



Reflections on Arrow's research program of social choice theory

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Published online: 26 February 2019
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Abstract

Despite the importance of pioneering work by such precursors as Jean-Charles de Borda and Marquis de Condorcet in the 18th century, it was Kenneth Arrow and his general impossibility theorem that elevated the scientific status of social choice theory into an unprecedented plateau. This paper tries to highlight several unique features of his research program of social choice theory vis-à-vis the classical contributions of Borda and Condorcet, on the one hand, and the “new” welfare economics à la Bergson and Samuelson, on the other hand, as well as to identify several channels through which his impossibility impasse could be circumvented. It is concluded with several personal reminiscences of Kenneth Arrow based on the author's own experiences with him over four decades.

JEL Classification B21 · D63 · D71

Any man's death diminishes me,
because I am involved in Mankind;
And therefore never send to know
for whom the bell tolls;
it tolls for thee.

John Donne

Devotions upon Emergent Occasions, Meditation XVII, 1624.

Paper prepared for the Special Issue of *Social Choice and Welfare* in Memory of Kenneth J. Arrow. Joint editors Marc Fleurbaey, Eric Maskin, Amartya Sen, and Kotaro Suzumura.

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Though our brother is on the rack, as long as we ourselves are at our ease, our senses will never inform us of what he suffers. ... It is by imagination that we can form any conception of what are his sensations.

Adam Smith

Theory of Moral Sentiments, 2nd edn., 1762.

1 Introduction

If it were not for Kenneth Arrow's seminal contribution to social choice theory, which culminates in his justly famous general impossibility theorem, the area of social choice theory as we know it now simply would not exist. Although there is no reason to undervalue pioneering contributions of such precursors as Jean-Charles de Borda and Marquis de Condorcet in the eighteenth century, it seems clear that Arrow and his general impossibility theorem elevated the scientific status of social choice theory into an unprecedented plateau. I should also testify to the fact that, in Arrow's own judgment, the creation of modern social choice theory and the proof of the general impossibility theorem rank the highest in his own evaluation ordering among his whole work in economics, which spans astonishingly wide fields of a theoretical and an applied nature.¹ On this occasion of writing a brief paper in his memory and honor, let me identify several facets of his path-breaking contributions in his magnum opus, *Social Choice and Individual Values*. In so doing, I wish to confirm that Arrow's problem and his general impossibility theorem are very much alive and kicking even after 70 years since their birth, and his insightful analyses keep on providing ample materials for fruitful further exploration.

2 Quantum leap in the history of social choice theory

At the outset, let me refute an often-made remark on the nature of Arrow's social choice theory and general impossibility theorem. It is casually alluded that Arrow's theorem is nothing other than a generalization of Condorcet's voting paradox. With the purpose of showing that this is at least misleading as a characterization of Arrow's seminal contribution, let us recollect that he was concerned with the process or rule of aggregating each profile of individual preference orderings on the set of social states into a social preference ordering on the same set of alternatives. For simplicity, consider the *minimal* society, where his theorem has a bite, viz. the society with two

¹ I asked Arrow over dinner at the San Antonio Meeting of the Public Choice Society held in 2001 to give me the first-best, the second-best and the third-best work of his own. The answer was immediate. His ranking was *social choice theory and general impossibility theorem* in the first place, followed by the *theory of contingent commodities and the role of securities in the theory of risk-bearing*, and thirdly the *existence of an equilibrium in a competitive economy*.

individuals, 1 and 2, and three social states, x , y , and z .² Assuming away all instances of indifference relations, there are six possible linear preference orderings on $X = \{x, y, z\}$, viz.,^{3,4}

$$\begin{aligned} \alpha : x > y > z & \quad \beta : x > z > y & \quad \gamma : y > x > z \\ \delta : y > z > x & \quad \varepsilon : z > x > y & \quad \theta : z > y > x. \end{aligned}$$

Let $\Gamma = \{\alpha, \beta, \gamma, \delta, \varepsilon, \theta\}$ be the set of all linear preference orderings on X . Arrow's aggregation rule is nothing other than a function f from the set $\Gamma \times \Gamma$ of all logically possible profiles of individual preference orderings to the set Γ of all logically possible linear preference orderings on X . The total number of logically possible profiles is given by $6 \times 6 = 36$. Since an Arrow aggregation rule f designates, for each logically possible profile, the corresponding social linear preference ordering, the number of the Arrow aggregation rules amounts to 6^{36} , which far exceeds the Avogadro constant.^{5,6}

The *simple majority decision rule* may be considered to be a candidate of Arrow's aggregation rules. In the context of the minimal society, however, it is disqualified as it may generate intransitivity involving social indifferences.⁷ Indeed, if voter 1 has $x >_1 y >_1 z$ and voter 2 has $y >_2 z >_2 x$, the simple majority decision rule asserts that x and y are socially indifferent, y is socially preferred to z , and x is socially indifferent to z , which is an obvious violation of the transitivity of social weak preference relation.⁸

Arrow is concerned with the whole class of preference aggregation rules, not just one specified aggregation rule, satisfying his deceptively simple axioms, which consist of the *unrestricted domain*, the *Pareto principle*, and the *independence of irrelevant alternatives*. The full set of potential aggregation rules is so large that it is totally out of the question to check each aggregation rule one by one for its eligibility in terms of Arrow's set of axioms. His tour de force was to use the axiomatic method of analysis and show that the set of all eligible aggregation rules may be reduced to the set $\{f_1, f_2\}$ for the minimal society, where f_1 and f_2 are aggregation rules that bestow on individual 1 and individual 2, respectively, the dictatorial power in the social decision-making procedure. Thus, if we add the axiom of *non-dictatorship* to the already listed axioms, we end up with the Arrow general impossibility theorem.

² As a matter of fact, Arrow's (1950) first exposition of his general impossibility theorem was for the minimal society in this sense.

³ If we allow individuals as well as society to express indifference relations, the total number of possible preference orderings on X becomes 13. This does not alter anything essential, and the crucial point we are making in what follows can be made, *mutatis mutandis*.

⁴ For example, α is a linear preference ordering that ranks x first, y second, and z third.

⁵ The Avogadro constant is defined by $N_A = 6.022\ 140\ 857(74) \times 10^{23} \text{ mol}^{-1}$ in the International System of Units.

⁶ More generally, if there are m social alternatives and n individuals in the society, where $3 \leq m < +\infty$ and $2 \leq n < +\infty$, the total number of the Arrow aggregation rules or processes amounts to $(m!)^{(m!)^n}$, which reduces to 6^{36} if $m = 3$ and $n = 2$, but becomes an astronomically large number for reasonably large values of m and n . The unreality of enumerating all possible rules and checking their eligibility one after another is obvious.

⁷ I owe thanks to Eric Maskin who brought me to this observation.

⁸ The Condorcet paradox requires at least three voters, so that it is *not* relevant in the context of social decision-making in the minimal society.

The gist of this theorem may be summarized in Arrow's own words as follows: "[I]f no prior assumptions are made about the nature of individual orderings, there is no method of voting which will remove the paradox of voting ..., neither plurality voting nor any scheme of proportional representation, no matter how complicated. Similarly, the market mechanism does not create a rational social choice."⁹

Thus, the Arrow general impossibility theorem and the Condorcet paradox are on different analytical plateaus altogether. The gulf that separates the Condorcet paradox on the simple majority decision rule and the Arrow general impossibility theorem should not be slighted, even though the importance of the Condorcet paradox in the evolution of social choice theory is clearly beyond any dispute.

There is another conspicuous contrast between Condorcet's work on the simple majority decision rule and Arrow's work on the whole class of preference aggregation rules.¹⁰ Condorcet starts from the space of individual *votes*, so that most properties of what Arrow calls "neutrality" are already incorporated in his informational framework. Given the same voting profile over (x, y) and (a, b) , we expect to treat (x, y) and (a, b) in a similar fashion. This is not assumed *ab initio* in the case of Arrow, as people's preferences over (x, y) and (a, b) may convey information that the simple rankings over alternatives may not capture. This is of crucial importance, as Arrow came into his social choice exercises partly through his interest in welfare economics, which extends far beyond the concerns of voting theory.

In the presence of the axioms of unrestricted domain, independence of irrelevant alternatives, and Pareto, however, Arrow demonstrated that a kind of the property of "neutrality" holds, which is a very substantial step towards his monumental general impossibility theorem. This is the "spread of decisiveness" lemma irrespective of the exact social alternatives involved. Once this lemma is established, Arrow is largely in the territory à la Condorcet, and the connection between Condorcet and his voting paradox, on the one hand, and Arrow and his general impossibility theorem, on the other hand, becomes transparent. But a lot of Arrow's original work, of which there is no parallel in Condorcet's formal writings, had been completed by the time the parallel becomes fitting.

3 Two scenarios of normative economics¹¹

As a preliminary step in identifying the nature of Arrow's social choice theory, let us observe that there are two sharply contrasting scenarios of normative economics in the

⁹ Arrow (1983a, p. 23).

¹⁰ I am grateful to Amartya Sen who emphasized the importance of this distinction between the informational disciplines within which Condorcet and Arrow pursued their respective exercises.

¹¹ In my perception, normative economics consists of two complementary parts, viz. welfare economics and social choice theory. On the one hand, welfare economics is concerned with the critical examination of the performance of actual or imaginary economic systems, and also with the critique, design, and implementation of alternative economic policies. On the other hand, social choice theory is concerned with the evaluation of alternative procedures of collective decision-making, and also with the logical foundations of welfare economics. This perception was expounded in my Introduction to the *Handbook of Social Choice and Welfare*, Vol. I, which I edited jointly with Kenneth Arrow and Amartya Sen.

historical evolution of this intellectual discipline, viz. the scenario of *grandeur social welfare maximization*, and the scenario of *piecemeal social welfare improvement*.¹²

The origin of the first scenario may be traced back to Jeremy Bentham (1776) and his *greatest happiness principle*, so-called, to the effect that *it is the greatest happiness of the greatest number that is the measure of right and wrong*. This dictum has been regarded as the flag mark of the “old” welfare economics attributed to Arthur Pigou (1920), which is commonly characterized by the *maximization of the social sum-total of individual utilities*. For the sake of later reference, this scenario of welfare economics may be dubbed the *Bentham–Pigou program*, the gist of which may be summed up as follows: Design an institutional framework of the society that motivates people to bring about the solution to the following constrained maximization problem:

$$\text{Max } \sum_{i=1}^n u_i(x) \text{ over all } x \in S, \quad (\text{B–P})$$

where n , $2 \leq n < +\infty$, denotes the number of individuals in the society, u_i is the utility function of individual $i \in N = \{1, 2, \dots, n\}$, and $S \subseteq X$ is the opportunity set of alternative social states.¹³

The attribution of the Bentham–Pigou program to Arthur Pigou may be disputed in view of his founding proclamation in *The Economics of Welfare* [Pigou (1920)]: “The complicated analysis which economists endeavor to carry through are not mere gymnastic. They are instruments for the bettering of human life.” Indeed, Pigou nowhere committed himself explicitly to the maximization program (B–P),¹⁴ and the declared task of his welfare economics was to design and implement the “instruments for the bettering of human life.” Thus, I am inclined to attribute the origin of the second—rather than the first—scenario of normative economics to Arthur Pigou, which seeks for the *piecemeal social welfare improvement*.

Recollect that the informational basis of the program (B–P) met Lionel Robbins’s (1932/1935) harsh criticism in the early 1930s on account of the “unscientific” nature of interpersonal comparability of individual utilities, and the “old” welfare economics was declared dead and buried. The vacuum thereby left open was soon filled up by Abram Bergson (1938) and Paul Samuelson (1947/1983) by means of their justly famous concept of the *social welfare function*.¹⁵ According to Samuelson (1947,

¹² The contrast between these two scenarios of normative economics is strongly reminiscent of Amartya Sen’s contrast between *transcendental institutionalism* and the *comparative assessment approach*, which Sen identified in the context of the ideas of justice [Sen (2009)]. Those who are interested in this contrast are referred to Suzumura (2018a, b) for more details in concrete contexts.

¹³ (B–P) stands for “Bentham and Pigou” for short, and individual utility functions are cardinally measurable and interpersonally unit-comparable.

¹⁴ Recollect that Arrow (1983b, p. 18) himself observed that “[s]urprisingly enough, there is only one mention of summing utilities [in Pigou’s *The Economics of Welfare*] and that is very incidental. Although the whole work is devoted to optimizing, there is no explicit formulation of a maximand. For the most part, the criterion is increase in the national income (“national dividend” in Pigou’s language). But he is at pains to point out national income is itself an imperfect approximation, though I am not clear what it was supposed to approximate.”

¹⁵ As a matter of fact, there is the second school of “new” welfare economics, which is based on the piecemeal welfare criteria of *hypothetical compensation principles*. It was Nicholas Kaldor (1939) and

p. 221), a Bergson–Samuelson social welfare function is “a function of all the economic magnitudes of a system which is supposed to characterize some ethical belief” that “we take as a starting point for our discussion ... [w]ithout inquiring into its origins.” The research program of the Bergson–Samuelson “new” welfare economics may be neatly summed up as follows: Design an institutional framework of the society that motivates people to bring about the solution to the constrained maximization problem:

$$\text{Max } h(u_1(x), \dots, u_i(x), \dots, u_n(x)) \text{ over all } x \in S, \quad (\text{B–S})$$

where h is the *Bergson–Samuelson social welfare function* of the individualistic type,¹⁶ u_i is an ordinally measurable and interpersonally non-comparable utility function of individual $i \in N = \{1, 2, \dots, n\}$, and (B–S) stands for “Bergson–Samuelson” for short.

On reflection, the transition from the program (B–P) of the “old” welfare economics to the program (B–S) of the “new” welfare economics was once considered a gigantic step towards salvaging the defunct program of “old” welfare economics and making use of the sound vestige of the upper structure thereof. However, the two programs (B–P) and (B–S) have a conspicuous family resemblance, which should not be overlooked. Indeed, these two scenarios capture in common the essence of welfare economics in terms of the *constrained maximization paradigm*, their crucial difference being neatly reducible to the difference in their respective objectives of constrained maximization.

Let us now proceed to what we believe is the crucial contrast between Arrow’s social choice theory and the Bergson–Samuelson “new” welfare economics. To begin with, it is worthwhile to recollect that Samuelson was fully uncompromising in his flat denial of asking the origin of the Bergson–Samuelson social welfare function, neither was he willing to ask whose ethical values it embodies. The reason of his insistence seems to be rooted in his strong desire to separate what belongs to the “scientific” realm of *objective facts* from what belongs to the “normative” realm of *ethical values*. It is doubtful, however, if indeed we can separate the realm of “what is” from the realm of “what should be,” even in principle. It was Hilary Putnam (2002, p. 44) who went as far as to assert that “[t]he worst thing about the fact/value dichotomy is that in practice it functions as a discussion-stopper, and not just a discussion-stopper, but a thought-stopper.” In sharp contrast, Arrow’s social choice theory asks whether there exists a social aggregation procedure embodying several normative desiderata, by means of

Footnote 15 continued

John Hicks (1940) who kicked off this school of “new” welfare economics. It is to be highlighted that there are two sharp distinctions between the social welfare function school led by Bergson and Samuelson, and the hypothetical compensation principles school led by Kaldor and Hicks. The first distinction is that the social welfare function school is a resurgence of the first scenario of welfare economics, viz. the constrained maximization paradigm, whereas the compensation principles school is a lineal descendent of the piecemeal welfare improvement paradigm that originates in Pigou. The second distinction is that the social welfare function school does not ask the origin and holder of the social value to be optimized, whereas the compensation principles school attempts to construct the piecemeal welfare criteria from within the economic system under scrutiny. See Suzumura (1999, 2018a, b) for more detailed contrast between these two schools of “new” welfare economics.

¹⁶ The Bergson–Samuelson social welfare function h is said to be *individualistic* if and only if it depends on the social state x through the mediation of the profile of individual utility functions $u = (u_1, \dots, u_i, \dots, u_n)$.

which a social value is endogenously constructed on the basis of individual values. Put differently, a social aggregation procedure à la Arrow is not exogenously given; it is an endogenous variable to be rationally designed and socially chosen.

Two further remarks on the nature of Arrow's social choice theory are in order. In the first place, Arrow's conceptualization of how a society makes its collective choice may well be construed as a variant of the first scenario of normative economics. Indeed, the social choice $C(S, \mathbf{R})$ from the opportunity set S when the profile $\mathbf{R} = (R_1, \dots, R_i, \dots, R_n)$ of individual values prevails is explained by Arrow through the optimization of the social value $R = f(\mathbf{R})$ subject to the feasibility constraint S , where f is the social welfare function in the sense of Arrow. Thus, there is a formal parallelism between the Bergson–Samuelson “new” welfare economics, which is captured by the constrained maximization problem (B–S), and the Arrow social choice theory, which describes social choice by means of the following constrained optimization scheme:

$$C(S, \mathbf{R}) = \{x^* \in S \mid \forall x \in S : x^* R x, \text{ where } R = f(\mathbf{R})\}. \quad (\text{A})$$

However, this formal parallelism cannot but be overshadowed by the conspicuous contrast between the *exogeneity* of the Bergson–Samuelson social welfare function and the *endogeneity* of the Arrow social welfare function.

In the second place, it is useful to remind ourselves that the title of Arrow's magnum opus is *Social Choice and Individual Values*, whereas the analytical core of his impossibility theorem is the general non-existence of a procedure for aggregating each profile of *individual values* into a *social value*. Furthermore, the concept of *social choice* and that of *social value* are linked through his assumption of *collective rationality*.¹⁷ This leads us to one of the widely held and long-sustained criticisms on Arrow's social choice theory to the following effect: “Social choice theory asks for too much out of the [social choice] process in that it asks for an entire *ordering* of the various social states The original question asked only for a ‘good’ allocation; there was no requirement to rank all allocations. The fairness criterion in fact limits itself to answering the original question. It is limited in that it gives no indication of the merits of two non-fair allocations, but by restricting itself in this way it allows for a reasonable solution to the original problem [Hal Varian (1974, p. 65)].”¹⁸

In the context of the first scenario of normative economics, viz. that of *grandeur social welfare maximization*, this criticism may have an intuitive appeal, but in the context of the second scenario, viz. that of *piecemeal social welfare improvement*, the exclusive concentration on the socially first-best choice in full neglect of sub-optimal alternatives for choice makes normative economics incapable of helping the practitioner of normative economics in his/her attempt to design and implement the “instruments for the bettering of human life.” In other words, Arrow's social welfare

¹⁷ Recollect that Arrow's concept of collective rationality is such that the social choice function is *rationalizable* through the optimization of an underlying social value ordering over the set of feasible social states.

¹⁸ Serge-Christophe Kolm (1996, p. 439) made the following observation to a similar effect: “The requirement of a social ordering is indeed problematic at first sight: Why would we want to know the 193th best alternative? Only the first best is required for the choice.”

function has an acute relevance in the context of the second scenario of normative economics in search of piecemeal social welfare improvement.

According to Sen (2009, p. 17), Arrow's social choice theory, which can be traced back to the pioneering work by Marquis de Condorcet in the eighteenth century, has been explored in recent decades in the form of methods and instruments of pursuing comparative assessments of social states on the basis of individual value orderings. Our understanding on the role played by Arrow's social choice theory in our context of normative economics is in conformity with Sen's understanding thereof in his context of the ideas of justice.

4 Escape from the social choice impossibility impasse

Once the general impossibility theorem is established, the next order of business is to grope for the possible escape routes from the impossibility impasse. Arrow tracked down the culprit of the Condorcet paradox so as to find a required escape route via the weakening of the axiom of unrestricted domain subject to his underlying assumption of collective rationality. His point of departure was the insight that "the possibility of social welfare judgments rests upon a similarity of attitudes toward social alternatives [Arrow (1951, p. 69)]." On reflection, the Condorcet paradox hinges squarely on the extreme dissimilarity of attitudes among individuals toward social alternatives.¹⁹ To circumvent the paradox, Black (1948; 1958, Chapter 4) and Arrow (1951, Chapter VII) identified the restriction on the profile of individual value orderings to be called the assumption of *single-peaked preferences*, and confirmed that, "if the Assumption of Single-Peaked Preferences is satisfied by the orderings of the various individuals, assumed odd in number, we could ... find the social choice by considering each pair of alternatives, having individuals vote between them, and then selecting that alternative which has a majority over every other in pairwise comparisons [Arrow (1951, p. 80)]." The Black–Arrow initiative, together with the subsequent contributions by Ken-Ichi Inada, Amartya Sen, Prasanta Pattanaik, and many others led to the elaborate edifice of the theory of majority decision rule under the assumption of similarity of individual attitudes toward social alternatives. Without any doubt, the Black–Arrow assumption of single-peaked preferences brought about one of the most prolific research areas in social choice theory.

Nevertheless, I am seriously concerned with the fact that human beings are thoroughly diverse, not only in their external circumstances, but also in their personal characteristics. According to Sen (1992, p. 1),

¹⁹ The simplest possible instance of the voting paradox consists of three candidates x , y , and z , and three voters 1, 2, and 3, where these voters have the following diametrically heterogeneous preference orderings on the set $\{x, y, z\}$ of candidates:

$$x \succ_1 y \succ_1 z, y \succ_2 z \succ_2 x, z \succ_3 x \succ_3 y \quad (\text{VP})$$

If we apply the simple majority decision rule to this profile $\succ = (\succ_1, \succ_2, \succ_3)$, we obtain the cyclic social preference $x \succ y \succ z \succ x$. It is clear that this cycle is caused by the extreme heterogeneity of voters' preferences (VP).

The powerful rhetoric of 'equality of man' often tends to deflect attention from [these through diversities of human beings]. Even though such rhetoric (e.g., 'all men are born equal') is typically taken to be part and parcel of egalitarianism, the effect of ignoring the interpersonal variations can, in fact, be deeply inegalitarian, in hiding the fact that equal consideration for all may demand very unequal treatment in favour of the disadvantaged.

The question to be asked, then, is as follows: Is there any way of identifying the channel through which the possibility of social welfare judgments rests upon the *diversity*, rather than the *similarity*, of individual attitudes toward social alternatives? An attempt was made to sound the depth of this possibility in a series of papers by Kotaro Suzumura and Yongsheng Xu (2001, 2003, 2004).

As an auxiliary step, let us introduce an extended preference ordering \succ^* on the pairs of social outcomes and social opportunity sets such that choosing an outcome x from an opportunity set S is at least as good as choosing an outcome y from an opportunity set T . This framework enables us to identify *consequentialism* and *non-consequentialism*: An individual is a *consequentialist* if $(x, S) \sim^* (x, T)$ holds true for all (x, S) and (x, T) , and an individual is a *non-consequentialist* if $(x, S) \succ^* (y, T)$ holds for all x and y if and only if $\#S > \#T$, where \sim^* and \succ^* stand, respectively, for the indifference relation and the strict preference relation corresponding to \succ^* .²⁰

For each profile $\succ^* = (\succ_1^*, \dots, \succ_i^*, \dots, \succ_n^*)$ of extended individual preference orderings, an *extended social welfare function* f^* maps the profile \succ^* into the social extended preference ordering: $\succ^* = f^*(\succ^*)$. On the basis of the work by Suzumura and Xu (2001, 2003, 2004), we may assert that:

- (a) If all individuals are consequentialists, there exists no extended social welfare function satisfying *unrestricted domain*, *strong Pareto*, *independence of irrelevant alternatives*, and *non-dictatorship*.²¹
- (b) If there exists at least one non-consequentialist along with consequentialists, there exists an extended social welfare function f^* that satisfies *unrestricted domain*, *strong Pareto*, *independence of irrelevant alternatives*, and *non-dictatorship*.

Thus, even a mild diversity of individual attitudes towards outcomes vis-à-vis opportunities may turn the Arrovian impossibility theorem (a) into the possibility theorem (b), where the axioms of strong Pareto, independence of irrelevant alternatives, and non-dictatorship are kept intact throughout the whole exercise. Lest we should be too pleased by this contrast between the impossibility theorem (a) and the possibility theorem (b), let us remind ourselves that the Arrovian axioms of *unrestricted domain*, *strong Pareto*, *independence of irrelevant alternatives*, and *non-dictatorship* are meant to be *necessary*, but not *sufficient*, conditions for social aggregation rules to be satisfactory. The modest purpose of our argument is to call the reader's attention to the fact

²⁰ This definition of non-consequentialism presupposes that the value of an opportunity set is entirely captured by its size measured by the number of alternatives within the set, which is admittedly restrictive. It requires elaborations in one way or the other in the future.

²¹ These four axioms for the extended social welfare function are counterparts to the four axioms that are required of the Arrow social welfare function. It should be obvious how these axioms for the extended social welfare function can be defined by modifying the original Arrow axioms for his social welfare function.

that dissimilarity of individual attitudes may serve as a potential key for the workability of social aggregation mechanisms.

5 Conceptual extensions of the framework of social choice theory

It may not be out of place to observe that my attempt in the previous section to shed a spotlight on the potential role of human diversity made use of an extended conceptual framework in the sense that not only individual preferences, but also social preferences are defined, not on the conventional space of consequences, but on the extended space of consequences-cum-opportunities. On reflection, Arrow also identified several directions for extending the conceptual framework of social choice theory. It includes the *extended sympathy approach*, which may be traced back to Adam Smith's (1759/2009) *Theory of Moral Sentiments*, and the *treatment of decision-making procedures as carriers of value*. A few brief comments on these extended conceptual frameworks may be in order.

Arrow's first conceptual extension is based on the counterfactual exchange of circumstances among individuals as a tool for extending the informational basis of social welfare judgments. Interpersonal comparisons of the extended sympathy type can be motivated as follows. In Arrow's (1963, pp. 114–115) own words,

People seem prepared to make comparisons of the form: State x is better (or worse) for me than y is for you. ... The ordinalist would ask what possible meaning the comparison could have to anyone; a comparison should represent at least a conceivable choice among alternative actions. Interpersonal comparisons of the extended sympathy type can be put in operational form; the judgment takes the form: It is better (in my judgment) to be myself in state x than to be you in state y .

This conceptual extension enables social choice theorists to build an interface between the theory of justice of John Rawls (1971), and social choice theory of Arrow (1951/1963/2012) and Sen (1970/2017).²² Likewise, thought experiments of the form that I gain more in going from state x to state y than you lose in going from x to y seem to permit us to connect Arrow's social choice theory to the Benthamite utilitarianism. It should also be pointed out that the extended sympathy approach seems to serve as a bridge between the Hicks–Tinbergen–Foley theory of *no-envy fairness*, on the one hand, and the Arrow–Sen social choice theory, on the other hand, thereby enriching both normative theories substantially.²³

Arrow's second conceptual extension is the combined treatment of decision-making procedures and consequential outcomes. According to Arrow (1951, pp. 89–90),

[N]o attempt has been made to find guidance by considering the components of the vector which defines the social state. One especially interesting analysis of this sort considers that, among the variables which taken together define the

²² See, among others, Sen (1977) who provided an axiomatic characterization of the Rawlsian principle of leximin justice within this extended framework of social choice theory.

²³ An attempt was made in Suzumura (1980) to explore this possibility.

social state, one is the very process by which the society makes its choice. This is especially important if the mechanism of choice itself has a value to the individuals in the society. ... If the decision process is interpreted broadly to include the whole socio-psychological climate in which social decisions are made, the reality and importance of such preferences, as opposed to preferences about the distributions of goods, are obvious.

In what follows, the decision-making procedures are described by means of game forms. Let X be the set of all *conventionally defined social alternatives*. A *game form* in the society $N := \{1, \dots, i, \dots, n\}$ is a triplet $\theta := (N; \Sigma = \{\Sigma_1, \dots, \Sigma_i, \dots, \Sigma_n\}; g)$, where Σ_i is the set of all *admissible strategies* for $i \in N$, and g is the *outcome function* that associates an outcome $g(\sigma) \in X$ with each strategy profile $\sigma = (\sigma_1, \dots, \sigma_i, \dots, \sigma_n) \in \times_{i \in N} \Sigma_i$, one strategy for each individual. Θ is the set of all feasible game forms in the society N . A pair (x, θ) , where $x \in X$ and $\theta \in \Theta$, stands for an extended social alternative, where a conventionally defined social alternative x is attained through the medium of a game form θ . An extended individual preference ordering ER_i is defined for all extended social alternatives $(x, \theta), (y, \omega) \in X \times \Theta$ by $(x, \theta)ER_i (y, \omega)$ holds if and only if, according to i 's judgments, attaining x through θ is at least as good as attaining y through ω .

It may be thought that the proposed concept of extended social alternatives is nothing other than a *will-o'-the-wisp*, as we can always redefine the concept of social alternatives by treating the pairs such as (x, θ) and (y, μ) as refined primitives of social choice theory. To defend the extended framework from this often-made criticism, let $S \subseteq X$ be the set of conventionally defined *feasible* alternatives. Recollect that, in the conventional social choice theory, the feasibility of a conventionally defined social alternative x is defined by $x \in S$, whereas the feasibility of an extended, viz. refined, social alternative (x, θ) is defined by $x \in g(\mathcal{E}(\theta, \text{pro}_X(\mathbf{ER})))$, where $\text{pro}_X(\mathbf{ER})$ stands for the projection of the profile $\mathbf{ER} := (ER_1, \dots, ER_i, \dots, ER_n)$ on the set X of all conventionally defined social alternatives, and $\mathcal{E}(\theta, \text{pro}_X(\mathbf{ER}))$ is the set of all equilibrium outcomes of the game $(\theta, \text{pro}_X(\mathbf{ER}))$. Thus, just to see whether (x, θ) is feasible, it is necessary to know the profile $\text{pro}_X(\mathbf{ER})$ and the equilibrium concept \mathcal{E} prevailing in the society. This is why we think that it is impossible to reduce the extended framework of social choice theory simply through the redefinition of the primitive concept of social choice theory.²⁴ No free lunch is available in this context, as is the case almost everywhere else.

These extended frameworks of social choice theory suggested by Arrow are awaiting further explorations in future research. Instead of indulging in premature speculations on the possible directions to try in the future research along these lines, however, let us now turn to some gleaning activities on the well-trodden ground in the traditional social choice theory. In this context, two problems immediately suggest themselves.

The first problem goes back to Arrow's requirement of *collective rationality*, which means that the *social choice function should be rationalizable through the optimization of an underlying social value ordering*. Several methods of weakening his original

²⁴ For more details of this extended social choice framework in the specific context of game-form rights, those who are interested are cordially referred to Prasanta Pattanaik and Kotaro Suzumura (1996).

requirement of collective rationality have been proposed in the literature, preserving *completeness* of the social preference relation. The best-known example of such attempts is due to Sen (1969), who introduced the concept of *quasi-transitivity* of social preference relations. Note that, if the *weak social preference* relation R is transitive, the associated *strict social preference* relation $P(R)$ and *social indifference* relation $I(R)$ are also transitive. R is said to be *quasi-transitive* if and only if $P(R)$ is transitive. To the extent that the quasi-transitivity of R *does not* require the transitivity of $I(R)$, Sen's quasi-transitivity of the weak social preference relation R is a logical weakening of Arrow's original requirement of collective rationality.²⁵ By means of this *prima facie* mild concession on the extent of collective rationality from full transitive rationality to quasi-transitive rationality, Sen could show that there exists a social aggregation function satisfying quasi-transitive rationality and all the other Arrovian axioms of unrestricted domain, Pareto, independence of irrelevant alternatives, and non-dictatorship.²⁶ As Andreu Mas-Colell and Hugo Sonnenschein (1972) have shown, however, this escape route eventually brings us back to the blind alley in the Arrovian labyrinth if only we are ready to strengthen Arrow's axiom other than that of collective rationality slightly. Nevertheless, there is an alternative coherence concept due to Suzumura (1976) that also weakens Arrow's transitivity axiom, which Walter Bossert (2008) christened *Suzumura consistency*. It simply rules out all cases of incoherent social preference cycles, not only of strict preference cycles, but also of all cycles with at least one instance of strict preference.²⁷ It is easy to confirm that the transitivity of a weak preference relation R implies the Suzumura consistency, but not necessarily vice versa, and if R satisfies the Suzumura consistency *and* completeness, it reduces to an ordering.²⁸ Compared with Sen's concept of quasi-transitivity, the logical fate of the Suzumura consistency in the arena of Arrow's impossibility impasse seems to be much brighter. Indeed, Bossert and Suzumura (2008) have succeeded in an axiomatic characterization of the Suzumura consistent and otherwise Arrovian social choice rules. It is our belief that this line of research is not at all a complete blind alley, and very much alive.

²⁵ Recollect that violations of transitive indifference are quite likely to surface in practical choice situations. Indeed, Duncan Luce's (1956) well-known coffee-sugar example provides a plausible argument against the transitivity of indifference relation: the inability of a decision-maker to perceive "small" differences in alternatives is bound to lead to intransitive overall preferences. Thus, full transitivity is often too strong to impose in the context of individual as well as social choice.

²⁶ Sen's *Pareto extension rule* f^E is a preference aggregation rule that aggregates each profile $\mathbf{R} = (R_1, R_2, \dots, R_n)$ of individual preference orderings into a social weak preference relation R^e such that, for each pair of social alternatives $x, y \in X$, $(x, y) \in R^e$ if and only if $(y, x) \notin P(R^p)$, where $R^p = \bigcap_{i \in N} R_i$. It is easy to verify that R^e , thus defined, is quasi-transitive, and f^E satisfies the axioms of unrestricted domain, Pareto principle, independence of irrelevant alternatives, and non-dictatorship.

²⁷ To be precise, a preference relation R on X is *Suzumura-consistent* if and only if there exists no S -cycle of any finite order, where a t -tuple of alternatives (x^1, x^2, \dots, x^t) is an S -cycle of order t ($3 \leq t < \infty$) if and only if $(x^1, x^2) \in P(R)$, $(x^h, x^{h+1}) \in R$ ($h = 2, \dots, t - 1$) and $(x^t, x^1) \in R$. See Suzumura (1983, 2009), Bossert and Suzumura (2010, Chapter 2), and Sen (2018) for several crucial properties of the concept of Suzumura-consistency.

²⁸ A crucial property of Suzumura-consistency is *Suzumura's ordering extension theorem* which asserts that a binary relation R on the universal set X of alternatives has an ordering extension R^* such that $R \subseteq R^*$ and $P(R) \subseteq P(R^*)$ if and only if R satisfies Suzumura-consistency. In many contexts of individual and social choice, Suzumura's ordering extension theorem has proved to be of crucial importance.

The origin of the second problem goes all the way back to the Benthamite “greatest happiness of the greatest number” principle, which is widely thought to be tantamount to the maximization of the social sum-total of individual utilities. Arrow (1951) eagerly tried to purge all the vestigial traces of cardinally measurable and interpersonally comparable individual utilities from welfare economics and social choice theory. It is interesting to recollect that Bentham himself observed that

‘Tis in vain to talk of adding quantities which after the addition will continue as they were before, one man’s happiness will never be another man’s happiness: a gain to one man is no gain to another: you might as well pretend to add 20 apples to 20 pears.²⁹

Taking Arrow as well as Bentham seriously, Bossert and Suzumura (2016, 2017) explored an alternative approach to the Benthamite dictum within the informational framework of ordinally measurable and interpersonally non-comparable individual utilities. Their attempt resulted in the alternative interpretation of the greatest happiness principle by means of the *plurality rule*. A similar analysis is applicable to the counterpart of the maximin principle of justice in the welfaristic interpretation of John Rawls’s (1971) theory of justice. This area is also very much alive and awaiting further explorations in the future.

6 Some personal reminiscences

Let me conclude with a few personal reminiscences of this great scholar with a cool head and warm heart, who left unforgettable impacts and lasting influences on my whole academic life.

My first encounter with Kenneth Arrow was in his office at Harvard University in 1978, which was exactly 10 years after my lonely struggle with *Social Choice and Individual Values*. That struggle took place during my third-year undergraduate days at my alma mater in Tokyo. I was enchanted by this great classic that taught me to think rationally and analytically about functions and values of democratic decision-making procedures. Although the level of technical sophistication of this book was so high that I could not fully understand all the subtle details of Arrow’s reasoning in my first attempt, I was overwhelmed by the grandiose vista that suddenly unfolded in front of me. This experience of intellectual intoxication almost decided my academic aspiration in one stroke.

When I visited Arrow at his Harvard office, I could not but be nervous. From the beginning, however, he treated me as if we were close year-long acquaintances over the years. To my surprise, he knew my small papers that I published on revealed preference theory, welfare economics, and social choice theory. I thought it was my great fortune that Arrow was so friendly and treated me as if we were long-time colleagues and friends in the social choice circle. I think it is an even greater fortune of mine that my subsequent experiences with him gave me no reason to discount even an iota of his benignity, modesty, and sincerity.

²⁹ This quote is from Wesley Mitchell (1937), which was cited by Arrow in *Social Choice and Individual Values*, p. 11.

Arrow also gave me several opportunities to collaborate with him. The first opportunity was that he invited me to the famous Summer School at the Hebrew University of Jerusalem in 1993 when he chose Social Choice Theory as the central focus of the summer school of the year. I was among the six invited lecturers along with Kenneth Arrow, Mathias Dewatripont, Leonid Hurwicz, Eric Maskin, and Bezalel Peleg. I gave six lectures on the Theory of Individual Rights and Social Welfare. I could also enjoy the lectures delivered by the other five speakers, along with a group of brilliant students who came from almost every part of the world. Most of them subsequently became excellent researchers themselves. All lecturers stayed in the historic guesthouse overlooking East Jerusalem, and my wife and I were enchanted by a grand view of Jerusalem under the setting sun. Having breakfast with the other lecturers, Kenneth Arrow and Leonid Hurwicz in particular, gave me most enjoyable moments, and I could feel how deep a friendship Arrow and Hurwicz had nourished over the years. The more I came to know them, the more I came to like and respect them. This was the first time I could enjoy my collaboration with Arrow, which was followed by our joint effort, together with Amartya Sen, for the 1994 IEA Roundtable Meeting on Social Choice Theory held at the old castle of Hernstein in Austria, and the *Handbook of Social Choice and Welfare* (2002; 2011). Both occasions were uphill battles for me, but I was overcompensated by my personal experiences with Arrow and Sen, from which I could learn how much weight we should attach to the importance of being honest, industrious, and responsible.

After the Jerusalem Summer School, there was a coach tour arranged for the invited lecturers that covered such places of historic importance as Qumran, Jericho, and Masada, which led us to the Dead Sea. On our way to get there, Hurwicz told me that he had a youthful dream of floating on the Dead Sea while reading a newspaper. To prepare for the fulfillment of this dream, Hurwicz brought a daily newspaper from Minneapolis. His attempt was successful until he floated on the Dead Sea. However, his round body betrayed him by rolling on the Dead Sea water, and his newspaper became soaking wet completely. Everybody laughed, and the loud laughter of Arrow and Hurwicz still echoes in my mind.

The third reminiscence is related to my joint effort with Kaushik Basu and Prasanta Pattanaik for the *Festschrift* in Honor of Amartya Sen.³⁰ Arrow gladly accepted our invitation to contribute to the *Festschrift*, but his paper could not make the first deadline by a margin of several weeks. In that year, Arrow was scheduled to deliver an important lecture (the Richard T. Ely lecture) at the Annual Meeting of the American Economic Association.³¹ All editors fully understood that he had to give priority to this significant commitment. When I entered the lecture theater to attend this important occasion just a few minutes early, Arrow was already standing on the platform. As soon as he noticed that I was in the room, he jumped down from the platform and rushed towards me to apologize for the delay. I was totally taken aback and said in reply that he had no reason to apologize. He was profuse in apology and blamed himself for having broken his promise. My respect for him had to be brought up to an even higher plateau for his

³⁰ This *Festschrift* was subsequently published by Basu et al. (1995). Arrow's contribution is Chapter 1 of this *Festschrift*, which is entitled "A Note on Freedom and Flexibility."

³¹ This lecture was subsequently published in *American Economic Review: Papers and Proceedings* as Arrow (1994).

sincere attitude at the time when he was about to deliver one of the most important lectures in the life of an economist. This is the man whom we cannot but respect and aspire to follow in his footsteps.

It is my belief that there are many scholars in the social choice circle who treasure innumerable, warm and unforgettable personal reminiscences of Kenneth J. Arrow in a similar vein. May his great soul rest in peace.

Acknowledgements I delivered a speech at the *Academic Tribute to Kenneth Arrow* held at the Frances C. Arrillaga Alumni Center of Stanford University on October 9, 2017, which was organized by Matthew Jackson, Alvin Roth, and John Shoven. This paper partly capitalizes on my Stanford speech in memory of Kenneth Arrow. I am most grateful to Eric Maskin, Amartya Sen, Walter Bossert, and Marc Fleurbaey for their helpful comments that enabled me to improve it. Needless to say, I am solely responsible for any defect and opaqueness that may remain. Financial support from a Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan for the Project on *Normative Economics with Extended Informational Bases, and the Reexamination of Its Doctrinal History* (Grant number 16H 03599) is gratefully acknowledged.

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