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ORIGINAL PAPER

Consistent updating of social welfare functions

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Abstract It is one of the central doctrines for 'rational' decision making that an agent should be forward-looking and not be bound by bygones. We argue that this is not an appealing principle for collective decision making, and that bygones have necessary and substantive roles. We consider an explicitly dynamic process of social welfare orderings, and propose a dynamic constraint which is acceptable even after rejecting the principle that one should be forward-looking. It is a conjunction of two assertions: (i) the process must be dynamically consistent, which means an ex ante welfare judgment must be respected by the ex post ones and there should be no contradiction between them; (ii) the structure of a welfare judgment should be recurrent under consistent updating, in the sense that if a postulate is satisfied by an ex ante welfare judgment, then it is also satisfied by any of the ex post ones that follow this ex ante judgment. Based on this standpoint, we present a set of axioms for social welfare orderings which are recurrent under consistent updating, and characterize a set of social welfare functions which are closed under updating. With such a class of social welfare functions, we characterize the roles that can be played in the updating stage by the past and things known not to have occurred.

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1 Introduction

1.1 Re-examining the principle of being forward-looking

This paper is concerned with the following questions. Should a collective decision criterion have the same form of rationality as those of individuals do? If not, what type of dynamic constraint should be imposed? What should be the decision-theoretic content of collective decision criteria?

It is one of the central doctrines of 'rational' decision making that an agent should be forward-looking and not be bound by bygones. This presumes that the past and things already known not to have occurred are 'sunk,' and have no role to play except purely informational ones to facilitate inference about uncertain or strategic worlds. Throughout this paper, we will call this principle *the principle of being forward-looking*.[1]

Most economic analyses presume the principle of being forward-looking for collective decision criteria as well. To evaluate streams of social outcomes, they assume that the social welfare function satisfies the discounted utility form which is standard for individual decision making: a stream of social outcomes $(c_0, c_1, \ldots)$, where c_τ denotes the social outcome at period τ, is evaluated in the form

$$\sum_{\tau=0}^{\infty} \beta^{\tau} u(c_\tau),$$

in which β is the discount factor and the function u evaluates the social outcomes at each period independently of history and time (see Koopmans 1960). Thus, after the passage of $t-1$ periods of time, the ex post social welfare function induced by the ex ante one evaluates a stream starting at period t, denoted by $(c_t, c_{t+1}, \ldots)$, in the form

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau),$$

which is independent of $(c_0, \ldots, c_{t-1})$, the history of social outcomes up to $t-1$, and independent even of when 'now' is.

Likewise, to evaluate uncertain social outcomes, these analyses assume that the social welfare function satisfies the expected utility theory which is standard for individual decision making: an uncertain social outcome f given as a mapping from the set of possible states of the world S into the set of social outcomes C is evaluated in the form

$$\int_S u(f(s))p(ds),$$

in which p is the prior belief adopted as a 'consensus' and the function u evaluates social outcomes independently of which states they are realized at. Thus, after learning

[1] In the literature of choice under uncertainty this term is called consequentialism (see for example Hammond 1996; Machina 1989), but since it is used for different meanings in different fields such as ethics, we adopt the current terminology.

that an event $E \subset S$ has occurred, or, equivalently, that $S \backslash E$ turned out not to have occurred, the ex post social welfare function induced by the ex ante one takes the form of the conditional expectation

$$\int_E u(f(s))p(ds|E),$$

where $p(\cdot|E)$ denotes the posterior conditional on E. Note that this ex post welfare judgment conditional on E is independent of what outcome would have been obtained if event E had not occurred, and when there is no more uncertainty, it even does not matter which event has been realized.

We argue, however, that the principle of being forward-looking is not an appealing principle for collective decision making, even if we accept it for individuals' evaluations of decisions. First, there are a handful of impossibility results. Zuber (2010) and Jackson and Yariv (2014) show that aggregate preferences over deterministic streams of outcomes can satisfy the Pareto principle, dynamic consistency, and independence of histories and time, which are assumed in the discounted utility form above, only if individual and aggregate preferences are stationary additive, i.e., the discount factors are the same for all individuals, and the aggregation rule is additive.

This means that under the principle of being forward-looking, any dynamically consistent social welfare function excludes (i) intertemporal trading due to differences in time perspectives, and (ii) equity concerns even when individuals have the same time perspective. To understand the impossibility of (i), imagine two individuals, one of whom is more patient and the other is less patient. Then the Pareto principle implies that the first one should get more consumption now and less in the future, and the opposite for the other. This, however, cannot be compatible with the social evaluation of outcomes per period being time invariant. To understand the impossibility of (ii), imagine that the society should equalize the lifetime utilities of the individuals. But once somebody is happier and somebody else is unhappier in some period, we cannot apply the same rule to sequences of social outcomes starting in the next period.

In the setting of uncertainty, Mongin (1995) shows that an aggregate preference over social outcomes (considered as random variables) satisfies the ex ante Pareto principle and the subjective expected utility theory only when individuals have the same beliefs and the aggregation is additive. Even if individuals are assumed to have the same beliefs, in earlier papers, Hammond (1996) shows that a social choice rule satisfies the Pareto principle, dynamic consistency, and is forward-looking only if it follows additive aggregation. More strongly, Zuber (2010) and Mongin and Pivato (2015) show that an aggregate preference can satisfy the Pareto principle, dynamic consistency, and the state-independence of ex post aggregate preferences, which are assumed in the subjective expected utility form above, only if individual and aggregate preferences have the subjective expected utility form, the individuals all have the same beliefs, and the aggregation rule is additive. These strengthen the result of Harsanyi (1955) that individual and aggregate preferences have the expected utility forms and satisfy the Pareto principle only if the aggregation is additive.

This means that under the principle of being forward-looking, any dynamically consistent social welfare function excludes (i) trading across states due to differences in beliefs, and (ii) equity concerns as considered in Diamond (1967), that the society should give equal chances to everybody for the sake of fairness, even when individuals have the same beliefs. To understand the impossibility of (i), imagine two individuals, one of whom believes it will rain tomorrow and the other believes it will be sunny. Then the Pareto principle implies that the first one should get more consumption if it rains and less if sunny, and the opposite for the other. This, however, cannot be compatible with the social evaluation of outcomes per period being state-independent, as was pointed out by Chambers and Hayashi (2009). To understand the impossibility of (ii), notice that a strict preference for randomization leads to a violation of the independence axiom due to von Neumann and Morgenstern, and also that under the principle of being forward-looking, the independence axiom is implied by dynamic consistency: this classic point has been extensively discussed by Machina (1989).

Not only that, we argue through examples below that the past and things already known not to have occurred have intrinsic roles to play. Let us first discuss a classical fable attributed to Aesop and one of its variations.

Example 1 ‘The Grasshopper and the Ant’
(This is also known as ‘The Cicada and the Ant’.) The ancient version, Babrius 140, in Perry (1965) runs:

> An ant in the winter-time was dragging out of his hole some grain which he had stored up in the summer, in order to air it. A cicada, dying of starvation, begged him to give him some of his food, to keep him alive. “What were you doing last summer?” asked the ant. “I was not loafing,” said the cicada, “I was busy singing all the time.” The ant laughed and barred up his grain, saying: “Dance in the winter, since you piped during the summer.”

A variation due to Disney (from the retelling by Brown 1993) is:

> .....When the ants came to the door, they found him there, half frozen. And ten of the kind and busy ants came out and carried the poor grasshopper into their house. They gave him hot corn soup. And they hurried about, making him warm. Then the Queen of All the Ants came to him. And the grasshopper was afraid, and he begged of her: “Oh, Madam Queen, wisest of ants, please, please give me another chance.”
> The Queen of All the Ants looked at the poor, thin frozen grasshopper as he lay shivering there. Then she spoke these words: “With ants, just those who work may stay. So take your fiddle – and PLAY!”
> The grasshopper was so happy that his foot began beating out the time in the old way, and he took up his fiddle and sang.....

To translate the above stories into a more formal setting, consider an economy with two periods, summer and winter, in which Ant and Grasshopper each initially own two units of the consumption good and has access to a storage technology. Assume that Ant saves one unit and Grasshopper consumes the entire two units during the summer. Thus in winter there is just one unit left, which was saved by Ant. If we accept the ending of the classic version, it reveals a judgment which ranks giving the entire one unit to Ant over splitting the remaining one unit. If we accept the ending of the modern variation, it reveals a judgment which prescribes splitting the remaining unit, let's say equally.[2,3]

Each of the judgments amounts to a problem. The second one leads to an obvious conflict between ex post justice and ex ante justice. The idea of ex post justice is quite prevalent, and it is indeed regarded as a basis for redistribution in many real life situations. In this example, it prescribes splitting the remaining unit equally. However, under the natural requirement of dynamic consistency, the ex ante welfare criterion must prescribe that Ant should receive a two-period consumption profile $(1, 0.5)$ and Grasshopper should receive $(2, 0.5)$, which is unfair in any sense from the ex ante viewpoint.

The first judgment is, surprisingly, not explained by any of the accepted arguments in welfare economics. We should notice that the resource allocation problem they face in winter is an ex post problem, which is different from the ex ante one. The problem here is not about sharing across periods, but about whether we should take some resource from Ant and move it to Grasshopper. The standard welfare economics argument, based on the the principle of being forward-looking, is silent about this.

Let us further illustrate our points with a different situation: risk and uncertainty.

Example 2 'Machina's Mom'
From Machina (1989):

> Mom has a single indivisible item – a 'treat' – which she can give to either daughter Abigail or son Benjamin. Assume that she is indifferent between Abigail getting the treat and Benjamin getting the treat, and strongly prefers either of these outcomes to the case where neither child gets it. However, in a violation of the precepts of expected utility theory, Mom strictly prefers a coin flip over either of these sure outcomes, and in particular, strictly prefers 1/2: 1/2 to any other pair of probabilities. This random allocation procedure would be straightforward, except that Benjie, who cut his teeth on Raiffa's classic Decision Analysis, behaves as follows:
>
> Before the coin is flipped, he requests a confirmation from Mom that, yes, she does strictly prefer a 50:50 lottery over giving the treat to Abigail. He gets her to put this in writing. Had he won the flip, he would have claimed the treat. As it turns out, he loses the flip. But as Mom is about to give the treat to Abigail, he reminds Mom of her preference for flipping a coin over giving it to Abigail (producing her signed statement), and demands that she flip again.

[2] There may be a different interpretation of the modern variation that Ant is willing to allow redistribution, even if not ethically prescribed or driven to do so, because of the hope for reciprocal compensation in the future, which is a story about reputation effects. Our focus is rather on a purely ethical argument on the ex post welfare judgment in the absence of reputation effects.

[3] One might give another interpretation, that the main problem here is rather whether Grasshopper is responsible for its lack or wrongness of its foresight about the food conditions in winter. This will be handled as a separate issue in a later section on updating welfare judgments under uncertainty with non-common prior beliefs.

> What would your Mom do if you tried to pull a stunt like this? She would undoubtedly say "You had your chance!" and refuse to flip the coin again. This is precisely what Mom does.

Machina continues, 'By replying "You had your chance", Mom is reminding Benjamin of the existence of the snipped-off branch (the original 1/2 probability of B) and that her preferences are not separable, so the fact that nature could have gone down that branch still matters. Mom is rejecting the property of consequentialism—and, in my opinion, rightly so.' Here the term 'consequentialism' is what the principle of being forward-looking is called in the literature of choice under uncertainty, which says, in particular, that the decision should be independent of events or outcomes which turned out not to have occurred (see Hammond 1996, for example).

Again, Mom's claim amounts to a problem, as does Benjamin's claim as well. If we accept Benjamin's claim, in so far as we want our welfare judgment to be dynamically consistent, this implies that the ex ante judgment must support Abigail's winning the item with probability $1/2 \times 1/2 = 1/4$ and Benjamin's winning with probability $1/2 + 1/2 \times 1/2 = 3/4$, which is unfair in any sense from the ex ante viewpoint. If we accept Mom's claim, rejecting consequentialism, then we are required to propose an alternative decision-theoretic restriction, since there is no welfare economic theory stepping outside the realm of the principle of being forward-looking, except that Epstein and Segal (1992) address issues arising in non-consequentialist updating of quadratic social welfare functions.

1.2 An alternative dynamic restriction

We have argued above that the principle of being forward-looking is not an appealing principle for collective decision making, even if it is acceptable for individuals' evaluations of decisions. This leads us to ask what is the alternative dynamic restriction that should be accepted even after rejecting the principle of being forward-looking.

It should be noted that we are not looking for a particular ethical standpoint, just as the principle of being forward-looking is not a particular ethical standpoint. What we are looking for is a form of rationality which should be satisfied by collective decisions.

It is presented in the form a *meta axiom*, or axiom-selection device in other words, which a reasonable axiom on social decisions is required to satisfy. This contrasts to that the principle of being forward-looking reduces to the form of a particular axiom, such as independence of history and time index.

To motivate our question, let us come back to the Ant and the Grasshopper. One may think of simply applying the welfare criterion which was accepted ex ante. Under the rationality assumption that both Ant and Grasshopper have correct foresight of the consequences of their (non-)saving, one may argue that Ant preferred a two-period consumption profile $(1, 1)$ to $(2, 0)$ ex ante and the opposite for Grasshopper, and deduce that since they have unanimously agreed to a plan, Ant's receiving $(1, 1)$ and Grasshopper's receiving $(2, 0)$, it is unreasonable to redistribute consumption in the winter.

However, if we support the execution of a plan merely on the ground that it was accepted in the past as being desirable, where is there any thinking in the present? This is essentially to say,

Grasshopper, you should die. Why? Because we decided that before.

Can't we say anything other than that? Anybody could say this just by mechanically following the projection of the ex ante decision. There is no autonomy. We believe that ex post decision making must be a deliberate recalibration, which should satisfy a certain level of rationality.

A practical argument may say that the execution of an ex ante agreed-to plan must be enforced (or self-enforcing) even if there is no intrinsic reason to do so ex post, because otherwise such a plan is null and will never be implemented. For example, consider why you should pay your debt. The practical argument is that you should pay your debt because otherwise you would be punished or nobody would finance you in the first place. Our argument is rather that the fulfillment of a plan (repayment of a debt) is not only practically necessary but also should be justifiable by the deliberate recalibration of welfare.[4]

To illustrate further, imagine any situation in which there is no prior contract or agreement explicitly made in the past, and one goes to court requesting compensation for a loss or lack of fairness due to somebody's wrongdoing; or imagine any situation in which a prior contract or agreement is violated by somebody and one goes to court requesting its fulfillment or compensation for the foregone benefit due to its violation. The judge then needs to make a decision as an independent thinker, and needs to track back the history in order to calculate the right compensation, by investigating what could have been done, what could have occurred, and what had been conceived as agreeable. This is a deliberate process of recalibration. What, then, is the judge maximizing?

Dynamic consistency is of course necessary, since otherwise the welfare judgment is not regarded as credible over time. We consider that it is not sufficient as a reasoning to derive an ex post welfare ranking, however. We would even say that dynamic consistency itself may be trivial unless there is a formal restriction imposed on the process of decision criteria. One can always have the process dynamically consistent trivially, by simply taking the projections of the initial decision criterion.

Consider the following example taken from Hayashi (unpublished). There is a cake and the problem is how much of it to eat over time, and the decision maker has the following process of dynamic decision criteria.

- Period 1: I would like to eat double today, and would like to eat equal amounts everyday from tomorrow on.
- Every period after Period 1: I would like to eat equal amounts everyday.

This process is dynamically consistent, but it is totally unnatural and absurd, both descriptively and normatively. When the decision maker says 'today is special' it is natural to expect that the same decision maker will say 'today is special' in the next

[4] This is different from an ethical viewpoint that fulfilling a promise or prior agreement is a virtue by itself regardless of its welfare consequences.

period again and in further future periods as well. In the literature of intertemporal choice, stationarity of preferences is usually referred to as a synonym of ‘time consistency’, and its violation is referred to as ‘time inconsistency’. It should be noted, however, that such an equivalence is an implication of the principle of being forward-looking.

To be precise, let $\succsim_{h^{t-1}}$ denote the ranking over streams of outcomes starting at period t, conditional on the history of the outcomes up to the previous period, denoted by h^{t-1}. Then, dynamic (time) consistency means that

$$\begin{aligned}&(c_t, c_{t+1}, c_{t+2}, \ldots) \succsim_{h^{t-1}} (c_t, c'_{t+1}, c'_{t+2}, \ldots)\\ &\iff (c_{t+1}, c_{t+2}, \ldots) \succsim_{(h^{t-1}, c_t)} (c'_{t+1}, c'_{t+2}, \ldots),\end{aligned}$$

where (h^{t-1}, c_t) denotes the updated history after having c_t. On the other hand, the stationarity of preference means that

$$(c_t, c_{t+1}, c_{t+2}, \ldots) \succsim (c_t, c'_{t+1}, c'_{t+2}, \ldots) \iff (c_{t+1}, c_{t+2}, \ldots) \succsim (c'_{t+1}, c'_{t+2}, \ldots)$$

where $\succsim$ is a ranking which is not indexed by history or time. Thus, what underlies the equivalence between the two is the conjunction of (i) independence of history, i.e.,

$$\succsim_{h^{t-1}} = \succsim_{\widetilde{h}^{t-1}} \equiv \succsim_t \quad \text{for all } h^{t-1} = \widetilde{h}^{t-1}$$

and (ii) independence of time, i.e.,

$$\succsim_t = \succsim_{t+1} \equiv \succsim \quad \text{for all } t.$$

The principle of being forward-looking, put into the context of intertemporal choice, is now understood as the conjunction of the independence of history with the independence of time. Since we are departing from the principle of being forward-looking, we have to give up these properties. However, unless we impose an alternative restriction, the dynamic consistency requirement would be trivial, and such an imposition is exactly what we are going to propose.

This point is important in the normative argument as well. It is unfair to give the special nature or status only to the current (self of the) decision maker. If there is a good reason to say ‘today is special’, there will naturally be a good reason to say ‘today is special’ in the next period again and in further future periods as well.

This point applies to decisions with chances as well. If the decision maker says ‘this is the last chance’, it is natural to expect that the same decision maker will say ‘this is the last chance’ next time, and again in future moments as well. Also, if there is a good reason to say ‘this is the last chance’, there will naturally be a good reason to say ‘this is the last chance’ next time, and again in further future moments as well. This is exactly the conundrum underlying the existing impossibility results.

Our proposal is that the structure of deliberate recalibration itself must be ‘dynamically consistent’. There is a distinction between the structure of a welfare judgment, i.e., the form of thinking, and a particular choice of welfare weights and priorities.

We hold that a normatively meaningful structure which appears in an ex ante welfare judgment should appear again in the ex post welfare judgment as its own property, and at the same time in a recurrent manner, while particular weights should depend on histories and vary over time. This is the dynamic constraint we propose, and will be presented in the form of a meta axiom. We call it *Recurrence under Consistent Updating*.

This meta axiom has been informally discussed by Epstein and Segal (1992), in order to show the merit of their quadratic social welfare function, which they axiomatically characterized in the setting of risk – the non-consequentialist update of a quadratic social welfare function is again quadratic. Also, Gumen and Savochkin (2013) have recently adopted this dynamic restriction (they call it dynamic stability) as the primary condition for characterizing a dynamic decision criterion under ambiguity, in particular, the variational model.

We did not have to be explicit about this meta axiom as long as we were staying with the principle of being forward-looking, since it was implicitly presumed there. It has to be made explicit now, since we are departing from the principle of being forward-looking. We believe that the alternative dynamic restriction still should be a decision-theoretically natural one. The recurrence condition is a natural weakening of the principle of being forward-looking in the sense that it still maintains the 'stationarity' property at a meta level, and avoids introducing an ad hoc exceptional feature of the initial decision node.

The meta axiom we impose is a requirement of formal rationality rather than substantive rationality. It is compatible with imposing substantive criteria that are particularly appealing in judgment at the initial stage. What should be recurrent under consistent updating is the structure of the judgment, not the particular choice of welfare weights. The axioms of horizontal equity, for example, are not recurrent under consistent updating and cannot be justified without introducing an ad hoc special role of the initial decision node. We can see this from the previous example: the criterion of equalizing lifetime utilities across individuals can be imposed only at the initial period, because in some period one may be happier and another may be unhappier and in order to be consistent over time we cannot apply the same criterion to sequences of social outcomes starting after such a history. However, such an axiom of horizontal equity is appealing when the initial node is agreed to be indeed 'special,' and our argument does not exclude it. We emphasize that being recurrent under consistent updating and being meaningful particularly at the initial stage are logically independent properties.

The fact that the constraint we propose is formal rather than substantive does not mean that it lacks relevance, since the relevance here is about the decision theoretic content of collective decision criteria rather than particular ethical standpoints. What form of rationality should be met by collective decisions is central and intrinsic particularly when time and risk/uncertainty are involved.

1.3 Outline

In this paper we consider a dynamic process of social welfare orderings explicitly. We then propose a dynamic constraint, which is a conjunction of two assertions: (i) the process must be dynamically consistent, which means an ex ante welfare judgment must be respected by ex post ones and there should be no contradiction between them; (ii) the structure of welfare judgment should be recurrent under consistent updating, in the sense that a postulate met by an ex ante welfare judgment is met by any of ex post ones as its own property.

Based on this standpoint, we present a set of axioms for social welfare orderings which are recurrent under consistent updating, and characterize a set of social welfare functions which are closed under updating. With such a class of social welfare functions, we characterize the roles for pasts and things known not to have occurred, which are played in the updating stage.

To give an overview of the results, consider that the objects of choice are streams of utility profiles. Ex-post decision at time t can depend on history of utility profiles up to period $t-1$. Thus it is conditional on $\mathbf{u}^{t-1} = (u_0, \ldots, u_{t-1})$, a history of utility profiles up to period $t-1$, where I denotes the set of individuals and $u_\tau = (u_{i\tau})_{i\in I}$ denote the utility profile at period τ. For simplicity, assume there is a common discount factor β, while in the main text we cover the case of heterogenous discounting. Then the characterized social welfare function Φ defined over streams of utility profiles starting at period t, typically denoted by $\mathbf{u}_t = (u_t, u_{t+1}, \ldots)$, takes either (i) the exponential form

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1}) \exp\left(-\lambda(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}\right)$$

in which the welfare weight vector $a(\mathbf{u}^{t-1})$ and the degree of inequality aversion $\lambda(\mathbf{u}^{t-1})$ follow the updating rule

$$\begin{aligned} a_i(\mathbf{u}^{t-1}) &= \frac{a_i(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{t-1}\beta^\tau u_{i\tau}\right)}{\sum_{j\in I} a_j(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{t-1}\beta^\tau u_{j\tau}\right)} \\ \lambda(\mathbf{u}^{t-1}) &= \lambda(\emptyset)\beta^t \end{aligned}$$

or (ii) the additive form

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = \sum_{i\in I} a_i(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}$$

in which $a(\mathbf{u}^{t-1})$ follows

$$a_i(\mathbf{u}^{t-1}) = a_i(\emptyset),$$

where $\emptyset$ denotes the empty history.

These formulae say that if the social ranking has equity concern by taking the exponential form then it must incorporate past utilities when updating welfare weights, while the equity concern must decrease over time according to the exponential order.

There, higher (lower) past utilities must be 'compensated' in the way that they decrease (increase) welfare weights in the exponential manner. This provides a bound on how much the society can or should be tolerant of redistribution based on the idea of ex post equity, so that it is compatible with the idea of ex ante equity. And it says that when the social ranking has no equity concern, by taking the additive form, welfare weights must be constant over time, and it does not take past utilities into account.

In the problem of updating under uncertainty the objects of choice are random utility profiles defined over the set of terminal states denoted by Ω, where information gradually is revealed over time. The ex-post decision at event E_t at period t can depend on utility profiles which would have been obtained if E_t had not occurred. Thus it is conditional on $\mathbf{u}_{-E_t}$, denoting a random utility profile defined over the *complement* of event E_t. For simplicity, let p be the common belief across individuals, while in the main text we cover the case of heterogenous beliefs. Then the characterized social welfare function Φ defined over random utility profiles over the realized event E_t, typically denoted by $\mathbf{u}_{E_t}$, takes either (i) the exponential form

$$\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t}) \exp\left(-\lambda(\mathbf{u}_{-E_t}) \int_{E_t} u_i(s)p(ds|E_t)\right)$$

in which the welfare weight vector $a(\mathbf{u}_{-E_t})$ and the degree of inequality aversion $\lambda(\mathbf{u}_{-E_t})$ follow the updating rule

$$a_i(\mathbf{u}_{-E_t}) = \frac{a_i(\emptyset)\exp\left(-\lambda(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p(ds)\right)}{\sum_{j\in I} a_j(\emptyset)\exp\left(-\lambda(\emptyset)\int_{\Omega\setminus E_t} u_j(s)p(ds)\right)}$$
$$\lambda(\mathbf{u}_{-E_t}) = \lambda(\emptyset)p(E_t)$$

or (ii) the additive form

$$\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = \sum_{i\in I} a_i(\mathbf{u}_{-E_t}) \int_{E_t} u_i(s)p(ds|E_t)$$

in which the welfare weight vector $a(\mathbf{u}_{-E_t})$ follows

$$a_i(\mathbf{u}_{-E_t}) = a_i(\emptyset)$$

where $\emptyset$ denotes the initial history at which nothing is known.

These formulae say that if the social ranking has equity concern, by taking the exponential form, it must incorporate utilities which turned out not to have occurred when updating welfare weights, while the equity concern must decrease as we acquire more and more information about the true state of nature. There, higher (lower) utilities which turned out not to have occurred must be 'compensated' in the way that they decrease (increase) welfare weights conditional on the present event. This explains Machina's Mom reply, "You had your chance!" It also says that when the social ranking has no equity concern, by taking the additive form, welfare weights must be constant, and it does not take utilities into account which turned out not to have occurred.

The paper proceeds as follows. In Sect. 2, we consider the problem of updating in the deterministic setting with intertemporal compensations, and propose a condition that an axiom imposed on social welfare ordering should be recurrent under consistent updating. Then we provide a set of axioms, hence a class of characterized social welfare functions, which is closed under consistent updating, and present the updating rule of welfare weights and equity attitudes, which explains the role of pasts. In Sect. 3, we consider the problem of updating along with resolution of uncertainty, and propose a condition that an axiom imposed on social welfare ordering be recurrent under consistent updating. Then we provide a set of axioms, hence a class of characterized social welfare functions, which is closed under updating, and present the updating rule of welfare weights and equity attitudes, which explains the role of things known not to have occurred. All proofs are relegated to the "Appendix". Also in the "Appendix" we present an axiomatic characterization of the class of static social welfare functions, which allows for the role of equity concern and is shown to be closed under consistent updating in each of Sects. 2 and 3.

2 Consistent updating with intertemporal compensations

2.1 Consistent updating under homogeneous discounting

In this section we consider the problem of aggregation and updating with intertemporal compensation in a deterministic setting. First we limit attention to the case where discounting is homogeneous across individuals. In a later section, we will extend the argument to the case of heterogeneous discounting.

Consider discrete time with an infinite horizon. Let I be the set of individuals. For technical reasons, we assume $|I| \geq 3$.[5]

For each t, let $u_t = (u_{it})_{i \in I}$ be the list of utilities received by the individuals at period t. For each t, let $\mathbf{u}^{t-1} = (u_0, \ldots, u_{t-1}) \in \mathbb{R}^{I \times (t-1)}$ be the history of utilities received by the individuals before period t. Denote the initial point, with a null history, by $\emptyset$. Also, for each t, let $L^{I \times [t,\infty]} = \{\mathbf{u}_t = (u_t, u_{t+1}, \ldots) \colon \max_i \sup_{\tau \geq t} |u_{i\tau}| < \infty\}$ be the set of bounded sequences starting at period t. For each possible history $\mathbf{u}^{t-1}$, let $\succsim_{\mathbf{u}^{t-1}}$ be the social welfare ordering over $L^{I \times [t,\infty]}$ conditional on $\mathbf{u}^{t-1}$. Given $\mathbf{u}_t, \mathbf{v}_t \in L^{I \times [t,\infty]}$, the ranking may be written for example as $\mathbf{u}_t \succsim_{\mathbf{u}^{t-1}} \mathbf{v}_t$. Let $\{\succsim_{\mathbf{u}^{t-1}} : t \in \mathbb{N} \text{ and } \mathbf{u}^{t-1} \in \mathbb{R}^{I \times (t-1)}\}$ be a process of such social welfare orderings. Also, let $\beta \in (0, 1)$ be the discount factor which is assumed to be the same for all individuals.

We will consider two kinds of axioms. One kind describes the properties of the social welfare criteria which are desirable to be satisfied at each time in each possible history, and are investigated as to whether they are recurrent under consistent updating

[5] This technical condition is required to establish the uniqueness of separable aggregation. It apparently excludes the two-person cases which are treated in the leading examples, but under separable aggregation one may without loss of generality add a dummy third individual.

over time. We call such axioms *intra-profile axioms*. The other kind describes relations between social welfare orderings indexed with different times and histories. We call such axioms *inter-profile axioms*.

Dynamic consistency is the inter-profile axiom we impose throughout the analysis.

Dynamic consistency for all t and $\mathbf{u}^{t-1} \in \mathbb{R}^{I\times(t-1)}$, for all $u_t \in \mathbb{R}^I$ and $\mathbf{u}_{t+1}, \mathbf{v}_{t+1} \in L^{I\times[t,\infty]}$,

$$(u_t, \mathbf{u}_{t+1}) \succsim_{\mathbf{u}^{t-1}} (u_t, \mathbf{v}_{t+1}) \Longleftrightarrow \mathbf{u}_{t+1} \succsim_{(\mathbf{u}^{t-1},u_t)} \mathbf{v}_{t+1}.$$

We define the condition that an intra-profile axiom is recurrent under consistent updating over time.

Definition 1 An intra-profile axiom is said to be *recurrent under consistent updating* if for any process of social welfare orderings $\{\succsim_{\mathbf{u}^{t-1}}\}$ satisfying dynamic consistency, for all $\mathbf{u}^{t-1} \in \mathbb{R}^{I\times(t-1)}$ and $u_t \in \mathbb{R}^I$, if it is true for $\succsim_{\mathbf{u}^{t-1}}$, then it is true for $\succsim_{\mathbf{u}^{t-1},u_t}$.

Example 3 The following intra-profile axioms are not recurrent under consistent updating.

Anonymity at a given history $\mathbf{u^{t-1}}$: for all $\mathbf{u}_t, \mathbf{v}_t \in L^{I\times[t,\infty]}$ and any permutation $\pi : I \to I$ it holds that

$$\mathbf{u}_t \succsim_{\mathbf{u}^{t-1}} \mathbf{v}_t \Longleftrightarrow \mathbf{u}_t^{\pi} \succsim_{\mathbf{u}^{t-1}} \mathbf{v}_t^{\pi},$$

where $\mathbf{u}_t^{\pi}$ is given by $u_{i\tau}^{\pi} = u_{\pi^{-1}(i)\tau}$ for all $i \in I$ and $\tau \geq t$.

Homogeneity at a given history $\mathbf{u^{t-1}}$: for all $\mathbf{u}_t, \mathbf{v}_t \in L^{I\times[t,\infty]}$ and for all non-negative numbers c,

$$\mathbf{u}_t \succsim_{\mathbf{u}^{t-1}} \mathbf{v}_t \Longleftrightarrow c\mathbf{u}_t \succsim_{\mathbf{u}^{t-1}} c\mathbf{v}_t.$$

To see that these axioms are not recurrent under updating, consider the social welfare function at history $\mathbf{u}^{t-1}$ in the form

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = \min_{i\in I} \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}.$$

This obviously satisfies anonymity and homogeneity at history $\mathbf{u}^{t-1}$. However, its consistent updating $\Phi(\mathbf{u}_{t+1}|\mathbf{u}^{t-1}, u_t)$ at history $(\mathbf{u}^{t-1}, u_t)$ must be ordinally equivalent to the function

$$\min_{i\in I}\left(u_{it} + \beta \sum_{\tau=t+1}^{\infty} \beta^{\tau-(t+1)} u_{i\tau}\right),$$

which does not satisfy anonymity or homogeneity as a ranking over $\mathbf{u}_{t+1}$.

Violating the first of these two axioms means something different from violating the other one.

Anonymity says that the identity of individuals does not matter in evaluating future utilities. From the ex ante viewpoint of horizontal equity, this is a natural and desirable axiom. However, anonymity cannot be maintained over time when we update social welfare orderings consistently over time.

This does not mean that we should abandon anonymity or axioms of horizontal equity in general just because they are not recurrent under consistent updating. It rather means that the axioms of horizontal equity are meaningful only when they are imposed on a judgment at the initial stage.

What the condition requires is that the structure of the judgment, not particular ethical attitudes, should be recurrent under consistent updating. Thus the second case has something more to bite. Homogeneity says that changing the common scale of individual utilities in the future does not matter. However, changing the scale of future utilities changes the impact of past utilities, which in general results in violating dynamic consistency.

To introduce intra-profile axioms, fix an arbitrary time t and a history $\mathbf{u}^{t-1}$. The first one is self-explanatory.

Order for all $t \in \mathbb{N}$ and for all $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$, $\succsim_{\mathbf{u}^{t-1}}$ is a complete and transitive ordering over $L^{I\times[t,\infty]}$.

Next we consider the axiom that the social ranking over utility streams depends only on individuals' evaluations of discounted utility (DU). This says that each individual is taken to be maintaining 'unity' over time, and their utilities at different periods are summarized by themselves, and the social ranking concerns only such summaries by individuals. One may see the non-triviality of this condition by thinking of a situation in which each individual has different 'selves' at different periods and there is a conflict among them, and the social choice may concern the resolution of such conflicts as well.

DU-Pareto for all $t \in \mathbb{N}$, $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$ and for all $\mathbf{u}_t, \mathbf{v}_t \in L^{I\times[t,\infty]}$, $\sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau} \geq \sum_{\tau=t}^{\infty} \beta^{\tau-t} v_{i\tau}$ for all $i \in I$ implies $\mathbf{u}_t \succsim_{\mathbf{u}^{t-1}} \mathbf{v}_t$, and the conclusion is strict if $\sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau} > \sum_{\tau=t}^{\infty} \beta^{\tau-t} v_{i\tau}$ for some $i \in I$ in addition.

Under DU-Pareto, one can define a social ranking induced over discounted utilities.

Definition 2 Given t and $\mathbf{u}^{t-1}$, the DU-welfare ordering $\succsim^*_{\mathbf{u}^{t-1}}$ over $\mathbb{R}^I$ induced by $\succsim_{\mathbf{u}^{t-1}}$ is defined by

$$U_t \succsim^*_{\mathbf{u}^{t-1}} V_t \Longleftrightarrow \mathbf{u}_t \succsim_{\mathbf{u}^{t-1}} \mathbf{v}_t$$

for $\mathbf{u}_t, \mathbf{v}_t \in L^{I\times\infty}$ with $U_t = \left(\sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}\right)_{i\in I}$ and $V_t = \left(\sum_{\tau=t}^{\infty} \beta^{\tau-t} v_{i\tau}\right)_{i\in I}$ respectively. The DU-Pareto axiom ensures that this is well-defined.

We consider the following four intra-profile axioms on the induced ranking. Although one can write them directly in terms of the original ranking, this helps simplifying the exposition.

DU-continuity for all $t \in \mathbb{N}$ and for all $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$, $\succsim^*_{\mathbf{u}^{t-1}}$ is a closed subset of $\mathbb{R}^I \times \mathbb{R}^I$.

DU-inequality aversion for all $t \in \mathbb{N}$ and for all $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$, for all $U_t, V_t \in \mathbb{R}^I$ and $c \in [0,1]$, $U_t \sim^*_{\mathbf{u}^{t-1}} V_t$ implies $cU_t + (1-c)V_t \succsim^*_{\mathbf{u}^{t-1}} U_t$.

DU-separability for all $t \in \mathbb{N}$ and for all $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$, for all $J \subset I$, for all $U_{Jt}, V_{Jt} \in \mathbb{R}^J$ and $U_{-Jt}, V_{-Jt} \in \mathbb{R}^{I\setminus J}$, $(U_{Jt}, U_{-Jt}) \succsim^*_{\mathbf{u}^{t-1}} (V_{Jt}, U_{-Jt})$ holds if and only if $(U_{Jt}, V_{-Jt}) \succsim^*_{\mathbf{u}^{t-1}} (V_{Jt}, V_{-Jt})$.

DU-shift covariance for all $t \in \mathbb{N}$ and for all $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$, for all $U_t, V_t \in \mathbb{R}^I$ and $c \in \mathbb{R}$, $U_t \succsim^*_{\mathbf{u}^{t-1}} V_t$ implies $U_t + c\mathbf{1} \succsim^*_{\mathbf{u}^{t-1}} V_t + c\mathbf{1}$.

This last, DU-shift covariance, needs explanation. In the static setting, the shift covariance axiom says that adding ‘equal utilities’ to everybody does not change the ranking. In the context of updating over time, this means that adding ‘equal lifetime utilities’ does not change the ranking. While Homogeneity, another prominent independence property, is not recurrent under consistent updating, (DU-)shift covariance can be shown to be recurrent under consistent updating. Note, however, that it relies on the assumption that the individual utility functions fall in the class of stationary discounted utility.[6]

Proposition 1 *Order, DU-Pareto, DU-continuity, DU-inequality aversion, DU-separability and DU-shift covariance are recurrent under consistent updating.*

The proof of this result and most of the other results are in the “Appendix”.

The following characterization result deals with each time and history separately, which has nothing to do with dynamic consistency yet. Let $\Delta^I = \left\{a \in \mathbb{R}^I_+ :\ \sum_{i\in I} a_i = 1\right\}$ and $int\,\Delta^I$ denote the relative interior of Δ^I.

Proposition 2 *Fix a time t and a history $\mathbf{u}^{t-1}$. The social welfare ordering $\succsim_{\mathbf{u}^{t-1}}$ satisfies order, DU-Pareto, DU-continuity, DU-inequality aversion, DU-separability and DU-shift covariance if and only if either of the following two cases holds:*

(i) *There exists $\lambda(\mathbf{u}^{t-1}) > 0$ and a vector $a(\mathbf{u}^{t-1}) \in int\,\Delta^I$ such that $\succsim_{\mathbf{u}^{t-1}}$ can be represented in the form*

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1}) \exp\left(-\lambda(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}\right).$$

We call this class of orderings and representations the exponential class.

(ii) *There exists a vector $a(\mathbf{u}^{t-1}) \in int\,\Delta^I$ such that $\succsim_{\mathbf{u}^{t-1}}$ can be represented in the form*

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = \sum_{i\in I} a_i(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}.$$

We call this class of orderings and representations the additive class.

[6] Continuity is a rather technical requirement but it excludes interesting rules such as the leximin rule. Allowing discontinuities and lexicographic arguments in the current framework would lead to considering how lexicographic priorities should change over time and depend on histories.

Moreover, for each t *and* $\mathbf{u}^{t-1}$*, in case* (i)*,* $a(\mathbf{u}^{t-1})$ *and* $\lambda(\mathbf{u}^{t-1})$ *are unique, and in case* (ii)*,* $a(\mathbf{u}^{t-1})$ *is unique.*

Theorem 1 *The process of social welfare orderings* $\{\succsim_{\mathbf{u}^{t-1}}\}$ *satisfies order, DU-Pareto, DU-continuity, DU-inequality aversion, DU-separability and DU-shift covariance at* $\emptyset$ *and dynamic consistency if and only if either*

(i) $\{\succsim_{\mathbf{u}^{t-1}}\}$ *falls in the exponential class and* $\{a(\mathbf{u}^{t-1})\}$ *and* $\{\lambda(\mathbf{u}^{t-1})\}$ *follow the updating rule*

$$a_i(\mathbf{u}^{t-1}) = \frac{a_i(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{t-1}\beta^\tau u_{i\tau}\right)}{\sum_{j\in I} a_j(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{t-1}\beta^\tau u_{j\tau}\right)} \tag{1}$$

$$\lambda(\mathbf{u}^{t-1}) = \lambda(\emptyset)\beta^t \tag{2}$$

for all $\mathbf{u}^{t-1}$*, or*

(ii) $\{\succsim_{\mathbf{u}^{t-1}}\}$ *falls in the additive class and* $(a_i(\mathbf{u}^t))_{i\in I}$ *follows*

$$a_i(\mathbf{u}^{t-1}) = a_i(\emptyset) \tag{3}$$

for all $\mathbf{u}^{t-1}$.

The theorem says that if the social ranking has a concern for equity, then it must incorporate past utilities when updating welfare weights as described by formula (1), while the equity concern must decrease over time according to the exponential order as described by formula (2). There, higher (lower) past utilities must be 'compensated' in the way that they decrease (increase) welfare weights in the exponential manner. This is the way how we maintain compatibility between the concept of ex ante equity and the concept of ex post equity. It also says that when the social ranking has no equity concern, welfare weights must be constant over time as in formula (3), and does not take past utilities into account.

This implies that when we require that social rankings should ignore the past, the only possibility is additive aggregation with weights being constant over time.

Independence of past utilities for all t and for all $\mathbf{u}^{t-1}$ and $\widetilde{\mathbf{u}}^{t-1}$, $\succsim_{\mathbf{u}^{t-1}}=\succsim_{\widetilde{\mathbf{u}}^{t-1}}$.

Corollary 1 *Suppose that the process of social welfare orderings* $\{\succsim_{\mathbf{u}^{t-1}}\}$ *satisfies dynamic consistency and falls in the class as characterized in Proposition 2. Then it satisfies the independence of past utilities if and only if it falls in the additive class with* $a(\mathbf{u}^{t-1}) = a(\emptyset)$ *for all* $\mathbf{u}^{t-1}$.

2.2 Consistent updating under heterogeneous discounting

If we attempt to extend the previous argument on aggregating discounted utility to the case of heterogeneous discounting described by $(\beta_i)_{i\in I}$, we have

$$\Phi(\mathbf{u}_0|\emptyset) = -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{\infty}\beta_i^\tau u_{i\tau}\right)$$

$$= -\sum_{i \in I} a_i(\emptyset) \exp\left(-\lambda(\emptyset) \sum_{\tau=0}^{t-1} \beta_i^{\tau} u_{i\tau}\right) \exp\left(-\lambda(\emptyset)\beta_i^t \sum_{\tau=t}^{\infty} \beta_i^{\tau-t} u_{i\tau}\right),$$

which in general cannot represent the same ranking over $\mathbf{u}_t$ as $\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1}) \exp\left(-\lambda(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta_i^{\tau-t} u_{i\tau}\right)$ does, because the term $\lambda(\emptyset)\beta_i^t$ is not the same for different individuals.

Therefore, the exponential class is not closed under updating under heterogeneous discounting, and we have the following result.

Proposition 3 *Consider the class of social welfare functions as obtained in Proposition 2. Under strictly heterogeneous discounting, it satisfies dynamic consistency if and only if the social welfare function is limited to the additive class with $a(\mathbf{u}^{t-1}) = a(\emptyset)$ for all $\mathbf{u}^{t-1}$.*

The reason is that DU-shift covariance is not in general recurrent under consistent updating when discounting is heterogenous. However, a weaker version of DU-shift covariance can be shown to be recurrent under consistent updating.

DU-general shift covariance for all $t \in \mathbb{N}$ and for all $\mathbf{u}_{t-1} \in \mathbb{R}^{I\times(t-1)}$, there exists a $W_{\mathbf{u}^{t-1}} \in \mathbb{R}^I_{++}$ such that for all $U_t, V_t \in \mathbb{R}^I$ and $c \in \mathbb{R}$, $U_t \succsim^*_{\mathbf{u}^{t-1}} V_t$ implies $U_t + cW_{\mathbf{u}^{t-1}} \succsim^*_{\mathbf{u}^{t-1}} V_t + cW_{\mathbf{u}^{t-1}}$.

Proposition 4 *Under heterogeneous discounting, order, DU-Pareto, DU-continuity, DU-inequality aversion, DU-separability and DU-general shift covariance are recurrent under consistent updating.*

Proposition 5 *For each fixed time t and history $\mathbf{u}^{t-1}$, the social welfare ordering $\succsim_{\mathbf{u}^{t-1}}$ satisfies order, DU-Pareto, DU-continuity, DU-inequality aversion, DU-separability and DU-general shift covariance if and only if either of the following two cases holds:*

(i) *there exists a vector $\lambda(\mathbf{u}^{t-1}) \in \mathbb{R}^I_{++}$ and a vector $a(\mathbf{u}^{t-1}) \in int\,\Delta^I$ such that $\succsim_{\mathbf{u}^{t-1}}$ can be represented in the form*

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1}) \exp\left(-\lambda_i(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta_i^{\tau-t} u_{i\tau}\right).$$

We call this class of orderings and representations the generalized exponential class.

(ii) *there exists a vector $a(\mathbf{u}^{t-1}) \in int\,\Delta^I$ such that $\succsim_{\mathbf{u}^{t-1}}$ can be represented in the form*

$$\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta_i^{\tau-t} u_{i\tau}.$$

We call this class of orderings and representations the generalized additive class.

Moreover, for each t and $\mathbf{u}^{t-1}$, in case (i) *$a(\mathbf{u}^{t-1})$ and $\lambda(\mathbf{u}^{t-1})$ are unique and in case* (ii) *$a(\mathbf{u}^{t-1})$ is unique.*

Theorem 2 *The process of social welfare orderings* $\{\succsim_{\mathbf{u}^{t-1}}\}$ *satisfies order, DU-Pareto, DU-continuity, DU-inequality aversion, DU-separability and DU-general shift covariance at* $\emptyset$ *and dynamic consistency if and only if either*

(i) $\{\succsim_{\mathbf{u}^{t-1}}\}$ *falls in the generalized exponential class and* $\{a(\mathbf{u}^{t-1})\}$ *and* $\{\lambda(\mathbf{u}^{t-1})\}$ *follow the updating rule*

$$a_i(\mathbf{u}^{t-1}) = \frac{a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\sum_{\tau=0}^{t-1}\beta_i^{\tau}u_{i\tau}\right)}{\sum_{j\in I}a_j(\emptyset)\exp\left(-\lambda_j(\emptyset)\sum_{\tau=0}^{t-1}\beta_j^{\tau}u_{j\tau}\right)} \tag{4}$$

$$\lambda_i(\mathbf{u}^{t-1}) = \lambda_i(\emptyset)\beta_i^t \tag{5}$$

for all $\mathbf{u}^{t-1}$*, or*

(ii) $\{\succsim_{\mathbf{u}^{t-1}}\}$ *falls in the generalized additive class and* $(a_i(\mathbf{u}^t))_{i\in I}$ *follows the updating rule*

$$a_i(\mathbf{u}^{t-1}) = \frac{a_i(\emptyset)\beta_i^t}{\sum_{j\in I}a_j(\emptyset)\beta_j^t} \tag{6}$$

for all $\mathbf{u}^{t-1}$.

The theorem says that if the social ranking has equity concern then it must incorporate past utilities when updating welfare weights as described by formula (4), while the equity concern on each individual decreases over time according to her discount factor as in formula (5). There, higher (lower) past utilities must be 'compensated' in the way that they decrease (increase) welfare weights in the exponential manner. It also says that when the social ranking has no equity concern welfare weights evolve according to individuals' discount factors as in formula (6), which depends on the time index but does not take past utilities into account.

Let us come back to the Ant/Grasshopper story here. Since Anonymity or the axiom of horizontal equity cannot be recurrent under consistent updating our ex post welfare judgment cannot support the idea of ex post redistribution in the complete sense. However, as the vector of initial degree of inequality aversion: $\lambda(\emptyset)$ is larger when the updated ones are larger as well, implying that we are more supportive of the idea of redistribution.

The above result implies that when we require that social rankings should ignore past utilities, the only possibility is additive aggregation.

Corollary 2 *Suppose that the process of social welfare orderings* $\{\succsim_{\mathbf{u}^{t-1}}\}$ *falls in the class as characterized in Proposition 5. Then it satisfies dynamic consistency and independence of past utilities hold if and only if it falls in the generalized additive class in which the updating of welfare weights follows*

$$a_i(\mathbf{u}^{t-1}) = \frac{a_i(\emptyset)\beta_i^t}{\sum_{j\in I}a_j(\emptyset)\beta_j^t}.$$

for all $\mathbf{u}^{t-1}$.

Notice that under heterogeneous discounting, even if there is no equity concern, the social welfare function still must depend on histories in the sense that when the history started should make a difference.

Independence of past utilities and time for all t, s and for all $\mathbf{u}^{t-1}$ and $\widetilde{\mathbf{u}}^{s-1}$, $\succsim_{\mathbf{u}^{t-1}} = \succsim_{\widetilde{\mathbf{u}}^{s-1}}$.

Corollary 3 *Under strictly heterogeneous discounting, there is no process of social welfare orderings which satisfies dynamic consistency and independence of past utilities and time.*

3 Consistent updating under uncertainty

3.1 Consistent updating under homogeneous beliefs

In this section, we consider the problem of aggregation and updating along with the resolution of uncertainty. Here we limit attention to the case of common beliefs held by all individuals. In a later section, we will extend the argument to the case of heterogeneous beliefs.

As before, let I be the set of individuals with $|I| \geq 3$. Let Ω be the set of states of the world, which is a measure space with a common prior p. Given a measurable subset $E \subset \Omega$ and a subset of individuals $J \subset I$, let $L^{J \times E}$ be the set of integrable functions from E to $\mathbb{R}^J$, which is interpreted as a set of random utility profiles for group J conditional on E.

Let $\{\mathcal{E}_t\}$ be a refining sequence of partitions of Ω, which describes how uncertainties are resolved over time. Assume that $p(E_t) > 0$ for all t and $E_t \in \mathcal{E}_t$. Given t and $E_t \in \mathcal{E}_t$, let $\mathbf{u}_{-E_t} \in L^{I \times (\Omega \backslash E_t)}$ be an integrable random utility profile defined over $\Omega \backslash E_t$, which describes what individuals' utilities would have been if E_t had not occurred. Denote the initial point with no such foregone utilities by $\emptyset$. For simplicity, we restrict attention to utility values at final states, which can easily be combined with the argument with intertemporal consumption as presented in the previous section. Given $E_t \in \mathcal{E}_t$, let $p(\cdot|E_t)$ be the Bayesian update of p conditional on E_t.

For each possible $\mathbf{u}_{-E_t}$, let $\succsim_{\mathbf{u}_{-E_t}}$ be the social welfare ordering over $L^{I \times E_t}$ conditional on $\mathbf{u}_{-E_t}$. Given $\mathbf{u}_{E_t}, \mathbf{v}_{E_t} \in L^{I \times E_t}$, the ranking may be written for example as $\mathbf{u}_{E_t} \succsim_{\mathbf{u}_{-E_t}} \mathbf{v}_{E_t}$. Let $\{\succsim_{\mathbf{u}_{-E_t}} : t \in \mathbb{N}, E_t \in \mathcal{E}_t, \text{and} \mathbf{u}_{-E_t} \in L^{I \times (\Omega \backslash E_t)}\}$ be a process of such social welfare orderings.

Note that when $\succsim_{\mathbf{u}_{-E_t}}$, the foregone utility profile $\mathbf{u}_{-E_t}$ that would have obtained if event E_t had not occurred is taken to be a *commitment plan*. To see this, imagine that Benjamin counterargues against Mom's claim 'You had your chance!' as follows: 'But if I had won the flip Abigail would have claimed the same thing as I did and you would have accepted her claim, so why not flip the coin again for me as well?' This will lead to a hierarchical argument that the decision at the current event depends on the decision at the counterfactual event, which again depends on the decision at the current event and so on, but we avoid this problem in the current paper.

We will consider two kinds of axioms. One kind describe the properties of social welfare criteria for which it is desirable that they be satisfied at each time and possible event, investigate whether they are recurrent under consistent updating with the passage of time and the realization of uncertainty. We call such axioms *intra-profile axioms*. The other kind describe the relations between social welfare orderings indexed with different times and possible events. We call such axioms *inter-profile axioms*.

Dynamic consistency is the inter-profile axiom we impose throughout the analysis. Given t and $E_t \in \mathcal{E}_t$, let $\mathcal{E}_{t+1} \cap E_t = \{E_{t+1} \in \mathcal{E}_{t+1} \colon E_{t+1} \subset E_t\}$.

Dynamic consistency for all t, E_t and $\mathbf{u}_{-E_t}$, for all $E_{t+1} \in \mathcal{E}_{t+1} \cap E_t$ and $\mathbf{u}_{E_t \setminus E_{t+1}} \in L^{I \times (E_t \setminus E_{t+1})}$, and for all $\mathbf{u}_{E_{t+1}}, \mathbf{v}_{E_{t+1}} \in L^{I \times E_{t+1}}$,

$$(\mathbf{u}_{E_{t+1}}, \mathbf{u}_{E_t \setminus E_{t+1}}) \succsim_{\mathbf{u}_{-E_t}} (\mathbf{v}_{E_{t+1}}, \mathbf{u}_{E_t \setminus E_{t+1}}) \Longleftrightarrow \mathbf{u}_{E_{t+1}} \succsim_{\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}}} \mathbf{v}_{E_{t+1}}.$$

We define the condition that an intra-profile axiom is recurrent under consistent updating over time.

Definition 3 An intra-profile axiom is said to be *recurrent under consistent updating* if for any process of social welfare orderings $\{\succsim_{\mathbf{u}_{-E_t}}\}$ satisfying dynamic consistency, for all t, E_t and $\mathbf{u}_{-E_t}$, for all $E_{t+1} \in \mathcal{E}_{t+1} \cap E_t$ and $\mathbf{u}_{E_t \setminus E_{t+1}} \in L^{I \times (E_t \setminus E_{t+1})}$, if it holds for $\succsim_{\mathbf{u}_{-E_t}}$, then it holds for $\succsim_{\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}}}$.

Example 4 The following intra-profile axioms are not recurrent under consistent updating.

Anonymity at given t, E_t and $\mathbf{u}_{-E_t}$: for all $\mathbf{u}_{E_t}, \mathbf{v}_{E_t} \in L^{I \times E_t}$, and any permutation $\pi : I \to I$,

$$\mathbf{u}_{E_t} \succsim_{\mathbf{u}_{-E_t}} \mathbf{v}_{E_t} \Longleftrightarrow \mathbf{u}^{\pi}_{E_t} \succsim_{\mathbf{u}_{-E_t}} \mathbf{v}^{\pi}_{E_t},$$

where $\mathbf{u}^{\pi}_{E_t}$ is given by $u^{\pi}_{i,E_t} = u_{\pi^{-1}(i),E_t}$ for all i.

Homogeneity at given t, E_t and $\mathbf{u}_{-E_t}$: for all $\mathbf{u}_{E_t}, \mathbf{v}_{E_t} \in L^{I \times E_t}$, and for all non-negative number c,

$$\mathbf{u}_{E_t} \succsim_{\mathbf{u}_{-E_t}} \mathbf{v}_{E_t} \Longleftrightarrow c\mathbf{u}_{E_t} \succsim_{\mathbf{u}_{-E_t}} c\mathbf{v}_{E_t}.$$

To see that these axioms are not recurrent under updating, consider the following social welfare function at $\mathbf{u}_{-E_t}$:

$$\Phi(\mathbf{u}_{E_t} | \mathbf{u}_{-E_t}) = \min_{i \in I} \int_{E_t} u_i(s) p(ds | E_t).$$

This obviously satisfies anonymity and homogeneity at $\mathbf{u}_{-E_t}$. However, its consistent updating $\Phi(\mathbf{u}_{E_{t+1}} | \mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}})$ at $(\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}})$ must be ordinally equivalent to the function

$$\min_{i \in I} \left(\int_{E_t \setminus E_{t+1}} u_i(s) p(ds) + \int_{E_{t+1}} u_i(s) p(ds) \right),$$

which does not satisfy anonymity or homogeneity as a ranking over $\mathbf{u}_{E_{t+1}}$.

Violating the first of these two axioms means something different from violating the other one.

Anonymity says that identity of an individual does not matter in evaluating still possible utilities. However, in the presence of events which did not occur but might have delivered different utilities to different individuals, the identity of the individuals is indispensable for consistent updating.

Again, this does not mean that we should abandon anonymity or axioms of horizontal equity in general just because they are not recurrent under consistent updating. It rather means that the axioms of horizontal equity are meaningful only when they are imposed on a judgment at the initial stage.

What the condition requires is that the structure of the judgment, not particular ethical attitudes, should be recurrent under consistent updating. Thus the second case has something more to bite. Homogeneity says that changing the common scale of individual utilities in the future does not matter. However, changing the scale of future utilities changes the impact of past utilities, which in general results in violating dynamic consistency.

To introduce intra-profile axioms, fix an arbitrarily time t, E_t and $\mathbf{u}_{-E_t}$. The first one is self-explanatory.

Order for all $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, $\succsim_{\mathbf{u}_{-E_t}}$ is complete and transitive ordering over $L^{I\times E_t}$.

Next we require that the social ranking over utility streams depends only on individuals' expected utility (henceforth EU) evaluations. This says that each individual is taken to be maintaining 'unity' over time, and their utilities at different periods are summarized by themselves, and the social ranking concerns only such summaries. One may see the non-triviality of this condition by thinking for example of treating inequalities across both individuals and states in tandem, which is excluded by the axiom (see the discussions in Ben-Porath et al. 1997).

EU-Pareto for all $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, for all $\mathbf{u}_{E_t}, \mathbf{v}_{E_t} \in L^{I\times E_t}$, $\int_{E_t} u_i(s)p(ds|E_t) \geq \int_{E_t} v_i(s)p(ds|E_t)$ for all $i \in I$ implies $\mathbf{u}_{E_t} \succsim_{\mathbf{u}_{-E_t}} \mathbf{v}_{E_t}$, and the conclusion is strict if $\int_{E_t} u_i(s)p(ds|E_t) > \int_{E_t} v_i(s)p(ds|E_t)$ for some $i \in I$ in addition.

Under EU-Pareto, one can define a social ranking induced over expected utilities.

Definition 4 Given $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, the EU-welfare ordering $\succsim^*_{\mathbf{u}_{-E_t}}$ over $\mathbb{R}^I$ induced by $\succsim_{\mathbf{u}_{-E_t}}$ is defined by

$$U_{E_t} \succsim^*_{\mathbf{u}_{-E_t}} V_{E_t} \Longleftrightarrow \mathbf{u}_{E_t} \succsim_{\mathbf{u}_{-E_t}} \mathbf{v}_{E_t}$$

for $\mathbf{u}_{E_t}, \mathbf{v}_{E_t} \in L^{I\times E_t}$ with $U_{E_t} = \left(\int_{E_t} u_i(s)p(ds|E_t)\right)_{i\in I}$ and $V_{E_t} = \left(\int_{E_t} v_i(s)p(ds|E_t)\right)_{i\in I}$ respectively. The EU-Pareto axiom ensures that this is well-defined.

We impose the following four intra-profile axioms on the induced ranking. Although one can write them down directly in terms of the original ranking, this helps simplifying the exposition.

EU-continuity for all $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, $\succsim^*_{\mathbf{u}_{-E_t}}$ is a closed subset of $\mathbb{R}^I \times \mathbb{R}^I$.

EU-inequality aversion for all $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, for all $U_{E_t}, V_{E_t} \in \mathbb{R}^I$ and $c \in [0, 1]$, $U_{E_t} \sim^*_{\mathbf{u}_{-E_t}} V_{E_t}$ implies $cU_{E_t} + (1-c)V_{E_t} \succsim^*_{\mathbf{u}_{-E_t}} V_{E_t}$.

EU-separability for all $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, for all $J \subset I$, for all $U_{J,E_t}, V_{J,E_t} \in \mathbb{R}^J$ and $U_{-J,E_t}, V_{-J,E_t} \in \mathbb{R}^{I\setminus J}$, $(U_{J,E_t}, U_{-J,E_t}) \succsim^*_{\mathbf{u}_{-E_t}} (V_{J,E_t}, U_{-J,E_t})$ holds if and only if $(U_{J,E_t}, V_{-J,E_t}) \succsim^*_{\mathbf{u}_{-E_t}} (V_{J,E_t}, V_{-J,E_t})$.

EU-shift covariance for all $t \in \mathbb{N}$, $E_t \in \mathcal{E}_t$ and $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, for all $U_{E_t}, V_{E_t} \in \mathbb{R}^I$ and $c \in \mathbb{R}$, $U_{E_t} \succsim^*_{\mathbf{u}_{-E_t}} V_{E_t}$ implies $U_{E_t} + c\mathbf{1} \succsim^*_{\mathbf{u}_{-E_t}} V_{E_t} + c\mathbf{1}$.

This last, EU-shift covariance, needs some explanation. In the static setting, the shift covariance axioms say that adding 'equal utilities' to everybody does not change the ranking. In the context of updating under uncertainty, it means that adding 'equal expected utilities' does not change the ranking. While Homogeneity, another prominent independence property, is not recurrent under consistent updating, (EU-)shift covariance can be shown to be recurrent under consistent updating. Note, however, that it relies on the assumption that the individual utility functions fall in the class of expected utility.

Proposition 6 *Order, EU-Pareto, EU-continuity, EU-inequality aversion, EU-separability and EU-shift covariance are recurrent under consistent updating.*

Proposition 7 *Fix t, E_t and $\mathbf{u}_{-E_t}$. The social welfare ordering $\succsim_{\mathbf{u}_{-E_t}}$ satisfies order, EU-Pareto, EU-continuity, EU-inequality aversion, EU-separability and EU-shift covariance if and only if either of the following two cases holds:*

(i) *there exists $\lambda(\mathbf{u}_{-E_t}) > 0$ and a vector $a(\mathbf{u}_{-E_t}) \in int\Delta^I$ such that $\succsim_{\mathbf{u}_{-E_t}}$ can be represented in the form*

$$\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t}) \exp\left(-\lambda(\mathbf{u}_{-E_t}) \int_{E_t} u_i(s)p(ds|E_t)\right).$$

We call this class of orderings and representations the exponential class.

(ii) *there exists a vector $a(\mathbf{u}_{-E_t}) \in int\Delta^I$ such that $\succsim_{\mathbf{u}_{-E_t}}$ can be represented in the form*

$$\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = \sum_{i\in I} a_i(\mathbf{u}_{-E_t}). \int_{E_t} u_i(s)p(ds|E_t)$$

We call this class of orderings and representations the additive class.

Moreover, for each t and $\mathbf{u}_{-E_t}$, in case (i) *$a(\mathbf{u}_{-E_t})$ and $\lambda(\mathbf{u}_{-E_t})$ are unique and in case* (ii) *$a(\mathbf{u}_{-E_t})$ is unique.*

Theorem 3 *The process of social welfare orderings $\{\succsim_{\mathbf{u}_{-E_t}}\}$ satisfies order, EU-Pareto, EU-continuity, EU-inequality aversion, EU-separability and EU-shift covariance at $\emptyset$ and dynamic consistency if and only if either*

(i) $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *falls in the exponential class and* $\{a(\mathbf{u}_{-E_t})\}$ *and* $\{\lambda(\mathbf{u}_{-E_t})\}$ *follow the updating rule*

$$a_i(\mathbf{u}_{-E_t}) = \frac{a_i(\emptyset)\exp\left(-\lambda(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p(ds)\right)}{\sum_{j\in I} a_j(\emptyset)\exp\left(-\lambda(\emptyset)\int_{\Omega\setminus E_t} u_j(s)p(ds)\right)} \tag{7}$$

$$\lambda(\mathbf{u}_{-E_t}) = \lambda(\emptyset)p(E_t) \tag{8}$$

for all $\mathbf{u}_{-E_t}$, *or*

(ii) $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *falls in the additive class and* $\{a(\mathbf{u}_{-E_t})\}$ *follows*

$$a_i(\mathbf{u}_{-E_t}) = a_i(\emptyset) \tag{9}$$

for all $\mathbf{u}_{-E_t}$.

The theorem says that if the social ranking has a concern for equity, then it must incorporate past utilities which turned out not to have occurred when updating welfare weights as described by formula (7), while the equity concern must decrease as we acquire more and more information about the true state of nature, according to formula (8). There, higher (lower) utilities which turned out not to have occurred must be 'compensated' for in the way that they decrease (increase) welfare weights conditional on the present event. This is the way how we maintain compatibility between the idea of fair chance ex ante and the idea of fair chance ex post. It also says that when the social ranking has no equity concern welfare weights must be constant as in formula (9), and it does not take utilities into account which turned out not to have occurred.

This implies that when we require that social rankings should ignore utilities which turned out not to have occurred, the only possibility is additive aggregation with weights' being constant.

Independence of counterfactual consequences for all t, for all E_t, for all $\mathbf{u}_{-E_t}$ and $\tilde{\mathbf{u}}_{-E_t}$, and for all $U, V \in \mathbb{R}^I$,

$$(U_i\mathbf{1}_{E_t})_{i\in I} \succsim_{\mathbf{u}_{-E_t}} (V_i\mathbf{1}_{E_t})_{i\in I} \Longleftrightarrow (U_i\mathbf{1}_{E_t})_{i\in I} \succsim_{\tilde{\mathbf{u}}_{-E_t}} (V_i\mathbf{1}_{E_t})_{i\in I}.$$

Corollary 4 *Suppose that the process of social welfare orderings* $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *satisfying Dynamic Consistency falls in the class as characterized in Proposition 7. Then it satisfies independence of counterfactual consequences if and only if it falls in the additive class with* $a(\mathbf{u}_{-E_t}) = a(\emptyset)$ *for all* $\mathbf{u}_{-E_t}$.

3.2 Consistent updating under heterogeneous beliefs

Now consider heterogeneous priors $(p_i)_{i\in I}$, assuming that $p_i(E_t) > 0$ for all i, t and $E_t \in \mathcal{E}_t$. If we attempt to extend the previous argument on aggregating expected utility to the case of heterogeneous beliefs, for the exponential class we have

$$
\begin{aligned}
\Phi(\mathbf{u}_\Omega|\emptyset) &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\int_\Omega u_i(s)p_i(ds)\right)\\
&= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p_i(ds)\right)\\
&\quad\times\exp\left(-\lambda(\emptyset)p_i(E_t)\int_{E_t} u_i(s)p_i(ds|E_t)\right),
\end{aligned}
$$

which in general cannot represent the same ranking over $\mathbf{u}_{E_t}$ as

$$
\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t})\exp\left(-\lambda(\mathbf{u}_{-E_t})\int_{E_t} u_i(s)p_i(ds|E_t)\right)
$$

does, because the term $\lambda(\emptyset)p_i(E_t)$ is not independent of the individual.

Therefore, the exponential class is not closed under updating with heterogeneous beliefs, and we have the following result.

Proposition 8 *Consider the class of social welfare functions as obtained in Proposition 7. Under strictly heterogeneous beliefs, it satisfies dynamic consistency if and only if the social welfare function is limited to the additive class with* $a(\mathbf{u}_{-E_t}) = a(\emptyset)$ *for all* $\mathbf{u}_{-E_t}$.

The reason is that EU-shift covariance is not in general recurrent under consistent updating when beliefs are heterogeneous. However, a weaker version of EU-shift covariance is shown to be recurrent under consistent updating.

EU-general shift covariance for all $t \in \mathbb{N}$, for all $E_t \in \mathcal{E}_t$ and for all $\mathbf{u}_{-E_t} \in L^{I\times(\Omega\setminus E_t)}$, there exists $W_{\mathbf{u}_{-E_t}} \in \mathbb{R}^I_{++}$ such that for all $U_{E_t}, V_{E_t} \in \mathbb{R}^I$ and $c \in \mathbb{R}$, $U_{E_t} \succsim^*_{\mathbf{u}_{-E_t}} V_{E_t}$ implies $U_{E_t} + cW_{\mathbf{u}_{-E_t}} \succsim^*_{\mathbf{u}_{-E_t}} V_{E_t} + cW_{\mathbf{u}_{-E_t}}$.

Proposition 9 *Under heterogeneous beliefs, order, EU-Pareto, EU-continuity, EU-inequality aversion, EU-separability and EU-general shift covariance are recurrent under consistent updating.*

Proposition 10 *Fix* t, E_t *and* $\mathbf{u}_{-E_t}$. *The social welfare ordering* $\succsim_{\mathbf{u}_{-E_t}}$ *satisfies order, EU-Pareto, EU-continuity, EU-inequality aversion, EU-separability and EU-general shift covariance if and only if either of the following two cases holds:*

(i) *there exists a vector* $\lambda(\mathbf{u}_{-E_t}) \in \mathbb{R}^I_{++}$ *and a vector* $a(\mathbf{u}_{-E_t}) \in int\,\Delta^I$ *such that* $\succsim_{\mathbf{u}_{-E_t}}$ *can be represented in the form*

$$
\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t})\exp\left(-\lambda_i(\mathbf{u}_{-E_t})\int_{E_t} u_i(s)p_i(ds|E_t)\right).
$$

We call this class of orderings and representations the generalized exponential class.

(ii) *there exists a vector* $a(\mathbf{u}_{-E_t}) \in int\Delta^I$ *such that* $\succsim_{\mathbf{u}_{-E_t}}$ *can be represented in the form*

$$\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t}) \int_{E_t} u_i(s)p_i(ds|E_t).$$

We call this class of orderings and representations the generalized additive class.

Moreover, for each t *and* $\mathbf{u}_{-E_t}$, *in case* (i) $a(\mathbf{u}_{-E_t})$ *and* $\lambda(\mathbf{u}_{-E_t})$ *are unique and in case* (ii) $a(\mathbf{u}_{-E_t})$ *is unique.*

Theorem 4 *The process of social welfare orderings* $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *satisfies order, EU-Pareto, EU-continuity, EU-inequality aversion, EU-separability and EU-general shift covariance at* $\emptyset$ *and dynamic consistency if and only if either*

(i) $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *falls in the generalized exponential class and* $\{a(\mathbf{u}_{-E_t})\}$ *and* $\{\lambda(\mathbf{u}_{-E_t})\}$ *follow the updating rule*

$$a_i(\mathbf{u}_{-E_t}) = \frac{a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p_i(ds)\right)}{\sum_{j\in I} a_j(\emptyset)\exp\left(-\lambda_j(\emptyset)\int_{\Omega\setminus E_t} u_j(s)p_j(ds)\right)} \tag{10}$$

$$\lambda_i(\mathbf{u}_{-E_t}) = \lambda(\emptyset)p_i(E_t) \tag{11}$$

for all $\mathbf{u}_{-E_t}$, *or*

(ii) $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *falls in the generalized additive class and* $\{a(\mathbf{u}_{-E_t})\}$ *follows*

$$a_i(\mathbf{u}_{-E_t}) = \frac{a_i(\emptyset)p_i(E_t)}{\sum_{j\in I} a_j(\emptyset)p_j(E_t)} \tag{12}$$

for all $\mathbf{u}_{-E_t}$.

The theorem says that if the social ranking has a concern for equity, then it must incorporate utilities which turned out not to have occurred when updating welfare weights as described by formula (10), while the equity concern on each individual decreases when acquiring more and more information about the true state of nature (as interpreted by prior beliefs) as in formula (11). There, higher (lower) utilities which turned out not to have occurred must be 'compensated' for in the way that they decrease (increase) welfare weights conditional on the present event. It also says that when the social ranking has no equity concern, welfare weights conditional on an event must be proportional to the agent's belief in the event as in formula (12), which depends on the event which occurred but does not take into account utilities which turned out not to have occurred.

This implies that when we require that a social rankings should ignore utilities which turned out not to have occurred, the only possibility is additive aggregation.

Corollary 5 *Suppose that the process of social welfare orderings* $\{\succsim_{\mathbf{u}_{-E_t}}\}$ *satisfying Dynamic Consistency falls in the class as characterized in Proposition 10. Then it* *satisfies independence of counterfactual consequences if and only if it falls in the generalized additive class with*

$$a_i(\mathbf{u}_{-E_t}) = \frac{a_i(\emptyset)p_i(E_t)}{\sum_{j\in I} a_j(\emptyset)p_j(E_t)}$$

for all $\mathbf{u}_{-E_t}$.

Notice that under heterogeneous beliefs, even if there is no equity concern, the social welfare function must depend on history in the sense of depending on what the individuals had believed. This is consistent with the result of Mongin (1995, 1998), which has recently been refined by Chambers and Hayashi (2009) and Zuber (2010), that under heterogeneous beliefs any social ranking respecting the Pareto principle cannot be state-independent.

Independence of counterfactual consequences and events for all t, for all E_t and $\widetilde{E}_t$, for all $\mathbf{u}_{-E_t}$ and $\widetilde{\mathbf{u}}_{-\widetilde{E}_t}$, and for all $U, V \in \mathbb{R}^I$,

$$(U_i\mathbf{1}_{E_t})_{i\in I} \succsim_{\mathbf{u}_{-E_t}} (V_i\mathbf{1}_{E_t})_{i\in I} \Longleftrightarrow (U_i\mathbf{1}_{\widetilde{E}_t})_{i\in I} \succsim_{\widetilde{\mathbf{u}}_{-\widetilde{E}_t}} (V_i\mathbf{1}_{\widetilde{E}_t})_{i\in I}.$$

Corollary 6 *Under strictly heterogeneous beliefs, there is no process of social welfare orderings which satisfies dynamic consistency and independence of counterfactual consequences and events.*

4 Conclusions

The principle of being forward-looking, a central principle in economic decision making that says that a rational agent should be forward-looking and not be bound by bygones, is not necessarily appealing for collective decision making, where bygones may have necessary and substantive roles to play.

Then we considered a dynamic process of social welfare orderings, and proposed a restriction which is acceptable even when rejecting the principle of being forward-looking: the process must be dynamically consistent and any meaningful normative postulate should be recurrent under consistent updating.

Based on this standpoint, we have presented a set of axioms for social welfare orderings which are recurrent under consistent updating, and characterized a set of social welfare functions which are closed under updating. With such a class of social welfare functions, we characterized the roles for pasts and things known not to have occurred, which are played in the updating stage.

Although we do not say that the class of social welfare functions is the only one which is closed under consistent updating, we see it as a sufficiently tight class which enables operational arguments, and at the same a sufficiently general class which allows all the important arguments to be carried out.

Our arguments are limited to the cases where the individual utility functions fall in the class of stationary discounted utilities in the setting of intertemporal compensations, and fall in the class of expected utilities in the setting of uncertainty. The shift covariance axiom in particular makes sense for such classes of individual utility functions, and we view these classes as important benchmarks to start with. However, we do not know what should be the right counterpart of such an axiom when individual utility functions take more general forms. It is reasonable to ask what form of rationality should be imposed on social welfare functions, or what should be the decision theoretic content of social welfare functions, when we go beyond those classes of individual utility functions.

Appendix 1: The class of static social welfare functions

Basic axioms and characterizations

This section presents a set of axioms, which in the main text are translated into a dynamic setting and shown to be recurrent under consistent updating with the passing of time and the resolution of uncertainty. It presents a class of social welfare functions characterized by that.

Let I be the set of individuals. For technical reason, we assume $|I| \geq 3$. We assume that individual utilities to are given as cardinal and interpersonally comparable objects. Let $\mathbb{R}^I$ be the domain of such individual utilities. We consider a social welfare ordering $\succsim$ defined over $\mathbb{R}^I$.

We consider the following axioms.

Order $\succsim$ is complete and transitive.
Continuity $\succsim$ is a closed subset of $\mathbb{R}^I \times \mathbb{R}^I$.
Pareto for all $U, V \in \mathbb{R}^I$, $U_i \geq V_i$ for every $i \in I$ implies $U \succsim V$, and the conclusion is strict if $U_i > V_i$ for some $i \in I$ in addition.
Inequality aversion for all $U, V \in \mathbb{R}^I$ and $c \in [0, 1]$, $U \sim V$ implies $cU + (1-c) V \succsim U$.
Separability for all $J \subset I$, for all $U_J, V_J \in \mathbb{R}^J$ and $U_{-J}, V_{-J} \in \mathbb{R}^{I \setminus J}$, $(U_J, U_{-J}) \succsim (V_J, U_{-J})$ holds if and only if $(U_J, V_{-J}) \succsim (V_J, V_{-J})$.
Shift covariance for all $U, V \in \mathbb{R}^I$ and $c \in \mathbb{R}$, $U \succsim V$ implies $U + c\mathbf{1} \succsim V + c\mathbf{1}$.

This last, the shift covariance axiom, is concerned with interpersonal comparison of utilities, and says that adding 'equal utilities' to everybody does not change the social welfare ranking. This means that the attitude toward inequality is independent of the absolute level of utilities. While homogeneity, another prominent independence property, is not recurrent under consistent updating when translated to dynamic settings, shift covariance can be shown to be recurrent under consistent updating. Note, however, that it relies on the assumption that individual utility functions fall in the class of additively separable ones.

Let $\Delta^I = \left\{a \in \mathbb{R}_+^I : \sum_{i \in I} a_i = 1\right\}$ and $int\Delta^I$ denote the relative interior of Δ^I.

Proposition 11 *A social welfare ordering* $\succsim$ *satisfies order, continuity, Pareto, inequality aversion, separability and shift covariance if and only if either of the following two cases holds:*

(i) *there exist a vector* $a \in int\Delta^I$ *and a number* $\lambda > 0$ *such that* $\succsim$ *can be represented in the form*

$$\Phi(U) = -\sum_{i \in I} a_i e^{-\lambda U_i}.$$

We call this class of orderings and representations the exponential class.

(ii) *there exists a vector* $a \in int\Delta^I$ *such that* $\succsim$ *can be represented in the form*

$$\Phi(U) = \sum_{i \in I} a_i U_i.$$

We call this class of orderings and representations the additive class.

Moreover, in case (i) *a and* λ *are unique and in case* (ii) *a is unique.*

Proof This is an asymmetric extension of the argument in Roberts (1980), Theorem 6 and Moulin (1989), Theorem 2.6.

The necessity of the axioms is routine. We prove sufficiency.

From order, continuity and separability, $\succsim$ allows the additive representation (see Debreu 1960)

$$\Phi(U) = \sum_{i \in I} \phi_i(U_i),$$

which is unique up to a positive affine transformation. From Pareto, each ϕ_i is strictly increasing.

From shift covariance, both $\sum_{i \in I} \phi_i(U_i)$ and $\sum_{i \in I} \phi_i(U_i + c)$ are additive representations of the same ranking for all $c \in \mathbb{R}$, hence they are cardinal equivalent: there exist real-valued functions ψ and ζ_i with ψ being positive such that

$$\phi_i(U_i + c) = \psi(c)\phi_i(U_i) + \zeta_i(c)$$

for all i.

This is the generalized Pexider equation, which has a strongly increasing and weakly concave solution either in the form

$$\phi_i(U_i) = -A_i e^{-\lambda_i U_i}$$

with $A_i >$ and $\lambda_i > 0$, or

$$\phi_i(U_i) = A_i U_i$$

with $A_i > 0$.

Because ψ is the same for all individuals, we must have

$$\phi_i(U_i) = -A_i e^{-\lambda U_i} \quad \text{for some } \lambda \text{ independent of } i$$

for all $i \in I$ or

$$\phi_i(U_i) = A_i U_i$$

for all $i \in I$.

By normalizing $a_i = A_i / \sum_{j \in I} A_j$ for each $i \in I$, we obtain the representation.

Uniqueness for the exponential case, suppose both $-\sum_{i \in I} a_i e^{-\lambda U_i}$ and $-\sum_{i \in I} a'_i e^{-\lambda' U_i}$ represent the same ranking. Since they are additive representations of the same ranking, we have cardinal equivalence: there exist constants C, D with $C > 0$ such that

$$-a'_i e^{-\lambda' U_i} = -C a_i e^{-\lambda U_i} + D$$

for all i.

Suppose $D \neq 0$. Then as $U_i \to \infty$, we have $-a'_i e^{-\lambda' U_i} \to 0$. But $-C a_i e^{-\lambda U_i} + D \to D \neq 0$, a contradiction. Hence $D = 0$.

By letting $U_i = 0$, we have $a'_i = C a_i$. Since $\sum_{i \in I} a_i = 1$, we obtain $C = 1$, which implies $a'_i = a_i$. Then it is immediate to see $\lambda' = \lambda$.

For the additive case, suppose both $\sum_{i \in I} a_i U_i$ and $\sum_{i \in I} a'_i U_i$ represent the same ranking. Since they are additive representations of the same ranking, we have cardinal equivalence: there exist constants C, D with $C > 0$ such that

$$-a'_i U_i = -C a_i U_i + D$$

for all i.

By letting $U_i = 0$, we obtain $D = 0$. By letting $U_i = 1$, we have $C = 1$, which implies $a'_i = a_i$. □

We will also consider a weaker version of Shift Covariance. This is because 'equality' does not necessarily mean 'equality of utilities', depending on the situation. This is particularly the case when the 'scaling' of utility is different for different individuals. As we have seen in the main text, when individuals' subjective weights on the future differ, it may be natural to say that their scalings of future utilities are treated as different too, and when individuals' beliefs on events differ it may be natural to say that their scalings of utilities contingent on events are treated as different too. The axiom below says that up to some scalings, adding 'equal conditions' to everybody does not change the social welfare ranking.

> *General shift covariance* there exists $W \in \mathbb{R}^I_{++}$ such that for all $U, V \in \mathbb{R}^I$ and $c \in \mathbb{R}$, $U \succsim V$ implies $U + cW \succsim V + cW$.

Proposition 12 *A social welfare ordering $\succsim$ satisfies order, continuity, Pareto, inequality aversion, separability and general shift covariance if and only if either of the following two cases holds:*

(i) *there exists a vector* $a \in int\Delta^I$ *and a vector* $\lambda \in \mathbb{R}^I_{++}$ *such that* $\succsim$ *can be represented in the form*

$$\Phi(U) = -\sum_{i \in I} a_i e^{-\lambda_i U_i}.$$

We call this class of orderings and representations the generalized exponential class.

(ii) *there exists a vector* $a \in int\Delta^I$ *such that* $\succsim$ *can be represented in the form*

$$\Phi(U) = \sum_{i \in I} a_i U_i.$$

We again call this class of orderings and representations the additive class.

Moreover, in case (i) *a and λ are unique and in case* (ii) *a is unique.*

Proof The necessity of the axioms is routine. We prove sufficiency.

Define $\succsim^\star$ by

$$U \succsim^\star V \iff (W_i U_i/\overline{W})_{i \in I} \succsim (W_i V_i/\overline{W})_{i \in I},$$

where $\overline{W} = \sum_{i \in I} W_i$.

Then $\succsim^\star$ satisfies order, continuity, Pareto, shift covariance, inequality aversion and separability, and it follows from Theorem 11 that one of the following two cases holds:

(i) there exists $\lambda^\star > 0$ and a vector $a^\star \in \mathbb{R}^I_{++}$ such that $\succsim^\star$ can be represented in the form

$$\Phi^\star(U) = -\sum_{i \in I} a_i^\star e^{-\lambda^\star U_i}$$

(ii) there exists a vector $a^\star \in \mathbb{R}^I_{++}$ such that $\succsim$ can be represented in the form

$$\Phi^\star(U) = \sum_{i \in I} a_i^\star U_i$$

In case (i), we have

$$\begin{aligned} U \succsim V &\iff (\overline{W} U_i/W_i)_{i \in I} \succsim^\star (\overline{W} V_i/W_i)_{i \in I} \\ &\iff -\sum_{i \in I} a_i^\star e^{-\lambda^\star \overline{W} U_i/W_i} \geq -\sum_{i \in I} a_i^\star e^{-\lambda^\star \overline{W} V_i/W_i}. \end{aligned}$$

Therefore, by letting $\lambda_i = \lambda^\star \overline{W}/W_i$ for each $i \in I$ and $a = a^\star$, we obtain the representation. In case (ii), we have

$$U \succsim V \iff (\overline{W} U_i/W_i)_{i \in I} \succsim^\star (\overline{W} V_i/W_i)_{i \in I}$$

$$\iff \sum_{i\in I} a_i^\star \overline{W} U_i / W_i \geq - \sum_{i\in I} a_i^\star \overline{W} V_i / W_i$$

Therefore, by letting $a_i = \frac{a_i^\star \overline{W}/W_i}{\sum_{j\in I} a_j^\star \overline{W}/W_j}$ for each $i \in I$ we obtain the representation.

Uniqueness for the exponential case, suppose both $-\sum_{i\in I} a_i e^{-\lambda_i U_i}$ and $-\sum_{i\in I} a_i'$ $e^{-\lambda_i' U_i}$ represent the same ranking. Since they are additive representations of the same ranking, we have cardinal equivalence: there exist constants $C > 0$ and $(D_i)_{i\in I}$ such that

$$-a_i' e^{-\lambda_i' U_i} = -C a_i e^{-\lambda_i U_i} + D_i$$

for all i.

Suppose $D_i \neq 0$. Then as $U_i \to \infty$, we have $-a_i' e^{-\lambda_i' U_i} \to 0$. But $-C a_i e^{-\lambda_i U_i} + D_i \to D_i \neq 0$, a contradiction. Hence $D_i = 0$.

By letting $U_i = 0$, we have $a_i' = C a_i$. Since $\sum_{i\in I} a_i = 1$, we obtain $C = 1$, which implies $a_i' = a_i$. Then it is immediate to see $\lambda_i' = \lambda_i$, which is true for all i.

Uniqueness for the additive case is immediate. □

Comparative inequality aversion

Here we discuss the normative content of the parameters in the social welfare function characterized above. For the exponential class and additive class, one can make a straightforward interpretation of the parameters, which is an analogue of the standard argument about risk aversion: a explains the welfare weights on the individuals and λ explains the degree of inequality aversion.

We extend this interpretation to the generalized exponential class, in which the notion of 'equality' may depend on the different scalings of utilities of different individuals, and hence the degree of inequality aversion may be depend on the individual.

We define comparative inequality aversion as follows.

Definition 5 $\succsim$ is more inequality averse than $\succsim'$ if there exists a vector $W \in \mathbb{R}^I_{++}$ such that for all $c \in \mathbb{R}$ and $U \in \mathbb{R}^I$, $U \succsim cW$ implies $U \succsim' cW$.

Here the ray spanned by W reflects what is regarded as 'equal' by the given social welfare judgment. This includes the standard definition of inequality aversion as the special case in which W is proportional to $\mathbf{1}$.

Proposition 13 *Suppose $\succsim$ and $\succsim'$ fall in the generalized exponential class, where (a, λ) describes $\succsim$ and (a', λ') describes $\succsim'$. Then $\succsim$ is more inequality averse than $\succsim'$ if and only if $a = a'$ and $\lambda = \mu\lambda'$ for some $\mu \geq 1$.*

Proof The 'if' part is routine. We prove the 'only if' part.

Consider the 'marginal rate of substitution' between individual utilities associated with $\succsim$, which is given by

$$MRS(U) = \left(\frac{\lambda_i a_i e^{-\lambda_i U_i}}{\lambda_1 a_1 e^{-\lambda_1 U_1}}\right)_{i\in I\setminus\{1\}}.$$

Note that

$$MRS(cW) = \left(\frac{\lambda_i a_i}{\lambda_1 a_1}\right)_{i\in I\setminus\{1\}}$$

for all c, where $W = (1/\lambda_i)_{i\in I}$. Do the same argument for MRS' and $W' = (1/\lambda_i')_{i\in I}$ associated with $\succsim'$.

For $\succsim$ and $\succsim'$ to be comparable, the indifference curves passing through the origin given by $\succsim$ and $\succsim'$ must be tangent to each other at the origin, for otherwise they cross and the comparison fails along any ray. Thus, we conclude that $MRS(\mathbf{0}) = MRS'(\mathbf{0})$. In other words,

$$\frac{\lambda_i a_i}{\lambda_1 a_1} = \frac{\lambda_i' a_i'}{\lambda_1' a_1'},$$

for all $i \in I\setminus\{1\}$. It follows that $MRS(cW) = MRS'(c'W')$ for all $c, c' > 0$.

Then it must be that W and W' span the same ray passing through the origin. Suppose not. Then the indifference curves given by $\succsim$ are parallel along the ray spanned by W yielding the same vector of MRS, and those given by $\succsim'$ are parallel along the ray spanned by W' yielding the same vector of MRS', while keeping $MRS = MRS'$. Hence, by convexity, they must cross somewhere between the two rays and cannot be tangent to each other anywhere.

Therefore, $\lambda = \mu\lambda'$ for some $\mu > 0$. Since MRS and MRS' must be the same along the ray, we have $a = a'$. By comparing the second-order derivatives, we obtain $\mu \geq 1$. □

Now it is immediate to see the following claim. Note that in the exponential class, λ reduces to a scaler.

Corollary 7 *Suppose $\succsim$ and $\succsim'$ fall in the exponential class, where (a, λ) describes $\succsim$ and (a', λ') describes $\succsim'$. Then $\succsim$ is more inequality averse than $\succsim'$ if and only if $a = a'$ and $\lambda \geq \lambda'$.*

Appendix 2: Proofs for Sect. 2

Proof of Proposition 1

Order obvious.

DU-Pareto let $\sum_{\tau=t+1}^{\infty} \beta^{\tau-t} u_{i\tau} \geq \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} v_{i\tau}$ for all $i \in I$. Then $u_{it} + \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau} \geq u_{it} + \sum_{\tau=t}^{\infty} \beta^{\tau-t} v_{i\tau}$ for all $i \in I$. Since DU-Pareto was assumed to hold for $\succsim_{\mathbf{u}^{t-1}}$, we have $(u_t, \mathbf{u}_{t+1}) \succsim_{\mathbf{u}^{t-1}} (u_t, \mathbf{v}_{t+1})$. By dynamic consistency, $\mathbf{u}_{t+1} \succsim_{(\mathbf{u}^{t-1}, u_t)} \mathbf{v}_{t+1}$. The strict case is proved similarly.

DU-continuity let $\{U_{t+1}^{\nu}\}$ be a sequence in $\mathbb{R}^I$ converging to U_{t+1}, and let $\{V_{t+1}^{\nu}\}$ be a sequence in $\mathbb{R}^I$ converging to V_{t+1}. Suppose $U_{t+1}^{\nu} \succsim^*_{\mathbf{u}^{t-1}, u_t} V_{t+1}^{\nu}$ for all ν. By dynamic consistency, we have $(u_{it} + \beta U_{i,t+1}^{\nu})_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta V_{i,t+1}^{\nu})_{i\in I}$ for all ν. Since DU-continuity was assumed to hold for $\succsim_{\mathbf{u}^{t-1}}$, we have $(u_{it} + \beta U_{i,t+1})_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta V_{i,t+1})_{i\in I}$. By dynamic consistency, $U_{t+1} \succsim^*_{\mathbf{u}^{t-1}, u_t} V_{t+1}$.

DU-inequality aversion let $U_{t+1} \sim^*_{\mathbf{u}^{t-1}, u_t} V_{t+1}$. By dynamic consistency, we have $(u_{it} + \beta U_{i,t+1})_{i\in I} \sim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta V_{i,t+1})_{i\in I}$. Since DU-inequality aversion was assumed to hold for $\succsim_{\mathbf{u}^{t-1}}$, we have $(u_{it} + \beta(cU_{i,t+1} + (1-c)V_{i,t+1}))_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta U_{i,t+1})_{i\in I}$. By dynamic consistency, $cU_{t+1} + (1-c)V_{t+1} \succsim^*_{\mathbf{u}^{t-1}, u_t} U_{t+1}$.

DU-separability let $(U_{J,t+1}, U_{-J,t+1}) \succsim^*_{\mathbf{u}^{t-1}, u_t} (V_{J,t+1}, U_{-J,t+1})$. By dynamic consistency, this holds if and only if $(u_{Jt} + \beta U_{Jt}, u_{-Jt} + \beta U_{-Jt}) \succsim^*_{\mathbf{u}^{t-1}} (u_{Jt} + \beta V_{Jt}, u_{-Jt} + \beta U_{-Jt})$. Since DU-separability was assumed to hold for $\succsim^*_{\mathbf{u}^{t-1}}$, this holds if and only if $(u_{Jt} + \beta U_{Jt}, u_{-Jt} + \beta V_{-Jt}) \succsim^*_{\mathbf{u}^{t-1}} (u_{Jt} + \beta V_{Jt}, u_{-Jt} + \beta V_{-Jt})$. By dynamic consistency, this holds if and only if $(U_{J,t+1}, V_{-J,t+1}) \succsim^*_{\mathbf{u}^{t-1}, u_t} (V_{J,t+1}, V_{-J,t+1})$.

DU-shift covariance let $U_{t+1} \succsim^*_{\mathbf{u}^{t-1}, u_t} V_{t+1}$. By dynamic consistency this holds if and only if $(u_{it} + \beta U_{it})_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta V_{it})_{i\in I}$. Since DU-shift covariance was assumed to hold for $\succsim_{\mathbf{u}^{t-1}}$, this holds if and only if $(u_{it} + \beta(U_{it} + c))_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta(V_{it} + c))_{i\in I}$. By dynamic consistency, this holds if and only if $U_{t+1} + c\mathbf{1} \succsim^*_{\mathbf{u}^{t-1}, u_t} V_{t+1} + c\mathbf{1}$.

Proof of Proposition 2

It follows from the fact that $\succsim^*_{\mathbf{u}^{t-1}}$ satisfies all the conditions in Proposition 11.

Proof of Theorem 1

Note that for the exponential class, we have

$$\begin{aligned}\Phi(\mathbf{u}_0|\emptyset) &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{\infty}\beta^{\tau}u_{i\tau}\right)\\ &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\sum_{\tau=0}^{t-1}\beta^{\tau}u_{i\tau}\right)\exp\left(-\lambda(\emptyset)\beta^{t}\sum_{\tau=t}^{\infty}\beta^{\tau-t}u_{i\tau}\right),\end{aligned}$$

which under dynamic consistency must yield the same ranking over $\mathbf{u}_t$ as $\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1})\exp\left(-\lambda(\mathbf{u}^{t-1})\sum_{\tau=t}^{\infty}\beta^{\tau-t}u_{i\tau}\right)$ does. Note that it is obviously impossible to switch from the exponential class to the additive class through updating, and vice versa.

For the additive class, we have

$$\begin{aligned}\Phi(\mathbf{u}_0|\emptyset) &= \sum_{i\in I} a_i(\emptyset)\sum_{\tau=0}^{\infty}\beta^{\tau}u_{i\tau}\\ &= \sum_{i\in I} a_i(\emptyset)\sum_{\tau=0}^{t-1}\beta^{\tau}u_{i\tau} + \beta^t\sum_{i\in I} a_i(\emptyset)\sum_{\tau=t}^{\infty}\beta^{\tau-t}u_{i\tau},\end{aligned}$$

which under dynamic consistency must yield the same ranking over $\mathbf{u}_t$ as $\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = \sum_{i\in I} a_i(\mathbf{u}^{t-1})\sum_{\tau=t}^{\infty}\beta^{\tau-t}u_{i\tau}$ does.

Proof of Proposition 4

We only prove that DU-general shift covariance is recurrent, since the rest is similar to the case of homogeneous discounting.

Let $U_{t+1} \succsim^*_{\mathbf{u}^{t-1},u_t} V_{t+1}$. By dynamic consistency this holds if and only if $(u_{it} + \beta_i U_{it})_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta_i V_{it})_{i\in I}$. Since DU-general shift covariance was assumed to hold for $\succsim_{\mathbf{u}^{t-1}}$, for some $W_{\mathbf{u}^{t-1}}$ this holds if and only if $(u_{it} + \beta_i(U_{it} + cW_{i,\mathbf{u}^{t-1}}/\beta_i)_{i\in I} \succsim^*_{\mathbf{u}^{t-1}} (u_{it} + \beta_i(V_{it} + cW_{i,\mathbf{u}^{t-1}}/\beta_i))_{i\in I}$. By dynamic consistency, this holds if and only if $U_{t+1} + cW_{\mathbf{u}^{t-1},u_t} \succsim^*_{\mathbf{u}^{t-1},u_t} V_{t+1} + cW_{\mathbf{u}^{t-1},u_t}$, where $W_{\mathbf{u}^{t-1},u_t} = (W_{i,\mathbf{u}^{t-1},u_t}/\beta_i)_{i\in I}$.

Proof of Proposition 5

It follows from the fact that $\succsim^*_{\mathbf{u}^{t-1}}$ satisfies all the conditions in Proposition 12.

Proof of Theorem 2

For the generalized exponential class, we have

$$\begin{aligned}\Phi(\mathbf{u}_0|\emptyset) &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\sum_{\tau=0}^{\infty}\beta_i^{\tau}u_{i\tau}\right)\\ &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\sum_{\tau=0}^{t-1}\beta_i^{\tau}u_{i\tau}\right)\exp\left(-\lambda_i(\emptyset)\beta_i^t\sum_{\tau=t}^{\infty}\beta_i^{\tau-t}u_{i\tau}\right),\end{aligned}$$

which under dynamic consistency yields the same ranking over $\mathbf{u}_t$ as $\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = -\sum_{i\in I} a_i(\mathbf{u}^{t-1})\exp\left(-\lambda_i(\mathbf{u}^{t-1})\sum_{\tau=t}^{\infty}\beta_i^{\tau-t}u_{i\tau}\right)$ does.

For the additive class, we have

$$\Phi(\mathbf{u}_0|\emptyset) = \sum_{i\in I} a_i(\emptyset)\sum_{\tau=0}^{\infty}\beta_i^{\tau}u_{i\tau}$$

$$= \sum_{i \in I} a_i(\emptyset) \sum_{\tau=0}^{t-1} \beta_i^\tau u_{i\tau} + \sum_{i \in I} a_i(\emptyset) \beta_i^t \sum_{\tau=t}^{\infty} \beta_i^{\tau-t} u_{i\tau},$$

which under dynamic consistency yields the same ranking over $\mathbf{u}_t$ as $\Phi(\mathbf{u}_t|\mathbf{u}^{t-1}) = \sum_{i \in I} a_i(\mathbf{u}^{t-1}) \sum_{\tau=t}^{\infty} \beta_i^{\tau-t} u_{i\tau}$ does.

Appendix 3: Proofs for Sect. 3

Proof of Proposition 6

Order obvious.

EU-Pareto let $\int_{E_{t+1}} u_i(s)p(ds|E_{t+1}) \geq \int_{E_{t+1}} v_i(s)p(ds|E_{t+1})$ for all $i \in I$ for all $i \in I$. Then we have

$$\int_{E_t \setminus E_{t+1}} u_i(s)p(ds|E_t) + \int_{E_{t+1}} u_i(s)p(ds|E_t)$$
$$\geq \int_{E_t \setminus E_{t+1}} u_i(s)p(ds|E_t) + \int_{E_{t+1}} v_i(s)p(ds|E_t)$$

for all $i \in I$. Since EU-Pareto is assumed to hold for $\succsim_{\mathbf{u}_{-E_t}}$, we have $(\mathbf{u}_{E_{t+1}}, \mathbf{u}_{E_t \setminus E_{t+1}}) \succsim_{\mathbf{u}_{-E_t}} (\mathbf{v}_{E_{t+1}}, \mathbf{u}_{E_t \setminus E_{t+1}})$. By dynamic consistency, $\mathbf{u}_{E_{t+1}} \succsim_{\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}}} \mathbf{v}_{E_{t+1}}$. The strict case is proved similarly.

EU-continuity let $\{U_{E_{t+1}}^\nu\}$ be a sequence in $\mathbb{R}^I$ converging to $U_{E_{t+1}}$, and let $\{V_{E_{t+1}}^\nu\}$ be a sequence in $\mathbb{R}^I$ converging to $V_{E_{t+1}}$. Suppose $U_{E_{t+1}}^\nu \succsim^*_{\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}}} V_{E_{t+1}}^\nu$ for all ν. By dynamic consistency, we have

$$\left(\int_{E_t \setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t) U_{i,E_{t+1}}^\nu \right)_{i \in I}$$
$$\succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t \setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t) V_{i,E_{t+1}}^\nu \right)_{i \in I}$$

for all ν. Since EU-continuity was assumed to hold for $\succsim_{\mathbf{u}_{-E_t}}$, we have

$$\left(\int_{E_t \setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t) U_{i,E_{t+1}} \right)_{i \in I}$$
$$\succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t \setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t) V_{i,E_{t+1}} \right)_{i \in I}.$$

By dynamic consistency, $U_{E_{t+1}} \succsim^*_{\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}}} V_{E_{t+1}}$.

EU-inequality aversion let $U_{E_{t+1}} \sim^*_{\mathbf{u}_{-E_t}, \mathbf{u}_{E_t \setminus E_{t+1}}} V_{E_{t+1}}$. By dynamic consistency, we have

$$\left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)U_{i,E_{t+1}}\right)_{i\in I} \sim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{i,E_{t+1}}\right)_{i\in I}.$$

Since EU-inequality aversion was assumed to hold for $\succsim_{\mathbf{u}_{-E_t}}$, we have

$$\left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)(cU_{i,E_{t+1}} + (1-c)V_{i,E_{t+1}})\right)_{i\in I} \succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{i,E_{t+1}}\right)_{i\in I}.$$

By dynamic consistency, $cU_{E_{t+1}} + (1-c)V_{E_{t+1}} \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} U_{E_{t+1}}$.

EU-separability let $(U_{J,E_{t+1}}, U_{-J,E_{t+1}}) \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} (V_{J,E_{t+1}}, U_{-J,E_{t+1}})$. By dynamic consistency, this holds if and only if

$$\left(\int_{E_t\setminus E_{t+1}} u_J(s)p(ds|E_t) + p(E_{t+1}|E_t)U_{J,E_{t+1}}, \int_{E_t\setminus E_{t+1}} u_{-J}(s)p(ds|E_t) + p(E_{t+1}|E_t)U_{-J,E_{t+1}}\right) \succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_J(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{J,E_{t+1}}, \int_{E_t\setminus E_{t+1}} u_{-J}(s)p(ds|E_t) + p(E_{t+1}|E_t)U_{-J,E_{t+1}}\right).$$

Since separability was assumed to hold for $\succsim_{\mathbf{u}_{-E_t}}$, this holds if and only if

$$\left(\int_{E_t\setminus E_{t+1}} u_J(s)p(ds|E_t) + p(E_{t+1}|E_t)U_{J,E_{t+1}}, \int_{E_t\setminus E_{t+1}} u_{-J}(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{-J,E_{t+1}}\right) \succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_J(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{J,E_{t+1}}, \int_{E_t\setminus E_{t+1}} u_{-J}(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{-J,E_{t+1}}\right)$$

By dynamic consistency, this holds if and only if $(U_{J,E_{t+1}}, V_{-J,E_{t+1}}) \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} (V_{J,E_{t+1}}, V_{-J,E_{t+1}})$.

EU-shift covariance let $U_{E_{t+1}} \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} V_{E_{t+1}}$. By dynamic consistency, this holds if and only if

$$\left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)U_{i,E_{t+1}}\right)_{i\in I}$$
$$\succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)V_{i,E_{t+1}}\right)_{i\in I}.$$

Since EU-shift covariance was assumed to hold for $\succsim_{\mathbf{u}_{-E_t}}$, this holds if and only if

$$\left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)(U_{i,E_{t+1}}+c)\right)_{i\in I}$$
$$\succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_i(s)p(ds|E_t) + p(E_{t+1}|E_t)(V_{i,E_{t+1}}+c)\right)_{i\in I}.$$

By dynamic consistency, this holds if and only if $U_{E_{t+1}}+c\mathbf{1} \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} V_{E_{t+1}}+c\mathbf{1}$.

Proof of Proposition 7

It follows from the fact that $\succsim^*_{\mathbf{u}_{-E_t}}$ satisfies all the conditions in Proposition 11.

Proof of Theorem 3

Note that for the exponential class, we have

$$\begin{aligned}\Phi(\mathbf{u}_\Omega|\emptyset) &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\int_\Omega u_i(s)p(ds)\right)\\ &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p(ds)\right)\\ &\quad\times\exp\left(-\lambda(\emptyset)p(E_t)\int_{E_t} u_i(s)p(ds|E_t)\right),\end{aligned}$$

which under dynamic consistency must yield the same ranking over $\mathbf{u}_{E_t}$ as $\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t})\exp\left(-\lambda(\mathbf{u}_{-E_t})\int_{E_t} u_i(s)p(ds|E_t)\right)$ does.

For the additive class, we have

$$\begin{aligned}\Phi(\mathbf{u}_\Omega|\emptyset) &= \sum_{i\in I} a_i(\emptyset)\int_\Omega u_i(s)p(ds)\\ &= \sum_{i\in I} a_i(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p(ds) + p(E_t)\sum_{i\in I} a_i(\emptyset)\int_{E_t} u_i(s)p(ds|E_t),\end{aligned}$$

which under dynamic consistency must yield the same ranking over $\mathbf{u}_{E_t}$ as $\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = \sum_{i\in I} a_i(\mathbf{u}_{-E_t}) \int_{E_t} u_i(s)p(ds|E_t)$ does.

Proof of Proposition 9

We only prove that EU-general shift covariance is recurrent, since the rest is similar to the case of homogeneous beliefs.

Let $U_{E_{t+1}} \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} V_{E_{t+1}}$. By dynamic consistency this holds if and only if

$$\left(\int_{E_t\setminus E_{t+1}} u_i(s)p_i(ds|E_t) + p_i(E_{t+1}|E_t)U_{i,E_{t+1}}\right)_{i\in I} \succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_i(s)p_i(ds|E_t) + p_i(E_{t+1}|E_t)V_{i,E_{t+1}}\right)_{i\in I}.$$

Since EU-general shift covariance was assumed to hold for $\succsim_{\mathbf{u}_{-E_t}}$, for some $W_{\mathbf{u}_{-E_t}}$ this holds if and only if

$$\left(\int_{E_t\setminus E_{t+1}} u_i(s)p_i(ds|E_t) + p_i(E_{t+1}|E_t)\left(U_{i,E_{t+1}} + c\frac{W_{\mathbf{u}_{-E_t}}}{p_i(E_{t+1}|E_t)}\right)\right)_{i\in I} \succsim^*_{\mathbf{u}_{-E_t}} \left(\int_{E_t\setminus E_{t+1}} u_i(s)p_i(ds|E_t)+p_i(E_{t+1}|E_t)\left(V_{i,E_{t+1}} + c\frac{W_{\mathbf{u}_{-E_t}}}{p_i(E_{t+1}|E_t)}\right)\right)_{i\in I}.$$

By dynamic consistency, this holds if and only if $U_{E_{t+1}}+cW_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} \succsim^*_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} V_{E_{t+1}} + cW_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}}$, where $W_{\mathbf{u}_{-E_t},\mathbf{u}_{E_t\setminus E_{t+1}}} = \left(W_{i,\mathbf{u}_{-E_t}}/p_i(E_{t+1}|E_t)\right)_{i\in I}$.

Proof of Proposition 10

It follows from the fact that $\succsim^*_{\mathbf{u}_{-E_t}}$ satisfies all the conditions in Proposition 12.

Proof of Theorem 4

For the generalized exponential class, we have

$$\begin{aligned}\Phi(\mathbf{u}_\Omega|\emptyset) &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\int_\Omega u_i(s)p_i(ds)\right)\\ &= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\left(\int_{\Omega\setminus E_t} u_i(s)p_i(ds) + p_i(E_t)\int_{E_t} u_i(s)p_i(ds|E_t)\right)\right)\end{aligned}$$

$$= -\sum_{i\in I} a_i(\emptyset)\exp\left(-\lambda_i(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p_i(ds)\right)$$
$$\times\exp\left(-\lambda_i(\emptyset)p_i(E_t)\int_{E_t} u_i(s)p_i(ds|E_t)\right),$$

which under dynamic consistency must yield the same ranking over $\mathbf{u}_{E_t}$ as $\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = -\sum_{i\in I} a_i(\mathbf{u}_{-E_t})\exp\big(-\lambda_i(\mathbf{u}_{-E_t})\int_{E_t} u_i(s)p_i(ds|E_t)\big)$ does.

For the additive class, we have

$$\Phi(\mathbf{u}_\Omega|\emptyset) = \sum_{i\in I} a_i(\emptyset)\int_\Omega u_i(s)p_i(ds)$$
$$= \sum_{i\in I} a_i(\emptyset)\left(\int_{\Omega\setminus E_t} u_i(s)p_i(ds) + p_i(E_t)\int_{E_t} u_i(s)p_i(ds|E_t)\right)$$
$$= \sum_{i\in I} a_i(\emptyset)\int_{\Omega\setminus E_t} u_i(s)p_i(ds) + \sum_{i\in I} a_i(\emptyset)p_i(E_t)\int_{E_t} u_i(s)p_i(ds|E_t),$$

which under dynamic consistency must yield the same ranking over $\mathbf{u}_{E_t}$ as $\Phi(\mathbf{u}_{E_t}|\mathbf{u}_{-E_t}) = \sum_{i\in I} a_i(\mathbf{u}_{-E_t})\int_{E_t} u_i(s)p_i(ds|E_t)$ does.

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