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Intertemporal poverty comparisons

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Abstract The paper deals with poverty orderings when multidimensional attributes exhibit some degree of comparability. The paper focuses on an important special case of this, that is, comparisons of poverty that make use of incomes at different time periods. The ordering criteria extend the power of earlier multidimensional dominance tests by making (reasonable) assumptions on the relative marginal contributions of each time dimension to poverty. *Inter alia*, this involves drawing on natural symmetry and asymmetry assumptions as well as on the mean/variability framework commonly used in the risk literature. The resulting procedures make it possible to check for the robustness of poverty comparisons to choices of intertemporal aggregation procedures and to areas of intertemporal poverty frontiers. The results are illustrated using a rich sample of 23 European countries over 2006–2009.

1 Introduction

This paper deals with the problem of making general comparisons of well-being when well-being is measured in multiple dimensions. We note at the outset that much of the literature on the measurement of well-being incorporates multiple dimensional indicators by adding them up, such as when food and non-food expenditures are aggregated to compute total expenditures and assess monetary poverty—essentially returning to a univariate analysis. In some cases, these procedures may be perfectly appropri-

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ate. In other cases, however, it could be that the specific aggregation rules used to sum up the dimensions may be deemed somewhat arbitrary or objectionable, especially when the dimensions cannot be considered evidently comparable or perfectly substitutable in generating overall well-being. This then leaves open the possibility that two equally admissible rules for aggregating across several dimensions of well-being could lead to contradictory rankings of well-being and/or conclusions for policy guidance.

One way to address this problem is through the use of multidimensional dominance procedures, as found in Atkinson and Bourguignon (1982), Bourguignon (1989), or Duclos et al. (2006). These are indeed useful procedures that make relatively few assumptions on the structure of the framework used to measure and compare well-being. Their first-order multidimensional dominance comparisons suppose, for instance, that overall well-being should increase with dimensional well-being, but that the importance of these increases cannot be ranked across dimensions. Such comparisons do not impose the assumption of full comparability on the dimensional indicators of well-being. Because of this, they can generate rather robust multidimensional comparisons of well-being from a normative point of view.

These weak assumptions come, however, at the cost of a limited power to order distributions of multidimensional well-being. It would seem that they could be strengthened in several settings. One such setting is when the dimensional indicators have values that are comparable. Examples include the measurement of household poverty, using the incomes of the members of the same household as dimensions, but without assuming perfect income pooling; the measurement of child well-being, using the health or the nutritional status of children of the same household as dimensions, but without assuming that there is perfect substitutability of such status across the children; or the measurement of household education, using the education of members of the same household as dimensions, but again without assuming that for measurement purposes we can impose perfect substitutability of educational achievements across members of the same household.

We build in this paper on the natural cardinality of multi-period incomes, which makes it possible to compare them in more specific ways than has been done until now. Thus, although the methods we develop have broader applicability (reference will be made to this from time to time), the paper focuses on intertemporal poverty comparisons, that is, comparisons of poverty over different time periods.[1]

In contrast to some of the earlier work,[2] the paper's objective is to develop procedures for checking for whether intertemporal poverty comparisons are robust to aggregation procedures and to choices of multi-period poverty frontiers. Intertemporal poverty comparisons can thereby be made "poverty-measure robust," namely, valid for broad classes of aggregation rules across individuals and also for broad

[1] Though intertemporal poverty is the most widely used name for that concept, it is sometimes also called "longitudinal poverty" (Busetta et al. 2011; Busetta and Mendola 2012) or "lifetime poverty" (Hoy et al. 2012) in the literature.

[2] See for instance Foster (2009), Calvo and Dercon (2009), Hoy et al. (2012), Duclos et al. (2010), Bossert et al. (2012), Busetta and Mendola (2012), and Dutta et al. (2013), but analogously to Hoy and Zheng (2011), though within a rather different—time-additive—framework.

classes of aggregation rules across time. The comparisons can thereby also be made "poverty-line robust," in the sense of being valid for *any* intertemporal poverty frontier over broad areas of poverty frontiers. Given the difficulty involved in choosing poverty frontiers and poverty indices, and given the frequent sensitivity of poverty comparisons to these choices, this would appear to be a potentially useful contribution.

One of the first conceptual challenges in analyzing intertemporal poverty is deciding who is "time poor." Measuring well-being across two time periods, say, a person can be considered intertemporally poor if her income falls below an income poverty line in *both* periods or in *either* period. This can be defined respectively as *intersection* and *union* definitions of intertemporal poverty. The procedures that we develop are valid for both definitions as well as for any intermediate definition for which the poverty line at one time period is a decreasing function of income at the other periods. Note that the procedures assume 'substitutability' of incomes, which says that an increase in one period's income makes poverty less sensitive to changes in another period's income. Though this is probably a natural assumption for intertemporal poverty, complementarity of temporal incomes cannot be entirely ruled out.

The paper also considers the role of mobility in the measurement of intertemporal poverty, both across time and across individuals. With the increased availability of longitudinal data sets, it is now well known that there are often significant movements in and out of poverty, as well as within poverty itself. Such income mobility has at least two welfare impacts.[3] The first is to make the distribution of "permanent" incomes across individuals more equal than the periodic distribution of incomes. Traditional measures of poverty that are averse to inequality across individuals will therefore generally tend to be lower when based on permanent incomes. Mobility also induces intertemporal variability. If individuals would prefer their incomes to be distributed as equally as possible across time (because they are risk averse or because they have limited access to credit and hence cannot smooth their consumption), then income mobility will also decrease well-being and thus increase poverty. This is most likely valid in an *ex ante* sense, but arguably also in an *ex post* one, if observed *ex post* variability is a good proxy for the *ex ante* risk borne by individuals. The framework developed below implicitly takes into account that possible trade-off between the benefit of across-individual mobility and the cost of across-time variability.

The rest of the paper is organized in the following manner. The next section elaborates on Duclos et al.'s (2006) multidimensional dominance criteria so as to extend the power of their procedures without making the usual higher-order dominance assumptions. Increases in the power of dominance tests are traditionally obtained by emphasizing the importance of attribute-specific inequality across individuals. Section 2 uses instead across-attribute symmetry and asymmetry properties and introduces assumptions on how permutations of multi-period income profiles should affect poverty.

[3] See, in the recent literature, Atkinson et al. (2002), Chaudhuri et al. (2002), Ligon and Schechter (2003), Cruces and Wodon (2003), Bourguignon et al. (2004), Christiaensen and Subbarao (2004), and Kamanou and Morduch (2004).

Since it is often supposed that individuals prefer smoothed income patterns, Sect. 3 also explicitly takes into account intertemporal inequalities. This is done by drawing, in a flexible measurement setting, on the popular mean/variability framework that is used in the risk literature to measure the cost of risk and assess behavior under such risk. The links between the classes of poverty indices described in Sects. 2 and 3 are highlighted in Sect. 4.

The results are illustrated in Sect. 5 using a rich set of data on 23 European countries drawn from the European Union Survey on Income and Living Conditions. The results indicate that about 63 % of the 253 possible pairs of European countries can be ordered using an assumption of normative neutrality towards intertemporal income variability (that is, perfect pooling of incomes at the individual level). An assumption of no aversion to intertemporal income variability is, however, a strong assumption. Relaxing it and allowing for intertemporal variability to matter in a flexible measurement framework reduces the proportion of ranked pairs to 46 %.

Strengthening the measurement framework by imposing early poverty or loss aversion sensitivity increases the ordering power to around 50 % of the pairwise comparisons. Adding intertemporal symmetry (which says that the cost of early poverty is no more or no less important than the cost of loss aversion) further increases the number of orderings to 55 % of the total number of pairs. This is not far from the 63 % power obtained in the initial context in which income variability is ignored, suggesting that the empirical ordering cost of using a flexible poverty measurement framework may not be large. It is also not far from the 62 % of pairwise comparisons that can be ordered using general second-order multidimensional dominance tests. These second-order dominance tests require, however, full comparability of the different attributes used to measure welfare, a requirement that is not needed for first-order symmetric/asymmetric dominance tests.

Section 5 also reports that the popular mean/variability framework for thinking about intertemporal welfare does not have empirical strength in our data. This suggests that this framework may not be as empirically useful for making intertemporal welfare comparisons as Duclos et al.'s (2006) basic framework. Section 6 concludes.

2 Intertemporal poverty

Let overall well-being be a function of two indicators, x_1 and x_2, and be given by $\lambda(x_1, x_2)$.[4] This function is a member of Λ, defined as the set of continuous and non-decreasing functions of x_1 and x_2. For our purposes, we will typically think of x_t as income at time t; the vector (x_1, x_2) is called an income profile. For instance, x_1 may denote an individual's income during his working life, while x_2 could be his income when retired. Without loss of generality, we assume that incomes are defined on the set of positive real numbers, so that $\lambda : \Re^2_+ \rightarrow \Re$.

Similarly to Duclos et al. (2006), we assume that an unknown poverty frontier separates the poor from the rich. We can think of this frontier as a set of points at

[4] For expositional simplicity, we focus on the case of two dimensions of individual well-being. Extensions to cases with more than two dimensions are discussed in footnotes.

which the well-being of an individual is precisely equal to a "poverty level" of well-being, and below which individuals are in poverty. This frontier is assumed to be defined implicitly by a locus of the form $\lambda(x_1, x_2) = 0$, and is analogous to the usual downward-sloping indifference curves in the (x_1, x_2) space. Intertemporal poverty is then defined by states in which $\lambda(x_1, x_2) \leq 0$, and the poverty domain is consequently obtained as:

$$\Gamma(\lambda) := \left\{(x_1, x_2) \in \Re^2_+ \,|\, \lambda(x_1, x_2) \leq 0\right\}. \tag{1}$$

Let the joint cumulative distribution function of x_1 and x_2 be denoted by $F(x_1, x_2)$. For analytical simplicity, we focus on classes of additive bidimensional poverty indices, which are the kernels of broader classes of subgroup-consistent bidimensional poverty indices.[5] Such bidimensional indices can be defined generally as $P(\lambda)$:

$$P(\lambda) = \iint_{\Gamma(\lambda)} \pi(x_1, x_2; \lambda)\, dF(x_1, x_2), \tag{2}$$

where $\pi(x_1, x_2; \lambda)$ is the contribution to overall poverty of an individual whose income at period 1 and 2 is respectively x_1 and x_2. The well-known "focus axiom" entails that:

$$\pi(x_1, x_2; \lambda) \begin{cases} \geq 0 & \text{if } (x_1, x_2) \in \Gamma(\lambda), \\ = 0 & \text{otherwise.} \end{cases} \tag{3}$$

This says that someone contributes to poverty only if his income profile is in the poverty domain.

Our definitions of both the poverty domain and the poverty indices are consistent with different types of aggregation procedures. In a recent paper, Ravallion (2011) contrasted two different approaches to aggregation at the individual level, that is, the "attainment aggregation" and the "deprivation aggregation" approaches. With the first approach, the values of the different attributes are blended together into a single well-being value,[6] the resulting value then being compared to some poverty threshold. In the context of intertemporal poverty, that approach is used for instance by Rodgers and Rodgers (1993a) and Jalan and Ravallion (1998) for the measurement of chronic poverty. With the second "deprivation aggregation" approach, the extent of deprivations in each dimension is first assessed and those deprivations are then aggregated into a composite index. This is exemplified by Foster (2009); Hoy and Zheng (2011), Duclos et al. (2010) and Bossert et al. (2012). The first approach generally allows deprivations in some dimension to be compensated by "surpluses" in some other dimension; such compensation effects are often not allowed with the deprivation aggregation approach.[7] The respective merits of each approach are discussed notably in Ravallion (2011) and Alkire and Foster (2011b). This paper's framework encompasses both approaches.

[5] For the unidimensional case, see Foster and Shorrocks (1991).

[6] Ravallion (2011) only deals with the case of linear aggregation using a fixed set of prices, but the use of well-being functions like λ could also be considered.

[7] Notable exceptions are Zheng (2012) and Dutta et al. (2013).

For ease of exposition, let the derivatives of π in (3) be defined as:

- $\pi^{(i)}(a, b)$, $i = 1, 2$, for the first-order derivative of π with respect to its ith argument,
- and as $\pi^{(a)}(a, b)$, for the first-order derivative of π with respect to the variable a, so that $\pi^{(u)}\big(a(u), b(u)\big) = \pi^{(1)}\big(a(u), b(u)\big)\frac{\partial a}{\partial u} + \pi^{(2)}\big(a(u), b(u)\big)\frac{\partial b}{\partial u}$.

Then, define the class $\ddot{\Pi}(\lambda^+)$ of poverty indices $P(\lambda)$ as:

$$\ddot{\Pi}(\lambda^+) = \left\{ P(\lambda) \left| \begin{array}{l} \Gamma(\lambda) \subset \Gamma(\lambda^+), \\ \pi(x_1, x_2; \lambda) = 0, \text{ whenever } \lambda(x_1, x_2) = 0, \\ \pi^{(1)}(x_1, x_2; \lambda) \leq 0 \quad \text{and } \pi^{(2)}(x_1, x_2; \lambda) \leq 0 \quad \forall x_1, x_2, \\ \pi^{(1,2)}(x_1, x_2; \lambda) \geq 0, \quad \forall x_1, x_2. \end{array} \right. \right\} \tag{4}$$

The class $\ddot{\Pi}(\lambda^+)$ includes *inter alia* the families of bidimensional poverty indices proposed by Chakravarty et al. (1998), Tsui (2002), and Chakravarty et al. (2008), as well as some members of the family of indices introduced by Bourguignon and Chakravarty (2003). The first condition in (4) indicates that the poverty domain $\Gamma(\lambda)$ for each $P(\lambda)$ should lie within the domain defined by λ^+ (λ^+ then representing the maximum admissible poverty frontier). The second condition in (4) says that the poverty measures are continuous along the poverty frontier. Continuity is often assumed in order to prevent small measurement errors from resulting in non-marginal variations of the poverty index.[8] The third condition in (4) is a monotonicity condition, *i.e.*, a condition that says that an income increment in any period should never increase poverty.[9]

The fourth and last condition in (4) says that poverty should not decrease after a "correlation increasing switch", an axiom introduced by Atkinson and Bourguignon (1982). It is thus supposed that the poverty benefit of an income increment at period 1 (2) decreases with the income level at period 2 (1). Intuitively, this also says that a permutation of the incomes of two poor individuals during a given period should not decrease poverty if one of them then becomes more deprived than the other in both periods. This can be seen on Fig. 1, where it is supposed that profile I moves to profile I', and profile J moves to profile J'. This permutation does not change the distribution of incomes at each time period, but it does increase the correlation of incomes across individuals. The axiom of "non-decreasing poverty after a correlation increasing switch" says that poverty should not fall after this permutation.

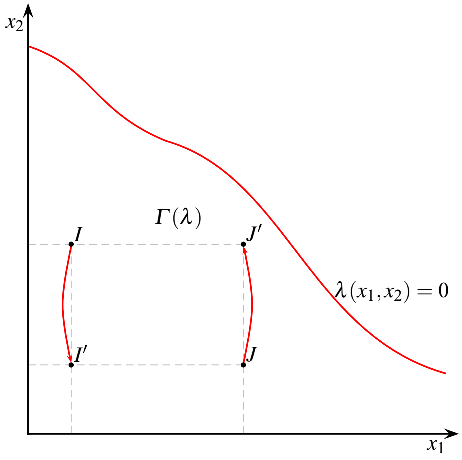
Note that the correlation increasing axiom implies that incomes at time 1 and 2 are substitutes in producing overall well-being, which would seem to be a natural assumption.[10] In an alternative context (say) of household poverty being a function

[8] This continuity assumption therefore precludes most members of the Alkire and Foster (2011a) family of poverty indices from being part of $\ddot{\Pi}(\lambda^+)$.

[9] As noted in Duclos et al. (2006), we must also have that $\pi^{(1)} < 0$, $\pi^{(2)} < 0$, and $\pi^{(1,2)} > 0$ over some ranges of x_1 and x_2 for the indices to be non-degenerate.

[10] To our knowledge, the literature has failed until now to provide robust rankings of poverty across areas of poverty frontiers when complementarity of dimensions is allowed—such robustness is, however, not needed in the context of social welfare comparisons, see Atkinson and Bourguignon (1982) for instance.

Fig. 1 An increase in periodic income correlation cannot decrease intertemporal poverty



of the levels of child education in each household, the correlation-increasing axiom says that total poverty is lowered when child education is more evenly spread across households.[11]

A bidimensional stochastic dominance surface can now be defined using:

$$P^{\alpha,\beta}(z_u, z_v) := \int_0^{z_u} \int_0^{z_v} (z_u - u)^{\alpha-1} (z_v - v)^{\beta-1} \, dF(u, v), \tag{5}$$

where α and β refer to the dominance order in each dimension and where z_u and z_v are thresholds analogous to the usual poverty lines in the poverty literature. The present paper focusses on first-order dominance, so that α and β are set equal to 1. The function $P^{1,1}(z_u, z_v)$ is the *intersection* bidimensional poverty headcount index: it is the population of individuals whose incomes at time 1 and 2 are below z_u and z_v, respectively.

Duclos et al. (2006) then show:

Proposition 1 (Duclos et al. 2006)

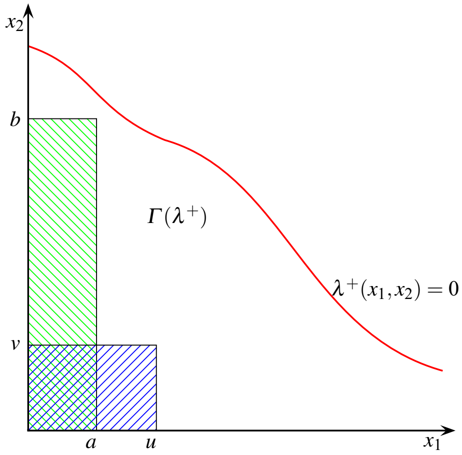
$$P_A(\lambda) \geq P_B(\lambda), \; \forall P(\lambda) \in \ddot{\Pi}(\lambda^+), \tag{6}$$

$$\textit{iff} \;\; P_A^{1,1}(x_1, x_2) \geq P_B^{1,1}(x_1, x_2), \; \forall (x_1, x_2) \in \Gamma(\lambda^+). \tag{7}$$

Proposition 1 says that poverty is unambiguously larger for population A than for population B for all poverty sets within $\Gamma(\lambda^+)$ and for all members of the class of bidimensional poverty measures $\ddot{\Pi}(\lambda^+)$ if and only if the bidimensional poverty headcount $P^{1,1}$ is greater in A than in B for all intersection poverty frontiers in $\Gamma(\lambda^+)$.

[11] Note also that, in Roberts (1980)'s terminology on interpersonal comparability, the assumptions made in (4) require only ordinality and non-comparability of the dimensions. This is equivalent to saying that strictly monotonically increasing transformations of the dimensional indicators should not affect social welfare rankings.

Fig. 2 Bidimensional poverty dominance



This is illustrated in Fig. 2, which shows both the position of the upper poverty frontier λ^+ and some of the rectangular areas over which $P_A^{1,1}$ and $P_B^{1,1}$ must be computed. If $P_A^{1,1}(x_1, x_2)$ is larger than $P_B^{1,1}(x_1, x_2)$ for all of the rectangles that fit within $\Gamma(\lambda^+)$, then (6) is obtained.

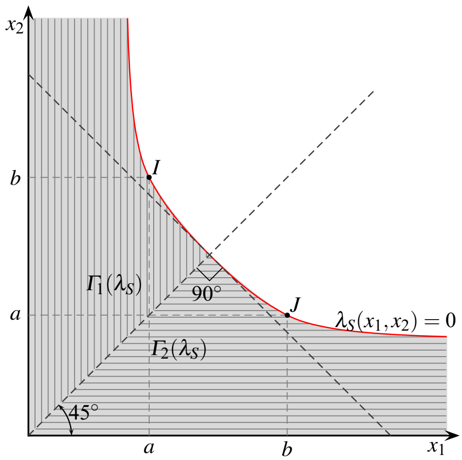
In the next pages, the power of the dominance criterion found in Proposition 1 is increased by adding assumptions on the poverty effects of income changes at each time period. For this, it is useful to distinguish between profiles with a lower first-period income and profiles with a lower second-period income. The poverty domain can be split into $\Gamma_1(\lambda) := \{(x_1, x_2) \in \Gamma(\lambda) | x_1 < x_2\}$, the set of poverty profiles whose minimal income is found in the first period, and $\Gamma_2(\lambda) := \{(x_1, x_2) \in \Gamma(\lambda) | x_1 \geq x_2\}$, the set of poverty profiles whose minimal income is found in the second period. Equation (2) can then be written as:

$$P(\lambda) = \iint_{\Gamma_1(\lambda)} \pi(x_1, x_2; \lambda)\, dF(x_1, x_2) + \iint_{\Gamma_2(\lambda)} \pi(x_1, x_2; \lambda)\, dF(x_1, x_2), \quad (8)$$

that is, the sum of relatively low-x_1 poverty and of relatively low-x_2 poverty. It is worth noting that the use of Eq. (8) makes sense only if incomes can be compared. Cost of living differences between the two periods and/or discounting preferences of the social evaluator may thus have to be taken into account before proceeding to (8) and to the symmetry and asymmetry properties that we are about to introduce.[12]

[12] Note that the need to compare the different values of x_1 and x_2 imposes the relatively weak assumption of "ordinality and level comparability" of the dimensions in a setting similar to that of Roberts (1980). An example of this is when x_1 and x_2 represent the health status of an individual at two points in time and when we need to be able to tell whether a value a at the first period is lower or larger than a value b at the second period. It would seem that such comparability is possible with many indicators of well-being, including those based on education or health and those involving different income sources and different recipients of these incomes in the same household.

Fig. 3 A poverty domain with symmetry



2.1 Symmetry

We now impose symmetry in the treatment of incomes, so that switching the values of the intertemporal income profile of any individual does not change poverty. This is a rather strong assumption since it means that the social evaluator is indifferent to the period at which incomes are enjoyed (again, after possibly adjusting for price differences and discounting preferences). Symmetry may, however, be regarded as reasonable for intertemporal poverty comparisons when the analysis focuses on a relatively short-time span. It may also be appropriate for snapshot analyses when one wishes to relax the assumption of perfect substitutability of different income sources (made when a univariate analysis focuses on the sum of periodic incomes) without imposing some particular form of asymmetry in the treatment of incomes. It is also in line with the intertemporal normative view sometimes expressed that the discount rate should be zero for welfare analysis (see for instance Ramsey 1928; Solow 1974). In a context of a measurement of household poverty based on the distribution of incomes across different members, the symmetry assumption says that it does not matter who earns a particular level of income, although earnings inequality across household members has normative importance.

The symmetry assumption implies that the poverty frontier is symmetric with respect to the line of perfect equality of periodic incomes. As a consequence, the poverty domain is defined with respect to the functions λ_S that are symmetric at the poverty frontier: $\lambda_S(x_1, x_2) = \lambda_S(x_2, x_1)\ \forall (x_1, x_2)$, such that $\lambda_S(x_1, x_2) = 0$. Figure 3 illustrates this in the case of two income profiles, $I := (a, b)$ and $J := (b, a)$, both on the poverty frontier. The poverty frontier that links I and J is symmetric along the 45-degree line, the line of periodic income equality; so is the straight line that is perpendicular to that same 45-degree line. That straight line is a special case of all of the symmetric poverty frontiers; it is a poverty frontier that assumes perfect substitutability of periodic incomes. As we will discuss later, the use of those symmetric and straight poverty frontiers is equivalent to measuring intertemporal poverty using the sum of periodic incomes.

Let Λ_S be the subset of Λ whose members are symmetric, and consider the class $\ddot{\Pi}_S$ of symmetric poverty measures defined as:

$$\ddot{\Pi}_S(\lambda_S^+) = \left\{P(\lambda_S) \in \ddot{\Pi}(\lambda_S^+) \,\middle|\, \pi(x_1, x_2; \lambda_S) = \pi(x_2, x_1; \lambda_S), \forall (x_1, x_2) \in \Gamma(\lambda_S)\right\}. \tag{9}$$

A restriction imposed by (9) is that the marginal effect of an income increment in the first period equals the marginal effect of the same increment in the second period, for two symmetric income profiles ($\pi^{(1)}(x_1, x_2; \lambda_S) = \pi^{(2)}(x_2, x_1; \lambda_S),\ \forall (x_1, x_2) \in \Gamma(\lambda_S)$). Similarly, (9) also says that the variation of the marginal contribution of an income increment is symmetric for symmetric income profiles ($\pi^{(1,2)}(x_1, x_2; \lambda_S) = \pi^{(1,2)}(x_2, x_1; \lambda_S),\ \forall (x_1, x_2) \in \Gamma(\lambda_S)$).

Proposition 2 shows how robust comparisons of bidimensional poverty can be made with symmetry.

Proposition 2

$$P_A(\lambda_S) > P_B(\lambda_S),\ \forall P(\lambda_S) \in \ddot{\Pi}_S(\lambda_S^+), \tag{10}$$

$$\textit{iff}\quad P_A^{1,1}(x_1, x_2) + P_A^{1,1}(x_2, x_1) > P_B^{1,1}(x_1, x_2) + P_B^{1,1}(x_2, x_1),\ \forall (x_1, x_2) \in \Gamma(\lambda_S^+). \tag{11}$$

Proof See Appendix 1. □

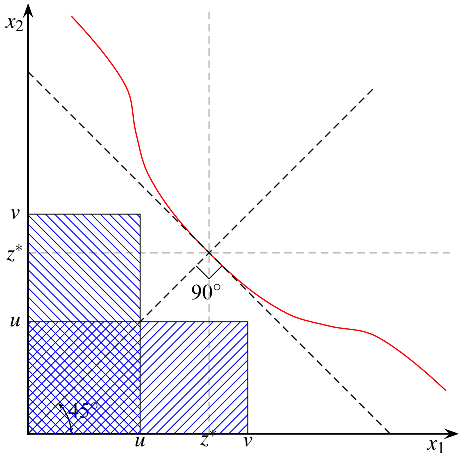
Proposition 2 says that poverty dominance can be checked by adding up two intersection headcounts, the first at a poverty line (x_1, x_2) and the second at (x_2, x_1).[13] With symmetric intertemporal poverty indices, we must therefore compare the sum of two intersection intertemporal headcounts that have symmetric poverty lines. Figure 4 shows graphically what this means: we must sum the proportions of income profiles found within two symmetric rectangular areas, each of them capturing the importance of those with low incomes in one time period. This effectively double counts the number of individuals that are highly deprived in both periods, as the double-slashed rectangle in Fig. 4 shows.[14]

Define z^* as the minimal permanent income value an individual should enjoy at each period in order to escape poverty, that is, $\lambda_S(z^*, z^*) = 0$. Chronic poverty is often

[13] Extending Proposition 2 to cases with more than two dimensions is relatively straightforward. For instance, if symmetry is assumed with three dimensions, one has to compare the sum of the joint distributions for the six permutations of each possible set of per-period poverty lines, that is $F(u, v, w) + F(u, w, v) + F(v, u, w) + F(v, w, u) + F(w, u, v) + F(w, v, u)$.

[14] In the tridimensional case mentioned in footnote 13, multiple counting also occurs but in a more complex manner. Those individuals whose incomes are less than z^* in each period are counted six times when checking dominance. Double counting occurs for those poor individuals whose incomes are below z^* during only two periods of time. The multidimensional dominance criterion thus introduces weights on poor households that depend on the number of periods of deprivations that they experience. Because of this, the social benefit of decreasing individual deprivation increases with the number of income shortfalls (with respect to z^*): a two-period-deprived person is twice as important as a single-period-deprived person, and a three-period-deprived person is thrice as important as a two-period-deprived person.

Fig. 4 Symmetric property dominance



defined in the literature (see for instance the "always poor" in Hulme and Shepherd 2003) as income being below z^* in both periods. The transient poor (according to the literature's usual definition) are those that are below the poverty frontier but that are not chronically poor. The double counting of Proposition 2 can be seen to weight the chronic poor twice as much as the transient ones.

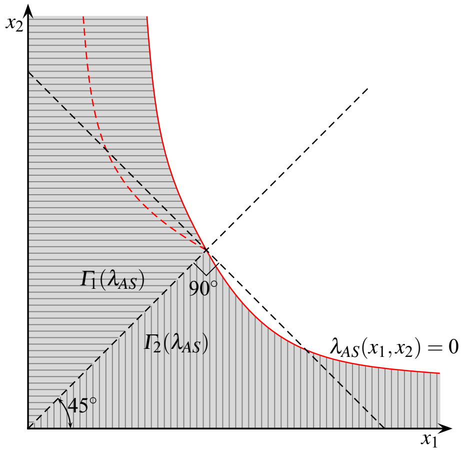
The power of Proposition 2 to order two distributions is larger than that of Proposition 1. This is because (11) gives greater importance to "more severe" intertemporal poverty, namely, poverty in both periods. To see this, consider two income distributions, A and B, made of profiles $\{(2, 1), (2, 1), (3, 4)\}$ and $\{(1, 2), (4, 3), (4, 3)\}$ respectively. Using Proposition 1, one would not be able to order these two distributions since Eq. (7) is larger for A when evaluated at $(2, 1)$ and larger for B when evaluated at $(1, 2)$. We would, however, observe dominance using Proposition 2 since Eq. (11) at $(1, 2)$ would now be larger for A. This is because the symmetry assumption makes it possible to compare $(1, 2)$ with $(2, 1)$, and that distribution A can thus be declared to have more severe poverty.

2.2 Asymmetry

Symmetry may not be appropriate, however, in those cases in which we may not be (individually or socially) indifferent to a permutation of periodic incomes. We may yet feel that poverty is higher with income profile (x_1, x_2) than with (x_2, x_1) whenever $x_1 < x_2$. For instance, we may think that low income is more detrimental to well-being during childhood than during adulthood, perhaps because low income as a child can lead to poorer health and lower educational outcomes over the entire lifetime.

Asymmetry can also be reasonable when there is uncertainty regarding the appropriate scaling up of incomes in a given period before applying symmetry. This may be the case when intertemporal price adjustments need to be made but when true inflation is unknown. If the purchasing power of money has decreased, but the extent of that fall is not known for sure, a prudent procedure may be to impose asymmetry on the

Fig. 5 Asymmetric poverty measurement



treatment of the components of the income profiles. Asymmetry is also the general case in the class of intertemporal poverty indices proposed by Hoy and Zheng (2011) and Calvo and Dercon (2009), where periodic weights decrease as the final period is approached. (Note that a reverse asymmetry setting, corresponding to loss aversion, would suppose that a decreasing income profile is undesirable; the ordering procedures for such a setting are analogous to those developed below, and are also empirically illustrated in Sect. 5 below.)

Without loss of generality, assume that income profiles within $\Gamma_1(\lambda)$ never yield less poverty than their symmetric image in $\Gamma_2(\lambda)$. The well-being functions λ_{AS} that are consistent with asymmetry are then members of the set Λ_{AS} of well-being functions defined by:

$$\Lambda_{AS} := \{\lambda \in \Lambda | \lambda(x_1, x_2) \leq \lambda(x_2, x_1) = 0, \quad \forall x_1 \leq x_2\}. \tag{12}$$

Figure 5 illustrates the possible shape of these functions. The asymmetry of $\lambda_{AS}(x_1, x_2)$ indicates that low x_1 is a source of greater poverty than low x_2. The poverty frontier ($\lambda_{AS}(x_1, x_2) = 0$, the continuous line) is chosen such that the poverty domain $\Gamma_1(\lambda_{AS})$ (the shaded area with vertical lines) is larger than $\Gamma_2(\lambda_{AS})$ (the shaded area with horizontal lines). In particular, the symmetric set of $\Gamma_2(\lambda_{AS})$ with respect to the line of perfect equality is a subset of $\Gamma_1(\lambda_{AS})$.

We can then consider the following class of asymmetric poverty measures:

$$\ddot{\Pi}_{AS}(\lambda^+_{AS}) = \left\{ P(\lambda) \in \ddot{\Pi}(\lambda^+_{AS}) \middle| \begin{array}{ll} \pi^{(1)}(x_1, x_2; \lambda) \leq \pi^{(2)}(x_2, x_1; \lambda) & \text{if } x_1 \leq x_2 \\ \pi^{(1,2)}(x_1, x_2; \lambda) \geq \pi^{(1,2)}(x_2, x_1; \lambda) & \text{if } x_1 \leq x_2. \end{array} \right\} \tag{13}$$

The conditions in (13) assume constant total income ($x_1 + x_2$). The first line to the right of (13) implies that changes in the lowest temporal income have a greater impact on poverty when the lowest income is in the first period. Consequently, for equal values of first-period and second-period incomes, changes in the first-period income have a greater impact on welfare. The second line states that the poverty bene-

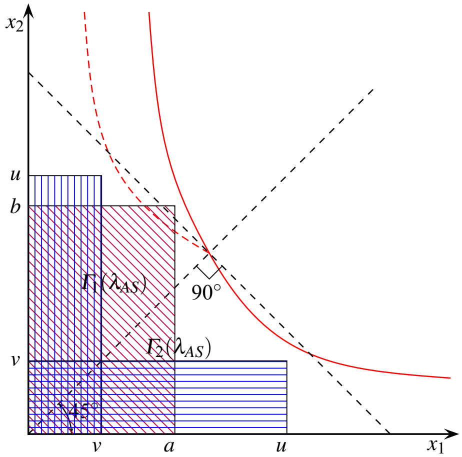


Fig. 6 Asymmetric poverty dominance

fit of an increase in the income of a low-income period income decreases faster when that low income is in the first period. This second line also says that a correlation-decreasing switch decreases poverty more when the first-period income is lower, for the same total income. Hence, it is in the presence of low first-period incomes that correlation-decreasing switches yield the greatest poverty benefits. Both lines emphasize the greater normative/poverty importance of those individuals with lower first-period incomes.

The necessary and sufficient conditions for robustly ordering asymmetric poverty measures are presented in Proposition 3:

Proposition 3

$$P_A(\lambda_{AS}) > P_B(\lambda_{AS}),\ \forall P(\lambda_{AS}) \in \ddot{\Pi}_{AS}(\lambda^+_{AS}), \tag{14}$$

$$\textit{iff}\quad P_A^{1,1}(x_1, x_2) > P_B^{1,1}(x_1, x_2),\ \forall (x_1, x_2) \in \Gamma_1(\lambda^+_{AS}) \tag{15}$$

$$\textit{and}\quad P_A^{1,1}(x_1, x_2) + P_A^{1,1}(x_2, x_1) > P_B^{1,1}(x_1, x_2) + P_B^{1,1}(x_2, x_1),\ \forall (x_1, x_2) \in \Gamma_2(\lambda^+_{AS}). \tag{16}$$

Proof See Appendix 1. □

The first condition in Proposition (3) says that dominance should first hold for each point in $\Gamma_1(\lambda_{AS})$. That condition is illustrated in Fig. 6. For any (a, b) with $a < b$, asymmetric poverty dominance implies that the share of the population whose incomes are simultaneously less than a and b respectively at period 1 and 2 (those in the rectangle with slanting lines on Fig. 6) should be lower in B than in A. Thus, contrary to symmetric dominance, poverty cannot be lower in B if the intersection headcount with a relatively low first-period threshold is higher in B. Condition (16) is the same as Condition (11) in Proposition 2, but for income profiles within

$\Gamma_2(\lambda_{AS})$. Since symmetric poverty indices can be regarded as limiting cases of asymmetric ones, dominance with asymmetry logically implies dominance with symmetry, so long as the set of symmetric poverty frontiers lie within the set of asymmetric ones.

The power of Proposition 3 is larger than that of Proposition 1. To illustrate the difference in ranking power, consider two income distributions, A and B, with distribution A made of profiles $\{(1, 2), (1, 2)\}$ and distribution B made of profiles $\{(2, 1), (6, 6)\}$, and with $z^* = 5$. Using Proposition 1, one would not be able to order these two distributions since Eq. (7) is larger for A when evaluated at $(1, 2)$ and larger for B when evaluated at $(2, 1)$; indeed, although A may look poorer than B at first glance, one of the profiles in B has the lowest income at time 2. We would, however, observe asymmetric dominance since Eq. (15) at $(2, 1)$ is larger for A.

Note, however, that with the example (used on page 11) of distributions A set to $\{(2, 1), (2, 1), (3, 4)\}$ and B set to $\{(1, 2), (4, 3), (4, 3)\}$ no asymmetric poverty ordering holds. The stronger symmetry assumptions of Proposition 2 are needed to rank these two distributions.

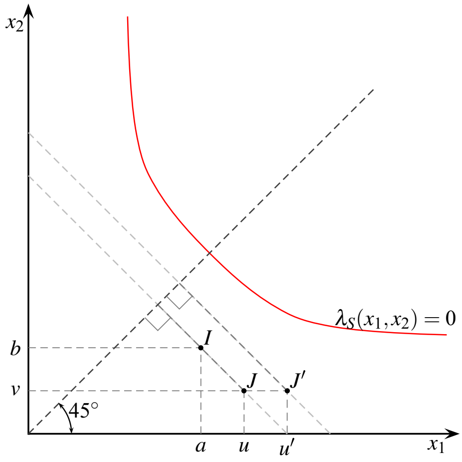
The conditions in Proposition 3 may thus hold even if B has a larger proportion of poor with low x_2, so long as this is compensated by a lower proportion with low x_1. This is reminiscent of the sequential stochastic dominance conditions found in Atkinson and Bourguignon (1987) and Atkinson (1992) and in subsequent work. Although apparently similar, the two frameworks and their respective orderings conditions are different. The literature on sequential dominance makes assumptions only on the *signs* of different orders of derivatives; the conditions in (13) *compare* the value of these derivatives *across* dimensions, a procedure that is possible only when the dimensions are level comparable (and a procedure that has also not been suggested or developed to our knowledge). Such comparability assumptions are not made in the sequential dominance literature since the dimensions involved (income and family size, for instance) typically do not need to be level comparable.

3 Aversion to intertemporal variability

Consider the income profiles $I := (a, b)$ and $J := (u, v)$ drawn in Fig. 7. By projecting these two profiles on the diagonal of perfect equality, it can be seen that both profiles are characterized by the same total income, so that the only difference between them is the way total income is allocated across the two periods. We may feel that individuals are better off when the distribution of a given total amount is smoothed across periods; we should then infer that poverty is unambiguously lower with income profile I than with J (since $|a - b| < |u - v|$). This, however, cannot be inferred with any of the previous propositions.

We can also compare two income profiles that differ in their total (or mean) income. For instance, let us assume that an income profile J sees an increase in its first-period income. Let the new income profile be $J' := (u', v)$, as in Fig. 7. Propositions 1, 2 and 3 would declare that movement to decrease poverty. Both intertemporal variability and average income have increased. A mean/variability evaluation framework does not therefore necessarily find that intertemporal poverty has fallen. To compare J and J', we could think of a lexicographic assumption that either mean income or distance

Fig. 7 Ranking income profiles with aversion to intertemporal inequalities



from the mean prevails on the other. We could also use results from the social welfare literature when both inequality and mean income differ.

In that regard and as noted by Kolm (1976) in a unidimensional context, views differ as to how additional income should be shared among different people so as to leave inequality unchanged. The relative view is that sharing this additional income according to the initial income shares of individuals would preserve the initial level of inequality; the absolute view is that inequality is maintained if the same absolute amount of income is distributed to everyone.

With this in mind, let us define poverty with respect to average income and income deviations from that average. An income profile (x_1, x_2) is then described by the coordinates (μ, τ), with μ being mean income and τ some measure of the distance of the lowest income to the mean.[15] One reasonable property to impose on τ is unit-consistency; this states that changing the income measurement scale (using euros instead of cents, for instance) should not change the ranking properties of the measure (Zheng 2007). Within our setting, unit consistency demands that multiplying each income profile element by the same scalar should not change the intertemporal inequality ranking of the income profiles.

We can then make use of a particular definition of τ, that is $\tau_\eta = \frac{\min\{x_1,x_2\}-\mu}{\mu^\eta}$, $\eta \in [0, 1]$, (Krtscha 1994; Zoli 2003; Yoshida 2005), so that $\tau_1 = \frac{\min\{x_1,x_2\}}{\mu} - 1$ for a relative inequality aversion view and $\tau_0 = \min\{x_1, x_2\} - \mu$ for an absolute inequality aversion view.[16] For a given μ, τ ranges from $-\mu^{1-\eta} \leq 0$ (extreme inequality) to 0 (perfect equality). Poverty is reasonably assumed to decrease with both μ and τ_η (which we term "variability", as a shorthand for intertemporal inequality).

Figure 8 illustrates the influence of η on the orderings of an income profile I with profiles with a lower mean income and located on the same side of the diagonal of

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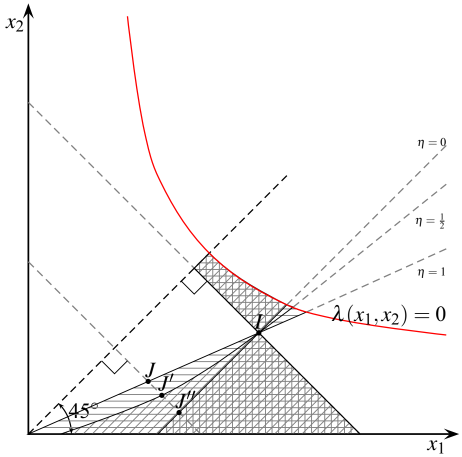
[15] These definitions of μ and τ mean that the two dimensions are necessarily fully comparable in the sense of Roberts (1980).

[16] See Zheng (2007) for more on this.

Fig. 8 Bidimensional poverty with relative, intermediate and absolute variability aversion views



equality. The areas below I with horizontal, slanting and vertical hatches correspond to the set of income profiles with unambiguously higher poverty than I when η is respectively set to 1, 0.5, and 0.[17] The areas above I but inside $\Gamma_2(\lambda)$ are those poor income profiles that are better than I for all values of η. Whatever the location of I, the relative view ranks more income profiles as worse than the intermediate and absolute views. For instance, income profiles J, J', and J'' exhibit the same distance τ_η as I with respect to the first diagonal when η is respectively set equal to 1, 0.5, and 0, but average income is lower. I is preferred to J, J', and J'' for $\eta = 1$, but cannot be ranked with J and J' when $\eta < 0.5$. Relative views also increase the set of income profiles that are preferred to I. In that sense, absolute views rely on the weakest measurement assumptions and also induce the weakest power for ranking income profiles.

We can also express the poverty frontier as a function of both μ and τ_η. Let $\tilde{\lambda}$ be defined as:

$$\tilde{\lambda}(\mu, \tau_\eta, j) = \begin{cases} \tilde{\lambda}(\mu, \tau_\eta, 1) & \text{if } x_1 < x_2, \\ \tilde{\lambda}(\mu, \tau_\eta, 2) & \text{otherwise.} \end{cases} \tag{17}$$

Recall that both μ and τ_η are functions of x_1 and x_2. We can also assume that $\tilde{\lambda}(\mu, \tau_\eta, j) = \lambda(x_1, x_2)$, namely, that each function $\tilde{\lambda}$ has a unique representation λ in the space (x_1, x_2), and that $\frac{\partial \tilde{\lambda}}{\partial \mu} > 0$ and $\frac{\partial \tilde{\lambda}}{\partial \tau_\eta} > 0$. Let $\tilde{\Lambda}$ be the set of mean-income increasing and variability-decreasing well-being functions. It is worth indicating that non-increasingness with respect to variability entails both that the poverty frontier is convex and that it is never below the straight line through (z^*, z^*) that is orthogonal

17 While the cases of η equal to 1 and 0 can easily be understood, intermediate cases are more difficult. For instance, with $\eta = 0.5$, inequality will be preserved when moving from μ_1 to μ_2 if each additional euro is distributed in the following manner between the two periods: fifty cents are distributed proportionally to the shares of each period in total income and the remaining fifty cents are equally shared; then fifty cents are allocated according to the new income shares and the remaining fifty cents are equally distributed, and so on until the individual's mean income is μ_2.

to the line of perfect equality.[18] For convenience, we can express the poverty domain in the space (x_1, x_2) as:

$$\Gamma(\tilde{\lambda}) := \left\{(x_1, x_2) \in \Re^2_+ \,\middle|\, \tilde{\lambda}(\mu, \tau_\eta, j) \le 0\right\}, \tag{18}$$

where, as previously, $\Gamma(\tilde{\lambda})$ can be divided into $\Gamma_1(\tilde{\lambda})$ and $\Gamma_2(\tilde{\lambda})$ to distinguish relatively low-x_1 income profiles from relatively low-x_2 income profiles.

3.1 The general case

To use the above setting for intertemporal poverty ranking, let $q := \text{prob}(x_1 < x_2)$ be the share of the population whose first-period income is lower than second-period income. Let ρ_1 (ρ_2) be the individual poverty measure when $x_1 < x_2$ ($x_1 \ge x_2$), and let F_1 (F_2) denote the joint cumulative distribution function of μ and τ_η conditional on $x_1 < x_2$ ($x_1 \ge x_2$). A variability-averse poverty measure is given by

$$\tilde{P}(\tilde{\lambda}) = q \iint_{\Gamma_1(\tilde{\lambda})} \rho_1(\mu, \tau_\eta; \tilde{\lambda})\, dF_1(\mu, \tau_\eta) + (1-q) \iint_{\Gamma_2(\tilde{\lambda})} \rho_2(\mu, \tau_\eta; \tilde{\lambda})\, dF_2(\mu, \tau_\eta). \tag{19}$$

As in Sect. 2, Eq. (19) corresponds to a general definition of additive intertemporal poverty measures, i.e., overall poverty is simply the average individual poverty level.[19] Let the class $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$ of mean/variability poverty indices be defined as:

$$\tilde{\Pi}_\eta(\tilde{\lambda}^+) = \left\{ P(\tilde{\lambda}) \,\middle|\, \begin{array}{l} \Gamma(\tilde{\lambda}) \subset \Gamma(\tilde{\lambda}^+) \\ \rho_t(\mu, \tau_\eta; \tilde{\lambda}) = 0, \text{ whenever } \tilde{\lambda}(\mu, \tau_\eta) = 0\ \forall t \\ \rho_1(\mu, 0, \tilde{\lambda}) = \rho_2(\mu, 0, \tilde{\lambda}) \forall \mu \\ \rho_t^{(1)}(\mu, \tau_\eta; \tilde{\lambda}) \le 0 \text{ and } \rho_t^{(2)}(\mu, \tau_\eta; \tilde{\lambda}) \le 0\ \forall \mu, \tau_\eta, t \\ \rho_t^{(1,2)}(\mu, \tau_\eta; \tilde{\lambda}) \ge 0, \forall \mu, \tau_\eta, \forall t. \end{array} \right\} \tag{20}$$

As in the case of the class $\ddot{\Pi}(\lambda^+)$ defined in Eq. (4), the first two conditions say that the chosen poverty frontier should be nowhere above the maximum admissible

[18] Were these conditions not met, it would be possible for some income profiles to leave the poverty domain by an increase in intertemporal variability.

[19] Although not as straightforward as with the poverty indices of Sect. 2, extending this mean/variability framework to $T > 2$ periods can be done. Let μ_k be the average value of the $k = 1, \ldots T$ lowest values of an income profile. μ_1 is thus the minimal value of the income profile and $\mu_T = \mu$ is average income. Then, define $\tau_{k,\eta} := \frac{\mu_k - \mu}{\mu^\eta}$, with $\tau_{k,\eta} \in [-\mu^{1-\eta}, 0]$. It can be seen that for each income profile of size T, only $T - 1$ observations of inequality are needed to describe all relevant intertemporal inequalities. So an income profile $(x_1, x_2 \ldots, x_T)$ can be fully described in terms of intertemporal inequalities and average income by the T-vector $(\tau_{1,\eta}, \tau_{2,\eta} \ldots, \tau_{T-1,\eta}, \mu)$.
If income timing matters for poverty assessment (as for asymmetric poverty indices), this vector will not be sufficient. For instance, in the three-period case, it would be necessary to make use of $3! = 6$ possibly different individual poverty indices $\rho_{s,t}$, where s indicates the period of the lowest income and t is the period for the second-lowest income. Once this is done, generalizing Propositions 4–7 is relatively straightforward.

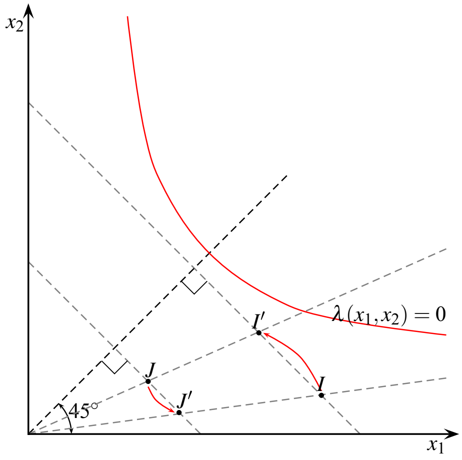


Fig. 9 A correlation-increasing switch in the space (μ, τ_1) (relative variability aversion)

poverty frontier $\tilde{\lambda}^+$, and that ρ_t is continuous at the poverty frontier. The third condition says that poverty measurement is continuous at the diagonal of perfect equality. The fourth condition states that intertemporal variability-preserving income increments and mean-preserving variability-reducing transfers should not increase poverty.

The last condition in (20) says that the greater the variability of income profiles, the more effective are variability-preserving income increments in reducing poverty. Similarly, the benefit of a mean-preserving variability-decreasing income change falls with mean income. This condition can also be interpreted in terms of correlation-increasing switches in the (μ, τ_η) space: permuting the values of either μ or τ_η of two poor individuals, so that one of them becomes unambiguously poorer than the other, cannot reduce poverty. Figure 9 illustrates this in the case of relative variability aversion. The permutation of τ_1 that moves I and J to I' and J', respectively, necessarily improves the situation of individual I and worsens that of J (who is then poorer than I'). The permutation does not affect the marginal distributions of μ and τ_1, but nevertheless results in an increased correlation between them. As the two different forms of deprivation are then cumulating over the same person, it seems natural to regard such a change as worsening overall poverty.

This leads to the following general result.

Proposition 4

$$P_A(\tilde{\lambda}) > P_B(\tilde{\lambda}),\ \forall P(\tilde{\lambda}) \in \tilde{\Pi}_\eta(\tilde{\lambda}^+), \tag{21}$$

$$\textit{iff}\quad q_A P_A^{1,1}(\mu, \tau_\eta | x_1 < x_2) > q_B P_B^{1,1}(\mu, \tau_\eta | x_1 < x_2),\quad \forall(\mu, \tau_\eta) \in \Gamma_1(\tilde{\lambda}^+), \tag{22}$$

$$\textit{and}\quad (1 - q_A) P_A^{1,1}(\mu, \tau_\eta | x_1 \geq x_2) > (1 - q_B) P_B^{1,1}(\mu, \tau_\eta | x_1 \geq x_2),\quad \forall(\mu, \tau_\eta) \in \Gamma_2(\tilde{\lambda}^+). \tag{23}$$

Proof See Appendix 2. □

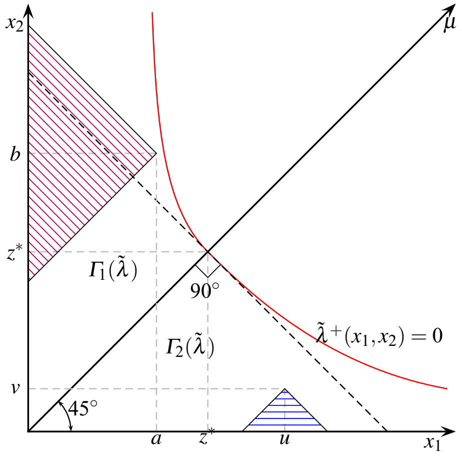


Fig. 10 Poverty dominance criteria with absolute variability aversion

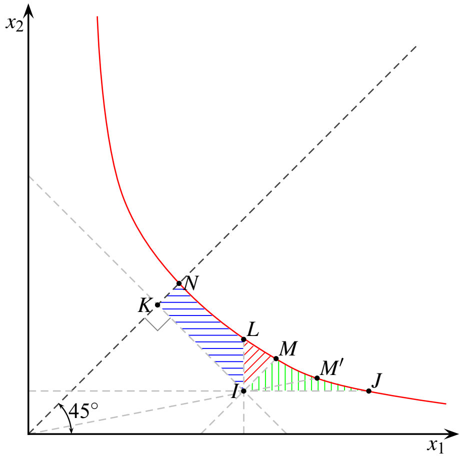
Proposition 4 says that distribution A exhibits more poverty than distribution B over the class of mean/variability poverty indices if and only if the share of the population with low mean income and high variability is greater in A than in B, whatever (μ, τ_η) within the poverty domain is used and considering separately each low-x_1 and low-x_2 region. Figure 10 illustrates the dominance criteria in the case of absolute variability aversion. Each income profile in $\Gamma(\tilde{\lambda}^+)$ defines a rectangular triangle whose hypotenuse is either the x_1 or the x_2-axis, for $x_1 > x_2$ and $x_1 < x_2$ respectively. Poverty is larger for the distribution that shows a larger population share within each one of those triangular areas that fit within $\Gamma(\tilde{\lambda}^+)$. It can be seen by inspection that a necessary (but not sufficient) condition for dominance of B over A is that the marginal distribution of μ for A is nowhere below that for B at each value of μ below z^*.

It is useful to compare the ability of Propositions 1 and 4 to rank distributions. Suppose that distributions A and B are respectively defined by the income profiles $\{(3, 1), (1, 5)\}$ and $\{(3, 1), (3, 4)\}$. These two distributions cannot be ordered by Proposition 1 if all profiles lie within the poverty domain $\Gamma(\lambda^+)$: the intersection headcount is larger for B when evaluated at $(3, 4)$, but lower when evaluated at $(1, 5)$. In the space (μ, τ_0), the ordinates of the two distributions become $\{(2, -1), (3, -2)\}$ and $\{(2, -1), (3.5, -0.5)\}$. It can then be seen that the joint distribution function of (μ, τ_0) is larger for A when evaluated at $(3, -2)$ and nowhere lower when evaluated at any other point of the poverty domain. Consequently, A exhibits more poverty than B by Proposition 4.

This does not mean that the overall ordering power of Proposition 4 is larger than that of Proposition 1. Proposition 1 orders $\{(3, 1), (3, 5)\}$ and $\{(3, 1), (3, 4)\}$), but Proposition 4 does not. This is also visible from Fig. 7. Profile I is judged better than J by Proposition 4 but not by Proposition 1; Profile J' is judged better than J by Proposition 1 but not by Proposition 4.

Figure 11 provides an alternative illustration of the differences in the measurement assumptions behind each of Proposition 1 and Proposition 4 in the case of absolute variability aversion. A movement from point I to point J (or to any other point in the

Fig. 11 Effect of changes in periodic incomes



area IJM) is deemed to decrease poverty according to the usual multidimensional poverty indices covered by Proposition 1, but not with respect to those of Proposition 4. A movement from point I to point K is deemed to decrease poverty according to Proposition 4, but not with respect to Proposition 1. This is also true of a movement from point I to any of the points in the area $IKNL$ with horizontal stripes. It is only to the points in the area ILM that a movement from point I will be judged to decrease poverty according to both Proposition 1 and Proposition 4. It is worth noting that, as η increases, that area increases and so does the probability of obtaining the same rankings from both propositions. In the limiting relative variability view, this area extends to ILM'.

Let $\tilde{\succsim}_{\eta,\lambda}$ denote dominance over the class $\tilde{\Pi}_\eta(\lambda)$, so that $A \tilde{\succsim}_{\eta,\tilde{\lambda}^+} B$ means that distribution A is preferred to distribution B according to Proposition 4. The next proposition considers how the dominance relationships $\succsim_{\eta,\tilde{\lambda}^+}$ are nested.

Proposition 5

$$\textit{If } A \tilde{\succsim}_{\eta,\tilde{\lambda}^+} B, \quad \textit{then} \quad A \tilde{\succsim}_{\eta',\tilde{\lambda}^+} B \quad \forall \eta' \in [\eta, 1]. \tag{24}$$

$$\textit{If } A \tilde{\not\succsim}_{\eta,\tilde{\lambda}^+} B, \quad \textit{then} \quad A \tilde{\not\succsim}_{\eta',\tilde{\lambda}^+} B \quad \forall \eta' \in [0, \eta]. \tag{25}$$

Proof The proof is straightforward since, for any couple of profiles (μ, τ_η) and (μ', τ'_η) from $\Gamma_i(\tilde{\lambda})$, $i = 1$ or 2, the first one is preferred iff $\mu' \leq \mu$ and $\tau'_\eta \leq \tau_\eta$. This implies that $\frac{x'_i-\mu'}{\mu'^\eta} \leq \frac{x_i-\mu}{\mu^\eta} \leq 0$; it can then be seen that $\frac{x'_i-\mu'}{\mu'^\eta\mu'^\varepsilon} \leq \frac{x_i-\mu}{\mu^\eta\mu'^\varepsilon} \leq \frac{x_i-\mu}{\mu^\eta\mu^\varepsilon}$ $\forall\varepsilon > 0$, since $\mu' \leq \mu$ and the variability measure is negative. Consequently, $(\mu, \tau_{\eta+\varepsilon})$ is preferred to $(\mu', \tau'_{\eta+\varepsilon})$. □

The first part of Proposition 5 states that, using our mean/variability framework, a sufficient condition for A to dominate B for some η is to observe such a dominance relationship for a lower value of η. An immediate consequence is that dominance holds for all values of η when dominance is observed for $\eta = 0$. This makes it possible to

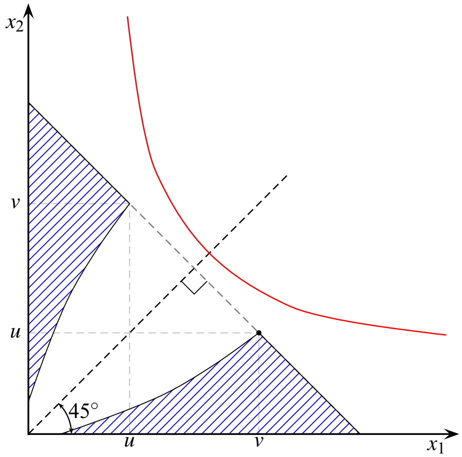


Fig. 12 Poverty dominance criteria with intermediate aversion to availability ($\eta = 0.5$) and symmetry

obtain poverty comparisons that are robust with respect to various views of variability aversion without having to perform dominance tests for all such views.

The second part of Proposition 5 is a corollary of the first part: it is useless to check for whether A dominates B for some η if dominance does not hold for a larger η. The inability to order two distributions with a relative variability aversion view means that there is no hope of obtaining dominance with intermediate or absolute views.

3.2 Symmetry

As in Sect. 2.1, symmetry can be assumed, so that poverty depends only on the gaps between incomes as well as on mean income. As a consequence, an income profile (x_1, x_2) is strictly equivalent to an income profile (x_2, x_1); both can be described by the same coordinates (μ, τ_η). We then have:

$$\tilde{\Pi}_{\eta S}(\tilde{\lambda}_S^+) = \left\{ P(\tilde{\lambda}_S) \in \tilde{\Pi}_\eta(\tilde{\lambda}_S^+) \mid \rho_1(\mu, \tau_\eta; \tilde{\lambda}_S) = \rho_2(\mu, \tau_\eta; \tilde{\lambda}_S), \forall(\mu, \tau_\eta) \in \Gamma(\tilde{\lambda}_S) \right\}. \tag{26}$$

Proposition 6

$$P_A(\tilde{\lambda}_S) > P_B(\tilde{\lambda}_S), \ \forall P(\tilde{\lambda}_S) \in \tilde{\Pi}_{\eta S}(\tilde{\lambda}_S^+), \tag{27}$$

$$\textit{iff} \quad P_A^{1,1}(\mu, \tau_\eta) > P_B^{1,1}(\mu, \tau_\eta), \quad \forall(\mu, \tau_\eta) \in \Gamma(\tilde{\lambda}_S^+). \tag{28}$$

Proof See Appendix 2. □

Dominance of A over B for all measures within $\tilde{\Pi}$ requires that the joint distribution of mean income and (the negative of) the distance of income to the mean for distribution A first-order dominates that for B, $\forall(\mu, \tau_\eta) \in \tilde{\Gamma}(\tilde{\lambda}^+)$. Figure 12 shows the two areas over which the joint distributions are assessed for $\mu = (u + v)/2$ and $\tau_{0.5} = (u - \mu)/\mu^{0.5}$. As in the case of the class of poverty indices studied in Sect. 2.1, symmetry implies that a larger share of the population in one area can be compensated by a lower share in the other.

3.3 Asymmetry

As in Sect. 2.2, we can relax the symmetry assumption and suppose that income profile (x_1, x_2), with $x_1 < x_2$, leads to greater poverty than (x_2, x_1). With our mean/variability framework, this says that the cost of variability depends on the timing of deprivations. Since profiles within $\Gamma_1(\tilde{\lambda})$ are then worse than their symmetric image within $\Gamma_2(\tilde{\lambda})$, we can consider the following class of intertemporal poverty measures:

$$\tilde{\Pi}_{\eta AS}(\tilde{\lambda}^+_{AS}) = \left\{ P(\lambda) \in \tilde{\Pi}_\eta(\tilde{\lambda}^+_{AS}) \,\middle|\, \begin{array}{l} \rho_1^{(2)}(\mu, \tau_\eta; \tilde{\lambda}) \le \rho_2^{(2)}(\mu, \tau_\eta; \tilde{\lambda}) \\ \rho_1^{(1,2)}(\mu, \tau_\eta; \tilde{\lambda}) \ge \rho_2^{(1,2)}(\mu, \tau_\eta; \tilde{\lambda}). \end{array} \right\} \tag{29}$$

The first condition says that, for given μ, shrinking risk reduces poverty most when income is lowest in the first period. The second condition says that the shrinking effect decreases more rapidly with μ when incomes are lower in the first period.

Proposition 7

$$P_A(\tilde{\lambda}_{AS}) > P_B(\tilde{\lambda}_{AS}),\ \forall P(\tilde{\lambda}_{AS}) \in \tilde{\Pi}_{\eta AS}(\tilde{\lambda}^+_{AS}), \tag{30}$$

$$\textit{iff}\quad q_A P_A^{1,1}(\mu, \tau_\eta | x_1 < x_2) > q_B P_B^{1,1}(\mu, \tau_\eta | x_1 < x_2), \quad \forall(\mu, \tau_\eta) \in \Gamma_1(\tilde{\lambda}^+_{AS}) \tag{31}$$

$$\textit{and}\quad P_A^{1,1}(\mu, \tau_\eta) > P_B^{1,1}(\mu, \tau_\eta), \quad \forall(\mu, \tau_\eta) \in \Gamma_2(\tilde{\lambda}^+_{AS}). \tag{32}$$

Proof See Appendix 2. □

Figure 13 illustrates the areas over which dominance tests are performed for asymmetric mean/variability poverty measures and relative variability aversion. Such tests first entail comparing the share of the population that belongs to each triangular area with a side along the x_2 axis and that fits within $\Gamma_1(\tilde{\lambda}_{AS})$. If that share is nowhere lower for each $(a, b) \in \Gamma_1(\tilde{\lambda}_{AS})$, then one turns to the second condition in Proposition 7 and compares the share of the population within the union of two triangular areas, such as those defined by (u, v) and (v, u), for each $(u, v) \in \Gamma_2(\tilde{\lambda}_{AS})$. If this never results in a lower share for A than for B, then A shows more poverty than B over the class of asymmetric mean/variability poverty indices and over the set of poverty frontiers lying within the maximum poverty domain $\Gamma\left(\tilde{\lambda}^+_{AS}\right)$. As in the case of the asymmetric poverty indices of Proposition 3, the dominance criteria of Proposition 7 have a greater ranking power than those for the general class of mean/variability poverty indices (Proposition 4). The power is weaker, however, than for the subclass of symmetric mean/variability indices considered in Proposition 6.

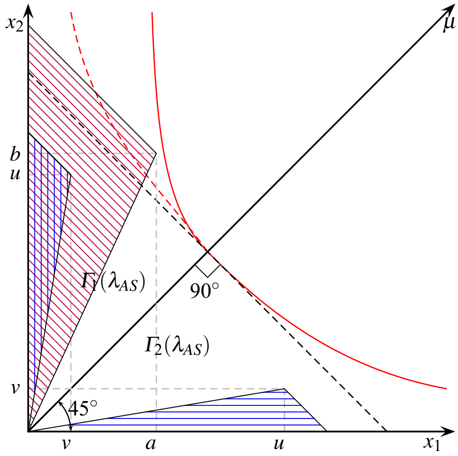


Fig. 13 Poverty dominance criteria with relative variability aversion and asymmetry

4 On the relationships between the dominance criteria

Each of the classes $\tilde{\Pi}(\tilde{\lambda}^+)$ and $\ddot{\Pi}(\lambda^+)$ (and their symmetric and asymmetric subclasses) of poverty measures displays appealing properties, but may not be individually regarded as fully satisfying. Take for instance an income profile (a, b), with $b > a$. If b increases, average income also increases but variability τ_η rises for all η, so that the net poverty effect is ambiguous over the class $\tilde{\Pi}(\tilde{\lambda}^+)$. Conversely, a transfer $\iota > 0$ that leads to $(a+\iota, b-\iota)$, with $2\iota < b-a$, reduces variability without affecting mean income, but cannot be regarded as favorable over the class $\ddot{\Pi}(\lambda^+)$ since it leads to a fall in one of the two incomes.

We may seek to address this difficulty by considering poverty indices $P(\lambda)$ that simultaneously belong to the above two classes. Define $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ as their intersection, that is:

$$\breve{\Pi}_\eta(\tilde{\lambda}^+) = \left\{ P(\tilde{\lambda}) \in \ddot{\Pi}(\tilde{\lambda}^+) \cap \tilde{\Pi}_\eta(\tilde{\lambda}^+) \right\}. \tag{33}$$

As an illustration of membership into the class $\breve{\Pi}_\eta(\tilde{\lambda}^+)$, we can consider some members of the family of union bidimensional poverty indices P_{BC} suggested by Bourguignon and Chakravarty (2003). For a population of size n, P_{BC} is defined as

$$P_{BC} = \frac{1}{n} \sum_{i=1}^{n} \left(a\,(1 - x_{1i})_+^\beta + (1-a)\,(1 - x_{2i})_+^\beta \right)^{\frac{\alpha}{\beta}}, \tag{34}$$

where x_{ji} denotes the income of the ith poor person at time j, $(y)_+ = \max(0, y)$, and where poverty lines have been normalized to 1 in each period. For P_{BC} to be a member of $\ddot{\Pi}(\lambda^+)$, it is necessary that $\beta \geq 1$ and $\alpha \geq \beta$. It can be shown that for $a = 0.5$,[20] one then obtains a family of measures $\breve{P}_{BC}$ that is included in $\breve{\Pi}_\eta(\tilde{\lambda}^+)$

[20] Equal weights for each deprivation are necessary in order to obtain individual poverty indices that are decreasing with respect to τ_0.

since the measure can be expressed as:

$$\breve{P}_{BC} = \frac{1}{n}\sum_{i=1}^{n}\left(0.5\,(1-\mu_i-\tau_{0i})_+^{\beta} + 0.5\,(1-\mu_i+\tau_{0i})_+^{\beta}\right)^{\frac{\alpha}{\beta}}. \quad (35)$$

Now consider the additional restrictions that need to be imposed on members of $\ddot{\Pi}(\tilde{\lambda}^+)$ for these also to be members of the subclass $\breve{\Pi}_\eta(\tilde{\lambda}^+)$. Since the elements of $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ also belong to $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$, the derivatives of π with respect to μ and τ_η have to obey the restrictions imposed on ρ. While condition $\pi^{(\mu)}(x_1, x_2) \leq 0$ is met with the restrictions imposed on $\pi^{(1)}$ and $\pi^{(2)}$ (see Appendix 3), conditions $\pi^{(\tau_\eta)}(x_1, x_2) \leq 0$ and $\pi^{(\mu,\tau_\eta)}(x_1, x_2) \leq 0$ respectively require (whenever $x_i \leq x_j$) that

$$\pi^{(i)}(x_1, x_2) - \pi^{(j)}(x_1, x_2) \leq 0, \quad (36)$$

$$\eta\left(\pi^{(i)}(x_1, x_2) - \pi^{(j)}(x_1, x_2)\right) + \left(\mu + \eta\tau_0\right)\left(\pi^{(i,i)}(x_1, x_2) - \pi^{(i,j)}(x_1, x_2)\right)$$
$$+\left(\mu - \eta\tau_0\right)\left(\pi^{(i,j)}(x_1, x_2) - \pi^{(j,j)}(x_1, x_2)\right) \geq 0. \quad (37)$$

Condition (36) says that the effect on the lower income of decreasing variability dominates the effect on the larger one. In the case of symmetric poverty measures, condition (36) can also be stated as $\pi^{(1,1)}(x_1, x_2) = \pi^{(2,2)}(x_2, x_1) \geq 0$, which is a well-known convexity property for poverty functions.[21] Since all second-order derivatives are then positive, it can be shown that members of $\breve{\Pi}_{\eta S}(\tilde{\lambda}^+)$ comply with a multidimensional extension of the Pigou-Dalton transfer, *i.e.* a progressive transfer at any period between two individuals that can unambiguously be ranked in terms of poverty do not increase poverty.

Consider now the members of $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$ that also belong to the subclass $\breve{\Pi}_\eta(\tilde{\lambda}^+)$. For these indices, ρ must be such that $\rho_t^{(x_1)}(\mu, \tau_\eta) \leq 0$, $\rho_t^{(x_2)}(\mu, \tau_\eta) \leq 0$, and $\rho_t^{(x_1,x_2)}(\mu, \tau_\eta) \geq 0\ \forall t = 1, 2$. The first two conditions are automatically respected when ρ_t is derived with respect to the lower value of the income profile; increasing that income simultaneously raises average income and reduces variability, so that such an income increment would undoubtedly decrease poverty. When the larger income increases, the conditions on the first-order derivatives of ρ_t are satisfied if and only if $\forall t$:

[21] Assuming $x_1 < x_2$, symmetry means that condition (36) can be expressed as:

$$\pi^{(1)}(x_1, x_2) \leq \pi^{(1)}(x_2, x_1). \quad (38)$$

At the same time, we know that $\pi^{(1,2)}(x_1, x_2) \geq 0$, *i.e.* $\pi^{(1)}(x_1, x_1) - \pi^{(1)}(x_1, x_2) \leq 0$. Combining this with (38) yields:

$$\pi^{(1)}(x_1, x_1) \leq \pi^{(1)}(x_2, x_1). \quad (39)$$

which implies that second-order derivatives of π are non-negative $\forall(x_1, x_2)$.

$$\rho_t^{(1)}(\mu, \tau_\eta) \leq \left(\mu^{-\eta} + \frac{\eta\tau_\eta}{\mu}\right)\rho_t^{(2)}(\mu, \tau_\eta), \tag{40}$$

which says that the mean effect dominates the risk effect, as would be the case for all members of $\ddot{\Pi}(\lambda)$. Regarding the cross-derivative condition, its sign is positive if and only if:

$$\begin{aligned}&\rho_t^{(1,1)}(\mu, \tau_\eta) - 2\frac{\eta\tau_\eta}{\mu}\rho_t^{(1,2)}(\mu, \tau_\eta) - \frac{\eta(\eta+1)\tau_\eta}{2\mu^2}\rho_t^{(2)}(\mu, \tau_\eta)\\ &+\left(\left(\frac{\eta\tau_\eta}{\mu}\right)^2 - \mu^{-2\eta}\right)\rho_t^{(2,2)}(\mu, \tau_\eta) \geq 0.\end{aligned} \tag{41}$$

In the case of absolute-variability aversion ($\tau = 0$), this condition simplifies to $\rho_t^{(1,1)}(\mu, \tau_\eta) \geq \rho_t^{(2,2)}(\mu, \tau_\eta)$, *i.e.* the poverty-reducing effect of mean increases should decrease more rapidly with mean income than the poverty-reducing effect of lowering variability with respect to variability.

Let $A \ddot{\succsim}_\lambda B$ indicate dominance of A with respect to B over $\ddot{\Pi}(\lambda)$ (*cf.* Proposition 1).

Proposition 8

$$P_A(\tilde{\lambda}) > P_B(\tilde{\lambda}),\ \forall P(\tilde{\lambda}) \in \breve{\Pi}_\eta(\tilde{\lambda}^+), \tag{42}$$

$$\textit{if}\quad A \ddot{\succsim}_{\tilde{\lambda}^+} B \tag{43}$$

$$\textit{or}\quad A \tilde{\succsim}_{\eta,\tilde{\lambda}^+} B. \tag{44}$$

Proof See Appendix 3. □

Proposition 8 highlights the complementary nature of the dominance relationships $\ddot{\succsim}_\lambda$ and $\tilde{\succsim}_{\eta,\tilde{\lambda}}$, shown through the "hybrid" class of intertemporal poverty indices $\breve{\Pi}_\eta(\tilde{\lambda}^+)$. If one fails to observe a dominance relationship using Proposition 1, dominance may still be obtained using Proposition 4, and vice-versa. Consider for instance a distribution A made of two poor income profiles $(3, 4)$ and $(7, 1)$. Suppose that a distribution B is obtained by changing the second income profile to $(6, 2)$ using some variability reducing transfer. The two distributions A and B cannot be compared using Proposition 1. However, whatever the value of η, the cumulative distribution functions at (μ, τ_η) are never larger for B than for A, so that it can be concluded that B has less poverty than A for all poverty indices in $\breve{\Pi}_\eta(\tilde{\lambda}^+)$, some of them members of $\ddot{\Pi}(\lambda^+)$.

Corollary 1 *Assuming* $\exists(x_1, x_2) \in \Gamma(\tilde{\lambda}^+)$ *such that* $P_A^{1,1}(x_1, x_2) \neq P_B^{1,1}(x_1, x_2)$, *the following result cannot be obtained:*

$$A \ddot{\succsim}_{\tilde{\lambda}^+} B \textit{ and } B \tilde{\succsim}_{\eta,\tilde{\lambda}^+} A. \tag{45}$$

Proof See Appendix 3. □

Corollary 1 says that if one observes that A is dominated by B over $\ddot{\Pi}(\tilde{\lambda}^+)$ ($\tilde{\Pi}_\eta(\tilde{\lambda}^+)$), one would try in vain to infer that B is dominated by A over $\tilde{\Pi}_\eta(\tilde{\lambda}^+)$ ($\ddot{\Pi}(\tilde{\lambda}^+)$). Checking dominance of the type $\ddot{\succcurlyeq}_{\tilde{\lambda}^+}$ ($\tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+}$) can thus provide information on dominance of type $\tilde{\succcurlyeq}_{\eta,\tilde{\lambda}^+}$ ($\ddot{\succcurlyeq}_{\tilde{\lambda}^+}$) since both dominance criteria apply to classes of poverty measures that include the set of "hybrid" indices $\breve{\Pi}_\eta(\tilde{\lambda}^+)$.

5 An illustration with European data

We illustrate the tools proposed in Sects. 2–4 for intertemporal poverty comparisons using intertemporal income data from 23 European countries.[22] These data come from the 2009 version of the EU-SILC (European Union Survey on Income and Living Conditions) database. For each country, we select individuals that were surveyed both in 2006 and 2009. The 2006–2009 period is interesting since the European crisis may have resulted in a greater variability of income, with an intensity that may, however, have been different across countries due in part to differences in social safety net systems.

Note that consumption might be deemed to be a better indicator of welfare than income. But comparable cross-country longitudinal consumption data are rare. The implicit assumption, therefore, is that all of the observed income variability may be costly to the individual, even though it may have been anticipated and may have generated borrowing, saving, and consumption smoothing.

The dominance checks are performed using adult-equivalent disposable income obtained with the OECD equivalence scale. Purchasing power differences are taken into account using Eurostat PPP indices, and CPI indices were also used to compare income across periods.[23] The maximum poverty domain is defined using a "union" approach; individuals are thus regarded as poor if they are suffering from monetary deprivation either in 2006 or 2009. The maximal deprivation line was set to 120 % of the overall median income, that is about €15,350 per person and per year.[24]

Note also that our primary objective is to assess the *relative* (and not the *absolute*) ranking power of the results provided by Propositions 1 to 7. For this reason, we do not proceed to statistical testing of the population orderings, preferring to focus on the sample orderings. This being said, many of the sample orderings observed with our data may not be statistically inferable: going beyond the illustrative purposes of this section would require developing an appropriate statistical inference setting.

[22] The countries are: Austria (AT), Belgium (BE), Bulgaria (BG), Cyprus (CY), Czech Republic (CZ), Denmark (DK), Estonia (EE), Spain (ES), Finland (FI), France (FR), Hungary (HU), Iceland (IS), Italy (IT), Latvia (LV), Lithuania (LT), Malta (MT), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Slovenia (SI), Sweden (SE), and United Kingdom (UK).

[23] Incomes were censored at the bottom, so that our results should be regarded as restricted dominance tests (see Davidson and Duclos 2012, for theoretical and practical arguments). Censoring was applied at the second centile of the pooled distribution of incomes in 2006 and 2009, that is, at around €2,100 per person and per year.

[24] This figure is almost exactly equal to Italy's median income over the period. This choice is quite conservative but would undoubtedly meet unanimous agreement as a value above which an individual cannot be considered as deprived in the European context.

5.1 Symmetric and asymmetric dominance within the Duclos et al. (2006) framework

Our first result is that about 46 % of the 253 pairwise comparisons performed using these samples can be made through Proposition 1.[25] Since asymmetry can be applied in different manners, the dominance checks use two rival versions of it, reflecting different attitudes with respect to the patterns of income profiles. The first version considers that an income profile (a, b) with $a > b$ has more poverty than a symmetric profile (b, a). Such a view can be supported by the concept of loss aversion. Loss aversion is prominent in prospect theory and suggests that losses (of a given magnitude) can outweigh gains (of the same magnitude) in terms of well-being, implying *inter alia* that individuals may prefer upward income profiles, everything else being the same. The second version of asymmetry supports the opposite view, that is, that an income profile (a, b) with $a < b$ has more poverty than (b, a). This view is consistent with aversion to early poverty. Earlier income deprivations have longer-lasting effects on people's abilities to enjoy a valuable life. Consequently, the earlier a deprivation occurs, the longer its effects may last. Both versions of asymmetry rely on reasonable and documented grounds, so that it cannot easily be said which one is necessarily more appropriate.

Asymmetry increases the ordering power from 46 to 52 % with loss aversion and from 46 to 49 % with aversion to early poverty. The increase in the ordering power is higher with loss aversion (an increase of 6/46 = 13 % in the ordering power), indicating that it is more difficult to compare our European countries with a concern for early income poverty. With symmetry, the ordering power increases from 46 to 55 % (an increase of 9/46 = 20 % in the ordering power). In most cases, the dominance relationships with symmetry correspond to comparisons that are also robust either with loss aversion or with aversion to early poverty. Indeed, symmetry is necessary to obtain a dominance relationship only in two cases, that is, when comparing Austria with Cyprus and France.

Since symmetry and asymmetry are ways of extending the ordering power for intertemporal poverty comparisons, we also contrast our first results with those obtained with second-order dominance. Increasing the order of dominance is a frequent procedure in the stochastic dominance literature for attempting to obtain more distributional rankings. In a multidimensional framework, second-order dominance means that poverty comparisons are made with respect to members from a subclass of $\ddot{\Pi}(\lambda^+)$ that are sensitive to inequalities between the poor (more details in Duclos et al. 2006). It is then supposed that a progressive within-period transfer between two individuals reduces poverty. Moreover, the second-order derivatives of the individual poverty index π are decreasing and convex with respect to the level of the other period's income.

Note that second-order dominance thus requires full comparability (see Roberts 1980 for an interpersonal setting) of the different attributes used to assess poverty, whereas ordinality and non-comparability are sufficient for first-order dominance

[25] Detailed results are provided in Appendix 4, Table 1.

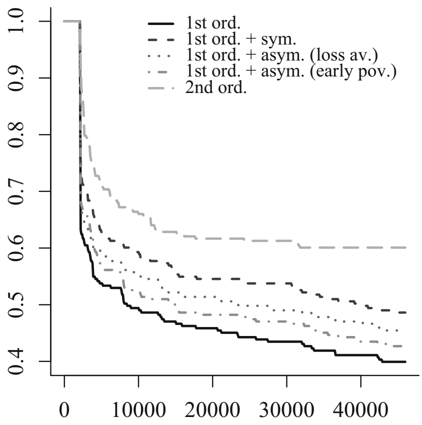


Fig. 14 Ordering power with different maximal deprivation lines. *Note* Each point gives the proportion of cross-country dominance relationships observed with the EU-SILC data for a maximal union poverty domain whose bounds are given by the x-axis

checks. This provides a rationale for preferring a search for first-order dominance rankings. Furthermore, if intertemporal poverty comparisons were performed for instance on health statuses at different periods, second-order dominance checks could not plausibly be used, and symmetry and asymmetry assumptions would be more natural avenues to extend the ordering power of first-order dominance tests. The results of those second-order dominance tests show that 62 % of the comparisons are now conclusive. For several pairwise comparisons, first-order dominance tests with asymmetry or symmetry yield robust comparisons that cannot be observed with second-order dominance, and *vice versa*. Such situations are observed for 28 pairwise comparisons, that is about 11 % of the possible pairwise comparisons, 22 out of these 28 additional orderings being observed only with the help of second-order dominance procedures.

Furthermore, even though our first-order dominance results with symmetry and asymmetry may look like those obtained at the second order without additional assumption, the two approaches should be considered as complements, not as substitutes. This is because some of the pairwise rankings can be made both with symmetric/asymmetric first-order and with second-order dominance tests. This means that two countries can sometimes be ranked over classes of poverty indices that are broader than the usual first-order and second-order classes. When this is observed, this has the effect of strengthening the degree of agreement on intertemporal poverty rankings across two populations.

The ordering power is likely to be contingent on the definition of the maximum poverty domain λ_S^+. To look into this, we estimate the share of pairwise comparisons yielding dominance relationships for different maximum values of the deprivation frontiers, up to €45,000. The results are reported in Fig. 14. Notice first that the absolute difference in ordering power between the different first-order dominance procedures does not significantly change with the value of the maximum deprivation line. In particular, asymmetry with loss aversion performs systematically better than asymmetry with aversion to early poverty. This result is likely due to heterogeneity in economic performance (and in the structure of social insurance and social assistance systems) observed in some of our countries during the 2006–2009 period: increasing unemployment and lower incomes yield joint distributions with a larger population within the relatively low second-period income domain $\Gamma_2(\lambda_S^+)$, that is, the set of poverty profiles on which greater emphasis is placed with loss aversion. Another interesting result is that the gap between the second-order dominance procedure and the different first-order dominance procedures is relatively constant, but widens significantly when the deprivation frontier increases above €30,000.

5.2 Mean/variability dominance

Our second set of dominance tests are made on the classes of mean/variability intertemporal poverty indices presented in Sect. 3. A potential problem with these classes deals with the choice of a value for the parameter η. Proposition 5 shows, however, that a useful start can be made by focussing on the absolute ($\eta = 0$) and relative ($\eta = 1$) bounds. The results (not reported in the appendix) are somewhat surprising: we are unable to obtain any robust comparison using absolute risk aversion, even when imposing symmetry. The rankings based on relative risk aversion should in principle be stronger, since the ordering power of relative risk aversion is theoretically greater (as shown by Proposition 5). Our results show, however, that the ordering power increases little with relative risk aversion since only one dominance relationship is obtained, between Cyprus and Spain. This limited ranking power is in large part due to the presence in each distribution of highly volatile income profiles, which make it difficult to establish dominance over large areas of mean/variability thresholds.

The role of income variability in explaining these results can be seen by comparing these results with those obtained when considering only the distribution of average incomes. Said differently, we keep the same value for the deprivation line but consider a subset of the class of intertemporal poverty indices $\tilde{\Pi}(\tilde{\lambda}_S)$ for which individual poverty is not affected by income variability; i.e. perfect individual-level income pooling is assumed, so that average temporal income is all that matters for assessing well-being at the individual level. The dominance criterion compares the distribution functions of mean intertemporal incomes, thus proceeding to a unidimensional analysis.[26] Our estimations confirm that income variability accounts for the low ranking power of the mean/variability dominance tests: the ordering power increases to 63 % when no variability aversion is assumed. Note that the ordering power is necessarily larger than the one observed with symmetry using the joint distribution of income (55 %), but that gain can be regarded as somewhat low considering the robustness loss caused by the assumption of perfect individual-level income pooling. Said differently, only 12 % (8/65) of the pairs of countries that can be ranked on the basis of the distributions of mean income cannot be ranked anymore if individual welfare can be affected (either positively or negatively) by income mobility.

26 More specifically, we consider poverty indices from $\tilde{\Pi}_S(\tilde{\lambda}_S)$ such that $\rho_t^{(2)}(\mu, \tau_\eta) = \rho_t^{(1,2)}(\mu, \tau_\eta) = 0$ $\forall t = 1, 2$. It can then be easily be seen from Eq. (81) in Appendix 2 that the poverty domain is necessarily defined as the set of income profiles such that $\mu < z^*$. Moreover, the corresponding dominance relationship compares the cumulative distribution of mean income up to a maximum value z^+ for the two populations.

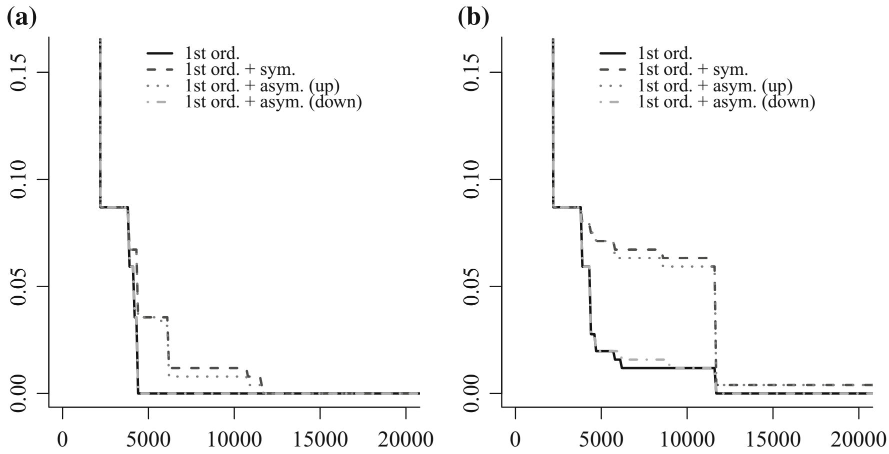


Fig. 15 Ordering power with different maximal deprivation lines for mean/variability indices. **a** Absolute view ($\eta = 0$). **b** Relative view ($\eta = 1$). *Note* Each point gives the proportion of cross-country dominance relationships observed with the EU-SILC data for a maximal union poverty domain with bounds given by the x-axis

The effect of income volatility can also be inferred from Figs. 15a, b. As for Fig. 14, the curves show the proportions of dominance relationships observed for different maximum deprivation thresholds (up to €20,000). The rapid collapse of the ordering power below €5,000, in particular when income variability is assessed in absolute terms, confirms that income volatility limits the power of mean/variability dominance relationships. This being said, comparing Fig. 15a, b shows nicely the gain in ordering power that can be attained by imposing relative as opposed to absolute variability aversion.

6 Conclusion

This paper proposes and applies procedures for making intertemporal poverty rankings. More generally, it considers comparisons of populations when multidimensional attributes of interest can be measured along comparable scales. The orderings are obtained with assumptions that do not require full comparability of the attributes, as would be required for instance with Pigou-Dalton-like transfer axioms. The role of symmetric and asymmetric assumptions is investigated. Symmetry supposes that the social evaluator is sensitive to the overall distribution of periodic deprivations, but not about the sequence of these deprivations, so that switching two incomes within an individual income profile is supposed not to affect overall well-being. The less demanding asymmetry assumptions suppose that the social evaluator prefers that incomes either increase (loss aversion) or decrease (early poverty aversion) over time. An empirical illustration on 23 European countries for the 2006–2009 period shows that such procedures can significantly improve the ranking power of dominance tests. The fact that the results assuming neutrality with respect to individual income variability are only slightly better further suggests that considerable ranking robustness can be obtained without having to suppose perfect intertemporal income pooling.

The paper also introduces classes of poverty indices that depend on mean intertemporal income and income variability. This framework is more demanding in terms of indicator comparability than the previous one as it requires full comparability of the indicators used to measure poverty (since it uses distances from mean income as a measure of income variability). The framework nevertheless makes it possible to incorporate a natural intertemporal "progressive transfer" assumption (without having to impose interpersonal progressive transfer assumptions, in the like of the popular Pigou-Dalton transfer principle). Moreover, it mirrors economists' common mean-and-variance framework often used to describe distributions and assess risk behavior. This framework can also incorporate symmetry and asymmetry axioms and can be applied to a continuum of different views of risk aversion, with the well-known relative and absolute views as limiting cases.

This common mean/variability framework for thinking about intertemporal welfare does not, however, have empirical strength when applied to our data. Dominance tests on a subset of indices that do not display variability–sensitivity show that this low ordering power is mostly due to the income variability observed in our distributions. This result suggests that the popular mean/variance approach may not be as useful for intertemporal poverty comparisons as some of the other recent income-based frameworks developed within welfare economics.

It is worth stressing that most families of intertemporal poverty indices have expressed a concern for the 'duration' of poverty and the 'bunching' of periods of deprivation—for a given set of income values, poverty is then supposed to be worse when deprivation periods are close or consecutive. These concerns have been prominent in the literature since the early papers by Bane and Ellwood (1986) and Rodgers and Rodgers (1993b). Recent examples include Bossert et al. (2012) (who attach importance to the damaging impact of long spells in poverty); Dutta et al. (2013) (who propose a mitigating effect of long spells of relative affluence on a subsequent poor period); Busetta and Mendola (2012) (who attach importance to the pairwise distances between waves of poverty) and Zheng (2012) (for 'gravitational' poverty measures that account for, among a number of other things, the combined effect of the distances between poor periods and the magnitude of the poverty in those poor periods).

In this paper's framework, poor income profiles are treated in the same manner by the symmetry and asymmetry axioms whatever the number of consecutive periods of deprivations. Future research in this area could thus develop an enriched version of this paper's framework, a framework that would be rich enough to accommodate other 'duration' and 'bunching' aspects of intertemporal poverty.

Note finally that this paper's classes of intertemporal poverty indices assume continuity with respect to income, while many of the literature's intertemporal poverty indices show discontinuities either at the poverty frontier or within the poverty domain—an exception is the family of indices proposed by Hoy and Zheng (2011). Future research might devise dominance criteria for classes of multidimensional poverty indices that exhibit income and/or intertemporal discontinuities.

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Appendix 1: proof of propositions from Sect. 2

Let $z_1(x_2)$ and $z_2(x_1)$ be respectively the value of the first and second-period income, such that $\lambda\big(x_1, z_2(x_1)\big) = 0$ and $\lambda\big(z_1(x_2), x_2\big) = 0$. Thus, $z_1\big(z_2(x_1)\big) = x_1$, and z_1 is then the inverse of z_2. Let z^* be the value of income such that $\lambda(z^*, z^*) = 0$. We then can define a two-period poverty index as a sum of low x_1 (with respect to x_2) and of low x_2 (with respect to x_1) time poverty:

$$P(\lambda) = \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_1 dx_2 + \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi(x_1, x_2; \lambda) f(x_1, x_2) dx_2 dx_1. \tag{46}$$

We first proceed with the first part of the right-hand term of (46). Integrating that expression by parts with respect to x_1, we find:

$$\begin{aligned}\int_0^{z^*} \int_{x_2}^{z_1(x_2)} & \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_1 dx_2 \\ &= \int_0^{z^*} \big[\pi(x_1, x_2) F(x_1|x_2)\big]_{x_1=x_2}^{x_1=z_1(x_2)} f(x_2)\, dx_2 \\ &\quad - \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2) F(x_1|x_2) f(x_2)\, dx_1 dx_2. \end{aligned} \tag{47}$$

Rearranging the first element of (47), we find

$$\int_0^{z^*} [\pi(x_1, x_2) F(x_1|x_2)]_{x_1=x_2}^{x_1=z_1(x_2)} f(x_2)\, dx_2$$

$$= \int_0^{z^*} \Big(\pi\big(z_1(x_2), x_2\big) F\big(z_1(x_2)|x_2\big) - \pi(x_2, x_2) F(x_1 = x_2|x_2)\Big) f(x_2)\, dx_2 \tag{48}$$

$$= -\int_0^{z^*} \pi(x_2, x_2) F(x_1 = x_2|x_2) f(x_2)\, dx_2, \tag{49}$$

since $\pi\big(z_1(x_2), x_2\big) = 0$.

To integrate the second part of the right-hand term of (47) by parts with respect to x_2, let $K(x_2) = \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2)\; F(x_1, x_2)\; dx_1$. We then get:

$$\frac{\partial K(x_2)}{\partial x_2} = z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)F\big(z_1(x_2), x_2\big) - \pi^{(1)}(x_2, x_2)F(x_2, x_2) + \int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\,dx_1 + \int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2)F(x_1|x_2)f(x_2)\,dx_1. \tag{50}$$

Integrating that expression along x_2 and over $[0, z^*]$ and rearranging, we have:

$$\int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1)}(x_1, x_2)F(x_1|x_2)f(x_2)\,dx_1dx_2 \tag{51}$$

$$= \big[K(x_2)\big]_0^{z^*} - \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)F\big(z_1(x_2), x_2\big)\,dx_2 + \int_0^{z^*} \pi^{(1)}(x_2, x_2)F(x_2, x_2)\,dx_2 - \int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\,dx_1dx_2, \tag{52}$$

$$= -\int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)F\big(z_1(x_2), x_2\big)\,dx_2 + \int_0^{z^*} \pi^{(1)}(x_2, x_2)F(x_2, x_2)\,dx_2 - \int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\,dx_1dx_2, \tag{53}$$

since $z_1(z^*) = z^*$ (hence $K(z^*) = 0$) and $F(x_1, 0) = 0\ \forall x_1$ (hence $K(0) = 0$). Using (49) and (53), we obtain:

$$\int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi(x_1, x_2, \lambda)f(x_1, x_2)dx_1dx_2 = -\int_0^{z^*} \pi(x_2, x_2)F(x_1 = x_2|x_2)f(x_2)\,dx_2 + \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)F\big(z_1(x_2), x_2\big)\,dx_2 - \int_0^{z^*} \pi^{(1)}(x_2, x_2)F(x_2, x_2)\,dx_2 + \int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\,dx_1dx_2. \tag{54}$$

Proceeding similarly with the second part of the right-hand term of (46) and adding the above, we obtain:

$$\begin{aligned}P(\lambda) = &-\int_0^{z^*} \pi(x_2, x_2)F(x_1 = x_2|x_2)f(x_2)\,dx_2 \\ &+\int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)F\big(z_1(x_2), x_2\big)\,dx_2 \\ &-\int_0^{z^*} \pi^{(1)}(x_2, x_2)F(x_2, x_2)\,dx_2 \\ &+\int_0^{z^*}\int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\,dx_1dx_2 \\ &-\int_0^{z^*} \pi(x_1, x_1)F(x_2 = x_1|x_1)f(x_1)\,dx_1 \\ &+\int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(x_1, z_2(x_1)\big)F\big(x_1, z_2(x_1)\big)\,dx_1 \\ &-\int_0^{z^*} \pi^{(2)}(x_1, x_1)F(x_1, x_1)\,dx_1 \\ &+\int_0^{z^*}\int_{x_1}^{z_2(x_1)} \pi^{(1,2)}(x_1, x_2)F(x_1, x_2)\,dx_2dx_1. \end{aligned} \tag{55}$$

It can be observed that $F(x_2 = x_1|x_1)f(x_1) = \frac{\partial F(x_1,x_1)}{\partial x_1} - F(x_1|x_2 = x_1)f(x_2 = x_1)$, so that:

$$\begin{aligned}&\int_0^{z^*} \pi(x_1, x_1)F(x_2 = x_1|x_1)f(x_1)\,dx_1 \\ &= \int_0^{z^*} \pi(x_1, x_1)\frac{\partial F(x_1, x_1)}{\partial x_1}\,dx_1 - \int_0^{z^*} \pi(x_1, x_1)F(x_1|x_2 = x_1)f(x_2 = x_1)\,dx_1 \end{aligned} \tag{56}$$

$$\begin{aligned}&= [\pi(x_1, x_1)F(x_1, x_1)]_0^{z^*} - \int_0^{z^*} \left(\pi^{(1)}(x_1, x_1) + \pi^{(2)}(x_1, x_1)\right) F(x_1, x_1)\,dx_1 \\ &\quad - \int_0^{z^*} \pi(x_1, x_1)F(x_1|x_2 = x_1)f(x_2 = x_1)\,dx_1 \end{aligned} \tag{57}$$

$$\begin{aligned}&= -\int_0^{z^*} \left(\pi^{(1)}(x_1, x_1) + \pi^{(2)}(x_1, x_1)\right) F(x_1, x_1)\,dx_1 \\ &\quad - \int_0^{z^*} \pi(x_2, x_2)F(x_1 = x_2|x_2)f(x_2)\,dx_2. \end{aligned} \tag{58}$$

Using that result and changing the integration variable in $\int_0^{z^*} \pi^{(2)}(x_2, x_2) F(x_2, x_2)\,dx_2$, we then have:

$$P(\lambda) = \int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(x_1, z_2(x_1)\big) F\big(x_1, z_2(x_1)\big)\, dx_1 + \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) F\big(z_1(x_2), x_2\big)\, dx_2 + \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_2 dx_1 + \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2. \quad (59)$$

Proof of Proposition 2

Symmetry implies the following properties:

$$\pi^{(1)}(x_1, x_2) = \pi^{(2)}(x_2, x_1) \quad \forall x_1, x_2, \quad (60)$$

$$\pi^{(1,2)}(x_1, x_2) = \pi^{(1,2)}(x_2, x_1) \quad \forall x_1, x_2. \quad (61)$$

At the poverty frontier, we also have $\lambda(x_1, x_2) = 0$ and $\pi^{(x_2)}\big(z_1(x_2), x_2\big) = 0$. Since:

$$\pi^{(x_2)}\big(z_1(x_2), x_2\big) = z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) + \pi^{(2)}\big(z_1(x_2), x_2\big), \quad (62)$$

we obtain:

$$z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) = -\pi^{(2)}\big(z_1(x_2), x_2\big). \quad (63)$$

Symmetry also leads to $z_1(x_2) = z_2(x_2)$ and $z_1'(x_2) = z_2'(x_2)$. Using (60), we find that:

$$z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big) = z_2'(x_2)\pi^{(2)}\big(x_2, z_2(x_2)\big). \quad (64)$$

From the expression of $P(\lambda)$ in (59), the symmetry assumptions therefore lead to:

$$P(\lambda) = \int_0^{z^*} z_1'(x_2)\pi^{(1)}\big(z_1(x_2), x_2\big)\Big(F\big(z_1(x_2), x_2\big) + F\big(x_2, z_1(x_2)\big)\Big)\, dx_2 \quad (65)$$

$$+ \int_0^{z^*} \int_{x_2}^{z_1(x_2)} \pi^{(1,2)}(x_1, x_2)\big(F(x_1, x_2) + F(x_2, x_1)\big)\, dx_1 dx_2 \quad (66)$$

The necessary and sufficient conditions for Proposition 2 follow upon inspection.

Proof of Proposition 3

With asymmetry, we assume that $z_2(x_1) \geq z_1(x_1)$ for all $x_1 \in [0, z^*]$. Equation (46) can then be rewritten as:

$$P(\lambda) = \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi(x_1, x_2; \lambda) f(x_1, x_2)\, dx_2 dx_1 + \int_0^{z^*} \int_{x_2}^{z_2(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2)\, dx_1 dx_2. \tag{67}$$

Equation (59) then becomes:

$$\begin{aligned} P(\lambda) = & \int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(x_1, z_2(x_1)\big) F\big(x_1, z_2(x_1)\big)\, dx_1 \\ & + \int_0^{z^*} z_2'(x_2)\pi^{(1)}\big(z_2(x_2), x_2\big) F\big(z_2(x_2), x_2\big)\, dx_2 \\ & + \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_2 dx_1 \\ & + \int_0^{z^*} \int_{x_2}^{z_2(x_2)} \pi^{(1,2)}(x_1, x_2) F(x_1, x_2)\, dx_1 dx_2. \end{aligned} \tag{68}$$

We obtain:

$$\begin{aligned} P(\lambda) = & \int_0^{z^*} z_2'(x_1)\pi^{(2)}\big(z_2(x_1), x_1\big) \left[F\big(z_2(x_1), x_1\big) + F\big(x_1, z_2(x_1)\big)\right]\, dx_1 \\ & + \int_0^{z^*} z_2'(x_1) \left[\pi^{(1)}\big(x_1, z_2(x_1)\big) - \pi^{(2)}\big(z_2(x_1), x_1\big)\right] F\big(x_1, z_2(x_1)\big)\, dx_1 \\ & + \int_0^{z^*} \int_{x_2}^{z_2(x_2)} \pi^{(1,2)}(x_1, x_2) \left[F(x_1, x_2) + F(x_2, x_1)\right] dx_1 dx_2 \\ & + \int_0^{z^*} \int_{x_1}^{z_2(x_1)} \left[\pi^{(1,2)}(x_1, x_2) - \pi^{(1,2)}(x_2, x_1)\right] F(x_1, x_2) dx_2 dx_1, \end{aligned} \tag{69}$$

with, by assumption, $\pi^{(1)}\big(x_1, z_2(x_1)\big) - \pi^{(2)}\big(z_2(x_1), x_1\big) \leq 0$ and $\pi^{(1,2)}(x_1, x_2) - \pi^{(1,2)}(x_2, x_1) \geq 0$. The second and fourth terms of the right-hand side of (69) account for the first condition of Proposition 3, while the first and third terms account for its second condition.

Appendix 2: proof of propositions from Sect. 3

Let the lowest value of mean income on the mean/variability poverty frontier be obtained for $\tau_\eta = 0$ at $\mu = z^*$, so that $\tilde{\lambda}(z^*, 0) = \lambda(z^*, z^*) = 0$. At this point, it is also necessary to differentiate between the cases of $x_1 < x_2$ and $x_1 > x_2$. Let $\tau_\eta^{z1}(\mu)$ ($\tau_\eta^{z2}(\mu)$) be the value of τ_η such that $\tilde{\lambda}\big(\mu, \tau_\eta^{z}(\mu)\big) = 0$ when $x_1 < x_2$ ($x_1 > x_2$). Since individuals are supposed to be poor $\forall \tau^\eta$ if $\mu \leq z^*$, $\tau_\eta^{z1}(\mu)$ and $\tau_\eta^{z2}(\mu)$ are defined on the intervals $[z^*, +\infty)$. Due to the monotonicity assumptions, $\frac{\partial \tau_\eta^{z1}}{\partial \mu} \leq 0$ and $\frac{\partial \tau_\eta^{z2}}{\partial \mu} \leq 0$.

Let $q := \text{prob}(x_1 < x_2)$ and ρ_1 (ρ_2) be the individual poverty measure to be applied when $x_1 < x_2$ ($x_1 > x_2$). Let f_1 (f_2) denote the joint density function of μ and τ_η, conditional on $x_1 < x_2$ ($x_1 > x_2$). The same notation applies for the cdf, conditional cdf and marginal cdf and marginal density functions. With the above, lifetime poverty defined in (19) can alternatively be defined as:

$$
\begin{aligned}
\tilde{P}(\tilde{\lambda}) = q \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_1(\mu, \tau_\eta, \tilde{\lambda}) f_1(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ q \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)} \rho_1(\mu, \tau_\eta, \tilde{\lambda}) f_1(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ (1-q) \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_2(\mu, \tau_\eta, \tilde{\lambda}) f_2(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ (1-q) \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{z2}(\mu)} \rho_2(\mu, \tau_\eta, \tilde{\lambda}) f_2(\mu, \tau_\eta)\, d\tau_\eta d\mu.
\end{aligned} \tag{70}
$$

For convenience, $\tilde{\lambda}$ is dropped from the expression of ρ. We first consider the first and third elements of the right-hand term of (70) and, integrating by parts, find $\forall j = 1, 2$:

$$
\begin{aligned}
&\int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_j(\mu, \tau_\eta) f_j(\mu, \tau_\eta) d\tau_\eta d\mu \\
&= \int_0^{z^*} \big[\rho_j(\mu, \tau_\eta) F_j(\tau_\eta|\mu)\big]_{\tau_\eta = -\mu^{1-\eta}}^{\tau_\eta = 0} f_j(\mu)\, d\mu \\
&- \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu, \tau_\eta) F_j(\tau_\eta|\mu) f_j(\mu)\, d\tau_\eta d\mu.
\end{aligned} \tag{71}
$$

As $F_j(\tau_\eta = -\mu^{1-\eta}|\mu) = 0$ and $F_j(\tau_\eta = 0|\mu) = 1$, the first element on the right-hand side of (71) can be expressed as:

$$
\begin{aligned}
&\int_0^{z^*} \big[\rho_j(\mu, \tau_\eta) F_j(\tau_\eta|\mu)\big]_{\tau_\eta = -\mu^{1-\eta}}^{\tau_\eta = 0} f_j(\mu)\, d\mu \\
&= \int_0^{z^*} \rho_j(\mu, 0) f_j(\mu)\, d\mu,
\end{aligned}
$$

$$= \left[\rho_j(\mu, 0)F_j(\mu)\right]_{\mu=0}^{\mu=z^*} - \int_0^{z^*} \rho_j^{(1)}(\mu, 0)F_j(\mu)\, d\mu$$
$$= -\int_0^{z^*} \rho_j^{(1)}(\mu, 0)F_j(\mu)\, d\mu, \tag{72}$$

since $F_j(\mu = 0) = 0$ and the function ρ_j is zero at the poverty frontier ($\rho_j(z^*, 0) = 0$).

We now can turn to the second element of the right-hand term of (71). Define $Q_j(\mu) = \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu, \tau_\eta)F_j(\mu, \tau_\eta)\, d\tau_\eta$. We have:

$$\frac{\partial Q_j}{\partial \mu} = (1-\eta)\mu^{-\eta}\rho_j^{(2)}\left(\mu, -\mu^{1-\eta}\right) F_j\left(\mu, -\mu^{1-\eta}\right)$$
$$+ \int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu, \tau_\eta)F_j(\mu, \tau_\eta)\, d\tau_\eta$$
$$+ \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu, \tau_\eta)F_j(\tau_\eta|\mu) f_j(\mu)\, d\tau_\eta$$
$$= \int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu, \tau_\eta)F_j(\mu, \tau_\eta)\, d\tau_\eta$$
$$+ \int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu, \tau_\eta)F_j(\tau_\eta|\mu) f_j(\mu)\, d\tau_\eta, \tag{73}$$

since $F_j(\mu, -\mu^{1-\eta}) = 0$. Integrating that expression along μ and over $[0, z^*]$ and rearranging, we have:

$$\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_j^{(2)}(\mu, \tau_\eta)F_j(\tau_\eta|\mu) f_j(\mu)\, d\tau_\eta d\mu$$
$$= \left[Q_j(\mu)\right]_0^{z^*} - \int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu, \tau_\eta)F_j(\mu, \tau_\eta)\, d\tau_\eta d\mu$$
$$= \int_{-z^{*1-\eta}}^{0} \rho_j^{(2)}(z^*, \tau_\eta)F_j(z^*, \tau_\eta)\, d\tau_\eta$$
$$- \int_0^{z^*}\int_{-\mu^{1-\eta}}^{0} \rho_j^{(1,2)}(\mu, \tau_\eta)F_j(\mu, \tau_\eta)\, d\tau_\eta d\mu. \tag{74}$$

We then consider the second and fourth elements on the right-hand side of (70) and, using once again integration by parts, find:

$$\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j(\mu, \tau_\eta) f_j(\mu, \tau_\eta)\, d\tau_\eta d\mu$$
$$= \int_{z^*}^{+\infty} \left[\rho_j(\mu, \tau_\eta)F_j(\tau_\eta|\mu)\right]_{\tau_\eta=-\mu^{1-\eta}}^{\tau_\eta=\tau_\eta^{zj}(\mu)} f_j(\mu)\, d\mu$$

$$
\begin{aligned}
&-\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\,d\tau_\eta d\mu\\
&=-\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)} \rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\,d\tau_\eta d\mu, \qquad (75)
\end{aligned}
$$

as $\rho_j\big(\mu,\tau_\eta^{zj}(\mu)\big)=0$ and $F_j(\tau_\eta=-\mu^{1-\eta}|\mu)=0$.

Let $R_j(\mu)=\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\,d\tau_\eta$. We have:

$$
\begin{aligned}
\frac{\partial R_j}{\partial\mu}&=\tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j\big(\mu,\tau_\eta^{zj}(\mu)\big)\\
&\quad+(1-\eta)\mu^{-\eta}\rho_j^{(2)}(\mu,-\mu^{1-\eta})F_j(\mu,-\mu^{1-\eta})\\
&\quad+\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\,d\tau_\eta+\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\,d\tau_\eta,\\
&=\tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j\big(\mu,\tau_\eta^{zj}(\mu)\big)\\
&\quad+\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\,d\tau_\eta+\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\,d\tau_\eta.
\end{aligned}
\qquad (76)
$$

Integrating that expression along μ and over $[z^*,+\infty]$ and rearranging, we have:

$$
\begin{aligned}
&\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(2)}(\mu,\tau_\eta)F_j(\tau_\eta|\mu)f_j(\mu)\,d\tau_\eta d\mu\\
&=\big[R_j(\mu)\big]_{z^*}^{+\infty}-\int_{z^*}^{+\infty}\tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j\big(\mu,\tau_\eta^{zj}(\mu)\big)\,d\mu\\
&\quad-\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\,d\tau_\eta d\mu\\
&=\int_{-\infty^{1-\eta}}^{\tau_\eta^{zj}(+\infty)}\rho_j^{(2)}(+\infty,\tau_\eta)F_j(+\infty,\tau_\eta)\,d\tau_\eta-\int_{-z^{*1-\eta}}^{0}\rho_j^{(2)}(z^*,\tau_\eta)F_j(z^*,\tau_\eta)\,d\tau_\eta\\
&\quad-\int_{z^*}^{+\infty}\tau_\eta^{zj\prime}(\mu)\rho_j^{(2)}\big(\mu,\tau_\eta^{zj}(\mu)\big)F_j\big(\mu,\tau_\eta^{zj}(\mu)\big)\,d\mu\\
&\quad-\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{zj}(\mu)}\rho_j^{(1,2)}(\mu,\tau_\eta)F_j(\mu,\tau_\eta)\,d\tau_\eta d\mu. \qquad (77)
\end{aligned}
$$

Using (72), (74), (77), and $\rho_1(\mu,0)=\rho_2(\mu,0)\,\forall\mu$, we finally obtain the following expression for $P(\lambda)$:

$$
\tilde P(\tilde\lambda)=-\int_0^{z^*}\rho_2^{(1)}(\mu,0)F(\mu)\,d\mu-q\int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)}\rho_1^{(2)}(+\infty,\tau_\eta)F_1(+\infty,\tau_\eta)\,d\tau_\eta \qquad (78)
$$

$$
\begin{aligned}
&+q\int_{z^*}^{+\infty}\tau_\eta^{z1'}(\mu)\rho_1^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)F_1\big(\mu,\tau_\eta^{z1}(\mu)\big)\,d\mu\\
&+q\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_1^{(1,2)}(\mu,\tau_\eta)F_1(\mu,\tau_\eta)\,d\tau_\eta d\mu\\
&+q\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)}\rho_1^{(1,2)}(\mu,\tau_\eta)F_1(\mu,\tau_\eta)\,d\tau_\eta d\mu\\
&-(1-q)\int_{-\infty^{1-\eta}}^{\tau_\eta^{z2}(+\infty)}\rho_2^{(2)}(+\infty,\tau_\eta)F_2(+\infty,\tau_\eta)\,d\tau_\eta\\
&+(1-q)\int_{z^*}^{+\infty}\tau_\eta^{z2'}(\mu)\rho_2^{(2)}\big(\mu,\tau_\eta^{z2}(\mu)\big)F_2\big(\mu,\tau_\eta^{z2}(\mu)\big)\,d\mu\\
&+(1-q)\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_2^{(1,2)}(\mu,\tau_\eta)F_2(\mu,\tau_\eta)\,d\tau_\eta d\mu\\
&+(1-q)\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z2}(\mu)}\rho_2^{(1,2)}(\mu,\tau_\eta)F_2(\mu,\tau_\eta)\,d\tau_\eta d\mu.
\end{aligned}
$$

Proposition 4 then follows directly from (78) by inspection.

Proof of Proposition 6

Letting $\rho_1(\mu,\tau_\eta)=\rho_2(\mu,\tau_\eta)\ \forall(\mu,\tau_\eta)\in\Gamma_S(\tilde{\lambda})$, it follows that $\forall(\mu,\tau_\eta)\in\Gamma_S(\tilde{\lambda})$:

$$\rho_1^{(2)}(\mu,\tau_\eta)=\rho_2^{(2)}(\mu,\tau_\eta), \tag{79}$$

$$\rho_1^{(1,2)}(\mu,\tau_\eta)=\rho_2^{(1,2)}(\mu,\tau_\eta). \tag{80}$$

Moreover, $\tau_\eta^{z1}(\mu)=\tau_\eta^{z2}(\mu)$, so that $\tau_\eta^{z1'}(\mu)=\tau_\eta^{z2'}(\mu)$. Equation (78) can then be rewritten as:

$$
\begin{aligned}
\tilde{P}(\tilde{\lambda})=&-\int_0^{z^*}\rho_1^{(1)}(\mu,0)F(\mu)\,d\mu-\int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)}\rho_1^{(2)}(+\infty,\tau_\eta)F(+\infty,\tau_\eta)\,d\tau_\eta\\
&+\int_{z^*}^{+\infty}\tau_\eta^{z1'}(\mu)\rho_1^{(2)}\big(\mu,\tau_\eta^{z1}(\mu)\big)F\big(\mu,\tau_\eta^{z1}(\mu)\big)\,d\mu\\
&+\int_0^{z^*}\int_{-\mu^{1-\eta}}^{0}\rho_1^{(1,2)}(\mu,\tau_\eta)F(\mu,\tau_\eta)\,d\tau_\eta d\mu\\
&+\int_{z^*}^{+\infty}\int_{-\mu^{1-\eta}}^{\tau_\eta^{z}(\mu)}\rho_1^{(1,2)}(\mu,\tau_\eta)F(\mu,\tau_\eta)\,d\tau_\eta d\mu.
\end{aligned}\tag{81}
$$

The rest of the proof follows by inspection.

Proof of Proposition 7

With asymmetry, it is assumed that $\tau_\eta^{z1}(\mu) \geq \tau_\eta^{z2}(\mu)$ for all $\mu \in [z^*, +\infty)$. As a consequence, Eq. (46) can be rewritten as:

$$
\begin{aligned}
\tilde{P}(\tilde{\lambda}) = q \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_1(\mu, \tau_\eta, \tilde{\lambda}) f_1(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ q \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)} \rho_1(\mu, \tau_\eta, \tilde{\lambda}) f_1(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ (1-q) \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_2(\mu, \tau_\eta, \tilde{\lambda}) f_2(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ (1-q) \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)} \rho_2(\mu, \tau_\eta, \tilde{\lambda}) f_2(\mu, \tau_\eta)\, d\tau_\eta d\mu.
\end{aligned}
\tag{82}
$$

Noting that $\rho_1^{(2)}(\mu, \tau_\eta) - \rho_2^{(2)}(\mu, \tau_\eta) \leq 0$ and $\rho_1^{(1,2)}(\mu, \tau_\eta) - \rho_2^{(1,2)}(\mu, \tau_\eta) \geq 0$, we obtain:

$$
\begin{aligned}
\tilde{P}(\tilde{\lambda}) = -\int_0^{z^*} \rho_2^{(1)}(\mu, 0) F(\mu)\, d\mu - \int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)} \rho_2^{(2)}(+\infty, \tau_\eta) F(+\infty, \tau_\eta)\, d\tau_\eta \\
- q \int_{-\infty^{1-\eta}}^{\tau_\eta^{z1}(+\infty)} \left(\rho_1^{(2)}(+\infty, \tau_\eta) - \rho_2^{(2)}(+\infty, \tau_\eta)\right) F_1(+\infty, \tau_\eta)\, d\tau_\eta \\
+ \int_{z^*}^{+\infty} \tau_\eta^{z1\prime}(\mu) \rho_2^{(2)}\left(\mu, \tau_\eta^{z1}(\mu)\right) F\left(\mu, \tau_\eta^{z1}(\mu)\right) d\mu \\
+ q \int_{z^*}^{+\infty} \tau_\eta^{z1\prime}(\mu) \left(\rho_1^{(2)}\left(\mu, \tau_\eta^{z1}(\mu)\right) - \rho_2^{(2)}\left(\mu, \tau_\eta^{z1}(\mu)\right)\right) F_1\left(\mu, \tau_\eta^{z1}(\mu)\right) d\mu \\
+ \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \rho_2^{(1,2)}(\mu, \tau_\eta) F(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ \int_0^{z^*} \int_{-\mu^{1-\eta}}^{0} \left(\rho_1^{(1,2)}(\mu, \tau_\eta) - \rho_2^{(1,2)}(\mu, \tau_\eta)\right) F_1(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)} \rho_2^{(1,2)}(\mu, \tau_\eta) F(\mu, \tau_\eta)\, d\tau_\eta d\mu \\
+ q \int_{z^*}^{+\infty} \int_{-\mu^{1-\eta}}^{\tau_\eta^{z1}(\mu)} \left(\rho_1^{(1,2)}(\mu, \tau_\eta) - \rho_2^{(1,2)}(\mu, \tau_\eta)\right) F_1(\mu, \tau_\eta)\, d\tau_\eta d\mu.
\end{aligned}
\tag{83}
$$

The rest of the proof follows by inspection.

Appendix 3: intersection of the different classes of poverty measures

The derivations of additional restrictions on the individual poverty measure π and ρ

We first consider the conditions that ρ must obey so that members of $\ddot{\Pi}(\lambda^+)$ are also members of $\tilde{\Pi}(\tilde{\lambda}^+)$. First, we have $\pi^{(x_i)}(x_1, x_2) \leq 0, \forall i = 1, 2$, so that we should also observe $\rho_t^{(x_i)}(\mu, \tau_\eta) \leq 0 \; \forall t$. When x_i is not the lowest income, an income increment increases variability ($|\tau_\eta|$ rises) while increasing mean income, so that the net effect of this is *a priori* not known. Assuming x_1 to be the lowest income, if that net effect is supposed to correspond to a decrease in the level of poverty, we should have:

$$\rho_1^{(x_2)}(\mu, \tau_\eta) = \rho_1^{(1)}(\mu, \tau_\eta)\mu^{(x_2)} + \rho_1^{(2)}(\mu, \tau_\eta)\tau_\eta^{(x_2)} \leq 0 \tag{84}$$

$$\Rightarrow \frac{1}{2}\rho_1^{(1)}(\mu, \tau_\eta) + \frac{1}{2}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2 - x_1) - \mu\right)\rho_1^{(2)}(\mu, \tau_\eta) \leq 0 \tag{85}$$

$$\Rightarrow \frac{1}{2}\rho_1^{(1)}(\mu, \tau_\eta) - \frac{1}{2}\left(\mu^{-\eta} + \frac{\eta\tau_\eta}{\mu}\right)\rho_1^{(2)}(\mu, \tau_\eta) \leq 0 \tag{86}$$

$$\Rightarrow \rho_1^{(1)}(\mu, \tau_\eta) \leq \left(\mu^{-\eta} + \frac{\eta\tau_\eta}{\mu}\right)\rho_1^{(2)}(\mu, \tau_\eta). \tag{87}$$

In the same manner, it would also be necessary to observe $\rho^{(x_1,x_2)}(\mu, \tau_\eta) \geq 0$. Still supposing x_1 to be the lower income, we have:

$$\begin{aligned}
\rho_1^{(x_1,x_2)}(\mu, \tau_\eta) = {} & \frac{1}{4}\rho_1^{(1,1)}(\mu, \tau_\eta) + \frac{1}{4}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2 - x_1) + \mu\right)\rho_1^{(1,2)}(\mu, \tau_\eta) \\
& - \frac{\eta(\eta+1)}{2}2^{\eta-1}(x_1 + x_2)^{-\eta-2}(x_2 - x_1)\rho_1^{(2)}(\mu, \tau_\eta) \\
& + \frac{1}{2}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2 - x_1) - \mu\right)\left(\frac{1}{2}\rho_1^{(1,2)}(\mu, \tau_\eta)\right. \\
& \left. + \frac{1}{2}\mu^{-\eta-1}\left(\frac{\eta}{2}(x_2 - x_1) + \mu\right)\rho_1^{(2,2)}(\mu, \tau_\eta)\right) \qquad (88) \\
= {} & \frac{1}{4}\rho_1^{(1,1)}(\mu, \tau_\eta) + \frac{\eta}{4}\mu^{-\eta-1}(x_2 - x_1)\rho_1^{(1,2)}(\mu, \tau_\eta) \\
& - \frac{\eta(\eta+1)}{2}2^{\eta-1}(x_1 + x_2)^{-\eta-2}(x_2 - x_1)\rho_1^{(2)}(\mu, \tau_\eta) \\
& + \frac{1}{4}\mu^{-2(\eta+1)}\left(\frac{\eta^2}{4}(x_2 - x_1)^2 - \mu^2\right)\rho_1^{(2,2)}(\mu, \tau_\eta) \qquad (89) \\
= {} & \frac{1}{4}\rho_1^{(1,1)}(\mu, \tau_\eta) - \frac{1}{2}\frac{\eta\tau_\eta}{\mu}\rho_1^{(1,2)}(\mu, \tau_\eta) - \frac{\eta(\eta+1)\tau_\eta}{8\mu^2}\rho_1^{(2)}(\mu, \tau_\eta) \\
& + \frac{1}{4}\left(\left(\frac{\eta\tau_\eta}{\mu}\right)^2 - \mu^{-2\eta}\right)\rho_1^{(2,2)}(\mu, \tau_\eta). \qquad (90)
\end{aligned}$$

Considering the case of absolute variability aversion ($\eta = 0$), ρ_t must exhibit the following two properties:

$$\begin{cases} \rho_t^{(1)}(\mu, \tau_0) \leq \rho_t^{(2)}(\mu, \tau_0), \\ \rho_t^{(1,1)}(\mu, \tau_0) \geq \rho_t^{(2,2)}(\mu, \tau_0). \end{cases} \tag{91}$$

With relative variability aversion ($\eta = 1$), the conditions become:

$$\begin{cases} \rho_t^{(1)}(\mu, \tau_1) \leq \frac{1+\tau_1}{\mu} \rho_t^{(2)}(\mu, \tau_1), \\ \rho_t^{(1,1)}(\mu, \tau_1) \geq \frac{2\tau_1}{\mu} \rho_t^{(1,2)}(\mu, \tau_1) + \frac{\tau_1}{\mu^2} \rho_t^{(2)}(\mu, \tau_1) + \frac{1-\tau_1^2}{\mu^2} \rho_t^{(2,2)}(\mu, \tau_1). \end{cases} \tag{92}$$

It can also be shown that $x_1 = \mu + \tau_\eta \mu^\eta$ and $x_2 = \mu - \tau_\eta \mu^\eta$ if $x_1 < x_2$. It is then possible to compute $\pi^{(\mu)}$, $\pi^{(\tau_\eta)}$ and $\pi^{(\mu,\tau_\eta)}$ to see what conditions have to be met so that π respects the conditions imposed on ρ. First, considering the derivatives of π with respect to mean income, we should observe:

$$\pi^{(\mu)}(x_1, x_2) = \pi^{(1)}(x_1, x_2) x_1^{(\mu)} + \pi^{(2)}(x_1, x_2) x_2^{(\mu)} \leq 0 \tag{93}$$

$$\Rightarrow (1 + \eta\tau_\eta \mu^{\eta-1}) \pi^{(1)}(x_1, x_2) + (1 - \eta\tau_\eta \mu^{\eta-1}) \pi^{(2)}(x_1, x_2) \leq 0. \tag{94}$$

That condition is always fulfilled since $1 + \eta\tau_\eta\mu^{\eta-1}$ and $1 - \eta\tau_\eta\mu^{\eta-1}$ are positive for $\eta \in [0, 1]$, and $\pi^{(1)}(x_1, x_2)$ and $\pi^{(2)}(x_1, x_2)$ are also non-negative. The result is intuitive. Increasing the mean without altering variability implies increasing income at both periods, so that poverty should logically fall.

Considering now a decrease in variability without a change in mean income, things are less clear since such a change raises the lower income but decreases the higher one. It is then necessary to consider the net sum of those opposite effects. Since $\rho_t^{(2)}(\mu, \tau_\eta) \leq 0$, we should obtain:

$$\pi^{(\tau_\eta)}(x_1, x_2) = \mu^\eta \pi^{(1)}(x_1, x_2) - \mu^\eta \pi^{(2)}(x_1, x_2) \leq 0 \tag{95}$$

$$\Rightarrow \pi^{(1)}(x_1, x_2) \leq \pi^{(2)}(x_1, x_2). \tag{96}$$

Finally, π has to be defined so as to respect $\pi^{(\mu,\tau_\eta)}(x_1, x_2) \geq 0$. We have:

$$\begin{aligned} &\pi^{(\mu,\tau_\eta)}(x_1, x_2) \\ &\quad = \eta\mu^{\eta-1}\pi^{(1)}(x_1, x_2) + \left(1 + \eta\tau_\eta\mu^{\eta-1}\right)\left(\mu^\eta \pi^{(1,1)}(x_1, x_2) - \mu^\eta \pi^{(1,2)}(x_1, x_2)\right) \\ &\qquad - \eta\mu^{\eta-1}\pi^{(2)}(x_1, x_2) + \left(1 - \eta\tau_\eta\mu^{\eta-1}\right)\left(\mu^\eta \pi^{(1,2)}(x_1, x_2) - \mu^\eta \pi^{(2,2)}(x_1, x_2)\right) \end{aligned} \tag{97}$$

$$
\begin{aligned}
&= \eta\mu^{\eta-1}\left(\pi^{(1)}(x_1,x_2)-\pi^{(2)}(x_1,x_2)\right)\\
&\quad+\mu^{\eta}\left(1+\eta\tau_\eta\mu^{\eta-1}\right)\left(\pi^{(1,1)}(x_1,x_2)-\pi^{(1,2)}(x_1,x_2)\right)\\
&\quad+\mu^{\eta}\left(1-\eta\tau_\eta\mu^{\eta-1}\right)\left(\pi^{(1,2)}(x_1,x_2)-\pi^{(2,2)}(x_1,x_2)\right).
\end{aligned} \tag{98}
$$

With absolute variability aversion ($\eta = 0$), π should be such that:

$$
\begin{cases}
\pi^{(1)}(x_1,x_2) \le \pi^{(2)}(x_1,x_2),\\
\pi^{(1,1)}(x_1,x_2) \ge \pi^{(2,2)}(x_1,x_2).
\end{cases} \tag{99}
$$

With relative variability aversion ($\eta = 1$), we obtain, for $x_1 < x_2$:

$$
\begin{cases}
\pi^{(1)}(x_1,x_2) \le \pi^{(2)}(x_1,x_2),\\
\pi^{(1)}(x_1,x_2)-\pi^{(2)}(x_1,x_2)+\mu(1+\tau_1)\left(\pi^{(1,1)}(x_1,x_2)-\pi^{(1,2)}(x_1,x_2)\right)\\
+\mu(1-\tau_1)\left(\pi^{(1,2)}(x_1,x_2)-\pi^{(2,2)}(x_1,x_2)\right) \ge 0.
\end{cases} \tag{100}
$$

Proof of Proposition 8 and Corollary 1

We have shown that it is possible to impose restrictions on the derivatives of both $\pi(x_1, x_2)$ and $\rho(\mu, \tau_\eta)$ to obtain measures that are included in both $\tilde{\Pi}(\tilde{\lambda}^+)$ and $\ddot{\Pi}(\lambda^+)$. Since the class of poverty measures $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ is not empty, any measure $P(\tilde{\lambda}) \in \breve{\Pi}_\eta(\tilde{\lambda}^+)$ can equally be expressed using equation (8) or equation (19). Consequently, both (59) and (78) are valid expressions for $P(\tilde{\lambda})$. For Proposition 8 not to hold, it would be necessary to show that one can find two distributions A and B such that $A \ddot{\succsim}_{\tilde{\lambda}^+} B$ and $B \tilde{\succsim}_{\eta,\tilde{\lambda}^+} A$. However, with the restrictions imposed on the classes $\ddot{\Pi}(\lambda^+)$ and $\tilde{\Pi}(\tilde{\lambda}^+)$, such a situation would imply that for any poverty measure in $\breve{\Pi}_\eta(\tilde{\lambda}^+)$, the difference $P_A(\tilde{\lambda}) - P_B(\tilde{\lambda})$ should simultaneously be non-negative and non-positive. This will happen if and only if $P_A^{1,1}(x_1, x_2) = P_B^{1,1}(x_1, x_2)$, $\forall (x_1, x_2) \in \Gamma(\tilde{\lambda})$. This proves Proposition 8.

The demonstration of Corollary 1 is straightforward. As long as the class of poverty measures $\breve{\Pi}_\eta(\tilde{\lambda}^+)$ is not empty, observing dominance with respect to either $\ddot{\Pi}(\lambda^+)$ or $\tilde{\Pi}(\tilde{\lambda}^+)$ precludes observing an opposite strong dominance relationship with the other class of poverty measures, as both classes include $\breve{\Pi}_\eta(\tilde{\lambda}^+)$.

Appendix 4: additional tables

See Tables 1, 2, 3.

Table 1 First-order dominance tests for intertemporal poverty indices (using income-defined indices)

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
AT	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}^{S}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\succcurlyeq}^{S}$	$\ddot{\succcurlyeq}^{+}$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}^{+}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}^{+}$
BE	–	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}^{+}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
BG	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^{+}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^{-}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
CY	–	–	–	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}^{+}$	$\ddot{\succcurlyeq}^{+}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}^{+}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$
CZ	–	–	–	–	$\emptyset$	$\ddot{\succcurlyeq}^{-}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
DK	–	–	–	–	–	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
EE	–	–	–	–	–	–	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
ES	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}^{+}$	$\ddot{\succcurlyeq}^{+}$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\emptyset$	$\ddot{\preccurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$
FI	–	–	–	–	–	–	–	–	$\emptyset$	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
FR	–	–	–	–	–	–	–	–	–	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
HU	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\preccurlyeq}^{+}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\preccurlyeq}^{-}$	$\emptyset$
IS	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\ddot{\succcurlyeq}^{-}$
IT	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^{-}$	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$
LT	–	–	–	–	–	–	–	–	–	–	–	–	–	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
LV	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\emptyset$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
MT	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}^{-}$	$\emptyset$	$\ddot{\succcurlyeq}^{+}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
NL	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\emptyset$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
NO	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\emptyset$	$\emptyset$	$\emptyset$
PL	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\emptyset$	$\ddot{\preccurlyeq}^{+}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}^{+}$
PT	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$

Table 1 continued

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
SE	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	Ø
SI	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø

$\ddot{\succcurlyeq}$ ($\ddot{\preccurlyeq}$) indicates that the first distribution dominates (is dominated by) the second distribution. Ø denotes a non-conclusive test. The "+", "-", and "S" subscripts indicate that dominance holds only when the asymmetry (loss aversion), asymmetry (early poverty), and symmetry assumptions are used. Remember that symmetry is a limiting case of asymmetry, so that observing dominance with asymmetry entails that dominance necessarily also holds with symmetry. As asymmetry is a special case of the general case covered by Proposition 1, dominance with asymmetry is observed when Proposition 1's general dominance is observed. Thus, for expositional simplicity, the table reports the broadest classes of intertemporal indices for which dominance is observed, if any

Table 2 Second-order dominance tests for intertemporal poverty indices (using income-defined indices)

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
AT	Ø	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$
BE	–	$\ddot{\succcurlyeq}$	Ø	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	Ø
BG	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
CY	–	–	–	$\ddot{\succcurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$
CZ	–	–	–	–	Ø	$\ddot{\succcurlyeq}$	Ø	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	$\ddot{\preccurlyeq}$	Ø	$\ddot{\succcurlyeq}$	Ø	Ø	Ø	Ø
DK	–	–	–	–	–	Ø	$\ddot{\succcurlyeq}$	Ø	Ø	Ø	$\ddot{\preccurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	Ø	Ø
EE	–	–	–	–	–	–	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	Ø	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
ES	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	Ø	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
FI	–	–	–	–	–	–	–	–	Ø	$\ddot{\succcurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$
FR	–	–	–	–	–	–	–	–	–	Ø	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	Ø
HU	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	Ø	Ø	Ø	$\ddot{\preccurlyeq}$	Ø
IS	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$
IT	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	$\ddot{\preccurlyeq}$	Ø	$\ddot{\preccurlyeq}$
LT	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
LV	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
MT	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	Ø
NL	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	$\ddot{\succcurlyeq}$
NO	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\succcurlyeq}$	$\ddot{\succcurlyeq}$	Ø	Ø	Ø
PL	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
PT	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$	$\ddot{\preccurlyeq}$
SE	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	Ø
SI	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø

$\ddot{\succcurlyeq}$ ($\ddot{\preccurlyeq}$) indicates that the first distribution dominates (is dominated by) the second distribution. Ø denotes a non-conclusive test

Table 3 First-order dominance tests with perfect intertemporal income pooling

Country	BE	BG	CY	CZ	DK	EE	ES	FI	FR	HU	IS	IT	LT	LV	MT	NL	NO	PL	PT	SE	SI	UK
AT	Ø	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	$\succcurlyeq_\mu$
BE	–	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	Ø
BG	–	–	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
CY	–	–	–	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$
CZ	–	–	–	–	Ø	Ø	Ø	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	Ø	Ø	Ø	Ø
DK	–	–	–	–	–	Ø	Ø	Ø	Ø	Ø	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	Ø	Ø	$\succcurlyeq_\mu$	Ø	Ø	Ø
EE	–	–	–	–	–	–	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
ES	–	–	–	–	–	–	–	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\preccurlyeq_\mu$
FI	–	–	–	–	–	–	–	–	Ø	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	Ø
FR	–	–	–	–	–	–	–	–	–	Ø	$\preccurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	Ø
HU	–	–	–	–	–	–	–	–	–	–	$\preccurlyeq_\mu$	Ø	Ø	Ø	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	Ø	Ø	$\preccurlyeq_\mu$	Ø
IS	–	–	–	–	–	–	–	–	–	–	–	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$
IT	–	–	–	–	–	–	–	–	–	–	–	–	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\preccurlyeq_\mu$
LT	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
LV	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
MT	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\preccurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	Ø
NL	–	–	–	–	–	–	–	–	–	–	–	–	-	–	-	–	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$
NO	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\succcurlyeq_\mu$	$\succcurlyeq_\mu$	Ø	Ø	Ø
PL	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
PT	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$	$\preccurlyeq_\mu$
SE	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø	Ø
SI	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	Ø

$\succcurlyeq_\mu$ ($\preccurlyeq_\mu$) indicates that the first distribution dominates (is dominated by) the second distribution. Ø denotes a non-conclusive test

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