have importance in other disciplines such as Political Science, Law, Operation Research and Electrical Engineering (see for instance, Austen-Smith and Banks 1996; Grofman 1978; Ladha 1992; Klevorick and Rothschild 1979 and Karotkin 1994). All these studies are restricted within the narrow limits of the dichotomous choice model where committees, boards of directors or teams are comprised of decision makers whose ultimate assignment is a choice of one out of

only two alternatives. Young (1983), who makes the first step to extend the model, is an exception. This paper extends the pairwise choice setting to a polychotomous choice model.

To the best of our knowledge the present paper is the first to derive optimal decision rules for a team with fixed size that has to select the best course of action from many alternatives.

Nitzan and Paroush (1982, 1985), Grofman et al. (1983) and Shapley and Grofman (1984) are the first works which lay the theoretical foundations of the binary choice model. Recently, Ben-Yashar and Nitzan (1997) formulated a general version of the dichotomous choice model and the present study extends this general version of the dichotomous model into a polychotomous one. Obviously, the extension of a feasible set that contains only two elements into a multiple-alternative choice set makes the entire issue of collective decision making more realistic and opens doors to additional applications.

Let us formulate our problem more specifically. Consider a choice set with k alternatives which are referred to as the k possible states of nature. Only one alternative, which is referred to as the correct alternative, is the best one with respect to the common interest of the n members of a given decision group. Often the choice of alternative in the present affects outcomes in the future and there is uncertainty as to which of k alternatives is “the best” in a sense that it will produce the best outcome for all the team’s members. The best alternative in this sense is called the correct alternative. We assume that each individual possesses a specific competence to identify the correct alternative that is parameterized as a single number between zero and one, i.e., the probability of a correct choice. This probability may also be a state-dependent probability and each alternative may also be associated with some prior likelihood to be the correct one. A decision rule is a function from the set of all possible judgments of the team’s members into the choice set. Given such a framework our analysis explicitly provides the optimal decision rule that maximizes the likelihood of the group’s correct decision. We shall show how exactly the optimal rule is a function of the matrix of state dependent members’ competence, the matrix of state dependent benefits and the vector of prior probabilities.

The rest of the paper is organized as follows. Section 2 presents the model, Sect. 3 derives the optimal rule, Sect. 4 takes account of special cases and related literature. Discussion and summary appear in Sect. 5.

2 The model

Consider a team of n members, $N = \{1, \dots, n\}$, which faces a choice set with k alternatives, $K = \{1, \dots, k\}$. Assume that one and only one of the k alternatives best suits the interest of each of the decision makers, to wit, it is assumed that there is no conflict of interest among the team’s members and they all share common tastes and possess identical preferences with regard to the elements of the choice set or the possible outcomes of the choice of dif-

ferent alternatives. We refer to this specific alternative as “the correct alternative”. However, because of the existence of a veil of uncertainty, choice is subject to human fallibility and none of the decision makers is endowed with complete competence and perfect foresight to identify the correct alternative. As an example, consider a group of medical doctors whose common objectives are the survival and the health condition of a patient, but they hold different opinions about the proper medical procedure that has to be taken, or consider a board of directors of a business firm where the ultimate goal of its members is the firm’s profit maximization, but the board’s members hold diversified views about the desired course of action that has to be taken in order to achieve their common goal. Suppose we can parameterize each individual’s competence by a single number that is bounded between zero and one and this number may also be a state-dependent, denote this competence (probability) by $P_i^r(r) \forall r$. In general, denote by $P_i^r(j)$ the competence (probability) of individual i to vote for alternative j while alternative r , the state parameter, is the correct one. The possibility of state dependent is reflected by the fact that the choice of alternative j by individual i may depend on which of the k alternatives is the correct one, i.e., alternative r . Also note that different individuals may possess different abilities to identify the correct alternative, i.e., $P_s^r(r)$ is not necessarily equal to $P_i^r(r)$ when $s \neq i$.

We assume that the decision-maker’s characteristics, $P_i^r(j)$, satisfy:

$$\forall_{i,r} \sum_{j=1}^k P_i^r(j) = 1. \quad (1)$$

We assume that individuals vote independently. The independence among votes can for instance be guaranteed by secret ballots. This assumption of independence is crucial and important and therefore raises discussions in the context of the dichotomous model. See for instance, Nitzan and Paroush (1984), Lahda (1992) and Berg (1993). This assumption is also plausible and rational, because the paper discusses the optimal collective rule and as shown by Austen-Smith and Banks (1996) and by Ben-Yashar (2001), independent decisions are rational behavior (i.e., constitute a Nash equilibrium) if and only if the optimal collective rule is used.¹ Denote the team’s voting profile by x , where $x = (x_1, \dots, x_n)$, $x_i \in K$ and denote the set of all the k^n possible profiles by Ω . A collective decision rule is a function $f(x)$ from Ω to K , i.e., $f: \Omega \rightarrow K$.

We next define and calculate the conditional probability to obtain the profile x given the correct alternative r , and the collective conditional probability to obtain alternative j , given the correct alternative r , while the decision

¹ Recently, several authors have examined the choice model under rational behavior, relaxing the assumption that decision makers vote non-strategically. For example, Feddersen and Pesendorfer (1996, 1997, 1998), McLenan (1998), Meyerson (1998) and Wit (1998), but as mentioned above, these discussions are not relevant when dealing with the optimal collective rule.

rule f is used by the team. Note that each profile x induces a partition on the set N into k subsets $N_j(x) = \{i \in N | x_i = j\}$ where $j = 1, \dots, k$.

Given the probabilities $P_i^r(j)$ and the assumption of independence, the probability to obtain x given the correct alternative r is denoted by $g(x/r)$:

$$g(x/r) = \prod_{i \in N_r(x)} P_i^r(r) \prod_{\substack{i \in UN_r(x) \\ j \neq r}} P_i^r(j). \tag{2}$$

Define $X(j|f) = \{x \in \Omega | f(x) = j\}$ and the collective probability to obtain j given r and f can be derived as $\prod^r(j|f) = P\{x \in X(j|f) | r\} = \sum_{x \in X(j|f)} g(x/r)$.

Obviously $\forall_{f,r} \sum_{j \neq r} \prod^r(j|f) = 1 - \prod^r(r|f)$, where $\prod^r(r|f)$ is the probability of the team's correct choice and the complement probability $1 - \prod^r(r|f)$ is the collective probability to err, when the correct alternative is r .

The team's benefit of correct choice r is denoted by $B(r/r)$ and that of incorrect choice j while alternative r is the correct one is $B(j/r)$. Note that the parameter on the right-hand side of the function $B(j/r)$ denotes the correct alternative and the argument on the left-hand side is the alternative actually chosen. We reasonably assume that the benefit of a correct choice is positive and provides higher benefit than an incorrect choice, i.e.,

$$\forall r: B(r/r) > 0 \quad \text{and} \quad \forall_{j \neq r} B(r/r) > B(j/r). \tag{3}$$

For a given f the team's expected net benefit when r is the correct alternative is:

$$E^r = \sum_{j \in K} B(j/r) \prod^r(j|f) \tag{4}$$

and the overall expected net benefit is

$$E = \sum_{r \in K} \alpha_r E^r \tag{5}$$

where α_r is some prior probability that alternative r is the correct alternative and not all α_r should necessarily be identical, $\sum_{i=1}^k \alpha_i = 1$.

3 The optimal decision rule

The problem of collective decision making under these circumstances is to find the optimal decision rule \hat{f} that maximizes E for a given prior vector α , a benefit matrix $B(j/r)$ and the $k^2 \cdot n$ probabilities $P_i^r(j)$.

Proposition. For any given decision profile x , if $f(x) = r$ when

$$\forall m \neq r \quad \sum_{s \in K} \alpha_s B(r/s) g(x/s) > \sum_{s \in K} \alpha_s B(m/s) g(x/s) \tag{6}$$

then $f \in \arg \max E$.

Proof. Define $E(x, m) = \sum_{s \in K} \alpha_s B(m/s)g(x/s)$ as the team's expected net benefit when m is the chosen alternative, for a given decision profile x . Note that

$$\begin{aligned} E &= \sum_{s \in K} \alpha_s E^s = \sum_{s \in K} \alpha_s \sum_{j \in K} \sum_{x \in X(j|f)} B(j/s)g(x/s) \\ &= \sum_{j \in K} \sum_{x \in X(j|f)} \sum_{s \in K} \alpha_s B(j/s)g(x/s) \\ &= \sum_{j \in K} \sum_{x \in X(j|f)} E(x, j). \end{aligned}$$

Recall that the rule f induces the partition of Ω into $X(j|f)$ and thus if this rule assigns r to x , i.e., $f(x) = r$, whenever $E(x, r) > E(x, m)$ ($\forall m \neq r$) then $f \in \arg \max E$. Q.E.D.

We can conclude that the optimal rule assigns for any profile the alternative that gives the highest expected net benefit when this alternative is the chosen one.²

Remark. $\forall m \neq r$ $E(x, r) > E(x, m)$, if the following two conditions are satisfied $\forall m \neq r$:

- (i) $\forall i \neq m, i \neq r$ $B(r/i) \geq B(m/i)$ ³
- (ii) $\alpha_r(B(r/r) - B(m/r))g(x/r) > \alpha_m(B(m/m) - B(r/m))g(x/m)$.

The following corollary specifies the optimal decision rule under the assumption of symmetric benefits, i.e., $\forall r, j \neq r, m \neq r$ $B(j/r) = B(m/r)$, and can be derived as a special case from the proposition.

Define the net benefit of the correct alternative r as follows: $\forall r$ $B(r) = B(r/r) - B(j/r)$, where $j \neq r$. Note that under the assumption of symmetric benefits, $B(r)$ is a well-defined term.

Corollary. For any given decision profile x , if $f(x) = r$ when

$$\begin{aligned} \forall m \neq r \quad \ln \frac{\alpha_r}{\alpha_m} + \ln \frac{B(r)}{B(m)} + \sum_{i \in N_r(x)} \ln \frac{P_i^r(r)}{P_i^m(r)} \\ > \sum_{i \in N_m(x)} \ln \frac{P_i^m(m)}{P_i^r(m)} + \sum_{\substack{i \in N_j(x) \\ j \neq m \\ j \neq r}} \ln \frac{P_i^m(j)}{P_i^r(j)} \end{aligned} \tag{7}$$

then $f \in \arg \max E$.

² Note that $E(x, r)$ represents the expected net benefit when r is the chosen alternative, for a given decision profile x and E^r represents the expected net benefit when r is the correct alternative.

³ Condition (i) cannot be satisfied for all i because under the model assumptions, $\forall j \neq r, B(r/r) > B(j/r)$.

Proof. The two conditions that were mentioned in the remark above are satisfied. The assumption of symmetric benefits ensure that condition (i) is satisfied. And we now show that (7) is in fact identical to condition (ii).

First note that (7) is equivalent to (8),

$$\forall m \neq r \quad \frac{\alpha_r}{\alpha_m} \frac{B(r)}{B(m)} \prod_{i \in N_r(x)} \frac{P_i^r(r)}{P_i^m(r)} > \prod_{i \in N_m(x)} \frac{P_i^m(m)}{P_i^r(m)} \cdot \prod_{\substack{i \in N_j(x) \\ j \neq m \\ j \neq r}} \frac{P_i^m(j)}{P_i^r(j)}. \tag{8}$$

Use (2) and substitute $g(x/r)$ and $g(x/m)$ in (8) to obtain the following inequality,

$$\forall m \neq r \quad \alpha_r B(r) g(x/r) > \alpha_m B(m) g(x/m). \tag{9}$$

Q.E.D.

Appendix A gives an alternative proof which is independent of the remark.

4 Special cases and related literature

In addition to the symmetry of the benefits assume symmetry of probabilities, i.e., $\forall i, r, m, j \neq m, j \neq r \ P_i^r(j) = P_i^m(j)$ and note that the second term of the right hand side in equation (7) disappears.⁴ Thus, the criterion for the optimal rule is: $\hat{f}(x) = r$ when

$$\forall m \neq r \quad \ln \frac{\alpha_r}{\alpha_m} + \ln \frac{B(r)}{B(m)} + \sum_{i \in N_r(x)} \ln \frac{P_i^r(r)}{P_i^m(r)} > \sum_{i \in N_m(x)} \ln \frac{P_i^m(m)}{P_i^r(m)}. \tag{10}$$

If in addition we assume that the probability to choose correctly is the same for all the alternatives, to wit, the correct choice is not state dependent, then the probability to err is independent of which alternative is chosen, i.e., to choose specific $j \neq r$ where r is the correct alternative is identical for all $j \neq r$. More formally, assume symmetry of probabilities

$$\text{if } \forall i, m \neq r \ P_i^r(r) = P_i^m(m), \text{ then } \forall i, r, m \neq r \ P_i^r(m) = \frac{1 - P_i^r(r)}{k - 1}.$$

Denote by P_i the correct probability $P_i^r(r)$. Thus, the optimal rule is: $\hat{f}(x) = r$ when

$$\forall m \neq r \quad \ln \frac{\alpha_r}{\alpha_m} + \ln \frac{B(r)}{B(m)} + \sum_{i \in N_r(x)} \beta_i > \sum_{i \in N_m(x)} \beta_i \tag{11}$$

where $\beta_i = \ln \frac{P_i}{(1 - P_i)/(k - 1)}$.

⁴ This term may also equal zero for a less restrictive assumption.

Assuming also symmetric environment that is $\forall m \neq r$ (i) $\alpha_r = \alpha_m$ and (ii) $B(r) = B(m)$ then the first two terms on the left-hand side of (11) disappear and the optimal rule is $\hat{f}(x) = r$ when

$$\sum_{i \in N_r(x)} \beta_i > \sum_{i \in N_m(x)} \beta_i \tag{12}$$

where $\beta_i = \ln \frac{P_i}{(1 - P_i)/(k - 1)}$.

Furthermore, if one assumes a homogeneous competence, that is $\forall i P_i = P$, then the optimal rule is $\hat{f}(x) = r$ when $\forall m \neq r$

$$\sum_{i \in N_r(x)} \beta > \sum_{i \in N_m(x)} \beta$$

or equivalently

$$|N_r(x)| > |N_m(x)| \tag{13}$$

which means that the optimal rule selects the most popular alternative as the correct one, i.e., $\hat{f}(x) = r$ if the number of members supporting this alternative is the highest.

The dichotomous case

A special case of the optimal rule in the more general framework (Proposition) is the dichotomous case, where we assume $k = 2$. Let us denote the two alternatives by 1 and -1 , and the two correct probabilities by P_i^1 and P_i^2 , the prior probability that alternative 1 is the correct one by α and the complement $1 - \alpha$ is the prior probability that alternative -1 is the correct one. In this case the optimal rule is: $\hat{f}(x) = 1$ when

$$\ln \frac{\alpha}{1 - \alpha} + \ln \frac{B(1)}{B(-1)} + \sum_{i \in N_1(x)} \ln \frac{P_i^1}{1 - P_i^2} > \sum_{i \in N_{-1}(x)} \ln \frac{P_i^2}{1 - P_i^1}. \tag{14}$$

This is the major result of Ben-Yashar and Nitzan (1997).

Referring now to the above assumptions of the symmetry of the benefits, symmetry of probabilities, the assumption that the probability to choose correctly is the same for all the alternatives and symmetric environment for which the optimal rule is given in (12), in the dichotomous case one obtains the same rule as firstly derived in Nitzan and Paroush (1982) and independently in Shapley and Grofman (1984), that is $\hat{f}(x) = 1$ when

$$\sum_{i \in N_1(x)} \beta_i > \sum_{i \in N_{-1}(x)} \beta_i \tag{15}$$

where $\beta_i = \ln \frac{P_i}{1 - P_i}$ or equivalently $\hat{f}(x) = 1$ when $\sum_{i \in N} \beta_i x_i > 0$ where $x_i \in \{-1, 1\}$.

5 Discussion and summary

This paper extends the dichotomous choice model to a polychotomous one. We specify the optimal rule and explicitly show how the optimal choice is determined by the vector of prior probabilities, the matrix of benefit and the state dependent probabilities.

The major conclusion of our analysis is that the number of the alternatives in the choice set matters. An extension of the choice set may change the optimal decision rule. Let us illustrate this last point with the simplest possible example. Consider a symmetric environment, i.e., the priors are identical and the net benefits of correct choice are identical for each alternative. Moreover, assume a team of only three members whose competence parameters are independent of states. Denote the competence vector by (P_1, P_2, P_3) where P_i are the probability of the choice of the correct alternative of member i and the probability of a choice of incorrect for each of the incorrect alternatives is assumed to be $(1 - P_i)/(k - 1)$ where k is the total number of alternatives in the choice set. Without loss of generality assume that $P_1 \geq P_2 \geq P_3$. Consider the case $k = 2$. The expert rule (dictator) and not the simple majority rule is the optimal rule if and only if $\frac{P_1}{1 - P_1} > \frac{P_2}{1 - P_2} \cdot \frac{P_3}{1 - P_3}$. However, if k is an integer larger than $k_0 = \frac{P_1}{(1 - P_1)} \frac{(1 - P_2)}{P_2} \frac{(1 - P_3)}{P_3} + 1$ then the optimal rule is changed and becomes the simple majority rule. To wit, the correct alternative is either the one with two votes or more or in case that such an alternative does not exist the one which is chosen by the most competent member. The reason for the above condition on k is that the criterion for the expert rule to be optimal as derived from (8) is

$$\frac{P_1}{(1 - P_1)/(k - 1)} > \frac{P_2}{(1 - P_2)/(k - 1)} \cdot \frac{P_3}{(1 - P_3)/(k - 1)}$$

and by definition this criterion is satisfied only if $k < k_0$.

It is worth mentioning that the optimal rule that is obtained above can be used also for ranking the alternatives. The importance of the ability to rank the various alternatives most notably arises when first best alternatives can not be implemented, or when a few alternatives should be selected.

When we try to extend our model to a situation where each individual has to rank the k alternatives instead of choosing only one, matters become more complicated. Ideally we would like to refer to the probabilities of each alternative being placed by an individual in a particular position in his ranking vector. However, a simplification would be to treat each of the $k!$ possible rankings as an alternative and then to choose the best one among the n available alternatives using our optimal rule.

In summary, this paper is concerned with the identification of the optimal decision rule for a fixed-size committee in a general uncertain polychotomous choice setting. Obviously, the extension of a feasible set that contains only

two elements into a multiple-alternative choice set makes the entire issue of collective decision making more realistic and opens doors for several more applications.

Appendix A

Proof. Use (4) to see that E^s , the expected net benefit when s is the correct alternative can be written as:

$$E^s = B(s/s)\prod^s(s|f) + \sum_{\substack{j \in K \\ j \neq s}} B(j/s)\prod^s(j|f)$$

Under the assumption of symmetric benefits

$$\sum_{\substack{j \in K \\ j \neq s}} B(j/s)\prod^s(j|f) = B(j/s) \sum_{\substack{j \in K \\ j \neq s}} \prod^s(j|f) = B(j/s)(1 - \prod^s(s|f)).$$

Thus,

$$E^s = (B(s/s) - B(j/s))\prod^s(s|f) + B(j/s)$$

and

$$E = \sum_{s \in K} \alpha_s B(s)\prod^s(s|f) + \sum_{s \in K} \alpha_s B(j/s)$$

Or,

$$E = \sum_{s \in K} \sum_{x \in X(s|f)} \alpha_s B(s)g(x/s) + C \tag{A1}$$

where C is a constant independent of f . Use the inequality (9) and the definition of (A1) to see that for any given profile x , if $f(x) = r$ when (9) holds, then $f \in \text{arg max } E$. And as we showed inequality (9) is the same as (7). Q.E.D.

References

Austen-Smith D, Banks JS (1996) Information aggregation, rationality and the Condorcet jury theorem. *Am Polit Sci Rev* 90(1): 34–45
 Ben-Yashar R (2001) Informative, sincere, rational voting and the optimal collective rule. Mimeo
 Ben-Yashar R, Nitzan S (1997) The optimal decision rule for fixed size committees in dichotomous choice situations, the general result. *Int Econ Rev* 38(1): 175–187
 Ben-Yashar R, Nitzan S (1998) Quality and structure of organizational decision making. *J Econ Behav Organiz* 36: 521–534
 Berg S (1993) Condorcet’s jury theorem, dependency among voters. *Soc Choice Welfare* 10: 87–96
 Berg S, Paroush J (1998) Collective decision making in hierarchies. *Math Soc Sci* 35: 233–244

- Feddersen TJ, Pesendorfer W (1996) The swing voter's curse. *Am Econ Rev* 86(3): 408–424
- Feddersen TJ, Pesendorfer W (1997) Voting behavior and information aggregation in elections and private information. *Econometrica* 65(5): 1029–1058
- Feddersen TJ, Pesendorfer W (1998) Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *Am Polit Sci Rev* 92: 23–35
- Fishburn PC, Gehrlein WV (1977) Collective rationality versus distribution of power of binary social choice functions. *J Econ Theory* 16: 72–91
- Gradstein M, Nitzan S, Paroush J (1990) Collective decision making and the limits on the organization's size. *Publ Choice* 66: 279–291
- Grofman B (1978) Judgemental competence of individuals and groups in a dichotomous choice situation: Is a majority of heads better than one? *J Math Sociol* 6: 47–60
- Grofman B, Owen G, Feld SL (1983) Thirteen theorems in search of the truth. *Theory Decision* 15: 261–278
- Karotkin D (1994) Effect of the size of the bench on the correctness of court judgments: The case of Israel. *Int Rev Law Econ* 14(3): 371–375
- Karotkin D (1998) The network of weighted majority rules and weighted majority games. *Games Econ Behav* 22(2): 299–315
- Karotkin D, Nitzan S (1996) Note on restricted majority rules: Invariance to rule selection and outcome distinctiveness. *Soc Choice Welfare* 13: 269–274
- Karotkin D, Paroush J (1995) Incentive schemes for investment in human capital by members of a team of decision makers. *Labour Econ* 2: 41–51
- Klevorick AK, Rothschild M (1979) A model of jury decision process. *J Legal Stud* 8: 141–164
- Ladha KK (1992) The Condorcet jury theorem, free speech and correlated votes. *Am J Polit Sci* 36: 617–634
- McLenan A (1998) Consequences of the Condorcet jury theorem for beneficial information aggregation by rational agents. *Am Polit Sci Rev* 92: 413–418
- Nitzan S, Paroush J (1980) Investment in human capital and social self protection under uncertainty. *Int Econ Rev* 21: 547–557
- Nitzan S, Paroush J (1982) Optimal decision rules in uncertain dichotomous choice situation. *Int Econ Rev* 23: 289–297
- Nitzan S, Paroush J (1984) The significance of independent voting under uncertain dichotomous choice situations. *Theory Decision* 17: 47–60
- Nitzan S, Paroush J (1985) *Collective Decision Making: An Economic Outlook*. Cambridge University Press, Cambridge, London, New York
- Paroush J (1985) Notes on partnerships in the services sector. *J Econ Behav Organiz* 6: 79–87
- Paroush J (1998) Stay away from fair coins: A Condorcet jury theorem. *Soc Choice Welfare* 15: 15–20
- Sah RK (1990) An explicit closed-form formula for profit-maximizing k -out-of- n systems subject to two kinds of failures. *Microelectronics Reliability* 30: 1123–1130
- Sah RK (1991) Fallibility in human organizations and political systems. *J Econ Perspectives* 5: 67–88
- Sah RK, Stiglitz JE (1988) Qualitative properties of profit-maximizing k -out-of- N systems subject to two kinds of failures. *IEEE Trans Reliabil* 37: 515–520
- Shapley L, Grofman B (1984) Optimizing group judgmental accuracy in the presence of interdependencies. *Publ Choice* 43: 329–343
- Wit J (1998) Rational choice and the Condorcet jury theorem. *Games Econ Behav* 22: 364–376
- Young P (1983) Ranking and choice: An axiomatic approach to maximum likelihood methods. Mimeo. Presented at the Pooling of Information Conference, University of California at Irvine
- Young P (1995) Optimal voting rules. *J Econ Perspectives* 9(1): 51–64